Design and optimization of a magnetic gravity compensator
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Design and Optimization of a Magnetic Gravity Compensator

PROEFSCHRIFT

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Preface

“Beyond the horizon of the place we lived,
when we were young
In a world of magnets and miracles
Our thoughts strayed constantly and without boundary
The ringing of the division bell had begun”

High Hopes by Pink Floyd

Electro-magnetic forces are nowadays considered as one of the four elementary forces\(^1\) in our universe. When I studied Mechatronic Design at the Eindhoven University my interest in magnetism and electro-magnetic forces was aroused by Prof. Compter, Prof. Kamerbeek and Prof. Vandenput. Although many scientists have found models to calculate and predict the behavior of electro-magnetic fields, the operating principle behind magnetism and its related forces have not been explicitly found until now. This dissertation will not deal with the questions related to these fundamentals, but the focus will be on the practical application of magnetism: the use of nature for daily life.

Without the help of the nice people around me, I would not have been able to complete this work. First of all, I wish to thank John Compter for his guidance during the past years; for the many inspiring discussions that helped me a lot during the start of my working career and the nice personal conversations we have. I would like to thank André Vandenput for his coaching and the accurate comments on the dissertation, for his encouraging and friendly approach. I owe many thanks to my direct colleagues at ASML (Veldhoven) for their interest

\(^{1}\)besides this force three other forces exist: gravity, the weak and the strong force.
in the progress, and to Rob Munnig Schmidt, who made it possible for me to work on my Ph D project at ASML. Many thanks also go to Berry for the design of the cover and to Camile and Funda for their help with \LaTeX.

I wish to thank my family for supporting me to become who I am now. Joost, thanks for the relaxing conversations we have. And last but not least, I wish to thank my beloved girlfriend, Patricia, for her remarks with respect to the dissertation and her help to set up the MatLab routines. Even more valuable, she always reminds me that there are even better things than enjoying my work.
Chapter 1

Introduction

1.1 Rationale

"Nature is the existence of things, so far as it is determined according to universal laws. Should nature signify the existence of things in themselves\(^1\), we could never know it either \textit{a priori} or \textit{a posteriori}. Not \textit{a priori}, for how can we know what belongs to things in themselves, since this never can be done by the dissection of our concepts (in analytical judgments)? We do not want to know what is contained in our concept of a thing (for the concept describes what belongs to its logical being), but what is in the actuality of the thing superadded to our concept, and by what the thing itself is determined in its existence outside the concept. Our understanding, and the conditions on which alone it can connect the determinations of things in their existence, do not prescribe any rule to things themselves; these do not conform to our understanding, but it must conform itself to them; they must therefore be first given us in order to gather these determinations from them, wherefore they would not be known \textit{a priori}.

A cognition of the nature of things in themselves \textit{a posteriori} would be equally impossible. For, if experience is to

\(^1\)Kant indicates this as ‘\textit{Ding an sich}’
teach us *laws*, to which the existence of things is subject, these laws, if they regard things in themselves, must belong to them of *necessity* even outside our experience. But experience teaches us what exists and how it exists, but never that it must necessarily exist so and not otherwise. Experience therefore can never teach us the nature of things in themselves.”


1.2 Workhorses

Life without motion is unimaginable. This statement is also valid in the production industry. Electromechanical actuators definitely form the workhorses within the industry, and the quality of industrial processes is strongly determined by the performance of actuators. In high-end industrial applications the accuracy of the production systems is increasing steadily, enabled by increasingly powerful, precise, efficient and cost-effective actuators.
1.3 Highly accurate industrial applications

1.3.1 Lithographic systems

A well-known high-precision production system, which incorporates highly accurate electromagnetic actuators, is a wafer scanner. Wafer scanners are used to project an image of a layer in an integrated circuit onto a substrate. In general, the substrate or so-called wafer is a silicon disc. Figure 1.2 shows a wafer scanner in which the fundamental and critical machine parts are indicated. Light is generated by a laser (1) and directed by several beam-manipulating systems to the so-called reticle (2) on which the structure of the layer is affixed. The lenses (3) project the image onto the wafer (4) that is treated with a photo resistant layer. For each new layer in the integrated circuit a new reticle will be used and the wafer obtains a new photo resistant layer.

Figure 1.2: Wafer scanner.
On one wafer, many (up to three hundred) integrated circuits are projected consisting of many (up to thirty) layers. When projecting a new layer, it is essential that the new layer is positioned very accurately with respect to the previous one, to ensure the quality of the future integrated circuit. When all layers are added, the integrated circuits are cut out of the wafer and connections to the outside world are made in the so-called bonding process.

Two different lithographic processes are distinguished: stepping and scanning. In Figure 1.3 both processes are shown. During the stepping process, each layer is projected at once on the substrate. The maximum image field size (die size) is restricted by the lens system. The maximum die size for steppers is approximately twenty-two millimeters by twenty-two millimeters. Larger die sizes would require extremely large and expensive lens systems. Nevertheless, the customer requires larger die sizes. To enable this, the scanning process is used. Both reticle and wafer are moved and only a small (moving) slit is projected onto the wafer. This process enables the manufacturing of integrated circuits with a die size of twenty-six millimeters by thirty-three millimeters. Both processes require very accurate positioning of the reticle, lenses and wafer with respect to each other. The reticle and wafer are positioned by the wafer stage and reticle stage respectively as shown in

![Figure 1.3: Stepping (left) and scanning (right).](image-url)
1.3. **HIGHLY ACCURATE INDUSTRIAL APPLICATIONS**

Figure 1.4. Stages.

Figure 1.4. Stages are the moving systems in a machine. These systems need to position with an accuracy of several nanometers, whereas strokes are several decimeters, because of the size of wafers (up to three hundred millimeters in diameter). The manufacturers of integrated circuits demand high production rates and an increased functionality per square centimeter of the integrated circuit. In Figure 1.5, Moore’s road map of the feature size on integrated circuits is shown. This road map roughly predicts the halving of the feature size every four year. In practice, the road map turns out to be more aggressive than Moore’s prediction. Nowadays the critical dimension is already less than one hundred nanometer. The accuracy of the stages restricts the feature size. The production rates are strongly determined by the required time of the lithographic process. Therefore, wafers and reticles need to be moved with high velocities (up to two meters per second) on the
one hand and high accuracy on the other hand. The previous considerations demonstrate the need for high-tech stages driven by high-tech actuators.

### 1.3.2 Nanometer precision

In order to illustrate the accuracy requirements that the high-tech stage actuators in lithographic equipment have to meet, a few ordinary comparisons are made here. First, if the map of Europe would be drawn on the palm of a hand and all streets should be indicated, the characteristic size of the features on the palm needs to be drawn with nanometer scale. For another illustrative example, which relates the stage velocity of one meter per second to its position accuracy of several nanometers, one considers two Boeing-747 aeroplanes flying at the speed of one thousand kilometers per hour maintaining a tail-to-nose distance
of ten micrometers. Finally, the growth of a human hair is approximately one centimeter per month. This means an average growth of four nanometers each second.

A more technical example is the extension of a bar due to a uniform temperature increase. A steel bar, being 500 millimeter long, will extend one nanometer if the temperature of the bar increases 0.0002 Kelvin.

The need for nanometer accuracy introduces severe problems. Vibrations of machine parts easily exceed these demands, amplitudes (due to floor vibrations or even acoustic waves for example) reach micrometers levels. Thermal gradients deflect constructions leading to the same difficulties, and over-constraint constructions result in unpredictable deformations. It is obvious that special precautions and measures are inevitable to achieve nanometer accuracy. The designer should be aware of the implications these precautions and measures have on the design process. This is expounded in Chapter 2.
Chapter 2

Objectives

2.1 Design considerations

As stated in Chapter 1, the need for fast and accurate production systems is increasing. The combination of high velocity, high accuracy, large stroke and (unfortunately) relatively high masses demands extreme efforts in the design of the mechanical construction, control strategy, sensors, amplifiers and last but not least the actuators. In general, high but accurate forces over a large stroke, e.g. displacements or rotations, are contradictory constraints. Therefore, special precautions are taken in order to meet these severe specifications: one could think of making use of a two-stroke concept, where a two-stroke actuator design is introduced, and suppressing disturbances.

2.1.1 Accuracy by the two-stroke concept

A popular approach to nano-positioning requirements in precision engineering in general and micro-lithography in particular is to subdivide the stage positioning architecture into a coarse positioning module with micrometer accuracy (long-stroke system), onto which is cascaded a fine positioning module (short-stroke system). The latter is responsible for correcting the residual error of the coarse positioning module to the last nanometers. In wafer scanners the typical ranges for the long-stroke systems are one meter, whereas the short-stroke system must be able to move several millimeters.
CHAPTER 2. OBJECTIVES

The principle of the two-stroke concept for a single degree of freedom system is illustrated in Figure 2.1. In practice, the long-stroke system is used to position in three degrees of freedom (DOF) in the horizontal plane: translations in two horizontal directions ($x, y$) and rotations around the vertical axis ($\theta_z$). For the suspension in the other degrees of freedom, air-bearing or magnetic bearing systems are commonly used. The short-stroke system however is controllable in six degrees of freedom, consequently actuating forces in six degrees of freedom are required (three forces along and torques around three axis, see

Figure 2.1: Master slave principle of stages.

Figure 2.2: Degrees of freedom.
2.1. DESIGN CONSIDERATIONS

Figure 2.2. The position \((x, y \text{ and } z)\) and the orientation \((\theta_x, \theta_y \text{ and } \theta_z)\) are measured and controlled with a high bandwidth feedback system. The stage, as illustrated in Figure 2.1, operates in a master/slave configuration. The long-stroke system operates as a slave, which means that the long-stroke tries to follow the short-stroke as well as possible (maintaining the position difference between long- and short-stroke system \((x_{ls} - x_{ss})\) as constant as possible). This two-stroke concept is used for both wafer stage and reticle stage.

2.1.2 Accuracy by suppressing disturbances

The previous section explained the motion control approach, where a fast and high precision short-stroke system is mounted onto a coarse long-stroke system. In order to obtain the required accuracy, any force other than the control force should be avoided. Cross talk forces or parasitic actuator forces between long- and short-stroke system will affect the accuracy. External disturbance forces on both systems may lead to a loss of accuracy as well.

In Figure 2.3, the combination of the long- and short-stroke system is shown schematically. Some cross talk exists between the systems. This cross talk is always present due to any mechanical or magnetic interaction, such as cables for powering, sensors or magnetic fields from permanent magnets or coils. The effect is that vibrations of the long-stroke system introduce forces and related (undesired) displacements at

![Figure 2.3: Coupling between long- and short-stroke systems.](image-url)
the short-stroke system. It is useful to discriminate between position-dependent and velocity-dependent cross talk that can be modelled as stiffness (a spring) and damping (a damper), respectively. For simplicity reasons the cross talk force in only one DOF is considered¹. Cross talk, as a displacement dependent force, is modelled as:

\[ F_{x,k} = k_x(x, y, z) \cdot (x_{ls} - x_{ss}) \]  

(2.1)

where \( F_{x,k} \) is the cross talk force acting on the short-stroke system, \( x_{ls} \) is the position of the long-stroke system, \( x_{ss} \) the position of the short-stroke and \( k_x(x, y, z) \) is the (position dependent) stiffness between the systems, defined as:

\[ k_x(x, y, z) = \frac{\partial F_x}{\partial (x_{ls} - x_{ss})} \]  

(2.2)

The reader must be aware of the fact that \( F_{x,k} \) is one of the components of the total force acting on the body \( F_x \). Beside the position dependent cross talk, the velocity dependent cross talk (damping) is modelled as:

\[ F_{x,d} = d_x(x, y, z) \cdot (\dot{x}_{ls} - \dot{x}_{ss}) \]  

(2.3)

where \( \dot{x} \) stands for the time derivative of \( x \) (i.e. \( \dot{x} = \frac{dx}{dt} \)). For the (position dependent) damping \( d_x(x, y, z) \) the following relation exists:

\[ d_x(x, y, z) = \frac{\partial F_x}{\partial (\dot{x}_{ls} - \dot{x}_{ss})} \]  

(2.4)

The stiffness and damping forces between the short- and long-stroke systems, which physical background is illustrated in Section 2.1.3, can be determined and used as feed forward information in the control loops, which is often realized by software (executed by a micro-processor). The effect is then highly reduced by this feed forward action. If we can minimize the influence of stiffness and damping by constructive means, the software effort can be used for minimizing unpredictable forces, called disturbances, caused by (for example) airflow or temperature gradients. The total of forces acting on the short-stroke mass is drawn in Figure 2.4. In general it consists of actuator force \( F_{x,act} \), gravity

¹In practice the cross talk will occur in six degrees of freedom.
2.1. DESIGN CONSIDERATIONS

Figure 2.4: Forces acting on the short-stroke mass.

force \( F_{x,g} \), unpredictable disturbance forces \( F_{x,\text{dist}} \), stiffness \( F_{x,k} \) and damping forces \( F_{x,d} \). Considering the latter category of forces, the effect of these forces on the short-stroke mass is investigated, using Newton’s law of motion:

\[
\sum F_x = F_{x,k} + F_{x,d} = m_{ss} \cdot \ddot{x}_{ss}
\]  
(2.5)

Substituting (2.1) and (2.3) in (2.5) yields:

\[
k_x(x, y, z) \cdot (x_{ls} - x_{ss}) + d_x(x, y, z) \cdot (\dot{x}_{ls} - \dot{x}_{ss}) = m_{ss} \cdot \ddot{x}_{ss}
\]  
(2.6)

Using the Laplace transform \( X_{ss} = \mathcal{L}(x_{ss}) \) (Appendix C.7.1) and re-ordering terms results into the Laplace expression for the system:

\[
X_{ss} \cdot (m_{ss} \cdot s^2 + d_x \cdot s + k_x) = X_{ls} \cdot (d_x \cdot s + k_x)
\]  
(2.7)

in which \( X_{ss} \) and \( X_{ls} \) is the Laplace transform of \( x_{ss} \) and \( x_{ls} \) respectively. The transfer function of the position of the short-stroke system, caused by displacements of the long-stroke system, due to the cross talk \( (k_x \) and \( d_x \)) is finally obtained from (2.7):

\[
\frac{X_{ss}}{X_{ls}}(s) = \frac{d_x \cdot s + k_x}{m_{ss} \cdot s^2 + d_x \cdot s + k_x}
\]  
(2.8)

Inserting \( s = j \cdot \omega \) in the former equation leads to the frequency response of \( X_{ss} \):

\[
\frac{X_{ss}}{X_{ls}}(\omega) = \frac{j d_x \cdot \omega + k_x}{-m_{ss} \cdot \omega^2 + j d_x \cdot \omega + k_x}
\]  
(2.9)
In the ideal case, where no stiffness and damping are present; i.e. $k_x = 0$ and $d_x = 0$, (2.9) reduces to $\frac{X_{ss}}{X_{ls}} = 0$, expressing that the short-stroke system does not experience any influence from displacements of the long-stroke. For the non-ideal cases shown in Table 2.1, Figure 2.5 shows the behavior of the short-stroke system\(^2\). The cases are listed in decreasing order of cross talk between long- and short-stroke system.

<table>
<thead>
<tr>
<th>Cross talk cases</th>
<th>$m_{ss}$ [kg]</th>
<th>$k_x$ [N/m]</th>
<th>$d_x$ [Ns/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>10</td>
<td>10000</td>
<td>10</td>
</tr>
<tr>
<td>case 2</td>
<td>10</td>
<td>1000</td>
<td>5</td>
</tr>
<tr>
<td>case 3</td>
<td>10</td>
<td>100</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.1: Cross talk cases.

Figure 2.5 shows that (in the absence of closed loop control) for low frequencies, the displacements of the long-stroke system are directly taken over by the short-stroke, since $\frac{X_{ss}}{X_{ls}} = 1$ if $\omega \to 0$. At low frequencies (from 0 to 10 Hz) the actuators driving the short-stroke (with closed loop control) can easily correct for errors. Difficulties arise at higher frequencies (above 10Hz). These actuators will not be able to correct for position errors of the long-stroke, due to bandwidth limitations of the actuating system (actuator including power amplifier, sensors and controller). As should be concluded from Figure 2.5, the short-stroke becomes less sensitive to errors of the long-stroke, when stiffness ($k_x$) and damping ($d_x$) are reduced. In other words, decreasing the cross talk between the systems will improve the accuracy of the short-stroke system.

### 2.1.3 Parasitic phenomena

The moral of the preceding section is that the system’s accuracy is improved by reducing the cross talk between several subsystems in the design. Also mentioned, is that cross talk is either caused by mechanical interaction, such as cables or bearings, or magnetic interaction [12]. In the previous section the cross talk was modelled only in one degree of freedom ($x$-direction), but in practice, cross talk occurs in all degrees of freedom and is therefore more complex and leads to complications for the designer. This holds for all other disturbance forces as well.

\(^2\)In this (open-loop) transfer the influence of a servo controller is not considered.
Figure 2.5: Transfer function: long-stroke displacement to short-stroke displacement.
Actuators form the (magnetic) coupling between moving modules and therefore influence the cross talk of the mentioned modules. This coupling then exists between mover (the moving part) and stator (the ‘static’ part\(^3\)) and four specific types of coupling forces are distinguished (the theoretical background of these forces is elaborated in Section 3.3.4):

**Cogging**

Cogging occurs when magnetic permeable material (like iron) is present in an actuating system with permanent magnets. This material experiences preferred positions with respect to the magnets. The coherent force to these positions is called cogging or detent force. Since this force is dependent on the position of the mover with respect to the stator, cogging is modelled as stiffness. For example, cogging is experienced in slotted synchronous servo actuators.

**Reluctance forces**

Reluctance forces occur in systems where the permanent magnets are replaced by current conducting coils. Again magnetic permeable material experiences preferred positions, due to the magnetic field generated by these coils. Reluctance forces are current and position dependent forces and therefore introduce current dependent stiffness between mover and stator.

**Magnetic damping**

Magnetic damping forces occur when eddy currents or current vortices are generated in electrically conducting materials by the relative movement of mover and stator, which are coupled by a magnetic field. These forces are dependent on the relative velocity of the mover with respect to the stator and introduce a damping force between stator and mover. In general, damping is present in iron-core machines and in machines

\(^3\)the reader should realise that the stator of the short-stroke system is mechanically coupled to the mover of the long-stroke, and in that sense the term stator may be confusing.
2.1. DESIGN CONSIDERATIONS

where electrically conducting materials are used for fixing and or cooling the coils (or magnets).

**Variations in motor constant**

In some types of actuators the motor constant, defined as the generated force as a function of the applied current, is dependent on the position of the mover with respect to the stator. In that case, the force will vary when the mover is displaced. This force variation is experienced as a current level dependent stiffness.

The strength of magnets is temperature dependent. For modern rare-earth magnets (such as NdFeB for example) the magnetic remanence and coercitive field strength decrease when the temperature is increased. This results is a temperature dependent reduction of the motor constant.

When the assumed motor constant in control loops deviates from the actual (real) motor constant, the effectiveness of feed forward signals is lost. This holds for both phenomena mentioned.

Besides the previously mentioned types of coupling forces there are other aspects that influence the accuracy of an application.

**Magnetic hysteresis**

Magnetic hysteresis is the non-linear magnetic behavior of materials characterized by a history dependent magnetic induction. This results into non-linear and current dependent motor constants. Hysteresis can also result in position-dependent and time history dependent forces.

**Amplifier ripples and output impedance**

If the amplifier’s output impedance is low, the coils of an actuator have a short circuit, through which a current will flow when the coils are moved relative to the magnetic field, resulting in damping forces. This is elaborated in detail in Appendix E. Besides, ripples and offsets present in the amplifier signal result in disturbance forces in the actuator.
CHAPTER 2. OBJECTIVES

The previous phenomena affect the actuator’s force. Two other aspects, that do not affect the force, have impact on the system’s accuracy.

Dissipation

The dissipation in the coils should be minimized, since thermal gradients deflect constructions. These deflections limit the system’s accuracy.

Mass

The mass of the actuator should be kept within reasonable limits, since large masses require high acceleration forces leading to construction deformations. Moreover, high masses introduce lower system eigenfrequencies limiting the bandwidth of a servo system. A low system bandwidth results in a poor disturbance suppression. Finally, high masses tend to increase the dissipation (because of increased required force).

In the design process of accurate actuators, all these performance determining properties should be specified. In [16] and [18] an effective magnet configuration for achieving a relatively low mass, low dissipation and low variations in the motor constant for actuators with a limited stroke is presented.

2.2 Problem description

In the previous paragraph, the complications for designing actuating systems with high precision was explained. In Section 2.1.1, the two-stroke principle of highly accurate machines was demonstrated. The need for attention to cross talk was illustrated in Section 2.1.2 and in Section 2.1.3, the physical background of cross talk forces caused by parasitic phenomena in actuators was elaborated. The short-stroke system, which has to be moved in six degrees of freedom, is responsible for positioning with nanometer accuracy. For this purpose, six actuators will be used: three for positioning in the $x$- and $y$-direction and rotations around the $z$-axis ($\theta_z$); and three for positioning in the
2.3 Organization of this dissertation

The study revealed in this dissertation concerns the design and optimization process for the dynamic gravity compensator.

Figure 2.6: Forces for positioning the short-stroke system in the $z$-, $\theta_x$- and $\theta_y$-direction.

$z$-direction and rotations around the $x$- and $y$-axis ($\theta_x$ and $\theta_y$) (see Figure 2.6). In this dissertation, the design process of the latter category of actuators is treated. Since this actuator has to generate a force to counteract earth’s gravitation field, this type of actuators is called: Dynamic Gravity Compensator.

The main objectives of this dissertation are the design and the optimization of this dynamic gravity compensator, that:

- should generate a constant force against gravity,
- is able to generate a controllable (dynamic) force, acting along the same line as the constant force,
- has low parasitic (cross talk) forces between the mover and stator,
- has a low mass and
- should dissipate a minimum amount of heat.
CHAPTER 2. OBJECTIVES

In Chapter 3, an overview of the electro-magnetic theory is given. All required tools for the electro-magnetic design are presented or derived in this chapter. The development of the concept of magnetic and electric fields in the nineteenth century is expounded in Paragraph 3.1. In the subsequent section, the generation of magnetic fields and the behavior of different materials to fields are explained. The use of scalar and vector potentials in calculations is illustrated. In Paragraph 3.3, the forces related to magnetic fields and the mathematical means to calculate them are described. Chapter 3 is finished with Earnshaw’s theorem (Paragraph 3.4) and the damping theorem (Paragraph 3.5).

Chapter 4 deals with the design process of the gravity compensator. Paragraph 4.1 presents the function and the specifications for the design process. In Paragraph 4.2, the geometric configuration is expounded. Moreover, the physical explanation is given: why does it act as it does? In Paragraph 4.3, the numerical and analytical models are presented: two numerical models (two- and three-dimensional boundary element models) and one analytical model. These models are used to calculate forces and torques, the dissipation and related thermal behavior. In Paragraph 4.4, the proto-type of the gravity compensator is shown, and in Paragraph 4.5, the measurements are compared to the models from Paragraph 4.3.

In Chapter 5, the validated numerical model is used to find an optimized design for the dynamic gravity compensator. In Paragraph 5.1, an introduction to optimization and the strategy for optimizing a physical design in general are presented. The criteria for optimizing the current design are found in Paragraph 5.2. A simplified optimization of the design under consideration is used to illustrate the optimization procedure, based on weight factors for the different criteria. The strategy for the final optimization is presented in Section 5.3.1 and the results of this procedure are discussed in Section 5.3.2. The influence of the weight factors on the final optimization results was investigated and is illustrated in Paragraph 5.4. The validation of the results of Paragraph 5.3 is done in Paragraph 5.5.

Finally, Chapter 6 presents the conclusions and recommendations.
Chapter 3

Electromechanics

3.1 History

Since the nineteenth century, scientists have investigated the concept of physical field, referring to the force distribution in the three dimensional space due to electricity and magnetism. Especially Michael Faraday (1791-1867) and James Clerk Maxwell (1831-1879) have delivered a very important contribution to the actual field theories. Both presented electrical and magnetic fields by making use of the so-called field lines. These field lines are a representation of the field, which required (according to Maxwell and Faraday) an ether for propagation through space. Although Faraday interpreted these lines as real physical lines, Maxwell only used them as an illustration in order to derive his famous mathematical relations. Around 1850, William Thomson (1804-1907), better known as Lord Kelvin, presented his view on the structure of the field: the functioning of the field could be explained by occurring vortices in the ether. Between 1850 and 1860, Maxwell came up with his famous mathematical relations for describing the forces between particles and bodies in the ether. His theory is known as one of the most important scientific achievements and gave new insights in magnetic and electrical phenomena, linked electromagnetic waves and light, and it turned out to be consistent with the Einstein’s theory on relativity.
CHAPTER 3. ELECTROMECHANICS

3.2 Fields and sources

3.2.1 Maxwell’s equations

Magnetic and electrical phenomena are described by the Maxwell equations. The equations explain the generation and the distribution of electromagnetic fields in space [1], [2], [3]:

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]  
(3.1)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]  
(3.2)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(3.3)

\[ \nabla \cdot \mathbf{D} = \rho_v \]  
(3.4)

The previous equations express the relations between the magnetic field strength \( \mathbf{H} \) [A/m] (\( \mathbf{H} = (H_x, H_y, H_z) \)), the magnetic field density or induction \( \mathbf{B} \) [T], the electric field strength \( \mathbf{E} \) [V/m], the electric field density \( \mathbf{D} \) [C/m\(^2\)], the current density \( \mathbf{J} \) [A/m\(^2\)] and the spatial charge distribution \( \rho_v \) [C/m\(^3\)] in differential form. Another familiar form of these equations is the integral approach:

\[ \oint \mathbf{H} \cdot \mathbf{\tau} dl = \iint \mathbf{J} \cdot \mathbf{n} dS + \frac{d}{dt} \iint \mathbf{D} \cdot \mathbf{n} dS \]  
(3.5)

\[ \oint \mathbf{E} \cdot \mathbf{\tau} dl = -\frac{d}{dt} \iint \mathbf{B} \cdot \mathbf{n} dS \]  
(3.6)

\[ \iint \mathbf{B} \cdot \mathbf{n} dS = 0 \]  
(3.7)

\[ \iint \mathbf{D} \cdot \mathbf{n} dS = Q \]  
(3.8)

for which \( Q \) [C] is the charge enclosed by the surface \( S \) [m\(^2\)]. Equation (3.1) and (3.2) or (3.5) and (3.6) describe the generation of magnetic and electric fields respectively, where (3.3) and (3.4) or (3.7) and (3.8) model the propagation of fields through space. When taking a close

\(^1\)\( \mathbf{D} \) is also known as the electric displacement.
look at (3.1), the charge conservation law can be derived from it, by applying the divergence operation to both parts of the equation and applying (3.4). Since it is known that $\nabla \cdot (\nabla \times A) = 0$ for any arbitrary vector $A$:

$$\nabla \cdot (\nabla \times H) = \nabla \cdot (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \Rightarrow$$

$$\nabla \cdot \mathbf{J} + \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} = 0 \Rightarrow \nabla \cdot \mathbf{J} = -\frac{d \rho_v}{dt} \quad (3.9)$$

Ironically, Maxwell appears to have been unaware of the connection between his equations and the charge conservation law. Had he known of this, his equations might have been universally accepted within his lifetime.

### 3.2.2 Field behavior of media

The constitutive relations describe the differences of materials to electric and magnetic fields:

$$\mathbf{D} = \varepsilon \cdot \mathbf{E} = \varepsilon_0 \cdot \mathbf{E} + \mathbf{P}_e \quad (3.10)$$

$$\mathbf{B} = \mu \cdot \mathbf{H} = \mu_0 \cdot \mathbf{H} + \mu_0 \cdot \mathbf{M} \quad (3.11)$$

$$\mathbf{J} = \sigma_e \cdot \mathbf{E} + \mathbf{J}_i \quad (3.12)$$

with the polarization $\mathbf{P}_e$ [C/m²], the magnetization $\mathbf{M}$ [A/m], the permeability $\mu$ [H/m] and permittivity $\varepsilon$ [F/m], the conductivity $\sigma_e$ [S/m] and the applied current density $\mathbf{J}_i$ [A/m²]. For most materials $\varepsilon$, $\mu$ and $\sigma_e$ are constants or tensors (in the anisotropic case). The dissipation related to the current density is given by:

$$P = \iiint \frac{\mathbf{J} \cdot \mathbf{J}}{\sigma_e} dV_c \quad (3.13)$$

where $V_c$ [m³] is the volume in which the current density $\mathbf{J}$ is present.

Soft and hard media are distinguished within the family of magnetic materials. In Figure 3.1 the relation between the magnetic polarization $\mathbf{J}_m$ and field strength $\mathbf{H}$ is drawn for soft magnetic materials. Soft
Figure 3.1: Magnetization of soft magnetic materials.
magnetic media show a small amount of hysteresis and are characterized in the linear region by:

\[ B = \mu_0 \cdot H + J_m(H) = \mu_r \cdot \mu_0 \cdot H \]  

(3.14)

where \( \mu_r \) is the relative permeability. Table 3.1 illustrates some examples of soft magnetic materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \mu_r ) for ( H=0 )</th>
<th>( J_{sat} ) [T]</th>
<th>( H_{cJ} ) [A/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron (Fe)</td>
<td>4000</td>
<td>2.05</td>
<td>-4</td>
</tr>
<tr>
<td>Cobalt (Co)</td>
<td>600</td>
<td>1.79</td>
<td>-800</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>0.999</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Silicon Iron (4% Si)</td>
<td>1000</td>
<td>2.03</td>
<td>-20</td>
</tr>
<tr>
<td>Cobalt Iron (50% Co)</td>
<td>1000</td>
<td>2.15</td>
<td>-140</td>
</tr>
<tr>
<td>( \mu )-Metal</td>
<td>300,000</td>
<td>0.80</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

Table 3.1: Soft magnetic materials.

Hard magnetic media, better known as permanent magnets, have relatively high hysteresis in their magnetization curve as shown in Figure 3.2. Figure 3.2 also shows the relation between the magnetic induction \( B \) and the electric field strength \( H \) inside the magnet. Permanent magnets are generally used in the second quadrant of the BH-curve, where the linear part of the relation between \( B \) and \( H \) is given by:

\[ B = \mu_0 \cdot \mu_r \cdot H + \mu_0 \cdot (\mu_r - 1) \cdot H + M_0 \]  

(3.15)

where \( M_0 \) [A/m] is the intrinsic magnetization of the permanent magnetic material (\( \mu_0M_0 = B_r \)). Table 3.2 illustrates some characteristic values of hard magnetic materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \mu_r ) for ( H=0 )</th>
<th>( B_r ) [T]</th>
<th>( H_{cJ} ) [kA/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>NdFeB</td>
<td>1.02...1.35</td>
<td>1.0...1.5</td>
<td>-400...-1000</td>
</tr>
<tr>
<td>SmCo</td>
<td>1.02...1.1</td>
<td>0.4...1.1</td>
<td>-300...-800</td>
</tr>
<tr>
<td>Alnico</td>
<td>1.7...3.7</td>
<td>0.6...1.4</td>
<td>-60...-170</td>
</tr>
</tbody>
</table>

Table 3.2: Hard magnetic materials.

The positive crossings of the \( J_m(H) \) and \( B(H) \)-curves (Figure 3.1 and Figure 3.2) with the \( H = 0 \) axis is called the magnetic remanence (\( B_r \)). The crossings of these curves with the \( J_m = 0 \) or the \( B = 0 \) axis
Figure 3.2: Magnetization of hard magnetic materials.
are known as the coercitive field strengths, indicated as $H_{cJ}$ and $H_{cB}$ respectively.

### 3.2.3 Vector and scalar potentials

Considering an arbitrary vector field $\mathbf{F}$ in a volume $V$ defined as $\mathbf{F} = \mathbf{F}(V)$ the Helmholtz’s theorems (Appendix C.2) can be used to quantify the vector potential $\mathbf{A}$ and scalar potential $f$. If $\nabla \times \mathbf{F} = 0$ (the curl of the vector field $\mathbf{F}$ equals zero) the scalar potential is defined as $f$:

$$\nabla \times \mathbf{F} = 0 \Rightarrow \mathbf{F} = \nabla f \quad (3.16)$$

for which $f = f(V)$ is a scalar quantity in the volume $V$. If $\nabla \cdot \mathbf{F} = 0$ (the divergence of the vector field $\mathbf{F}$ equals zero) the vector potential is defined as $\mathbf{A}$:

$$\nabla \cdot \mathbf{F} = 0 \Rightarrow \mathbf{F} = \nabla \times \mathbf{A} \quad (3.17)$$

for which $\mathbf{A} = \mathbf{A}(V)$ is a vector field (as $\mathbf{F}$ is) in the volume $V$. Both vector and scalar potentials simplify magnetic and electrical calculations.

From basic physics theory it is known that the electric potential $V_e$ at a distance $R$ from one single (non moving) point charge $Q$ is defined as:

$$V_e = \frac{Q}{4\pi \varepsilon_0 \cdot R} \quad (3.18)$$

Extending that equation to the potential $V_e$ in point $\mathbf{r}$ due to a volume charge in volume $V_e(\mathbf{r})$ leads to:

$$V_e = \frac{1}{4\pi \varepsilon_0} \iiint \frac{\rho_e(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV_e(\mathbf{r}') \quad (3.19)$$

where $\mathbf{r}'$ is the vector from the point $\mathbf{r}$ to a given charge element. $V_e$ is the electric scalar potential for the static case (no time dependency), since from (3.2) the condition $\nabla \times \mathbf{E} = 0$ is obtained. Equation (3.16) states that if:

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = \nabla V_e \quad (3.20)$$
Since $\nabla \cdot \mathbf{B} = 0$ is stated by (3.3) for the general case a magnetic vector potential exists. When replacing $\mu_0$ for $\frac{1}{\varepsilon_0}$ and $\mathbf{J}$ for $\rho_0$ in (3.19) (see Appendix D.2 for the justification of this replacement), the magnetic vector potential $\mathbf{A}$ is obtained:

$$\mathbf{A} = \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV_c(\mathbf{r}')$$  \hspace{1cm} (3.21)$$

and

$$\nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B} = \nabla \times \mathbf{A}$$  \hspace{1cm} (3.22)$$

where $V_c(\mathbf{r}')$ indicates the volume in space conducting the current density $\mathbf{J}(\mathbf{r}')$. Recalling (3.16) leads to the existence of a magnetic scalar potential if the condition $\nabla \times \mathbf{B} = 0$ is met. This is the case for volumes for which $\mathbf{J} = 0$ in magneto static analysis (see (3.1)). This is not as severe a restriction as one might think, since we are usually more interested in the fields at positions that are removed from their sources. Finally, the magnetic scalar potential $V_m$ is determined:

$$\mathbf{B} = -\mu \nabla V_m$$  \hspace{1cm} (3.23)$$

or $\mathbf{H} = -\nabla V_m$. Since $\nabla \cdot \mathbf{B} = 0$ the magnetic scalar potential is found by solving Laplace’s equation:

$$\nabla^2 V_m = 0$$  \hspace{1cm} (3.24)$$

### 3.2.4 Biot-Savart’s law

Starting point for obtaining an analytical expression for the magnetic induction $\mathbf{B}$ due to the current density $\mathbf{J}$ through a volume $V_c$ in space is the magnetic vector potential $\mathbf{A}$ in (3.21). By recalling (3.22) and applying some vector algebra (Appendix D.3) the Biot-Savart’s law for steady volumetric currents in free space is deduced:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV_c(\mathbf{r}')$$  \hspace{1cm} (3.25)$$
This equation is a powerful means for calculating fields in actuators. This relation forms an important tool for the work done in this dissertation.

3.3 Forces

Electromechanical actuators convert electrical energy into mechanical energy or vice versa. Three different methods are described to calculate the forces. In Section 3.3.4, the forces present in electromechanical actuators are expounded.

3.3.1 Lorentz’s law

Electromechanical forces can be related to charges. The physician Weber assumed that the action between two particles of electricity (charges) only depends on their relative distance and its first two time derivatives, which he erroneously called relative velocity and acceleration:

\[
F = \frac{Q \cdot Q'}{r^2} \left[ 1 - \frac{1}{2c^2} \left( \frac{dr}{dt} \right)^2 + \frac{r}{c^2} \frac{d^2r}{dt^2} \right] 
\]

(3.26)

where \( F \) is the force between two particles with a mutual distance \( r \), \( Q \) and \( Q' \) their charges, and \( c \) the light speed constant. Faraday refined this expression, claiming that the force is linear depending on the velocity (Paragraph 21 in [4]):

\[
F = \frac{Q \cdot Q'}{4\pi \varepsilon_0 \varepsilon_r} \left[ \frac{e_r}{r^2} - \frac{r}{c \, dt} \left( \frac{e_r}{r^2} \right) + \frac{1}{c^2 \, dt^2} (e_r) \right] 
\]

(3.27)

where \( e_r \) is the unit vector in the direction of \( r \). The first term in (3.26) and (3.27) represents Coulomb’s force. The other terms include relativistic effects, which are the basis of magnetic effects\(^2\). More recent theories show that the force on a particle is linear depending on the value of its charge and velocity on the one hand and the value of the external electric and magnetic field on the other hand.

\(^2\)The third term describes the effect due to inductance.
A usual expression, in accordance with the generally accepted Maxwell equations, for the action between electric and magnetic fields and a charge is Lorentz’s law, which expresses the specific force \( f \) [N/m\(^3\)] (or force density) on a charge distribution \( \rho_v \) [C/m\(^3\)] moving with speed \( v \) [m/s] due to the electric field strength \( E \) [V/m] and magnetic induction \( B \) [T] (due to another moving charge):

\[
f = \rho_v \cdot |E + v \times B|
\]  

(3.28)

In the case where more charges are present, the resulting force on a charge is determined by adding the effects of all individual sources (or fields due to sources) present in the environment of the charge. If the force due to the electric field can be neglected, the relation is simplified to the well-known Lorentz-force equation:

\[
f = J \times B
\]  

(3.29)

for \( J \) [A/m\(^2\)] (since \( J = \rho_v \cdot v \)).

### 3.3.2 Maxwell stresses

Using the famous Maxwell equations ((3.1) - (3.4)) in combination with the constitutive relations ((3.10) - (3.12)) the magnetic fields through a space are determined as function of the coordinate system. For a Cartesian system, \( B_x = B_x(x, y, z) \), \( B_y = B_y(x, y, z) \) and \( B_z = B_z(x, y, z) \) are derived in this way. If the force on a volume \( V \), enclosed by the bounding surface \( S \), is required, the Maxwell stress tensor \( \Lambda \) [N/m\(^2\)] is used [5], [30]:

\[
\Lambda_{i,j} = \frac{1}{\mu_0} \left( B_i \cdot B_j - \delta_{ij} \frac{B_k \cdot B_k}{2} \right)
\]  

(3.30)

in which \( i, j \) and \( k \) represent the coordinate directions in a Cartesian system and for \( \delta_{ij} \), the Kronecker delta function, holds \( \delta_{ij} = 1 \) for all \( i = j \) and \( \delta_{ij} = 0 \) in all other cases. The force on the surface is obtained by integrating the stresses over the surface \( S \):

\[
F = \iiint S \Lambda_{i,j} \cdot dS
\]  

(3.31)
3.3. FORCES

Normal stresses or shear stresses can be obtained directly from (3.30):

\[
\sigma_m = \frac{1}{2\mu_0} (B_n^2 - B_t^2) \quad (3.32)
\]

\[
\tau_m = \frac{1}{\mu_0} (B_n \cdot B_t) \quad (3.33)
\]

where \(\sigma_m\) and \(\tau_m\) [N/m\(^2\)] are the mechanical stresses acting on the volume due to the local normal \(B_n\) and tangential \(B_t\) components of the magnetic induction. Integrating these tensions over the bounding surface \(S\) results in the normal (\(F_n\)) and shear force (\(F_t\)) respectively.

### 3.3.3 Energy method

The main assumption used by the energy method is the energy conservation law [5]:

\[
dw = dw_{\text{mech}} + dw_{\text{elec}} + dw_{\text{magn}} = 0 \quad (3.34)
\]

stating that the total amount of specific energy (\(w\) [J/m\(^3\)]) for an isolated, conservative system will not vary. In (3.34) all losses are left out, but could be included. If electrical energy (\(w_{\text{elec}}\)) is supplied to an actuator, it will be stored as mechanical (\(w_{\text{mech}}\)) and magnetic (\(w_{\text{magn}}\)) energy. Increase of energy (\(dw \geq 0\)) is defined as an energy flow to the actuator. So when the actuator converts electrical energy into mechanical and magnetic energy, the energy conservation law is rewritten as:

\[
dw_{\text{mech}} = dw_{\text{elec}} - dw_{\text{magn}} \quad (3.35)
\]

In terms of power (3.35) can be rewritten as:

\[
\dot{p}_{\text{mech}} = \dot{p}_{\text{elec}} - \dot{p}_{\text{magn}} \quad (3.36)
\]

expressing the specific power balance (\(p\) [W/m\(^3\)]). Hence \(\dot{p}_{\text{mech}} = \textbf{f} \cdot \textbf{v}\), \(\dot{p}_{\text{elec}} = \textbf{E} \cdot (\textbf{J} + \frac{\partial \textbf{D}}{\partial t})\) and \(\dot{p}_{\text{magn}} = \textbf{H} \cdot \frac{\partial \textbf{B}}{\partial t}\), in which \(\textbf{f}\) [N/m\(^3\)] expresses the force per unit volume of the body, \(\textbf{v}\) [m/s] the velocity, \(\textbf{E}\) [V/m] the electric field strength, \(\textbf{J}\) [A/m\(^2\)] the current density, \(\textbf{D}\) [C/m\(^2\)] the electric displacement, \(\partial \textbf{B}\) [T] is the incremental change of magnetic
induction (time dependent) and $\mathbf{H}$ [A/m] the magnetic field strength. The velocity is rewritten as $\mathbf{v} = \frac{\partial \mathbf{r}}{\partial t}$. Substituting the specific powers in (3.36) yields the expression for the force:

$$f \cdot \partial \mathbf{r} = (\mathbf{E} \cdot \mathbf{J}) \partial t + \mathbf{E} \cdot \partial \mathbf{D} - \mathbf{H} \cdot \partial \mathbf{B}$$ (3.37)

Another way to derive (3.37) is to make use of Poynting’s theorem. This is done in Appendix C.6. Note that in Poynting’s theorem the magnetic term ($\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}$) differs in polarity, expressing the magnetic power stored at the system, where $-\mathbf{H} \cdot \partial \mathbf{B}$ in (3.37) expresses the amount of magnetic energy converted into mechanical energy.

Considerations about the energy method

It is interesting to take a closer look at (3.37). When forces due to electric displacement ($\mathbf{D}$) are neglected, which is often a valid assumption in practical magnetic systems, the power balance (3.37) can be rewritten as:

$$f \cdot \mathbf{v} = \mathbf{E} \cdot \mathbf{J} - \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}$$ (3.38)

In the case shown in Figure 3.3, a homogeneous and time invariant

![Figure 3.3: Energy method and Lorentz’s law.](image-url)
field $B_z$ (so $\frac{\partial B_z}{\partial t} = \frac{\partial B_z}{\partial x} = 0$) is present. In this conservative system\textsuperscript{3}, the movable conductor (placed in this field and having a velocity $v_x$) is conducting the current density $J$. Since $\frac{\partial B_z}{\partial x} = 0$ the latter term in (3.38) vanishes:

$$\mathbf{f} \cdot \mathbf{v} = \mathbf{E} \cdot \mathbf{J}$$

(3.39)

We consider the surface $S$. $S$ is bounded by the contour $C$, having the same dimension in $y$-direction as the infinite small subvolume $V_c$ in the moving conductor. From (3.2) it is known, that the electric field strength $\mathbf{E}$ is induced for a time-varying $\mathbf{B}$-field. But in this case $\frac{dB_z}{dt} = 0$. However, an electric field is induced due to the velocity of the conductor. Leibnitz’s theorem (C.9), where the considered vector field is $\mathbf{B}$, is used to prove the existence of the electric field:

$$\frac{d}{dt} \int_S \mathbf{B} \cdot \mathbf{n} dS = \int_S \left\{ \frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) + \mathbf{v} \nabla \cdot \mathbf{B} \right\} \cdot \mathbf{n} dS$$

(3.40)

Applying (3.40) on the surface $S$, making use of (3.3) results in:

$$U_{EMF} = -\frac{d}{dt} \int_S \mathbf{B} \cdot \mathbf{n} dS = \int_S \left\{ -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) \right\} \cdot \mathbf{n} dS$$

(3.41)

From (3.40) and (3.41) is concluded that an electric field is induced by a time variant magnetic field ($\frac{\partial \mathbf{B}}{\partial t}$) on the one hand and the movement through the magnetic field ($\mathbf{v} \times \mathbf{B}$) on the other hand. Application of Stokes’ theorem (C.8) on the right term of (3.41) and applying $\frac{\partial \mathbf{B}}{\partial t} = 0$ leads to:

$$U_{EMF} = -\frac{d}{dt} \int_S \mathbf{B} \cdot \mathbf{n} dS = \oint_C (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{\tau} dl$$

(3.42)

Combining (3.6) and (3.42) results in:

\textsuperscript{3}wire resistances are left out of the consideration.
\[
\frac{d}{dt} \oint_S \mathbf{B} \cdot \mathbf{n} dS = \oint_C \mathbf{E} \cdot \mathbf{t} dl = \oint_C (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{t} dl
\]  
\tag{3.43}

So the induced electric field strength due to the velocity is \( \mathbf{E} = \mathbf{v} \times \mathbf{B} \). In the moving subvolume \( V_c \) we recognize that: \( E_y = -v_x \cdot B_z \). The external voltage \( U \) must compensate for this induced electric field strength. The electric field strength experienced by the electrons\(^4\): \( E_y^* = -E_y \). Combining this result with (3.39) results in:

\[
\mathbf{f} \cdot \mathbf{v} = \mathbf{E} \cdot \mathbf{J} = E_y^* \cdot J_y
\]

\[
f_x = \frac{v_x \cdot B_z}{v_x} \cdot J_y = J_y \cdot B_z \tag{3.44}
\]

The same analysis can be carried out for magnetic fields in the other directions (i.e. \( B_x \) and \( B_y \)), leading to the famous Lorentz expression of (3.29):

\[
\mathbf{f} = \mathbf{J} \times \mathbf{B} \tag{3.45}
\]

Although (3.45) is the most straightforward relation for calculating the force, it must be used with care. In the particular case of Figure 3.3, the entire magnetic flux \( \lambda = \oint \mathbf{B} \cdot \mathbf{n} dS \), passing through the surface \( S \), will increase when the movable conductor is displaced in \( x \)-direction. As was shown, this increase of flux would cause an electric field \( E_y \) in the wires (better known as back emf) if the conductor would have a horizontal velocity. With (3.44) was proven, that a force will be generated when a current (density) is applied to the wires (also when the movable conductor has zero velocity): Lorentz’s law is valid. But we must be aware that in any configuration with magnets and coils, where no induced voltage is caused by a relative motion, there will be no net force when a current is applied to the coils.

This is illustrated by the example of a non functioning motor shown in Figure 3.4. The mover consists of a permanent magnet attached to a back iron, which is movable in \( y \)-direction. The stator consists of an

\(^4\) No losses are considered in this conservative system. Therefore, no electric field due to resistive losses has to be accounted for.
3.3. FORCES

iron yoke, through which the flux from the permanent magnet passes. Around this yoke a coil is wound, having its windings in $x$-direction. The reader might suggest that a force in $y$-direction will act between the magnet and coil yoke, if a current is present in the coil wires. Lorentz’s law predicts: $f_y = -J_x \times B_z$. However, this is not the case, since the entire magnetic flux ($\lambda = \int \mathbf{B} \cdot \mathbf{n}dS$) through the coil does not vary if the magnet would move in $y$-direction. So there will be no force interaction between the movable magnet and the coil yoke. The force interaction exists between the coil and the iron yoke, since the entire magnetic flux ($\lambda = \int \mathbf{B} \cdot \mathbf{n}dS$) through the coil varies if the coil is shifted in $y$-direction over the iron core. This example illustrates how careful one must be with the use of Lorentz’s law.

Additionally, in a lot of magnet configurations, the wires of the coils are only subjected to relatively low fields, but currents through the coils do generate high forces. This is the case in iron core actuators for example, where the coils are wound on teeth of highly permeable material. Also in this case, the force generation is explained by the change of magnetic flux linkage due to relative motion, even though

Figure 3.4: Example of a non functioning motor.
the magnetic flux density at the coil’s wires is low (or zero).

So Lorentz’s law must be used with care. A safe approach for force calculations is to make use of the energy balance of (3.37). The usual and always valid expression for the energy balance is the general form of (3.37):

\[ F \cdot dr = i_{\text{coils}} \partial \lambda \varphi - \partial W_{\text{magnetic}} = i_{\text{coils}} \partial \lambda \varphi - \partial W_{\text{magnets}} - \partial W_{\text{coils}} \]  

(3.46)

in which \( i_{\text{coils}} \) [A] is the current through the coils, \( \lambda \varphi \) [Wb] the magnetic flux linkage with the coils due to the magnets, \( W_{\text{magnets}} \) the stored magnetic energy due to the magnets and \( W_{\text{coils}} \) the stored magnetic energy due to the coils. This relation is found frequently in literature [5], [27] and [29].

### 3.3.4 Forces present in actuators

In most of the actuators electrical energy is supplied in order to generate mechanical energy. In such electromechanical actuators, the field from current conducting coils interferes with the field generated by other coils or permanent magnets. All possible forces occurring in electromechanical actuators, as listed in Section 2.1.3, are represented by (3.46). The actuator force is represented by \( F_{\text{actuator}} = i_{\text{coils}} \frac{\partial \lambda \varphi}{\partial r} \). The coils interfere with the field from the magnets. The cogging force is determined by \( F_{\text{cogging}} = - \frac{\partial W_{\text{magnets}}}{\partial r} \), expressing the force related to permeable material and magnets. Finally, the reluctance force, \( F_{\text{reluct}} = - \frac{\partial W_{\text{coils}}}{\partial r} \), expresses the force experienced by current conducting coils due to permeable materials. Losses due to damping forces and magnetic hysteresis should be subtracted from the effective actuator force \( F_{\text{actuator}} \).

### 3.4 Earnshaw’s theorem

In the first half of the nineteenth century, Earnshaw contributed to the development of the understanding of the physical field as Maxwell and Faraday did. In 1842, he published his famous article *On the nature of molecular forces which regulate the constitution of the luminiferous ether* [10]. He stated:
3.4. EARNSHAW’S THEOREM

“It is impossible to have a position of stable equilibrium for a pole placed in a static field of force.”

Successors refined Earnshaw’s postulate: Stable levitation is impossible in magnetostatic (or electrostatic) fields when all present materials have $\mu_r \geq 1$ (or $\varepsilon_0 \geq 1$), but possible if materials with $\mu_r < 1$ (or $\varepsilon_0 < 1$) are used. From this refinement, it is concluded that stable levitation in a static field is impossible, but if diamagnetic materials ($\mu_r < 1$) or even super-conducting materials ($\mu_r = 0$) are used, stable levitation in a static magnetic field is possible. A non-stable situation exists if a body experiences an increasing force in the direction of the displacement (see Figure 3.5), so:

$$\frac{\partial f_x}{\partial x} > 0 \quad (3.47)$$

The condition for stability is in that sense:

$$\frac{\partial f_x}{\partial x} < 0 \quad (3.48)$$

The statement of Earnshaw and successors can be quantified starting with the energy conservation law (3.37). Assume a system with permanent magnets where no current density is present ($\mathbf{J} = 0$). Equation (3.37) reduces to:

$$\mathbf{f} = -\mathbf{H} \frac{\partial \mathbf{B}}{\partial \mathbf{r}} = -\mathbf{H} \cdot \nabla \mathbf{B} = -\nabla w_{\text{magn}} \quad (3.49)$$
Applying the divergence to (3.49), we obtain:

\[ \nabla \cdot f = \nabla \cdot (-\nabla w_{magn}) = -\nabla^2 w_{magn} \quad (3.50) \]

Successively, three cases are analyzed: only materials with \( \mu_r = 1 \) are present, materials with \( \mu_r > 1 \) are present (as well) and finally materials with \( \mu_r < 1 \) are present (as well).

In the first case, when only materials with \( \mu_r = 1 \) are present, no field energy is used to magnetize the present material (other than vacuum), so \( \nabla^2 w_{magn} = 0 \) and we arrive at:

\[ \nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 0 \quad (3.51) \]

When a ferromagnetic body with \( \mu_r > 1 \) is placed in the environment of a constant magnetic source, the total magnetic energy will decrease when the body is moved towards the source (\( \int w_m dV \) decreases). According to (3.49) the body experiences an attracting force. This effect becomes stronger when approaching the source, so \( \nabla^2 w_{magn} < 0 \) and \( \nabla \cdot f > 0 \) (3.50).

\[ \nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} > 0 \quad (3.52) \]

Finally, the case in which the source is approached by a diamagnetic material (\( \mu_r < 1 \)). The body experiences a repellent force since \( \nabla w_{magn} > 0 \). This repellent force is stronger if the body moves towards the magnetic source, so \( \nabla^2 w_{magn} > 0 \) and therefore \( \nabla \cdot f < 0 \):

\[ \nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} < 0 \quad (3.53) \]

Equation (3.51) - (3.53) are the so-called Earnshaw relations for stability.

### 3.5 Damping theorem

Damping is nature’s tendency to suppress motion. This short proposition states that when a conducting body (\( \sigma_e > 0 \)) is placed in an external magnetic field and is subjected to motion, forces are induced.
3.5. DAMPING THEOREM

To clarify this, we first rewrite (2.4) in a general form, defining damping:

\[ d_x = -\frac{\partial f_x}{\partial \dot{x}} \]  

(3.54)

In general, the damping \( d_x \) [N·s/m] is a function of magnetic and geometric properties. An induced electric field is generated by a time varying magnetic field \( (\nabla \times \mathbf{E}_t = -\frac{dB}{dt}) \) and/or motion of the body in a magnetic field \( (\mathbf{E}_m = \mathbf{v} \times \mathbf{B}) \). Both contribute to the total induced electric field \( (\mathbf{E}) \) in a body. In bodies with non-zero conductivity \( (\sigma_c) \), the electric field leads to vorticing currents: eddy currents. The determination of the eddy currents in finite bodies is not trivial. The trajectory of eddy currents is strongly determined by the geometry and the conductivity of the body. An example is illustrated in Figure 3.6. A bar moves with a constant velocity in a homogeneous, time-invariant magnetic field \( B_z \). In Section 3.3.3, it was expounded that an electric field \( (E_y) \) is induced due to this motion. From the left case (1) in Figure 3.6 can now be seen that \( E_y \) is homogeneous in the bar if \( B_z \) is homogeneous, and therefore currents will not flow. Currents start flowing in the \( xy \)-plane when \( E_y \) is non-homogeneous as illustrated in the right case (2). Note that currents also start flowing when a conducting connection outside the magnetic field between the front end and back end of the bar is made (as was shown in Figure 3.3). Eddy currents lead finally to damping forces, because they interfere with the magnetic field according to (3.45). The outer product in (3.45) indicates that the damping force will always be directed in opposite direction of the velocity and therefore the trivial relation for damping can be deduced:

\[ \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \leq 0 \]  

(3.55)

Since the damping force is always directed opposite to the velocity, the even stronger expressions hold for the damping:

\[ \frac{\partial f_x}{\partial x} \leq 0, \frac{\partial f_y}{\partial y} \leq 0, \frac{\partial f_z}{\partial z} \leq 0 \]  

(3.56)
In terms of energy, damping forces dissipate and transfer mechanical energy \((\mathbf{f} \cdot \mathbf{dr})\) to electrical losses. Once the eddy currents \((\mathbf{J})\) within a volume \(V_c\) are known, by application of (3.12), the dissipative losses due to eddy currents can be obtained, making use of (3.13):

\[
P = \iiint \frac{\mathbf{J}(\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')}{\sigma_e(\mathbf{r'})} dV_c(\mathbf{r'})
\]  

(3.57)

in which the conductivity \(\sigma_e \, [\text{S/m}]\) \((\sigma_e = \frac{1}{\rho_e})\) and the current density \(\mathbf{J} \, [\text{A/m}^2]\) are functions of the coordinates in the considered body. A more straightforward method of determining the heat generation in a
3.5. DAMPING THEOREM

body due to damping is found when we reconsider that:

\[ d\dot{w}_{\text{mech}} + d\dot{w}_{\text{elec}} = 0 \Rightarrow \dot{w}_{\text{elec}} = -\mathbf{f} \cdot \mathbf{r} \quad (3.58) \]

If (3.54) and (3.58) are combined, the conclusion that electrical losses (and so material heating, \( P \)) are linear with the damping (\( d \)) and the squared velocity of the body (\( \mathbf{r}^2 \)). In finite element and boundary element solvers, the electrical losses due to eddy currents can be calculated quite easily. The amount of damping is then obtained by:

\[
\begin{align*}
    d_x &= \frac{P}{\dot{x}^2} \\
    d_y &= \frac{P}{\dot{y}^2} \\
    d_z &= \frac{P}{\dot{z}^2}
\end{align*}
\quad (3.59)
\]

In this chapter, the theory on Electromechanics was expounded. The tools needed for the design process were derived or elaborated. The design process is described in the subsequent chapter.
CHAPTER 3. ELECTROMECHANICS
Chapter 4

Dynamic Gravity Compensator

In Chapter 2 the need for highly accurate actuators was demonstrated by considering the effect of cross talk between different machine parts as the long- and short-stroke systems. In Section 2.1.3 was subsequently expounded what difficulties the designer has to cope with when designing accurate actuators. Chapter 3 gave an overview of the electromechanic theory and tools that are available for the design process. In this chapter the entire design and validating process of the dynamic gravity compensator are presented. In Paragraph 4.1 the function of the dynamic gravity compensator is clarified and a list of specifications is introduced. The magnetic and geometric basic configuration is shown in Paragraph 4.2. Three models were used for designing the dynamic gravity compensator: two (numerical) boundary element models, which are presented in Section 4.3.1, and an analytical model, derived in Section 4.3.2. The test bench, manufacturing aspects and measurement results of the proto-type are added in Paragraph 4.4. Finally the discussion, comparison and conclusion of the results of the models and the proto-type are expounded in Paragraph 4.5.
CHAPTER 4. DYNAMIC GRAVITY COMPENSATOR

4.1 Function

In Chapter 2 was clarified that the dynamic gravity compensator has to generate a force to compensate for the gravity force. In Figure 4.1 the short-stroke and long-stroke systems are schematically redrawn in their coordinate system. Gravity acts in the vertical direction ($z$) as indicated. The long-stroke system is suspended by a (passive) bearing system. The short-stroke system however will be positioned vertically by the dynamic gravity compensators as shown in Figure 4.2. Since three dynamic gravity compensators are used for positioning in the vertical direction, each system will only need to carry one third of the entire weight of the short-stroke system’s mass. In addition to the static force, needed for compensating gravity, a controllable dynamic force is needed for compensating four types of forces:

- **Levelling forces:** the wafer is in general not a perfectly flat disc. Variations in its thickness and curvature are compensated by the short-stroke system. The dynamic gravity compensators generate the required forces for this compensation.

- **Acceleration forces:** the actuators for generating forces in the $xy$-plane are not perfectly aligned with the short-stroke’s centre of
Figure 4.2: Gravity compensators between short-stroke and long-stroke systems.
gravity. Therefore torques around the $\theta_x$- and $\theta_y$-axis occur when accelerating in $x$- and $y$-direction (see Figure 2.6). The dynamic gravity compensators have to exert vertical forces to counteract these torques.

- Disturbance forces: unwanted displacements of the short-stroke system occur due to all unknown and known forces that are not accounted for in the controller’s feed forward strategy. The dynamic gravity compensators need to generate forces to keep these displacements as small as possible.

- Gravity forces: the static gravity force on the short-stroke mass will be counteracted by the gravity compensator only partly, due to tolerances in short-stroke mass (1%), tolerances in magnetic remanence and coercitive field strengths (5%) or mechanical tolerances of the gravity compensator. This so-called mass-mismatch must be compensated by the dynamic part of the gravity compensator.

As mentioned previously, these controllable forces should be added to the static, gravity compensating force. Therefore, we can distinguish two functions for the dynamic gravity compensator: one is to generate the static force against gravity (the so-called static gravity compensator function) and the other is to generate a controllable, dynamic force in addition to the static force (the $z$-actuator function). The configuration of the three dynamic gravity compensators, as shown in Figure 4.2, is able to generate forces in vertical direction ($z_{ss}$) and perform torques around the horizontal axes ($\theta_x$ and $\theta_y$) on the short-stroke mass. Since a limited volume is available for this function, the volume of the dynamic gravity compensators has to be restricted. Moreover the mass should be minimized and last but not least the dissipation should be kept low (see also Section 2.1.3). The following table lists the relevant specifications for each dynamic gravity compensator.
4.1. FUNCTION

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static force (F_{z,static})</td>
<td>85</td>
<td>N</td>
</tr>
<tr>
<td>Dynamic force range (F_{z,dynamic})</td>
<td>-20...+20</td>
<td>N</td>
</tr>
<tr>
<td>Maximum mover mass (m_{mover})</td>
<td>0.50</td>
<td>kg</td>
</tr>
<tr>
<td>Maximum stator mass (m_{stator})</td>
<td>1.0</td>
<td>kg</td>
</tr>
<tr>
<td>Motion range ((x_{ss}, y_{ss}, z_{ss})) ((Figure 4.1))</td>
<td>(2,2,2)</td>
<td>mm</td>
</tr>
<tr>
<td>Maximum stiffness ((k_x, k_y, k_z))</td>
<td>(500,500,500)</td>
<td>N/m</td>
</tr>
<tr>
<td>Maximum damping ((d_x, d_y, d_z))</td>
<td>(0.2,0.2,0.2)</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Maximum dissipation(^3)</td>
<td>40</td>
<td>W</td>
</tr>
<tr>
<td>Maximum bounding volume ((x, y, z))</td>
<td>(100,100,50)</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 4.1: Specifications for the dynamic gravity compensator.

Related studies on passive magnetic bearings were presented in several studies. In [20] and [21] the behavior of a passive, repulsive axial bearing consisting of two axially magnetized rings is analyzed. Another type of passive magnetic axial bearing for flywheel energy storage systems is presented in [22]. The use of numerical methods (2D and 3D FEM) for the design of a radial bearing with permanent magnets is discussed in [23]. A repulsive axial micro-bearing is presented in [24] and [26]. In [25] a repulsive magnetic bearing based on eddy currents is discussed. A magnetically levitated actuator in six degrees of freedom is expounded in [27], [28], [32], [33], [40] and [41]. The method for calculating the radial stiffness in a passive axial magnetic bearing, consisting of 2 coaxial magnetic rings is presented in [38]. In [55] different proposals for magnet configurations in passive magnetic bearings are presented. Finally, in [39] a magnet bearing consisting of a passive part and an active part is elaborated.

The work presented in this dissertation is different from other studies, since the dynamic gravity compensator generates a high (constant) axial force with low stiffness. This is realized by the magnet configuration (see Paragraph 4.2), which was not found elsewhere in literature. Moreover, the dynamic gravity compensator is able to exert a controllable force. These two functions have been united in a single design.

\(^1\)Required stroke of mover with respect to stator.
\(^2\)Cross talk between stator and mover.
\(^3\)Dynamic force equals 20 [N].
4.2 Basic configuration

Figure 4.3: Three dimensional view of the gravity compensator.

In Figure 4.3 a three dimensional view of the basic configuration of the dynamic gravity compensator is shown. The design, which was also presented in [13], [14], [15] and [17], is rotationally symmetric since it then becomes less sensitive to manufacturing inaccuracies and variations of magnetic properties in comparison to non rotationally symmetric configurations on the one hand and the fact that the current gravity compensator systems\(^4\) are rotationally symmetric on the other hand. The electro-magnetic configuration consists of four rings of permanent magnet material and one coil. The utmost inner and outer magnet rings are attached to the stator of the dynamic gravity compensator, which is connected to the long-stroke system (see Figure 4.2). The same holds

\(^4\)The current gravity compensators operate with pressured air.
for the coil. There are two reasons why the coil is located on the stator: On the one hand heat generation at the mover (which is connected to the short-stroke system) should be minimized, since the slightest temperature gradients in the mechanics connected to the mover will lead to unwanted deformations. On the other hand wiring from the stator to the mover (for powering and cooling the coil) should be restricted to minimize cross talk forces (see Section 2.1.2 for a detailed explanation).

The magnet rings close to the coil are attached to the mover, the short-stroke system.

Figure 4.4: Dynamic gravity compensator.

In Figure 4.4 the axi-symmetric 2D view of the configuration is presented. For simplicity reasons only one half of the configuration is shown. The inner and outer (stator) magnets have a vertical magnetization, indicated by a vertical arrow. The horizontally magnetized, moving magnets experience an upwards directed force by the vertical magnetization of the stator magnets. Intuitively this is known since opposing magnetic poles are attracted to each other and similar poles repel each other. So the moving magnets experience an attractive force with respect to the upper surfaces of the stator magnets and a repulsive one with respect to their lower surfaces. The radial distance between
the moving magnets and stator magnets is an important parameter for the static force magnitude. The smaller the distance, the larger the force. This is intuitively understood when noticing that the magnetic fields are stronger close to a magnetic source. The stronger the magnetic fields, the stronger the force caused by these fields (this is known from (3.32) and (3.33)). For this reason a magnet material with high remanence is chosen, to reduce the weight, since the force magnitude is also determined by the magnet volume of both types of magnet. Adding magnet material results into a stronger field as well.

The vertical force decreases when the moving magnets are moved upwards from the centre position (as shown in Figure 4.4). This is experienced as a positive stiffness of the moving magnets in the vertical direction as will be expounded in Section 4.3.1. The moving magnets experience a negative stiffness with respect to the lower poles of the stator magnets, since the vertical force is decreased as well when the moving magnets are moved downwards. The net vertical stiffness is the sum of both contributions. Previous considerations lead therefore to the expectation that in a certain vertical interval the vertical stiffness will be low and even zero at a certain vertical position (when the moving magnets are vertically located in the middle of the stator magnets). In Paragraph 3.4 was derived that \( \nabla \cdot \mathbf{f} = 0 \) for the case where \( \mu_r = 1 \). In a system defined in an \((r, z)\)-coordinate system this means (see Appendix D.4 for the derivation):

\[
\frac{\partial f_z}{\partial z} + 2 \frac{\partial f_r}{\partial r} = 0 \tag{4.1}
\]

Since \( \mu_r = 1.05 \) approximately for the magnets and \( \mu_r = 1.00 \) for the copper of the coil, the system nearly meets the condition \( \mu_r = 1 \). So for the positions of the moving magnets where the vertical stiffness \( \left( \frac{\partial f_z}{\partial z} \right) \) is low, a low horizontal stiffness \( \left( \frac{\partial f_r}{\partial r} \right) \) is expected as well. However \( \mu_r \) of the magnets is slightly larger than 1 and therefore the sum in (4.1) will be slightly larger than zero; stating that zero stiffness for both the vertical direction \( (z) \) and the radial direction \( (r) \) will not occur simultaneously.

In the design we must be aware of the implication of (3.52). If highly permeable materials \( (\mu_r \gg 1) \) such as iron are placed in the environment of the magnetic configuration, the stiffness will not meet the
4.2. BASIC CONFIGURATION

specifications anymore. From this point of view materials like copper, aluminium, titanium or ceramics (AlN, AlO or SiC) are applicable.

Since the stiffness between the moving magnets and the stator magnets is defined as the force variation for applied displacements (according to (2.2)), small distances between magnet poles also result into high stiffness in general. So for low stiffness values, the radial distance between the moving magnets and stator magnets should be kept relatively large.

Another complication is the damping (see Paragraph 3.5) between stator and mover. Since both stator and mover consist of permanent magnets a relative motion of the stator with respect to the mover induces eddy currents with damping as consequence. Highly conductive materials like aluminium ($\sigma_e = 3.817 \cdot 10^7$ S/m 5) or copper ($\sigma_e = 5.8 \cdot 10^7$ S/m) definitely should be avoided, but materials like titanium ($\sigma_e = 2.3 \cdot 10^6$ S/m) or carbon-fibre based materials ($\sigma_e = 7.1 \cdot 10^4$ S/m) can be used. Eddy currents are also present in the permanent magnets themselves, since the typical value of the conductivity of sintered neodymium iron boron magnets $\sigma_e = 7.1 \cdot 10^5$ S/m 6.

The magnetic field between the moving magnets will mainly be directed in radial direction. The interaction of this field with the tangential current density through the coil will generate a vertical force. According to (3.29) the force on the coil is found as:

$$F = \int \int \int \mathbf{J} \times \mathbf{B} \cdot dV_c$$

(4.2)

where $V_c$ is the current conducting volume of the coil. The required dynamic force was specified in Section 4.1. From (4.2) can be concluded that the current density $\mathbf{J}$ (and so the dissipation according to (3.13)) is kept low as long as the magnetic induction $\mathbf{B}$ due to the magnets at the coil is relatively high. This is achieved when magnets with relatively high remanence are used and when the radial distance between the moving magnets and the coil is kept as small as possible. This minimum

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5Conductivity at 293 K.
6Eddy currents in magnets occur due to varying magnetic field densities generated by other, neighbouring magnets or coils.
radial distance however is prescribed by the required stroke in radial direction of the moving magnets.

In Paragraph 4.3 the force generation by the permanent magnets and the coil is explained in detail.

4.3 Models

Three models were used for the synthesis of the dynamic gravity compensator design. Two boundary element models were used to design a configuration meeting the specifications presented in Section 4.1. A two-dimensional modeler was used to find the gravity compensator dimensions in an iterative process. The two-dimensional solver is relatively fast, but only calculates forces in the vertical direction ($F_z$). For radial forces and torques the three-dimensional modeler was required.

Parallel to these boundary element models an analytical model was derived. This analytical model will be used for the numerical optimization process as described in Chapter 5.

In Figure 4.5 the dimensions of the dynamic gravity compensator are shown in an $(r, z)$-coordinate system. The height of the static magnets is $2 \cdot h_s$, $2 \cdot h_m$ is the height of the moving magnets. The coil height is defined as $2 \cdot h_{coil}$. The inner and outer radii of the inner and outer static magnets are known as $R_{ii}$, $R_{io}$, $R_{oi}$ and $R_{oo}$ respectively. The width of the moving magnets is $b_m$ and the coil has a width $b_{coil}$.

4.3.1 Boundary element model

The Boundary Element Method was used to develop a configuration of the design, which meets the specifications. This method is based on the integral Maxwell expressions (3.5)-(3.8), where the Finite Element Method is based on the differential expression (3.1)-(3.4). A detailed explanation of the boundary element method is found in the user guide [11].
4.3. MODELS

Figure 4.5: Dimensions of the gravity compensator.

Force and stiffness

In Figure 4.6 the boundary element model is shown. After an iterative process the dimensions (according to Figure 4.5) were found for a design meeting the specifications. The dimensions are listed in Table 4.2.

<table>
<thead>
<tr>
<th>quantity</th>
<th>value</th>
<th>quantity</th>
<th>value</th>
<th>quantity</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{im}$</td>
<td>9</td>
<td>$R_{oi}$</td>
<td>5.5</td>
<td>$R_{im}$</td>
<td>19</td>
</tr>
<tr>
<td>$h_s$</td>
<td>16</td>
<td>$R_{io}$</td>
<td>10</td>
<td>$R_{om}$</td>
<td>33</td>
</tr>
<tr>
<td>$h_{coil}$</td>
<td>7</td>
<td>$R_{oi}$</td>
<td>46</td>
<td>$b_m$</td>
<td>4</td>
</tr>
<tr>
<td>$R_{coil}$</td>
<td>26</td>
<td>$R_{oo}$</td>
<td>48.5</td>
<td>$b_{coil}$</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4.2: Dimensions (mm) of the dynamic gravity compensator.

The following considerations played a role in this process. The static magnet’s height was chosen considerably larger ($h_s$) than the moving magnets ($h_{im}$) to obtain a low stiffness between the static and
moving magnets. The top and bottom surface of the static magnets were initially chosen equal, so that the total amount of magnetic flux, generated by these magnets, would be more or less equal. However, manufacturing limitations require a minimum thickness of 2.5 mm for the outer magnet ring \( R_{oo} - R_{oi} \), so the top and bottom surface of the outer static magnet ring is larger than that of the inner static magnet ring. The outside radius of the outer static magnet ring was limited to \( R_{oo} = 48.5 \) mm, in order to make the design fit in the current dimensions of the wafer scanner. In Section 4.2, it was mentioned that large distances between the static magnets and moving magnets result into low stiffness values. These distances were kept as large as possible.

The radial distance between the moving magnets and the coil was minimized in order to limit the dissipation on the one hand (see Paragraph 4.2) and to obtain a homogeneous magnetic field between the moving magnets on the other hand. A homogeneous magnetic field at the coil limits the variation of the vertical force (due to a current density through the coil), since \( \mathbf{J} \times \mathbf{B} \) in (4.2) is kept as constant as possible over the coil volume, when the magnets are displaced in vertical direction. The coil height \( h_{\text{coil}} \) was therefore chosen slightly smaller than
the moving magnet height \( h_m \). If the coil height would be similar to the magnet height, the fringing field at the upper and lower region of the moving magnets would result in force variations.

The magnet material was modelled with \( B_r = 1.2 \) T and \( H_{CB} = -955 \) kA/m. Note that \( \mu_r = 1 \) consequently holds for the permanent magnets. In general \( \mu_r > 1 \), but for validating the analytical model \( \mu_r = 1 \) is chosen (the effects of this assumption are discussed in Paragraph 4.5).

The ortho-cyclic wound copper coil [34] was assumed to have a filling factor of \( f = 0.65 \), where the filling factor \( f \) is known as the fraction of the coil’s cross section \( A_{coil} \) that effectively conducts the current \( A_c \) [34]:

\[
f = \frac{A_c}{A_{coil}}
\]  

In Figure 4.7 the magnetic field lines (flux lines) in the configuration are shown. The vertically directed force, as expounded in Paragraph 4.2, is well illustrated by the field lines, if one would imagine a tight sur-
face around the moving magnets. Integrating the Maxwell stress tensor (3.30) over the imagined surface clarifies the vertical force. In simple terms: imagine the field lines as elastic strings which tend to shorten in the longitudinal direction and repel each other in the transversal direction.

The moving magnets were moved from 1 mm below the centre position in vertical direction over a distance of 2 mm upwards (this represents the allowable stroke in vertical direction, as shown in Figure 4.4) in several steps ($z_{ss} = -1, \ldots, 1$ mm). The stator magnets were considered fixed ($z_{ls} = 0$). For each step the vertical force ($F_z$) on the moving magnets was calculated for the case no current was flowing through the coil. When knowing $F_z$, the stiffness between mover and stator $k_z$ is calculated by recalling (2.2):

$$k_z(z_{ss}) = \frac{\partial F_z}{\partial (z_{ls} - z_{ss})} = -\frac{\partial F_z}{\partial z_{ss}}$$  \hspace{1cm} (4.4)

In Figure 4.8 the static force ($F_z$) and the stiffness ($k_z$) are shown for the different vertical positions of the mover with respect to the stator. The static force is about $F_z = 85$ N and the stiffness does not exceed $k_z = 260$ N/m which is within the specifications. From the results is also concluded that $k_z = 0$ at the vertical position $z_{ss} = 0$ as was predicted in Paragraph 4.2. For $z_{ss} < 0$ the moving magnets experience a negative stiffness and for values $z_{ss} > 0$ they experience a positive stiffness as was elaborated as well in Paragraph 4.2.

The same analysis is done for the case, where the total current through the coil is $i = 655$ Aturns. Figure 4.9 shows the result. From these results, the motor constant is derived, defined as:

$$K_z(z_{ss}) = \frac{\partial F_z(z_{ss}, i)}{\partial i_{coil}}$$  \hspace{1cm} (4.5)

where $F_z(z_{ss}, i)$ [N] represents the vertical force on the moving magnets due to the current through the coil. In order to calculate the force due to the coil, the static force (values in Figure 4.8) should be subtracted from the values in Figure 4.9. At $z_{ss} = 0$ the motor constant $K_z(z_{ss})$ of the design is $K_z(z_{ss} = 0) = \frac{F_z(i_{coil})}{655} = \frac{104.52 - 84.43}{655} = 0.0307$ N/Aturns. At $z_{ss} = \pm 1 \Rightarrow K_z(z_{ss}) = \frac{104.15 - 84.30}{655} = 0.0303$ N/Aturns: a variation
Figure 4.8: Static force $F_z$ and stiffness $k_z$ (2D BEM).
Figure 4.9: Force $F_z(z_{ss})$ with current of 655 Aturns through coil (2D BEM).
of only \( \left| \frac{K_z(z_{ss}=\pm 1)-K_z(z_{ss}=0)}{K_z(z_{ss}=0)} \right| = 1.3\% \).

### Three dimensional analysis

By the two-dimensional rotationally symmetric model, as shown in Figure 4.6, we are able to predict the vertical force, stiffness and motor constant for any vertical displacement of the moving magnets. However, for the determination of parasitic forces in radial direction and the torques around a radial axis on the moving magnets, a three-dimensional model of the system is required [11]. In Figure 4.3 the three-dimensional model of the dynamic gravity compensator is shown. In this model \( \mu_r = 1 \) is chosen as was the case for the two-dimensional model. The moving magnets were displaced from the centre position, \((r_{ss}, z_{ss}) = (0,0)\), in both vertical and radial direction. Negative radial displacements correspond to negative displacements along the horizontal axis through the center of the design.

The vertical force \( F_z \) as function of the radial and vertical displacements is shown in Figure 4.10. The similarity with Figure 4.8 is immediately seen, since Figure 4.8 represents the curve in Figure 4.10 \( F_z(r_{ss}, z_{ss}) = F_z(0, z_{ss}) \): no radial displacement of the moving magnets. The vertical stiffness \( k_z \) is calculated again with (4.4). The result is shown in Figure 4.11. The similarity with the two-dimensional case (Figure 4.8) is evident. The maximum vertical stiffness equals \( k_{z,\text{max}} = 250 \) N/m. An extensive comparison between the two-dimensional model and three-dimensional model is given in Paragraph 4.5.

The radial force on the moving magnets is displayed in Figure 4.12, where negative radial forces indicate that the force is directed in the negative direction of the horizontal axis through the center of the design (along which the displacements were made). Since the design is rotationally symmetric, no radial force exists for \( r_{ss} = 0 \). The radial stiffness is derived from the data presented in Figure 4.12.

\[
k_r(r_{ss}, z_{ss}) = -\frac{\partial F_r(r_{ss}, z_{ss})}{\partial r_{ss}} \quad (4.6)
\]

The results are shown graphically in Figure 4.13.
Figure 4.10: Vertical force $F_z$ on moving magnets (3D BEM).
Figure 4.11: Vertical stiffness $k_z$ (3D BEM).
Figure 4.12: Radial force $F_r$ on moving magnets (3D BEM).
Figure 4.13: Radial stiffness $k_r$ (3D BEM).
Figure 4.14: Torque around radial axis $T_r$ on moving magnets (3D BEM).
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Figure 4.15: Motor constant $K_z$ (3D BEM).
Figure 4.16: Parasitic motor constant $K_r$ (3D BEM).
When taking a closer look to Figure 4.13 we notice that the radial stiffness \( \frac{\partial F_r}{\partial r_{ss}} \) is almost only dependent on the vertical displacement \( z_{ss} \). In a mathematical expression this implies:

\[
\frac{\partial F_r}{\partial r_{ss}} = C_0 \cdot z_{ss}
\]  

(4.7)

in which \( C_0 [N/m^2] \) is a constant to be determined later. In (4.1) was found that \( \frac{\partial F_z}{\partial z_{ss}} = -2 \frac{\partial F_r}{\partial r_{ss}} \). Substituting this in (4.7) and integrating with respect to \( z_{ss} \) results into:

\[
F_z(r_{ss}, z_{ss}) = -C_0 \cdot z_{ss}^2 + C_1
\]  

(4.8)

Hence \( C_1 [N] \) might be a function of the radial displacement \( r_{ss} \). Integrating (4.7) with respect to \( r_{ss} \) results into:

\[
F_r(r_{ss}, z_{ss}) = C_0 \cdot z_{ss} \cdot r_{ss} + C_2
\]  

(4.9)

(Unfortunately Earnshaw only relates \( \frac{\partial F_z}{\partial z_{ss}} \) to \( \frac{\partial F_r}{\partial r_{ss}} \), and not cross talk behavior of the radial and vertical force: \( \frac{\partial F_z}{\partial r_{ss}} \) and \( \frac{\partial F_r}{\partial z_{ss}} \). So the constants \( C_0 [N/m^2] \), \( C_1 [N] \) and \( C_2 [N] \) are derived from Figure 4.10 and Figure 4.12. Since

\[
F_r(0, 0) = 0 \Rightarrow C_2 = 0
\]  

(4.10)

and

\[
F_r(0.001, 0.001) = 0.125 \Rightarrow C_0 = 125000
\]  

(4.11)

Assuming a second order polynomial fit for \( C_1(r_{ss}) \):

\[
C_1(r_{ss}) = K_0 \cdot r_{ss}^2 + K_1
\]  

(4.12)

From Figure 4.10 is seen: \( F_z(0, 0.001) = 84.290 \) N and \( F_z(0.001, 0.001) = 84.365 \) N so \( K_0 = 75000 \) N/m\(^2\) and \( K_1 = 84.4 \) N. Therefore one yields finally:

\[
F_z(r_{ss}, z_{ss}) = -125000 \cdot z_{ss}^2 + 75000 \cdot r_{ss}^2 + 84.4
\]  

(4.13)

and

\[
F_r(r_{ss}, z_{ss}) = 125000 \cdot z_{ss} \cdot r_{ss}
\]  

(4.14)

Note that (4.13) and (4.14) satify the Earnshaw relation for stability (4.1).
If the moving magnets are displaced in radial direction, a torque around the axis through the centre of the moving magnets perpendicular to the direction of movement occurs. This torque [Nm] is shown in Figure 4.14. The torque is only dependent on the radial displacement.

The motor constant of the coil $K_z$ [N/Aturn] is shown in Figure 4.15. From the figure is seen that $K_z(0, 0) = 0.0307$ N/Aturns. The function of the coil is to generate a controllable vertical force. But due to fringing of the fields between the moving magnets, a parasitic radial force exists as well which is dependent on the coil’s current. The parasitic radial motor constant relates the coil’s current to the radial force on the moving magnets:

$$K_r(r_{ss}, z_{ss}) = \frac{\partial F_r(r_{ss}, z_{ss}, i)}{\partial i_{coil}}$$ (4.15)

This parasitic motor constant of the coil $K_r$ [N/Aturn] is shown in Figure 4.16. This force is position dependent as seen in Figure 4.16. Compared to the motor constant in vertical direction it is rather low ($K_r$ is approximately 1.3 % of $K_z$).

In the previous results, only the effect on the forces and torques due to vertical and radial displacements ($r_{ss}, z_{ss}$) were discussed. But also rotations around the horizontal axis ($\theta_x$ and $\theta_y$) affect these forces and torques. However, in practice those rotations will be less than 10 mrad, hardly having any impact on the forces and torques.

**Dissipation**

In Chapter 3 was shown that the dissipation is determined by (3.13). We may assume that the current density $J$ is uniformly distributed over the current conducting area $A_c$ since the frequencies of the current are low. The dissipation is then, according to (3.13):

$$P = \frac{V_c \cdot J^2}{\sigma_e}$$ (4.16)

in which $\sigma_e$ [S/m] is the electric conductivity and $V_c$ [m$^3$] the current conducting volume of the coil. If $A_{coil}$ is the cross-section area of the coil, the current conducting area ($A_c$) is found with (4.3): $A_c = f \cdot A_{coil}$. If the total ampere-turns in the coil is designated as $N \cdot i_{coil}$, the current
density in the copper wires is: \( J = \frac{N \cdot i_{coil}}{f \cdot A_{coil} \cdot K_z} \). Since the motor constant is known (relating vertical force to the total ampere-turns of the coil, \( F_z = K_z \cdot N \cdot i_{coil} \)), it is possible to relate force and current density:

\[
J = \frac{F_z}{f \cdot A_{coil} \cdot K_z} \tag{4.17}
\]

The bounding coil volume \( V_{coil} \) is also related to the current conducting volume \( V_c \) by \( f \): \( V_c = f \cdot V_{coil} \) The geometric properties of the coil are shown in Figure 4.5:

\[
A_{coil} = 2 \cdot h_{coil} \cdot b_{coil} \tag{4.18}
\]

\[
V_{coil} = 2 \cdot h_{coil} \cdot \pi \cdot ((R_{coil} + b_{coil})^2 - R_{coil}^2) \tag{4.19}
\]

In general the steepness of an actuator (as the \( z \)-actuator function of the dynamic gravity compensator) is known as:

\[
S_z = \frac{F_z^2}{P} \tag{4.20}
\]

This formula relates force and dissipation. The force is squared to make the steepness \( (S_z) \) independent of the current through the coil. Substituting (4.16) and (4.17) in (4.20) results in the steepness of the dynamic gravity compensator:

\[
S_z = \frac{\sigma_e \cdot f \cdot A_{coil}^2 \cdot K_z^2}{V_{coil}} \tag{4.21}
\]

Inserting the numerical values for \( f = 0.65 \), \( A_{coil} = 70 \cdot 10^{-6} \text{ m}^2 \), \( K_z = 0.03 \text{ N/Aturns} \), \( \sigma_e = 5.0 \cdot 10^7 \text{ S/m}^7 \) and \( V_{coil} = 1.25 \cdot 10^{-5} \text{ m}^3 \) \( \Rightarrow S_z = 11.5 \text{ N}^2/\text{W} \). So for generating a force of \( F_z = 20 \text{ N} \) a dissipation of \( P = \frac{F_z^2}{S_z} = 34.8 \text{ W} \) in the coil occurs.

The reader must be aware that the electric conductivity of materials \( \sigma_e \) and electric resistivity \( (\rho_e = \frac{\sigma_e}{\rho_e}) \) are temperature dependent. In general the resistivity increases with increasing temperature:

\[
\rho_e(T) = \frac{1}{\sigma_e} = \rho_{e,293} \cdot (1 + \alpha \cdot \Delta T) \tag{4.22}
\]

\(^7\text{Electric conductivity of copper at 293 K.}\)
where $\alpha$ [K$^{-1}$] is the temperature coefficient of the electric resistivity (for copper $\alpha = 4.0 \cdot 10^{-3}$ K$^{-1}$). The steepness $S_z = 11.5$ N$^2$/W holds for a temperature of $T = 293$ K. However, due to heating, the coil temperature will increase, affecting the electric conductivity (and resistivity) according to (4.22). The steepness $S_z$ will decrease. So if the mean temperature of the coil would be for example $T = 313$ K the steepness is decreased to $S_{z,T=313} = 10.6$ N$^2$/W. For the same force in vertical direction 8% more heat is dissipated compared to the case where the coil would be at room temperature.

Once the dissipation in the coil is known the thermal behavior of the actuator can be analyzed. The thermal behavior strongly depends on the so called thermal duty cycle ($\delta_t$) of the actuator. The current density is in general a time dependent property: $J = J(t)$. If the maximum current density in a period $T$ is $J_{\text{max}}$ the thermal duty cycle is defined as:

$$\delta_t = \frac{\int T J^2 dt}{\int T J_{\text{max}}^2 dt} = \frac{\int T J^2 dt}{T J_{\text{max}}^2 \cdot T}$$

(4.23)

For the application of the dynamic gravity compensator the thermal duty cycle turns out to be $\delta_t = 0.4$. For thermal calculations the mean dissipation is related to the thermal duty cycle:

$$P_\delta = \delta_t \cdot P$$

(4.24)

where $P$ was found by (4.16). In this particular case: $P_\delta = 0.4 \cdot 34.8 = 13.9$ W.

**Thermal behavior**

Special measures are taken for cooling the coil during operation. Therefore a cooling circuit was designed. In Figure 4.17 the cooling strategy for the coil is shown in a two dimensional, rotationally symmetric view. The copper coil (with thermal conductivity $\lambda_{\text{coil}} = 1.0$ W/mK) is wound on a ceramic aluminium nitride cylinder of thickness 1 mm. Aluminium nitride has a high thermal conductivity ($\lambda_{\text{AlN}} = 180$ W/mK) and an extremely low electric conductivity ($\sigma_{e,\text{AlN}} \approx 1.0 \cdot 10^{-5}$ S/m, preventing
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Figure 4.17: Cooling configuration.

Eddy currents and the related damping forces (see Paragraph 3.5)). An additional requirement for applicable materials is a relative permeability \( \mu_r \) of (nearly) one, to prevent additional, unwanted, reluctance forces and stiffness between stator and moving magnets. This requirement holds for AlN as well. Both coil and cylinder are glued on a Titanium housing \( (\lambda_{Ti} =13 \text{ W/mK} \text{ and } \sigma_{e,Ti} =2.3 \cdot 10^6 \text{ S/m}) \) in which two cooling channels are made. The glue is assumed to have a thickness of 0.2 mm and a thermal conductivity of \( \lambda_{\text{glue}} =1.0 \text{ W/mK} \). The thermal convection coefficient to the cooling water is \( h_{\text{water}} =5000 \text{ W/m}^2\text{K} \) (see (4.45) in Section 4.3.2). The cooling water temperature may assumed to be constant at \( T =293 \text{ K} \). In practice the cooling water is heated by the dissipated power, but as long as the water flow is considerably large \( (\dot{V}_{\text{water}} =1.0 \text{ l/min}) \) related to the dissipated heat \( (P_s =13.9 \text{ W}) \), the water temperature increase along the cooling channel can be neglected. This is proven by the specific heat of water \( (c =4180 \text{ J/kgK}) \) [6]:
\[ \Delta T = \frac{P_\delta}{\rho \cdot c \cdot V} \]  

(4.25)

where \( \rho \) [kg/m\(^3\)] is the density and \( V \) [m\(^3\)/s] is the flow of the water. Inserting the numerical values: \( \Delta T = 0.20 \) K.

Figure 4.18: Temperature distribution [K] in the configuration.

In Figure 4.18 the temperature distribution is displayed for the parts as shown in Figure 4.17, solved by a thermal boundary element solver [11]. Only conduction (through the copper wires, aluminium nitride, glue and titanium housing) and convection to the water channels are assumed to occur. Convection and radiation to the environment are not considered\(^8\). The hot spot of the coil, which is located at the outside

\(^8\)These assumptions imply a worst case scenario for the thermal behavior of the system.
of the coil at the upper surface (see Figure 4.18), will become 330 K; a
temperature rise of 37 K with respect to room temperature. From Fig-
ure 4.18 is concluded that the mean coil temperature is approximately
increased with 20 K. The steepness of the actuator has then decreased
by 8% as shown previously by (4.21) and (4.22). The total dissipated
amount of heat will therefore not be $P_\delta = 13.9$ W but $P_\delta = 15.0$ W. This
result was found by making use of the derivation in Appendix D.6. The
thermal analysis is redone for this amount of dissipation. The mean
coil temperature increases with 21.5 K.

### 4.3.2 Analytical model

Besides the numerical analyses (as expounded in the previous section),
analytical models were derived. These analytical models consist of
equations for calculating the force, stiffness, dissipation, mass and vol-
ume, which quantities are determined by the dimensions of the design
and the material choice. The analytical models are subsequently used
for the numerical optimization process clarified in Chapter 5.

#### Force and stiffness

The derivations in this dissertation are mainly based on the work pre-
sented in [42] and [43], which derives an analytical expression for the
magnetic field due to a current conducting coil. Comparable expres-
sions are given in [45], [46], [47], [48], [50] and [51]. The levitation
force between two magnetic discs (opposing each other), making use of
Biot-Savart’s law, is described in [49]. Powerful algorithms for elliptic
integrals required in field and potential computations were presented
in [52] and [54].

In Section 3.2.4 was demonstrated that Biot-Savart’s law is used
to determine the magnetic field density $\mathbf{B}$ due to a current density $\mathbf{J}$
flowing through a region $V_c$. If the permanent magnets are modelled as
current density distributions (as was presented in [5] and [31]), Biot-
Savart’s law enables us to derive the magnetic field distributions due to
these magnets. The equivalent current density model of the dynamic
gravity compensator is shown in Figure 4.19. The magnets are mod-
elled by circular currents. Each magnet with vertical magnetization is
represented by two current sheets, whose axes are parallel with respect to the magnetization direction of the magnets and have infinite small thickness (dimension in radial direction). The horizontally magnetized magnets are modelled by a radial current distribution as shown in Figure 4.19. The validity of this is proven in Appendix D.5. The equivalent linear current density \( J_S [\text{A/m}] \) of these sheets is: \( J_S = -H_{cB} \). The coil is modelled as a certain number of vertical current sheets as well. The magnetic field density due to the static magnets and the coil at the location of the current distributions of the radially magnetized magnets is determined by Biot-Savart’s law (3.25). Knowing these field density contributions, the force on the moving magnets due to the static magnets and the coil is calculated by Lorentz’s law (3.29).

First the field components due to a vertical current sheet is considered. In Figure 4.20 a vertical current sheet is shown with height \( 2 \cdot h \) and radius \( R \). The linear current density is \( J_S \). The magnetic field density is now considered in an arbitrary point \( P(r, z) \).

Starting from the magnetic vector potential we recall (3.21) for the case the current is contained by a sheet (surface \( S \)) as shown in Figure 4.20 and expressed by the linear current density \( \mathbf{J}_S \ [\text{A/m}] \) as \( \mathbf{J}_S(\mathbf{r'}) = \)
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\[ A = \frac{\mu_0}{4\pi} \int_S \frac{J_S(r')}{|r - r'|} dr' \]  

(4.26)

In Figure 4.21 the cylinder sheet is redrawn and the top view of this sheet is shown as well. The vector \( \mathbf{r} \) is pointing to the point \( P(r, z) \) located in the \( xz \)-plane (\( \varphi = 0 \)). The vector \( \mathbf{r}' \) describes the sheet \( S \) \((r' = R, \varphi' = [0, 2\pi]\) and \( z' = [-h, h] \)) containing the linear current density \( J_S [A/m] \). Since \( J_S \) is uniformly distributed within the sheet \( S \) and only consists of a component in the tangential direction \( (J_{S,\varphi'}) \), we express \( J_S = (0, J_S, 0) \). Due to symmetry, only the tangential component of \( \mathbf{A} \) \((A_\varphi) \) exists in \( P(r, z) \): \( A_\varphi(r, z) = 0 \) due to symmetry of the linear current distribution and \( A_z(r, z) = 0 \), since no vertical component in the linear current distribution \( J_S \) is present.

\[ J_S(r') dr' = J_S \cos \varphi' R d\varphi' dz' \]  

(4.27)
CHAPTER 4. DYNAMIC GRAVITY COMPENSATOR

Figure 4.21: Derivation of the magnetic induction.

For obtaining \(|\mathbf{r} - \mathbf{r}'|\) we express \(\mathbf{r}\) and \(\mathbf{r}'\) in cylindrical coordinates: 
\[
\mathbf{r}(x, y, z) = (r, 0, z) \quad \text{and} \quad \mathbf{r}'(x, y, z) = (R \cos \varphi', R \sin \varphi', z').
\]
From these results the length of the difference vector \(\mathbf{r} - \mathbf{r}'\) is derived:
\[
|\mathbf{r} - \mathbf{r}'| = \sqrt{(r - R \cos \varphi')^2 + (-R \sin \varphi')^2 + (z - z')^2} = \sqrt{(z - z')^2 + R^2 + r^2 - 2R \cdot r \cos \varphi'}
\]
(4.28)

Substituting the results of (4.27) and (4.28) in (4.26) yields:
\[
A_\varphi(r, z) = \frac{\mu_0 \cdot J_\varphi}{4\pi} \int_{z'=-h}^{h} \int_{\varphi'=0}^{2\pi} \frac{R \cos \varphi'}{\sqrt{(z - z')^2 + R^2 + r^2 - 2R \cdot r \cos \varphi'}} \, dz' \, d\varphi'
\]
(4.29)

Substituting \(u = z - z'\) and introducing \(a = \frac{\mu_0 J_\varphi}{2\pi}, u_+ = z + h\) and \(u_- = z - h\) (4.29) reduces to:
\[
A_\varphi(r, z) = a \int_{u=-h}^{u_+} \int_{\varphi'=0}^{\pi} \frac{R \cos \varphi'}{\sqrt{u^2 + R^2 + r^2 - 2R \cdot r \cos \varphi'}} \, du \, d\varphi'
\]
(4.30)

Once knowing the expression for the magnetic vector potential \(\mathbf{A}\) the magnetic field density \(\mathbf{B}\) is found by applying (3.22). In cylinder coordinates this yields:
\[ (B_r, B_\varphi, B_z) = \nabla \times (A_r, A_\varphi, A_z) \]
\[ = \nabla \times (0, A_\varphi, 0) = (\frac{\partial A_\varphi}{\partial z}, 0, \frac{1}{r} \frac{\partial (r \cdot A_r)}{\partial r}) \] (4.31)

Substituting (4.30) into (4.31) results in the radial and vertical components of the magnetic induction \( B \) in \( P(r, z) \). For \( B_r \) this yields:

\[ B_r(r, z) = -\frac{\partial A_\varphi}{\partial z} = -a \cdot R \int_{\varphi'}^{\pi} \frac{\cos \varphi'}{\sqrt{u^2 + R^2 + r^2 - 2R \cdot r \cos \varphi'}} d\varphi' \bigg|_{u^+}^{u^-} \] (4.32)

Introducing \( \delta = \frac{u}{\sqrt{2Rr}}, \beta = \frac{R^2 + r^2}{2R^2} \) and \( k^2 = \frac{2}{1 + \beta + \delta^2} \) the expression for the radial field component (4.32) is rewritten as:

\[ B_r(r, z) = -a \cdot \frac{R}{\sqrt{2R \cdot r}} \int_{\varphi'}^{\pi} \frac{\cos \varphi'}{\sqrt{\delta^2 + \beta - \cos \varphi'}} d\varphi' \bigg|_{k^-}^{k^+} \]
\[ = -a \cdot \frac{R}{\sqrt{2R \cdot r}} \int_{\psi}^{\pi/2} \frac{-2 \cos 2\psi}{\sqrt{\delta^2 + \beta + 1 - 2 \sin^2 \psi}} d\psi \bigg|_{k^-}^{k^+} \]
\[ = a \sqrt{\frac{R}{r}} \cdot k \int_{\psi}^{\pi/2} \frac{1 - 2 \sin^2 \psi}{\sqrt{1 - k^2 \sin^2 \psi}} d\psi \bigg|_{k^-}^{k^+} \] (4.33)

where the substitution \( \psi = \frac{\pi - \varphi'}{2} \) is made. Introducing the standard elliptic integrals \( E(k) \) and \( K(k) \) as described in Appendix C.5 we finally arrive at:

\[ B_r(r, z) = a \cdot \sqrt{\frac{R}{r}} \cdot \frac{2}{k} \left[ (1 - \frac{k^2}{2}) K(k) - E(k) \right]_{k_-}^{k_+} \] (4.34)

For the vertical component of the magnetic induction \( B_z \) can be deduced:
\[ B_z(r, z) = \frac{1}{r} \frac{\partial(r \cdot A_\phi)}{\partial r} \]
\[ = \frac{1}{r} \frac{\partial}{\partial r} \left( r \cdot \int_{u=u_-}^{u_+} \int_{\varphi'=0}^{\pi} \frac{R \cos \varphi'}{\sqrt{u^2 + R^2 + r^2 - 2Rr \cos \varphi'}} du d\varphi' \right) \]

which simplifies to:

\[ B_z(r, z) = a \cdot \frac{k \cdot \delta}{\sqrt{2}} \left[ K(k) + \frac{R - r}{R + r} \Pi\left( -\frac{2}{1 + \beta}, k \right) \right]^{\delta_+}_{\delta_-} \]  

In (4.34) and (4.36) the elliptic integrals \( E(k) \), \( K(k) \) and \( \Pi(\rho, k) \) appear. The expressions for \( B_r \) and \( B_z \) were written in these simplified forms to quicken the numerical calculation. This makes numerical optimization possible in a manageable period of time as will be executed in Chapter 5.

When having an expression for \( B_r(r, z) \) and \( B_z(r, z) \) a vector graph of the magnetic field around the sheet is easily made. In Figure 4.22 the magnetic vector field plot is shown for a current sheet with \( J_S = -H_{cB} = 955 \text{ kA/m} \), height \( 2h = 20 \text{ mm} \) and radius \( R = 5 \text{ mm} \). The absolute value of the magnetic induction \( |B(r, z)| = \sqrt{B_r^2(r, z) + B_z^2(r, z)} \) is shown in Figure 4.23.

Equation (4.34) and (4.36) enable us to calculate the magnetic induction due to vertical current sheets. Each vertically magnetized (stator) magnet of the dynamic gravity compensator is modelled as two vertical sheets as shown in Figure 4.19. Since the superposition principle holds for magnetic induction, we simply may add the magnetic induction due to the inner and outer sheet. The magnetic induction due to each stator magnet ring is calculated by adding the contributions of both sheets. For the inner stator magnet, the inner sheet is modelled by inserting \( R = R_{ii} \) and \( J_S = -955 \text{ kA/m} \) \( (u_+ = z + h_s, u_- = z - h_s, \delta_+ = \frac{u_+}{\sqrt{2R_{ii}\pi}}, \delta_- = \frac{u_-}{\sqrt{2R_{ii}\pi}}, \text{ and so on for } \beta, k_+ \text{ and } k_-) \) into (4.34) and (4.36). The outer sheet is modelled by inserting \( R = R_{io} \) and \( J_S = 955 \text{ kA/m} \) \( (u_+ = z + h_s, u_- = z - h_s, \delta_+ = \frac{u_+}{\sqrt{2R_{io}\pi}}, \delta_- = \frac{u_-}{\sqrt{2R_{io}\pi}}, \text{ and so on for } \beta, k_+ \text{ and } k_-) \).
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Figure 4.22: Magnetic vector field plot (analytical).
Figure 4.23: Magnetic induction (analytical).
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for $\beta$, $k_+$ and $k_-$ into (4.34) and (4.36). For the outer stator magnet the same is repeated with $R = R_{oo}$ and $R = R_{oo}$ respectively.

In this way $B_r(r, z)$ and $B_z(r, z)$, generated by the axially magnetized magnets (stator magnets), are obtained at the location of the upper and lower current sheets of the moving (horizontally magnetized) magnets (see Figure 4.19). In Figure 4.5 is shown that these current sheets are located at $z = \pm h_m + z_{ss}$ and $r \in [R_{im}, R_{im} + b_m]$ for the inner moving magnet and at $z = \pm h_m + z_{ss}$ and $r \in [R_{om}, R_{om} + b_m]$ for the outer moving magnet, in which $z_{ss}$ represents the displacement of the moving magnets with respect to the centered vertical position as shown in Figure 4.4. The vertical force on the inner moving magnet due to the field of the stator magnets is found by applying (3.29):

$$ F_z(z_{ss}) = 2\pi \int_{\varphi=0}^{\varphi=0} \int_{r=R_{im}}^{r=R_{im}+b_m} B_r(r, h_m + z_{ss}) \cdot J_S \cdot r \cdot d\varphi \cdot dr + $$

$$ + 2\pi \int_{\varphi=0}^{\varphi=0} \int_{r=R_{im}}^{r=R_{im}+b_m} B_r(r, -h_m + z_{ss}) \cdot J_S \cdot r \cdot d\varphi \cdot dr $$

$$ = 2\pi \cdot J_S \int_{r=R_{im}}^{r=R_{im}+b_m} (B_r(r, -h_m + z_{ss}) - B_r(r, h_m + z_{ss})) \cdot r \cdot dr $$

(4.37)

In a similar way the vertical force on the outer moving magnet is found:

$$ F_z(z_{ss}) = 2\pi \int_{\varphi=0}^{\varphi=0} \int_{r=R_{om}}^{r=R_{om}+b_m} B_r(r, h_m + z_{ss}) \cdot J_S \cdot r \cdot d\varphi \cdot dr + $$

$$ + 2\pi \int_{\varphi=0}^{\varphi=0} \int_{r=R_{om}}^{r=R_{om}+b_m} B_r(r, -h_m + z_{ss}) \cdot J_S \cdot r \cdot d\varphi \cdot dr $$

$$ = 2\pi \cdot J_S \int_{r=R_{om}}^{r=R_{om}+b_m} (B_r(r, -h_m + z_{ss}) - B_r(r, h_m + z_{ss})) \cdot r \cdot dr $$
Finally, the static force on the moving magnets is the sum of the results of (4.37) and (4.38).

As for the numerical boundary element model the moving magnets were moved from 1 mm below the centre position in vertical direction over a distance of 2 mm. For each step the vertical force on the moving magnets was determined according to (4.37) and (4.38). The stiffness is (again) calculated by (4.4). In Figure 4.24 the vertical force on the moving magnets and the derived stiffness are shown for the different vertical positions of the moving magnets ($z_{ss}$).

As expounded before the coil is modelled as a finite number of vertical current sheets (see Figure 4.19). It is modelled consisting of 10
Figure 4.25: Force with current through coil (analytical).
vertical current sheets. Equation (4.34) is first used to determine the radial component of the magnetic induction $B_r$ due to each vertical current sheet. The contribution of all sheets to the induction is added, since the super-position theorem holds. When having added the contribution of each vertical current sheet, (4.37) and (4.38) are used to calculate the force on the moving magnets due the field of the coil. The results are shown in Figure 4.25. The figure shows the force on the moving magnets due to the stator magnets and the current carrying coil together as was done for the numerical case in Figure 4.9. The results are discussed in Paragraph 4.5.

**Thermal behavior**

In the previous section the thermal behavior of the dynamic gravity compensator was analyzed by a numerical boundary element model. In this section a simplified analytical thermal model is derived.

Heat flowing through a material causes a temperature gradient across the material, according to the thermal conduction law [6]:

$$ q'' = -\lambda \frac{dT}{dz} $$  \hspace{1cm} (4.39)

where $q''$ is the heat flux through the material [W/m²], $\lambda$ the thermal conductivity [W/mK] and $\frac{dT}{dz}$ the temperature gradient [K/m].

In Figure 4.26 the coil, dissipating a power $P$, the aluminium nitride cylinder and the titanium cooling housing are shown in the left configuration (1). One fraction of the total heat ($P1$) is flowing in radial direction into the ceramic aluminium nitride cylinder and then flowing downwards through a glue layer. Another fraction ($P2$) flows directly downwards through the glue layer (configuration 2). The equivalent electric model is shown in configuration 3. Resistance $r1$ represents the ceramic cylinder, $r2$ and $r3$ represent the glue layer, $r4$ is the titanium housing and $r5$ is the heat resistance to the water channel. The temperature drop across a resistance is found by:

$$ \Delta T = r \cdot P $$  \hspace{1cm} (4.40)

in which the resistance $r = \frac{l}{\lambda A}$ (where $l$ the length of the heat path, $A$ the cross section area and $\lambda$ the thermal conductivity) and $P$ is the heat flux.
4.3. MODELS

It is quite straightforward to determine the resistances $r_2$, $r_3$ and $r_4$, but the determination of $r_1$ is more complicated, since not all heat $P_1$ is flowing entirely through $r_1$. To determine into what fractions ($P_1$ and $P_2$) the total amount of heat $P$ is split, an equivalent resistance $r_{1,\text{eq}}$ of $r_1$ is required.

In Figure 4.27 is illustrated that the downwards heat flow through the ceramic cylinder $P_{\text{ceramic}}(z)$ decreases with increasing height $z$; in an expression:

$$P_{\text{ceramic}}(z) = \frac{h_{\text{ceramic}} - z}{h_{\text{ceramic}}} P_1 \quad (4.41)$$

The temperature drop $dT(z)$ across an infinite small element $dz$ is according to (4.40):

$$dT(z) = d (r \cdot P_{\text{ceramic}}(z)) = \frac{dz}{\lambda \cdot A} \cdot \frac{h_{\text{ceramic}} - z}{h_{\text{ceramic}}} P_1$$

$$= \frac{P_1}{\lambda \cdot A} \left( 1 - \frac{z}{h_{\text{ceramic}}} \right) dz \quad (4.42)$$
Integrating (4.42) leads to the temperature increase as function of \( z \):

\[
\Delta T(z) = \frac{P_1}{\lambda \cdot A} \left( z - \frac{z^2}{2 \cdot h_{\text{ceramic}}} \right) \tag{4.43}
\]

The equivalent resistance of \( r_1 \), \( r_{1eq} \), is found by the definition in (4.40):

\[
r_{1eq} = \frac{\Delta T(z = h_{\text{ceramic}})}{P_1} = \frac{h_{\text{ceramic}}}{2 \cdot \lambda \cdot A} \tag{4.44}
\]

The numerical values of the resistances are: \( r_{1eq} = \frac{h_{\text{ceramic}}}{2 \cdot \lambda_{1-A_1}} = \frac{14 \cdot 10^{-3}}{2 \cdot 180 \cdot 160 \cdot 10^{-6}} = 0.24 \text{ K/W} \), \( r_2 = \frac{l_2}{\lambda_{2-A_2}} = \frac{0.2 \cdot 10^{-3}}{1.0 \cdot 160 \cdot 10^{-6}} = 1.25 \text{ K/W} \), \( r_3 = \frac{l_3}{\lambda_{3-A_3}} = \frac{0.2 \cdot 10^{-3}}{1.0 \cdot 895 \cdot 10^{-6}} = 0.22 \text{ K/W} \). The temperature drop across the titanium housing is found by applying (4.40): \( \Delta T_{ti} = \frac{l_4}{\lambda_{4-A_4}} \cdot P = \frac{h_{ti}}{\lambda_{ti-A_{ti}}} \cdot P = \frac{5.0 \cdot 10^{-3}}{13.0 \cdot 1055 \cdot 10^{-6}} \cdot 15.0 = 5.5 \text{ K} \) (\( r_4 = \frac{h_{ti}}{\lambda_{ti-A_{ti}}} = 0.36 \text{ K/W} \)). All dimensions according to the configuration in Figure 4.17.

For determining the heat flow to the cooling channels the law of thermal convection is used:

\[
q'' = h \cdot (T - T_\infty) \tag{4.45}
\]

where \( h \) is the thermal convection coefficient [W/m\(^2\)K], \( T \) the temperature of the material and \( T_\infty \) the temperature of the cooling fluid [K]. The resistance in terms of (4.40) is then: \( r = \frac{1}{h \cdot A} \). The total cooling area
of the water channels in Figure 4.26 (or Figure 4.17) is $A = 3000 \cdot 10^{-6}$ m$^2$. The resistance $r_5 = 0.07$ K/W, since $h = 5000$ W/m$^2$K.

For determining the fractions of heat ($P_1$ and $P_2$), we need to take a closer look into the model of the coil itself. It is not valid anymore to establish a one-dimensional heat flow model. In Figure 4.28 the model is redrawn (note that the entire height of the coil is indicated as $2 \cdot h_{coil}$ in accordance with the convention at the beginning of Section 4.3.2). At any position $z$ in the coil we assume a fraction of the heat flowing in horizontal direction and a fraction flowing in vertical direction. We assume that the total amount of horizontally directed heat flow is forced through the entire width of the coil ($h_{coil}$) and that the vertically directed heat flow will flow through the entire height of the coil ($2 \cdot h_{coil}$); this simplification is a worst case approach, since this implies that all heat is forced through the entire coil width and height respectively. If there was only one-dimensional flow in the horizontal direction, the equivalent resistance would be, according to (4.44): $r_{h,eq} = \frac{h_{coil}}{2 \cdot A} = \frac{5.10^{-3}}{2 \cdot 1.0 - 2287.10^{-6}} = 1.1$ K/W. If there was only one-dimensional flow in the vertical direction, the equivalent resistance would be: $r_{v,eq} = \frac{2 \cdot h_{coil}}{2 \cdot A} = \frac{14 \cdot 10^{-3}}{2 \cdot 1.0 \cdot 895.10^{-6}} = 7.8$ K/W.

The simplified model of configuration 1 in Figure 4.26 is shown in Figure 4.29. One fraction ($P_1$) of the heat generated by the coil

\footnote{\textit{h} is dependent on the flow properties of the water as velocity and hydraulic diameter of the cooling channels (\textit{h} =5000 is a valid value for the current design).}

Figure 4.28: Modelling the coil.
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flows through the horizontal equivalent resistance \( (r_{h,eq}) \), through the equivalent resistance of the ceramic cylinder \( (r_{1,eq}) \) and the glue layer \( (r2) \). The other fraction \( (P2) \) flows through the vertical equivalent resistance \( (r_{v,eq}) \) and the glue layer \( (r3) \). Both heat flows then join and flow through the titanium housing \( (r4) \) to the water channels \( (r5) \). The division of the entire heat \( (P) \) into the fractions \( (P1 \text{ and } P2) \) is (as in the electric equivalent) proportional to the resistances, so:

\[
P1 = P \frac{r_{v,eq} + r3}{r_{h,eq} + r_{1,eq} + r2 + r_{v,eq} + r3} \tag{4.46}
\]

\[
P2 = P \frac{r_{h,eq} + r_{1,eq} + r2}{r_{h,eq} + r_{1,eq} + r2 + r_{v,eq} + r3} \tag{4.47}
\]

Inserting the numerical values for the resistances we arrive at \( P1 = 11.3 \) W and \( P2 = 3.7 \) W. We conclude that a substantial amount of heat \( (80\%) \) is flowing through the ceramic cylinder. To determine the hot spot temperature of the coil all temperature drops across the resistances in Figure 4.29 need to be added, by using (4.40): \( \Delta T_{coil} = 36 \) K. In the numerical analysis a temperature rise of the hot spot of 37 K was found.

Figure 4.29: Simplified thermal model.
4.3. MODELS

This result demonstrates that the derived analytical thermal model is adequate for the determination of the temperatures in the actuator.

Masses

In Section 2.1.3 the importance of light weight constructions for accurate applications was emphasized. The masses of the moving and stator parts are calculated by the volumes (see Figure 4.5):

\[
V = 2 \cdot h \cdot \pi \cdot (R_o^2 - R_i^2) \tag{4.48}
\]

where \(2 \cdot h\) [m] is the height of the magnets, \(R_o\) [m] is the outer radius of the magnets and \(R_i\) [m] is the inner radius. The mass follows by:

\[
m = \rho \cdot V \tag{4.49}
\]

in which \(\rho\) [kg/m\(^3\)] is the density of the material. The density of the used magnet material (NdFeB) is 7800 kg/m\(^3\). The density of the copper is 8900 kg/m\(^3\). The total mass of the stator magnets is \(m_s = 0.239\) kg. From these numbers is calculated that the moving magnets weigh \(m_m = 0.197\) kg. The coil’s mass in the current design (filling factor \(f = 0.65\)) \(m_{coil} = 0.073\) kg.

The moving magnets are glued in an aluminium nitride housing as shown in Figure 4.30. The dimensions are \(h_{house} = 26\) mm, \(t_{top} = 5\) mm and \(t_{side} = 2.5\) mm (\(R_{im} = 19\) mm, \(R_{om} = 33\) mm and \(b_m = 4\) mm were presented previously). From these numbers the volume of the ceramic housing is calculated with:

\[
V_{house} = (h_{house} - t_{top}) \cdot \pi \cdot ((R_{im})^2 - (R_{im} - t_{side})^2) + (h_{house} - t_{top}) \cdot \pi \cdot ((R_{om} + b_m + t_{side})^2 - (R_{om} + b_m)^2) + t_{top} \cdot \pi \cdot ((R_{om} + b_m + t_{side})^2 - (R_{im} - t_{side})^2) \tag{4.50}
\]

which results into \(V_{house} = 43279\) mm\(^3\) and \(m_{house} = 0.274\) kg (density of 6340 kg/m\(^3\)). The total moving mass (magnets and housing) is \(m_{moving} = 0.472\) kg for each dynamic gravity compensator.
4.4 Proto-type results

The numerical and analytical models as presented in Section 4.3.1 and Section 4.3.2 are verified by the proto-type of the dynamic gravity compensator design. In Figure 4.31 a photo is shown of the static part of the dynamic gravity compensator. The outer magnet ring consists of 24 segments of permanent magnets and the inner magnet ring consists of 2 cylindrically shaped magnets. This for manufacturing reasons. Both magnet configurations are magnetized in vertical direction according to Figure 4.4. The magnet housing is made of titanium ($\mu_r = 1.0$). The coil and the aluminium nitride coil housing ($\mu_r = 1.0$) are clearly visible in between the stator magnet rings. Figure 4.32 displays the moving magnets. The radially magnetized rings each consist of 12 segments incorporated in an aluminium nitride housing ($\mu_r = 1.0$). For the dimensions of the magnets and coil is referred to Table 4.2. The magnet material for both stator and moving magnets is Neorem 476a: $B_r = 1.2$ T and $H_{cB} = -910$ kA/m (manufactured by Neorem, Finland). Note
Figure 4.31: Stator part of dynamic gravity compensator.
Figure 4.32: Moving magnets and housing.
that \( \mu_r = \frac{B}{B_{r,m}} = 1.05 \). The mass of the moving magnets including their aluminium nitride housing is \( m_{\text{moving}} = 0.504 \text{ kg} \). The moving mass according to the model equals \( m_{\text{moving}} = 0.472 \text{ kg} \). This difference is explained by the fact that the mass of the glue, for fixating the magnets, and the Ni-coating of the magnets was neglected in the model. The ortho-cyclic wound coil consists of \( N=105 \) turns insulated copper windings with a copper diameter of 0.71 mm and an average insulation thickness of 30 \( \mu \text{m} \). The coil resistance is \( R=0.79 \Omega \) at 293 K.

![Figure 4.33: Entire gravity compensator.](image)

Figure 4.33 shows the entire dynamic gravity compensator mounted in a separately manufactured force test bench, where the moving magnet rings are shifted over the coil, in between the stator magnets. The moving magnets are attached to a sensor, which measures the forces \( (F_x, F_y \text{ and } F_z) \) and torques \( (T_x, T_y \text{ and } T_z) \). The sensor consists

\(^{10}\)The magnetization curve is assumed to be linear in the second quadrant.
CHAPTER 4. DYNAMIC GRAVITY COMPENSATOR

of flexible elements on which strain gauges are mounted. The strain gauges measure the deflections of the flexible elements. These deflections are converted to forces and torques by software. The sensor itself is connected to the static part of the test bench. The test bench, on which the dynamic gravity compensator is mounted, is shown in Figure 4.34. The sensor, at the lower side connected to the housing of the moving magnets, was connected to the test bench at the upper side. The housing of the stator magnets was attached to a manipulator. The manipulator allows the stator magnets to move both in vertical and radial direction with respect to the moving magnets. Also rotations around the radial axis are possible. In this manner, the dependency on translations and rotations of the forces and torques on the moving magnets was measured.

Figure 4.34: Gravity compensator test bench.
4.4. PROTO-TYPE RESULTS

4.4.1 Force and stiffness

For the analytical and numerical models the forces and torques on the moving magnets were analyzed for different vertical and radial positions of the moving magnets. The test bench allowed us to do the same analysis for the proto-type. The moving magnets were displaced from the centre position \((r_{ss}, z_{ss}) = (0,0)\) in both vertical and radial direction. Figure 4.35 shows the vertical force \((F_z)\) on the moving magnets as a function of a vertical and radial displacement relative to the center position. From Figure 4.35 the vertical stiffness \((k_z)\) is derived with (4.4). The result is shown in Figure 4.36. The radial force \((F_r)\) and derived radial stiffness \((k_r)\) are visualized in Figure 4.37 and 4.38. The torque around a radial axis \((T_r)\) on the moving magnets was analyzed as
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Figure 4.36: Vertical stiffness $k_z$ (proto).
Figure 4.37: Radial force $F_r$ on moving magnets (proto).
Figure 4.38: Radial stiffness $k_r$ (proto).
Figure 4.39: Torque around radial axis $T_r$ on moving magnets (proto).
well (Figure 4.39). In Section 4.3.1 the torque was determined around the axis through the centre of the moving magnets. In Figure 4.34 is noticed that the sensor is located above the moving magnets. This offset affects the torque measurement. The torque around the radial axis ($T_r$) on the moving magnets is calculated with:

$$T_r = T_{r,s} - F_r \cdot d_s$$

(4.51)

where $T_{r,s}$ [Nm] is the torque measured by the sensor, $F_r$ [N] the radial force on the moving magnets and $d_s$ [m] the vertical offset of the sensor with respect to the centre of the moving magnets. The offset in the test bench is $d_s=60.0$ mm.

The current through the coil ($i_{\text{coil}}$) varied from -7 to +7 A while the vertical force ($F_z$) was analyzed for ($r_{ss}, z_{ss}$) = (0, 0). The results are
4.4.  PROTO-TYPE RESULTS

illustrated in Figure 4.40. The figure discloses that the vertical force
$F_z(i, r_{ss}, z_{ss})$ is $F_z = 82.1$ N for $i_{coil} = 0$ A and $F_z = 103.5$ N for $i_{coil} = 7$ A.
The motor constant $K_z$ for $(r_{ss}, z_{ss}) = (0, 0)$ is calculated with (4.5):

$$K_z = \frac{\partial F_z(i, r_{ss}, z_{ss})}{\partial i_{coil}}$$  \hspace{1cm} (4.52)

leading to $K_z = \frac{103.5 - 82.1}{7} = 3.06$ N/A or $K_z = 0.029$ N/Aturn since the
number of turns $N=105$.

The comparison of the previous results with the results of the models
is discussed in Paragraph 4.5.

4.4.2  Thermal behavior

In order to determine the thermal behavior of the proto-type, a tem-
perature sensor (PTC)\textsuperscript{11} was connected to the hot spot of the coil.
From Figure 4.18 was concluded that the coil’s hot spot is located at
the right upper corner. The sensor was thermally connected to the
hot spot, by gluing it on a copper foil, which was shifted between the
last two (outer) winding layers. In Figure 4.41 the temperature of the
hot spot is shown as function of the time for different current levels.
The current was switched on at $t=0$ s, when the coil was at room tem-
perature. The water flow through the cooling ducts was 1.0 l/min.
The previous measurements can be approximated by a function. The
general expression for the coil heating is:

$$T(i, t) = T_\infty + C_0 \cdot i^2 \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$  \hspace{1cm} (4.53)

in which $T_\infty$ [K] the temperature of the environment, $i$ [A] is the coil
current\textsuperscript{12}, $t$ [s] the time, $\tau$ [s] the thermal time constant and $C_0$ [K/A\textsuperscript{2}]
a constant (related to the thermal resistance of the model). An approx-
imating fit occurs for $T_\infty = 22.8$ K, $C_0 = 0.72$ K/A\textsuperscript{2} and $\tau = 60$ s. Since
$F_z = K_z \cdot i$ the thermal relation (4.53) can be rewritten as:

\textsuperscript{11}PTC: positive temperature coefficient; the resistance increases with increasing

\textsuperscript{12}$i$ is the root mean square value of the current.
Figure 4.41: Hot spot heating of the coil (proto).
4.5. COMPARISON OF MODELS AND PROTO-TYPE

\[ T(F_z, t) = T_\infty + \frac{C_0}{K_z^2} \cdot F_z^2 \cdot \left(1 - e^{-\frac{t}{\tau}}\right) \]  \hspace{1cm} (4.54)

The hot spot temperature of the coil for a force of \( F_z = 20 \text{ N} \) for \( t \to \infty \) is \( T(20, \infty) = 50.9 \text{ K} \); an increase of 28.1 K with respect to the environment. The reader must be aware that the heating of the coil occurs faster than cooling. The thermal time constant for heating (as derived from Figure 4.41) is shorter than thermal time constant for cooling. This is understood when noticing that the dissipation increases during heating, since the coil’s resistance increases due to the increased temperature. The temperature will increase faster due to this increased dissipation during the heating process. The time constant for cooling will therefore be \( \tau > 60 \text{ s} \).

4.5  Comparison of models and proto-type

In the previous paragraph several models and a proto-type were derived respectively measured to describe the behavior of the dynamic gravity compensator. A two-dimensional boundary element model (2D-BEM) and a three-dimensional boundary element model (3D-BEM) were evaluated in Section 4.3.1. An analytical model was derived in Section 4.3.2. These models and proto-type have similarities and differences. In this paragraph the comparison between the most important results are expounded.

4.5.1 Force and stiffness

The static vertical force \( F_z \) on the moving magnets is exactly the same for the 2D-BEM, 3D-BEM and analytical case (compare Figure 4.8, Figure 4.10 and Figure 4.24). Differences are less than 0.01 % and are attributed to numerical rounding differences. When comparing the 3D-BEM results (Figure 4.10) with the proto-type measurements (Figure 4.35) the general resemblance of the saddle shape of the diagram is seen. The location of the saddle point is shifted in radial direction in the proto-type case. This could be attributed to the fact that the magnets on one side of the gravity compensator (the \( r_{ss} < 0 \) region)
have a stronger coercitive field strength ($H_{cB}$). The vertical force in
the saddle point $(r_{ss}, z_{ss}) = (0, 0)$ of the 3D-BEM model is $F_z = 84.4$
N and in the proto-type case, the vertical force in the saddle point
$(r_{ss}, z_{ss}) = (0.9, 0.1)$ is $F_z = 81.2$ N. This difference of 3.8% is explained
by the possible 5% tolerance on magnetization strength, the possible
$5^\circ$ deviation in magnetization orientation and geometrical tolerances of
$\pm 0.2$ mm on the dimensions of the magnets in the proto-type. More-
over the Ni-coating of the magnets is magnetically permeable (approximately $\mu_r = 500$). A small fraction of the magnetic field has therefore
a short-cut, not contributing to the force.

Since the vertical stiffness ($k_z$) is derived from the vertical force, any
resemblance in force is seen as resemblance in stiffness. From Figure 4.8
and Figure 4.11 the similarity is immediately noticed, when knowing
that the curve in Figure 4.8 is represented by the curve $r_{ss} = 0$ in
Figure 4.11. The maximum stiffness in the 2D-BEM case is $|k_z| = 260$
N/m, where the 3D-BEM case has a maximum stiffness of $|k_z| = 250$
N/m. The maximum vertical stiffness for the analytical model (shown
in Figure 4.24) is $|k_z| = 260$ N/m. Both in Figure 4.11 and Figure 4.36
the dependency on the vertical displacement of the vertical stiffness is
comparable. For both cases the dependency on the radial displacement
of the vertical stiffness is negligible. Zero vertical stiffness occurs for
$z_{ss} = 0.1$ mm for the proto-type (see Figure 4.36). This could refer to
the fact that the moving magnets were located 0.1 mm below the middle
position. The minimum vertical stiffness in the case of the proto-type
is $|k_z| = 140$ N/m. The difference in stiffness values in comparison to
the numerical (BEM) and analytical models is, besides tolerances on
magnets, also explained by the fact that the relative permeability of the
magnets $\mu_r > 1$, namely $\mu_r = 1.05$, so Earnshaw’s theorem (as
described in Paragraph 3.4) does not hold exactly anymore.

For both the 3D-BEM and the proto-type case the radial force ($F_r$)
is linearly dependent on both radial and vertical displacements of the
moving magnets, as should be concluded from Figure 4.12 and Figure
4.37. From the figures is concluded that the differences of the extreme values ($F_{r,\text{max}} - F_{r,\text{min}}$) are comparable for both cases (The value
($F_{r,\text{max}} - F_{r,\text{min}}$) is slightly smaller in case of the proto-type). The fact
that $F_r < 0$ for all $(r_{ss}, z_{ss})$ in Figure 4.37 is again an indication that
the magnet strength on one side of the dynamic gravity compensator is higher. The mean radial force for the proto-type is \( F_r = -1.5 \) N.

The radial stiffness \( (k_r) \) is negative for all \((r_{ss}, z_{ss})\) in the case of the proto-type as shown in Figure 4.38. In both the 3D-BEM model and the proto-type case, the radial stiffness is nearly independent on radial displacements and linearly dependent on vertical displacements. Integrating (4.1) over the volume of the moving magnets \( (V_{magnets}) \) results into:

\[
\int \int \int \nabla \cdot \mathbf{f} dV_{magnets} = \nabla \cdot \mathbf{F} = 2 \frac{\partial F_r}{\partial r} + \frac{\partial F_z}{\partial z} > 0 \tag{4.55}
\]

for the case \( \mu_r > 1 \). Inserting \( \frac{\partial F_r}{\partial r} = -k_r \) and \( \frac{\partial F_z}{\partial z} = -k_z \) from (2.2) results into:

\[
2k_r + k_z < 0 \tag{4.56}
\]

Since \( \mu_r \geq 1 \) in this application, it is possible to verify (4.56) with the data of Figure 4.36 and Figure 4.38.

In Figure 4.42 the values of Figure 4.36 and Figure 4.38 are added according to (4.56). It can be concluded that (4.56) indeed holds for all \((r_{ss}, z_{ss})\). If the values of Figure 4.11 and Figure 4.13 are added according to (4.56), it is found that the result equals zero for all \((r_{ss}, z_{ss})\). In this case (three dimensional boundary element model) \( \mu_r = 1 \) exactly: so \( 2k_r + k_z = 0 \) was predicted by Earnshaw’s theorem.

The torque around the radial axis \( (T_r) \) for both the 3D-BEM case (Figure 4.14) and the proto-type case (Figure 4.39) is mostly dependent on the displacement in radial direction. The general slope \( \left( \frac{\partial T_r}{\partial r} \right) \) for both cases is comparable. For the proto-type case, it has a negative offset as was the case for the radial force. This is explained by the previously mentioned difference in magnet strengths for the magnets. This torque offset points on stronger magnets on the \( r_{ss} < 0 \) region, which is consistent with the previous conclusion with regard to differences in magnet strengths.

The motor constant \( K_z \) at \((r_{ss}, z_{ss}) = (0, 0)\) for the numerical analyses (Figure 4.9, Figure 4.15 and Figure 4.25) is \( K_z = 0.0307 \) N/Aturns for all cases. For the proto-type the motor constant at \((r_{ss}, z_{ss}) = (0, 0)\) was calculated (4.52): \( K_z = 0.029 \) N/Aturns; about 5.5% less than ac-
Figure 4.42: Verification of Earnshaw’s theorem (proto).
4.5. COMPARISON OF MODELS AND PROTO-TYPE

cording to the models. This is attributed to the magnet strength of the moving magnets and their geometrical tolerances.

In general the accuracy of the force sensor influences the comparison of the models and the proto-type. The force sensor has an absolute accuracy of about 1.2 N in the vertical direction (=1.5% of the static vertical force $F_z$) and 0.4 N in the radial direction (=27% of the radial force $F_r$). The resolution is 0.01 N in both directions. The sensor (based on the strain gage principle) suffers from cross talk errors: if for example a pure force in vertical direction is exerted, the sensor will indicate forces and torques in the other directions as well. This is due to mechanical manufacturing tolerances of the sensor. Although this cross talk is present, an absolute accuracy of 1.2 N and 0.4 N is achieved and errors will be less than these values. The offset in radial force and torque may be (partly) caused by the cross talk of the sensor. Since the absolute accuracy is rather poor, the level of the measured forces and torques may be misleading. The resolution is quite high, so the derived stiffnesses will be more accurate.

Moreover, the moving magnets were located in the centre of the stator magnets by manual adjustment of the manipulator (Figure 4.34), with a limited accuracy of 0.25 mm. Finally, the universal meter used for measuring the coil’s current has a negligible error (10 mA).

4.5.2 Thermal behavior

The numerical and analytical models predicted an increase of 36 K and 37 K respectively for the hot spot of the coil, when the coil generates a force of 20 N. These results demonstrate that the derived analytical model is adequate for the determination of the temperatures in the coil and its housing. For the proto-type was found that the hot spot temperature increase was only about 28 K. The difference between the models and the proto-type is attributed to the free convection of the heat to the air, that was neglected in both models. On the other hand the assumption of the coil’s thermal conductivity of $\lambda_{coil} = 1.0$ W/mK is probably too conservative. In practice, ortho-cyclic wound coils with higher thermal conductivity occur. The value $\lambda_{coil} = 1.0$ W/mK was chosen as a worst case assumption. If the thermal conductivity would
have been modelled higher, the hot spot temperature increase of the coil in the models would be lower.

4.5.3 Conclusion

All the results obtained with the boundary element models, the derived analytical model and measured proto-type have resemblance. Differences were explained by numerical rounding differences, tolerances in magnetization, geometric tolerances of the magnets, material choices ($\mu_r \geq 1$), assumptions on material properties (i.e. the thermal conductivity of the coil $\lambda_{coil}$) and accuracy of measuring equipment. The analytical model is accurate enough and suitable for the optimization process of the dynamic gravity compensator (Chapter 5). The measurement results prove that the specifications, as listed in Table 4.1, are met.

In Chapter 4 the design of the dynamic gravity compensator was expounded. Different models were derived and compared. The test bench for the proto-type measurements was described and the results were presented and compared to the expectations based on the models. In Chapter 5 the analytical model is used in an optimization procedure to find an optimal design.
Chapter 5
Optimization

In Chapter 4 the design process was expounded. As was proven in Paragraph 4.4 this resulted into a design satisfying the specifications as listed in Paragraph 4.1. The question that arises subsequently: is the design the most optimal? In this chapter is investigated how the analytical model (which was derived in Chapter 4) can be used in a numerical optimization process. In Paragraph 5.1 an introduction to optimization is presented. In Paragraph 5.2 and 5.3 the conditions are stated and the optimization process is expounded. The sensitivity and stability of the optimization process is investigated in Paragraph 5.4. Finally, the validation of the optimization process is done in Paragraph 5.5.

5.1 Introduction to optimization

In Section 5.1.1 the method for finding optimal values in functions is elaborated. The strategy for finding an optimal design by defining a weight function is illustrated in Section 5.1.2.

5.1.1 Function optimization

Consider a continuous differentiable function $f : x \to \mathbb{R}$, defined for a certain one-dimensional region $x \in [x_-, x_+]$. The extreme values for this function are found when solving [7]:

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CHAPTER 5. OPTIMIZATION

\[
\frac{df}{dx} = 0 \quad (5.1)
\]

Extreme values occur for those solutions of (5.1) \( x = \{x_1, x_2, x_3, \ldots\} \) for which holds:

\[
\frac{d^2f}{dx^2} \neq 0 \quad (5.2)
\]

Since only the region \( x \in [x_-, x_+] \) is considered, extreme values may (also) appear at the boundaries of the region: \( x = x_- \) and \( x = x_+ \). Depending on the values of \( f(x_1), f(x_2), f(x_3), \ldots \) and \( f(x_-) \) and \( f(x_+) \) we speak of local and absolute extreme values.

For multi-variable functions, a comparable strategy is used to find extreme values. Consider the multi-variable continuous differentiable function \( g : x \rightarrow \mathbb{R} \), defined for a certain multi-dimensional region \( x \in ([x_-, x_+], [y_-, y_+]) \). The extreme values are determined by simultaneous solution of:

\[
\frac{\partial g}{\partial x} = 0 \quad (5.3)
\]
\[
\frac{\partial g}{\partial y} = 0 \quad (5.4)
\]

Moreover the determinant of the following matrix must be positive for the solutions of (5.3) and (5.4):

\[
\begin{vmatrix}
\frac{\partial^2 g}{\partial x^2} & \frac{\partial^2 g}{\partial x \partial y} \\
\frac{\partial^2 g}{\partial x \partial y} & \frac{\partial^2 g}{\partial y^2}
\end{vmatrix} = \frac{\partial^2 g}{\partial x^2} \cdot \frac{\partial^2 g}{\partial y^2} - \left( \frac{\partial^2 g}{\partial x \partial y} \right)^2 > 0 \quad (5.5)
\]

If the determinant of the matrix is negative, a so-called saddle point occurs. For the case the determinant equals zero, no unambiguous conclusion concerning extreme values can be made.

5.1.2 Design optimization

In Section 5.1.1 the optimization of numerical functions was shortly introduced. In this section the design optimization is expounded. In [9]
multi-objective optimization is presented, to find the trade-offs among the various characteristics to obtain the best possible design parameters. This strategy is used in [56] to find an optimal design for a wire-bonder actuating mechanism, making use of a so-called objective function and weighting quotients. Optimization methods for magnetic designs are presented in several other publications as well. In [61] the pole teeth dimensions in a synchronous machine are optimized by a penalty function. Also [62] and [63] illustrate the use of an objective function for shape optimization in a synchronous motor. In [59] and [64] the use of intelligent systems for the design optimization of electromagnetic devices is described. The optimization techniques used for the dynamic gravity compensator are comparable to those in [56].

Starting point in the design process in general is determining a set of specifications. In Paragraph 4.1 the main functions and the specifications for the dynamic gravity compensator were listed. Such a list represents the boundary conditions the final design has to fit in. The specifications determine or even restrict the designer’s working space so to say. In general the specifications are no strict values and often may conflict with each other. For example, in case of the dynamic gravity compensator: reducing the mover mass by reducing the moving magnets’ dimensions leads to a higher coil current (for achieving the required dynamic force), which affects the dissipation. Another example: Reducing the design’s bounding volume by minimizing the distances between the magnets would result into higher stiffnesses (all these effects were expounded in Paragraph 4.2).

The different (and possibly conflicting) specifications may have a different importance. The designer may for example emphasize the need for a low dissipation value at the expense of a higher moving mass. In this (subjective) way all specifications could be ranked and rated with a weight factor, determining their importance. The designer makes the final design, for which a certain geometric configuration was invented, materials and dimensions were chosen, to meet the initial specifications as good as possible. In general this is achieved with an iterative process. But as the proto-type results show (in Paragraph 4.4), there will be a difference between the initial specification (the design target) and the final design’s behavior. For example the static force may be less than was specified or the dissipation is higher than specified.
Consider that a design was specified by several specifications \( (S_1, S_2, S_3, \ldots) \) (in which \( S_1 \) is for example the specification on the static force, \( S_2 \) is the specification on the mover mass, and so on). After the design process, the designer found a certain configuration and made choices on dimensions and applied materials. A prototype is built and several measurements are done to investigate its behavior. It turns out that the final design deviates from the initial specifications \( (S_1, S_2, S_3, \ldots) \) in magnitude: \( (\Delta S_1, \Delta S_2, \Delta S_3, \ldots) \). If the specifications were rated with the weight factors \( (w_1, w_2, w_3, \ldots) \) we could assign a single number \( W \), expressing the degree at which the design deviates from the specifications. This number is calculated by the so-called weight function:

\[
W = w_1 \cdot \frac{|\Delta S_1|}{S_1} + w_2 \cdot \frac{\Delta S_2}{S_2} + w_3 \cdot \frac{\Delta S_3}{S_3} + \ldots
\]

In this expression \( W \), the deviations \( (\Delta S_1, \Delta S_2, \Delta S_3, \ldots) \) are divided by the initial specifications \( (S_1, S_2, S_3, \ldots) \) in order to free \( W \) from dimensions. The higher \( W \), the worse the design with respect to the initial specifications. The weight factors \( (w_1, w_2, w_3, \ldots) \) indicate how severely the deviations are ‘punished’. If \( S_1 \) is the specification on the static force and \( S_2 \) on the mover mass, the quotients \( \frac{w_1}{S_1} \) and \( \frac{w_2}{S_2} \) indicate how severely 1 N deviation from the initial specification on the static force is punished in relation to 1 kg deviation in the mover mass. In (5.6) absolute numbers appear for the cases where deviations can be both positive and negative, e.g. for the specification on the static force. In case where the specifications prescribe a maximum value, the preferred value may be zero. This is the case for the mover mass or the maximum occurring stiffness between stator and mover for example.

The final design’s behavior, or in other words the way it meets the specifications, is determined by the design’s configuration, the dimensions of all parts and the material choice. Once the basic configuration is chosen, as was done in Paragraph 4.2, and the materials are selected, the design’s behavior is determined by the dimensions of the different parts. The deviations from the initial specifications \( (\Delta S_1, \Delta S_2, \Delta S_3, \ldots) \) are determined by the dimensions. So the weight function \( W \) is dependent on the dimensions. Referring to Table 4.2 and
Figure 4.5 the conclusion is made that in case of the dynamic gravity compensator $W$ is a function of $R_{ii}$, $R_{io}$, $R_{oi}$, $R_{oo}$, $h_s$, $h_m$, $h_{coil}$, $b_{coil}$, $R_{im}$, $R_{om}$, and the respective weight factors ($w_1$, $w_2$, $w_3$, ...)

specifying the importance of the different specifications. The weight factors are constants, i.e. not dependent of the final design. Once these factors are determined, the optimal design of the gravity compensator is found by minimizing the weight function $W$. In Section 5.1.1 was shown how multi-variable functions are minimized. In case of $W$ one yields:

$$\frac{\partial W}{\partial R_{ii}} = \frac{\partial W}{\partial R_{io}} = \frac{\partial W}{\partial R_{oi}} = ... = ... = 0 \quad (5.7)$$

and

$$\begin{bmatrix}
\frac{\partial^2 W}{\partial R_{ii}^2} & \frac{\partial^2 W}{\partial R_{ii} \partial R_{io}} & \frac{\partial^2 W}{\partial R_{ii} \partial R_{oi}} & \frac{\partial^2 W}{\partial R_{ii} \partial R_{oo}} & \ldots \\
\frac{\partial^2 W}{\partial R_{io} \partial R_{ii}} & \frac{\partial^2 W}{\partial R_{io}^2} & \frac{\partial^2 W}{\partial R_{io} \partial R_{oi}} & \frac{\partial^2 W}{\partial R_{io} \partial R_{oo}} & \ldots \\
\frac{\partial^2 W}{\partial R_{oi} \partial R_{ii}} & \frac{\partial^2 W}{\partial R_{oi} \partial R_{io}} & \frac{\partial^2 W}{\partial R_{oi}^2} & \frac{\partial^2 W}{\partial R_{oi} \partial R_{oo}} & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{\partial^2 W}{\partial R_{oo} \partial R_{ii}} & \frac{\partial^2 W}{\partial R_{oo} \partial R_{io}} & \frac{\partial^2 W}{\partial R_{oo} \partial R_{oi}} & \frac{\partial^2 W}{\partial R_{oo}^2} & \ldots \\
\end{bmatrix} > 0 \quad (5.8)$$

Solving (5.7), checking (5.8) and investigating the boundaries of $R_{ii}$, $R_{io}$, $R_{oi}$,... result into a set of extreme values of $W$ in which one specific combination of $R_{ii}$, $R_{io}$, $R_{oi}$,... describes the absolute minimum value of $W$: this combination is the optimal design of the dynamic gravity compensator according to the chosen weight factors.

## 5.2 Criteria for optimization

Optimizing the dynamic gravity compensator according to the strategy in Section 5.1.2, results into a set of 12 equations (since we have 12 independent variables) of (5.7) for which the condition of (5.8) has to hold. Besides, the value of $W$ at the boundaries of all dimensions ($R_{ii}$, $R_{io}$, $R_{oi}$, $R_{oo}$, $h_s$, ...) has to be evaluated. But first the function for $W$

\footnote{The dimensions of the housings (as introduced in Section 4.3.2) and cooling ducts are left out, since these parameters do not strongly affect the electromagnetic behavior of the design.}
has to be defined. The list of specifications of Table 4.1 (Paragraph 4.1) is taken:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static force (F_{z,\text{static}})</td>
<td>85</td>
<td>N</td>
</tr>
<tr>
<td>Dynamic force range (F_{z,\text{dynamic}})</td>
<td>-20...+20</td>
<td>N</td>
</tr>
<tr>
<td>Maximum mover mass (m_{\text{mover}})</td>
<td>0.50</td>
<td>kg</td>
</tr>
<tr>
<td>Maximum stator mass (m_{\text{stator}})</td>
<td>1.0</td>
<td>kg</td>
</tr>
<tr>
<td>Motion range ((x_{ss}, y_{ss}, z_{ss}))</td>
<td>(2,2,2)</td>
<td>mm</td>
</tr>
<tr>
<td>Maximum stiffness ((k_x, k_y, k_z))</td>
<td>(500,500,500)</td>
<td>N/m</td>
</tr>
<tr>
<td>Maximum damping ((d_x, d_y, d_z))</td>
<td>(0.2,0.2,0.2)</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Maximum dissipation(^2)</td>
<td>40</td>
<td>W</td>
</tr>
<tr>
<td>Maximum bounding volume ((x, y, z))</td>
<td>(100,100,50)</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 5.1: Specifications for the dynamic gravity compensator.

The static force \(F_{z,\text{static}}\) on the moving magnets is important in the performance of the design, since compensating gravity is one of the main functions of the design. The dynamic force range \(F_{z,\text{dynamic}}\) is important as well, since a controllable force is required. However, it will be left out of the weight function \((W)\), since the dynamic force is strongly coupled to the dissipation, since heat is dissipated in the wires when a current is applied. The dynamic force is restricted by the acceptable dissipation level. The mover mass \(m_{\text{mover}}\) and stator mass \(m_{\text{stator}}\) are important parameters as was mentioned in Section 2.1.3. They affect the dynamic behavior of the short-stroke and long-stroke systems. The mover mass has more impact on the performance than the stator mass: the first will therefore have a higher weight factor. The motion range \((x_{ss}, y_{ss}, z_{ss})\) is considered as a high demand. The design must be able to make the required strokes. This specification is thus left out of the weight function, since only designs will be considered which are able to make the strokes. The stiffness \((k_x, k_y, k_z)\) is important as well as was expounded in Section 2.1.2. In Paragraph 3.4 and Paragraph 4.4 was proven that the vertical stiffness \((k_z)\) and radial stiffness \((k_r)\) are strongly related to each other. Therefore, it is sufficient to weight the vertical stiffness only. The dissipation \((P)\) for a dynamic force of \(F_{z,\text{dynamic}} = 20\) N appears in the weight function, since heat generation

\(^2\)for generating a force of \(F_{z,\text{dynamic}} = 20\) N.
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affects the accuracy as well (see Section 2.1.3). The actuator bounding volume \( (V_{\text{bound}}) \) is weighed, since the machine restricts the total volume available for the gravity compensator.

In general the costs play an important role in the design process. They strongly affect the choices for a certain design. However, the costs are left out of the weight function, since in this particular case the optimization process affects the dimensions of the electro-magnetic components (magnets and coil). The total costs are hardly influenced by these components, but rather determined by the material choice of the cooling ducts for example.

The previous considerations result into the following weight function for the dynamic gravity compensator:

\[
W = w_1 \cdot \left| \frac{F_{z,\text{static}} - F_{z,\text{static}}^{\text{spec}}}{S_1} \right| + w_2 \cdot \frac{m_{\text{mover}}}{S_2} + w_3 \cdot \frac{m_{\text{stator}}}{S_3} + w_4 \cdot \frac{|k_{z,\text{max}}|}{S_4} + w_5 \cdot \frac{P_{F_z,\text{dynamic}=20}}{S_5} + w_6 \cdot \frac{V_{\text{bound}}}{S_6}
\]

(5.9)

where the importance of the different terms is indicated by the quotients in Table 5.2. (as elaborated in Section 5.1.2). This mutual importance is usually determined by the engineers in the design team.

<table>
<thead>
<tr>
<th>Property</th>
<th>Weight quotient ( \frac{w_i}{S_i} )</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Force ( (F_{z,\text{static}}) )</td>
<td>( \frac{w_1}{S_1} =1 )</td>
<td>N(^{-1})</td>
</tr>
<tr>
<td>Mover mass ( (m_{\text{mover}}) )</td>
<td>( \frac{w_2}{S_2} =10 )</td>
<td>kg(^{-1})</td>
</tr>
<tr>
<td>Stator mass ( (m_{\text{stator}}) )</td>
<td>( \frac{w_3}{S_3} =5 )</td>
<td>kg(^{-1})</td>
</tr>
<tr>
<td>Cross talk (stiffness) ( (k_{z,\text{max}}) )</td>
<td>( \frac{w_4}{S_4} =0.01 )</td>
<td>m(/N)</td>
</tr>
<tr>
<td>Dissipation ( (P) )</td>
<td>( \frac{w_5}{S_5} =1 )</td>
<td>W(^{-1})</td>
</tr>
<tr>
<td>Bounding volume ( (V_{\text{bound}}) )</td>
<td>( \frac{w_6}{S_6} =50000 )</td>
<td>m(^{-3})</td>
</tr>
</tbody>
</table>

Table 5.2: Weight factor quotients.

In Section 4.3.2 the analytical model of the gravity compensator was derived for any design according to the configuration from Figure 4.5. The static force is calculated with (4.37) and (4.38), the mass of the mover and stator with (4.48) and (4.49). For the maximum occurring vertical stiffness, the static force at two different vertical positions \( (z_{\text{ss}}) \) is determined and used in (4.4). The dissipation for a dynamic force of
20 N is calculated with (4.20), (4.21) and (4.22). The bounding volume of the design is determined by:

\[ V_{\text{bound}} = 2 \cdot h_s \cdot \pi \cdot R_{\text{oo}}^2 \]  

(5.10)

if the stator magnet height \(2 \cdot h_s\) is larger than the moving magnet height \(2 \cdot h_m\) and the coil height \(2 \cdot h_{\text{coil}}\). For the case \(h_m > h_s \land h_m > h_{\text{coil}}\):

\[ V_{\text{bound}} = 2 \cdot h_m \cdot \pi \cdot R_{\text{oo}}^2 \]  

(5.11)

In the other cases the bounding volume is calculated by:

\[ V_{\text{bound}} = 2 \cdot h_{\text{coil}} \cdot \pi \cdot R_{\text{oo}}^2 \]  

(5.12)

The previously mentioned properties can be calculated for any realistic set of dimensions \((R_{ii}, R_{io}, R_{oi}, R_{oo}, h_s, b_m, h_m, h_{\text{coil}}, b_{\text{coil}}, R_{im}, R_{oi}, R_{om})\).

As an illustrative example all dimensions were taken from Table 4.2 in Section 4.3.1 except for \(h_{\text{coil}}\) and \(h_m\). These two dimensions were varied: \(h_{\text{coil}} = [5, 5.5, ..., 11] \text{ mm}\) and \(h_m = [6, 6.5, ..., 13] \text{ mm}\). The effect of these dimensions on the mentioned properties (static force, mover mass, and so on) was calculated using the derived analytical models set up in MatLab. In Figure 5.1 the static force is shown as function of the coil height \((h_{\text{coil}})\) and the height of the moving magnets \((h_m)\): \(F_{z,\text{static}} = F_{z,\text{static}}(h_{\text{coil}}, h_m)\). The other dimensions were kept constant.

From Figure 5.1 is concluded that the static force is dependent on the height of the moving magnets \((h_m)\), but independent on the coil height \((h_{\text{coil}})\) as expected. Figure 5.2 illustrates how the (maximum) vertical stiffness for \((r_{ss}, z_{ss}) = (0, 1) \text{ mm}\) depends on the parameters \(h_{\text{coil}}\) and \(h_m\). The figure clearly shows that the vertical stiffness is negative for \(h_m < 7 \text{ mm}\) and positive for \(h_m > 8 \text{ mm}\). What happens for \(7 \leq h_m \leq 8 \text{ mm}\)? The arising question is: is there a configuration for which the maximum vertical stiffness is really zero, or does the \(k_{z,\text{max}}(h_{\text{coil}}, h_m)\)-function have a discontinuity? In other words: is there a configuration for which the vertical force maintains the same value in the interval \(z_{ss} = [-1, 1]\)? A two-dimensional BEM model was used to verify this. In this model \(h_m\) was varied. For each value of \(h_m\) the maximum occurring stiffness in the interval \(z_{ss} = [-1, 1]\) was monitored.
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Figure 5.1: Static force for different values of $h_{\text{coil}}$ and $h_m$. 
Figure 5.2: Maximum vertical stiffness $k_{z, \text{max}}$ at $(r_{ss}, z_{ss}) = (0, 1)$ [mm] for different values of $h_{\text{coil}}$ and $h_m$. 
5.2. **CRITERIA FOR OPTIMIZATION**

It was found that the maximum occurring vertical stiffness is $k_z = -1.5$ N/m when $h_m = 7$ mm. This value is considered as an extreme low value (almost zero). There is no practical use to investigate whether a configuration exists that has a maximum stiffness of exactly zero. Production tolerances of a proto-type (of magnet dimensions, magnet strengths and mechanical tolerances) will cause a deviation from this low stiffness value. Figures 5.3, 5.4, 5.5 and 5.6 show the effect of $h_{coil}$ and $h_m$ on the dissipation, the moving mass, the stator mass and the bounding volume, respectively.

The dissipation is both dependent on the coil height ($h_{coil}$) and the height of the moving mass ($h_m$). A high coil in combination with small moving magnets results in useless dissipation in the coil regions where

Figure 5.3: Dissipation ($P_{Fz,dynamic=20}$) for different values of $h_{coil}$ and $h_m$. 

![Dissipation, Analytical](image-url)
Figure 5.4: Moving mass for different values of $h_{\text{coil}}$ and $h_{m}$. 
5.2. CRITERIA FOR OPTIMIZATION

Figure 5.5: Stator mass for different values of $h_{\text{coil}}$ and $h_{\text{m}}$. 
Figure 5.6: Bounding volume for different values of $h_{coil}$ and $h_m$. 
no radial field is present due to the ‘absence’ of magnets. Generating
the required force therefore leads to a high dissipation. Extreme heights
of the moving magnets in combination with a small coil height result
in low magnet field densities in the coil, since the magnet field lines
tend to be located at the bounding edge parallel to the magnetization
direction (this is well illustrated in Figure 4.7 showing the field lines
in the 2D BEM case). For generating a vertical force a relatively high
current density \( J \) is required, leading to a high dissipation as well. The
figure illustrates that the dissipation is low for cases where \( h_{coil} \) and \( h_m \)
are comparable. The higher both values, the lower the dissipation can
be concluded from the figure as well.

The moving mass is not dependent on \( h_{coil} \). The stator mass is
independent on \( h_m \). Since both \( h_{coil} \) and \( h_m \) were maintained smaller
than \( h_a \) during the analysis, the bounding volume is constant for all
the considered values of \( h_{coil} \) and \( h_m \).

The values at each point \((h_{coil}, h_m)\) in Figure 5.1 to Figure 5.6 can
be inserted in the weight function (5.9) using the weight quotients from
Table 5.2. This results into a weight function \( W = W(h_{coil}, h_m) \), which
is graphically displayed in Figure 5.7. The optimal design, for the
specific set of weight quotients, is found for \( h_{coil} = 8 \) mm and \( h_m = 9 \)
mm, since the weight function has reached its minimum value \( (W =
49.2) \). This is a local optimum of the weight function. Other optimal
designs will possibly be found when also the other dimensions \( (R_{ai}, R_{ia},
R_{oi}, R_{oo}, h_a, b_m, b_{coil}, R_{cm}, R_{coli} \) and \( R_{com} \) \) are varied. The possibility
exists that the weight function is even lower for other combinations of
the dimensions. This is investigated in Paragraph 5.3.

5.3 Optimization process

In Section 5.3.1 an efficient procedure for the optimization is elaborated
and in Section 5.3.2 the results of this procedure are discussed.

5.3.1 Optimization procedure

The applied strategy for finding the optimal design was introduced in
Paragraph 5.2. Only two dimensions \( (h_{coil}, h_m) \) were varied in sensible
Figure 5.7: Weight function for different values of $h_{\text{coil}}$ and $h_{m}$. 
5.3. OPTIMIZATION PROCESS

intervals in \( n_{\text{coil}} = 13 \) and \( n_{\text{magnets}} = 15 \) steps, respectively. For that exercise \( n_{\text{coil}} \cdot n_{\text{magnets}} = 195 \) configurations were examined. The calculation of each configuration took approximately one second. Consider now the variation of all 12 dimensions in \( n = 10 \) steps each. This results in \( n^{12} = 1.0 \cdot 10^{12} \) steps. The total calculation time requires \( 1.0 \cdot 10^{12} \) seconds or 31709 years. This is unacceptable. The optimization strategy must be more efficient.

In Figure 5.8 the dimensions of the dynamic gravity compensator are shown. The horizontal dimensions are defined as a chain, since this is required by the optimization procedure for sorting out impossible configurations (this will be clarified later in this section). For the optimization process these dimensions will be varied from a minimum value to a maximum value with a certain step size. Table 5.3 shows these values and the step size.
From previous discussions in Paragraph 4.2 was known that larger gaps between the moving magnets and the stator magnets resulted in a low stiffness between the stator and mover in general. Therefore, the outer radius ($R_{oo}$) was chosen as large as possible, taking the available diameter in the machine into account. $R_{oo}$ was varied in three steps: $R_{oo} = \{46, 48, 50\}$. The inner radius of the inner magnet ($R_{ii}$) was for the same reason (low stiffness) chosen as small as possible and barely varied as well.

The thicknesses of the inner (stator) magnet ($b_{mi}$) and outer (stator) magnet ($b_{mo}$) are varied over a relatively large interval. The variation of the thickness of the inner magnet was considerably larger, since the upper and lower surfaces of the inner and outer magnets were assumed to be more or less equal (see the considerations in Section 4.3.1).

An equal value is applied to the thickness of the inner and outer moving magnet ring ($b_{mm}$). The corresponding step size is based on the fact that smaller moving magnets would result into low static forces ($F_{z,\text{static}}$) and larger moving magnets would result into high static forces. The width of the coil ($b_{coil}$) was altered such that the coil thickness is comparable to the width of the moving magnets ($b_{mm}$). Too small values for $b_{coil}$ and $b_{mm}$ would in general result into high dissipation levels at the required dynamic force. Too large values would result into a large and heavy design.

The gaps between the coil and the moving magnet rings ($g_c$) were kept as small as possible and hardly varied. Since $g_c$ determines the horizontal motion range ($x_{ss} = 2$ mm and $y_{ss} = 2$ mm) the minimum value would be $g_c = 1$ mm. Due to mechanical tolerances a minimum

<table>
<thead>
<tr>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Step size</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ii}$</td>
<td>4</td>
<td>7</td>
<td>1 mm</td>
</tr>
<tr>
<td>$R_{oo}$</td>
<td>46</td>
<td>50</td>
<td>2 mm</td>
</tr>
<tr>
<td>$b_{mi}$</td>
<td>3</td>
<td>10</td>
<td>1 mm</td>
</tr>
<tr>
<td>$b_{mo}$</td>
<td>0</td>
<td>4</td>
<td>1 mm</td>
</tr>
<tr>
<td>$b_{mm}$</td>
<td>2</td>
<td>5</td>
<td>1 mm</td>
</tr>
<tr>
<td>$b_{coil}$</td>
<td>3</td>
<td>6</td>
<td>1 mm</td>
</tr>
<tr>
<td>$g_c$</td>
<td>1.5</td>
<td>2</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>$g_m$</td>
<td>5</td>
<td>-</td>
<td>- mm</td>
</tr>
</tbody>
</table>

Table 5.3. Variations of horizontal dimensions.
value of \( g_c = 1.5 \text{ mm} \) is chosen. Finally, the gap between the moving magnets and stator magnets \( (g_m) \) is only restricted to a minimum value. This value is determined by the choice of all other horizontal dimensions (see Figure 5.8). From Table 5.3 and Figure 5.8 is concluded that not every combination of horizontal dimensions is a possible solution. For example: if \( R_{ii} = 7, R_{oo} = 46, b_{mi} = 10, b_{mo} = 4, b_{mm} = 5, b_{coil} = 6 \) and \( g_c = 2 \) the resulting dimension for \( g_m \) would be \( g_m = \frac{1}{2}(R_{oo} - R_{ii} - b_{mi} - b_{mo} - 2b_{mm} - b_{coil} - 2g_c) = 2.5 \text{ mm} \). This is smaller than the minimum required value from Table 5.3. So this configuration does not need to be evaluated in the optimization process. It is therefore sensible to use an optimization procedure that sorts out impossible configurations.

The flow chart for this procedure is shown in Figure 5.9. The horizontal dimensions are varied in for-loops. But each time before a next for-loop is entered, the validity of the configuration is checked. Using the values of Table 5.3 this strategy results in 15060 possible horizontal configurations. For each of these configurations the vertical dimensions \( (h_s, h_{coil}, \text{ and } h_m) \) still have to be varied. This variation strategy is shown in Figure 5.10. Each time the height of the stator and moving magnets \( (h_s \text{ and } h_m) \) is altered, the static force \( (F_{z,\text{static}}) \) is calculated. Only if the static force has a sensible value \( (80 < F_{z,\text{static}} < 90 \text{ N}) \) the coil height \( (h_{coil}) \) is varied. This is done to prevent the dissipation in the coil being too high, since the coil has to compensate the mismatch of the static force. The vertical dimensions are altered according to Table 5.4.

<table>
<thead>
<tr>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Step size</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_s )</td>
<td>12</td>
<td>20</td>
<td>2 mm</td>
</tr>
<tr>
<td>( h_m )</td>
<td>7</td>
<td>13</td>
<td>2 mm</td>
</tr>
<tr>
<td>( h_{coil} )</td>
<td>5</td>
<td>11</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

Table 5.4. Variations of vertical dimensions.

The vertical dimensions are varied over a relatively large range to study the effect of different values of the heights on the behavior of the gravity compensator. The total number of calculated configurations according to the strategy shown in the flow charts of Figure 5.9 and Figure 5.10 is 386790 of which only 85609 satisfy the \( F_{z,\text{static}} \in [80, 90] \text{ N} \) restriction. For all 85609 the weight function is calculated according to (5.9). Since twelve dimensions are varied, it is not possible to visualize
Figure 5.9: Flow chart for finding horizontal configurations.
Figure 5.10: Flow chart for finding vertical configurations and calculating properties.
the results in the way it was done in Paragraph 5.2. However, the weight results are sorted in ascending order. In Figure 5.11 the weight value for all 85609 configurations is shown. The minimum value for the weight function is 28.8. So according to the chosen weight factors, that configuration is the most optimal design. The dimensions are listed in Table 5.5. (according to the nomenclature of Section 4.3). The dimensions of the proto-type are mentioned in brackets.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_m$</td>
<td>9 (9)</td>
<td>$R_{ii}$</td>
<td>7 (5.5)</td>
<td>$R_{im}$</td>
<td>20 (19)</td>
</tr>
<tr>
<td>$h_s$</td>
<td>18 (16)</td>
<td>$R_{io}$</td>
<td>13 (10)</td>
<td>$R_{om}$</td>
<td>34 (33)</td>
</tr>
<tr>
<td>$h_{coil}$</td>
<td>9 (7)</td>
<td>$R_{oi}$</td>
<td>46 (46)</td>
<td>$b_m$</td>
<td>5 (4)</td>
</tr>
<tr>
<td>$R_{coil}$</td>
<td>26.5 (26)</td>
<td>$R_{oo}$</td>
<td>46 (48.5)</td>
<td>$b_{coil}$</td>
<td>6 (5)</td>
</tr>
</tbody>
</table>

Table 5.5: Dimensions [mm] of the dynamic gravity compensator.

The given dimensions of stator magnets, moving magnets and coil
represent the design with an optimal combination of static force, stiffness, dissipation, mass and bounding volume. The static force for this optimal configuration is $F_{z, \text{static}} = 83.7 \, \text{N}$, the mover mass $m_{\text{mover}} = 0.25 \, \text{kg}$, stator mass $m_{\text{stator}} = 0.22 \, \text{kg}$, maximum stiffness $k_{z, \text{max}} = 12.3 \, \text{N/m}$, dissipation $P_{Fz, \text{dynamic}=20} = 15.1 \, \text{W}$ and the bounding volume $V_{\text{bound}} = 0.000172 \, \text{m}^3$. If these values and the weight quotients from Table 5.2 are inserted in (5.9) the minimum value for the weight function is found ($W = 28.8$).

### 5.3.2 Optimization results

Taking a closer look to the dimensions of the optimal design (Table 5.5), we must conclude that one of the most remarkable results of the optimization process is the disappearance of the outer magnet ring of the stator: $R_{oi} = R_{oo}$, since $R_{oi} = R_{oo}$ implies that the design is improved when omitting the outer stator magnet ring. The assumption that the top surface of the inner stator magnet ring has to be more or less equal to the top surface of the outer stator magnet ring (Section 4.3.1) turns out to be not valid. The assumption was made from the idea that all magnetic field lines (flux) leaving from the inner stator magnet ring top surface should enter the left surface of the moving magnets and finally arrive at the outer stator magnet ring top surface as shown in Figure 4.7. By omitting the outer stator magnet ring, the field lines, leaving the right surface of the moving magnets, were expected to enter the bottom surface of the inner stator magnet ring leading to zero vertical force on the moving magnets. If the number of magnetic field lines, entering the imaginary bounding surface around the moving magnets at the top surface, is equal to the number of magnetic field lines leaving at the bottom surface, the Maxwell stresses (Section 3.3.2) would result in a zero vertical force (see Figure 5.12). However, the direction of the field lines, crossing the imaginary surface, is essential for the resulting force, since the normal and tangential direction of the field lines strongly determine the direction of the force according to (3.32) and (3.33). As will be illustrated later (in Figure 5.16), omitting the outer stator magnet ring will still result into the majority of the field lines entering the imaginary surface at the upper side and leaving the outer radial surface.
Figure 5.12: Maxwell stress surface for the optimized design.
5.4. SENSITIVITY TO WEIGHT FACTORS

From Table 5.3 is concluded that the inner diameter of the inner stator magnet ring ($R_{ii}$) should be maximized (in the given range). Since the outer stator magnet ring is omitted entirely, the magnet volume of the inner stator magnet had to be increased to achieve the required static force. This is accomplished by increasing the outer radius of the inner stator magnet ($R_{io}$) and increasing the magnet’s height ($h_s$).

The coil height ($h_{coil}$) and moving magnet height ($h_m$) should be designed equal according to Table 5.3. Large differences between these parameters lead to high dissipation levels as elaborated in Paragraph 5.2. When taking a closer look at Figure 5.3 we conclude that the dissipation is minimal when $h_{coil} \approx h_m$. Nevertheless, in practice $h_{coil}$ is often chosen slightly larger or smaller than $h_m$: the variations of the motor constant ($K_z$) will then be limited, since the flux linkage variations ($\frac{d\lambda \phi}{dz}$ from Section 3.3.3) will be rather constant within the required vertical stroke. The effect of these dimensions on the variations of the motor constant is not considered in the optimization procedure.

Moreover the dissipation is affected by the gap between the coil and the moving magnets ($g_c$). It should be minimized (as was expounded in Section 4.3.1). The radial dimensions of the coil and moving magnets ($b_{coil}$ and $b_m$) are slightly increased with respect to the design presented in Paragraph 4.3. In general this also affects the dissipation. When the coil’s volume ($V_c$) is increased, while the magnetic induction ($B$) is kept constant, the motor constant ($K_z$) increases, according to (4.2). The required current density ($J$) for the required dynamic force level is then decreased according to (4.17). From (4.16) is then concluded that the dissipation will be lower. However, if the coil’s width ($b_{coil}$) is increased, the width of the moving magnets ($b_m$) must increase. Otherwise the magnetic induction at the coil will reduce, which (in this case) will result into a higher dissipation.

5.4 Sensitivity to weight factors

The numerical values of the weight quotients ($\frac{w_i}{S_i}$) from Table 5.2 strongly determine the value of the weight function ($W$) from (5.9). These quotients determine the mutual importance of the specifications. Another set of weight quotients will result in a different optimum design. In
other words: what happens to the weight function, when one of the weight quotients is altered? This is expressed by the weight quotient sensitivity function ($\Psi$), expressed by:

$$\Psi = dW = \frac{F_{z,static} - F_{z,static}^{spec}}{S_1} \, dw_1 + \frac{m_{mover}}{S_2} \, dw_2 + \frac{m_{stator}}{S_3} \, dw_3 + \frac{|k_{z,max}|}{S_4} \, dw_4 + \frac{P_{Fz,dynamic=20}}{S_5} \, dw_5 + \frac{V_{bound}}{S_6} \, dw_6$$

(5.13)

Each weight quotient ($\frac{w_i}{S_i}$) from Table 5.2 is once doubled to investigate the sensitivity on the weight function. In total 7 cases occur. In case 1 the weight quotients were taken from Table 5.2. In case 2 all these values were maintained, except for $\frac{w_1}{S_1} = 2$ was chosen. In case 3: $\frac{w_2}{S_2} = 20$, case 4: $\frac{w_3}{S_3} = 10$, case 5: $\frac{w_4}{S_4} = 0.02$, case 6: $\frac{w_5}{S_5} = 2$ and finally case 7: $\frac{w_6}{S_6} = 100000$. In Figure 5.13 the weight function for the best hundred configurations is shown for case 1, the so-called default situation. In

![Sorted weights for default situation](image)
Figure 5.14: Effect of weight quotients on weight function (case 2, ..., case 7).
Figure 5.14 the weight functions for the best hundred configurations are shown for the cases 2 to 7. Remarkable for the different cases is the different shape of the weight function. Especially for case 6, where the dissipation is weighed more severely (increased w5), the shape of the weight function changes. The reader must be aware of the fact the best hundred configurations for each case will consist of a different set of designs. When the static force is weighed more severely (increased w1, case 2), the best configurations will mainly consist of designs that will meet the specification $F_{z, \text{static}} = 85$ N as good as possible; when the dissipation is weighed more severely (increased w5, case 6), the best configurations will consist of designs with a relative low dissipation level. In Table 5.6 the properties of the optimal design for each case are presented.

<table>
<thead>
<tr>
<th>Case</th>
<th>$F_{z, \text{static}}$</th>
<th>$m_{\text{mover}}$</th>
<th>$m_{\text{stator}}$</th>
<th>$k_{z, \text{max}}$</th>
<th>$P^3$</th>
<th>$V_{\text{bound}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83.7</td>
<td>0.25</td>
<td>0.22</td>
<td>12.3</td>
<td>15</td>
<td>0.000172</td>
</tr>
<tr>
<td>2</td>
<td><strong>84.9</strong></td>
<td>0.19</td>
<td>0.20</td>
<td>118</td>
<td>17</td>
<td>0.000153</td>
</tr>
<tr>
<td>3</td>
<td><strong>84.9</strong></td>
<td><strong>0.19</strong></td>
<td>0.20</td>
<td>118</td>
<td>17</td>
<td>0.000153</td>
</tr>
<tr>
<td>4</td>
<td>83.7</td>
<td>0.25</td>
<td><strong>0.22</strong></td>
<td>12.3</td>
<td>15</td>
<td>0.000172</td>
</tr>
<tr>
<td>5</td>
<td>84.5</td>
<td>0.27</td>
<td>0.27</td>
<td><strong>11.5</strong></td>
<td>13</td>
<td>0.000216</td>
</tr>
<tr>
<td>6</td>
<td>84.5</td>
<td>0.27</td>
<td>0.27</td>
<td><strong>11.5</strong></td>
<td><strong>13</strong></td>
<td><strong>0.000216</strong></td>
</tr>
<tr>
<td>7</td>
<td><strong>84.9</strong></td>
<td>0.19</td>
<td>0.20</td>
<td>118</td>
<td>17</td>
<td><strong>0.000153</strong></td>
</tr>
</tbody>
</table>

Table 5.6: Properties for optimal configuration for different cases.

From the results is concluded that increasing one of the weight quotients affects the result of the optimization procedure. If the static force is weighed more severely (case 2), the optimal design will achieve a static force quite close to $F_{z, \text{static}} = 85$ N. For all cases holds: increasing the weight quotient results into an optimal design with a better performance with respect to the corresponding specification, except for case 4. Increasing the weight on the stator mass ($m_{\text{stator}}$, increased w3) does not affect the design: the optimal designs of case 1 and case 4 are exactly identical. So, although the weight factor for the stator mass was increased, the optimization algorithm came up with the identical design. The explanation is found in the fact that the stator mass is weighed rather weakly in comparison to the other properties (see $^{3}$at $F_{z, \text{dynamic}} = 20$ N.)
5.5. COMPARISON TO BOUNDARY ELEMENT MODEL

Paragraph 5.2). Even doubling the weight factor does not affect the optimization result.

When looking at Table 5.6 we notice that the differences in static force for the different cases are limited. The static force varies only from $F_{z,\text{static}} = 83.2$ N to $84.9$ N (1.4%). The differences are larger for the other properties: for example the mover mass varies $m_{\text{mover}} = 0.19$ to 0.27 kg or 30%, the stator mass $m_{\text{stator}} = 0.20$ to 0.27 kg or 26% and the dissipation $P_{F_{z,\text{dynamic}} = 20} = 13$ to 17 W or (24%). This is explained by the fact that the static force is relatively weighed severely. From Table 5.2 is known that 1 N deviation from $F_{z,\text{static}} = 85$ N is weighed as severe as 0.1 kg for $m_{\text{mover}}$ (since $\frac{w_1}{S_1} = 1$ and $\frac{w_2}{S_2} = 10$). In other words a deviation of $\frac{1}{85.7} \times 100\% = 1.2\%$ from $F_{z,\text{static}} = 83.7$ N is ‘punished’ in the same way as 0.1 kg or $\frac{0.1}{0.25} \times 100\% = 40\%$ of $m_{\text{mover}} = 0.25$ kg.

5.5 Comparison to boundary element model

A two-dimensional boundary element model was made to verify the results of the optimization process. In Figure 5.15 the two-dimensional model of the optimized model is shown. Figure 5.16 reveals the field lines calculated by 2D BEM. The vertical force ($F_z$) and the stiffness ($k_z$), as function of the vertical displacement of the moving magnets, according to the boundary element model are shown in Figure 5.17. The vertical force at $z_{ss} = 0$ is $F_z = 83.2$ N. The (analytical) model used for the optimization process calculated $F_z = 83.7$ N (see Section 5.3.1): a difference of 0.6%. Although this is low, in Section 4.5.1 was shown that the difference between the analytical model and the boundary element model was even smaller (0.01%). The relatively large difference is attributed to the fact that the number of integration steps in the analytical model for the optimization procedure was reduced to decrease the calculation time. So the field (and force) calculations are less accurate.

The maximum stiffness according to the boundary element model is $k_{z,\text{max}} = 18$ N/m, where the analytical model determined: $k_{z,\text{max}} = 12.3$ N/m. This difference might seem rather large, but both stiffness values are considered to be very low in comparison to the specification ($k_{z,\text{max}} = 500$ N/m).
Figure 5.15: Boundary element model of optimized model (2D)
Figure 5.16: Magnetic field lines in the optimized model (2D BEM)
Figure 5.17: Static vertical force $F_z$ and stiffness $k_z$ for optimized model (2D BEM).
Figure 5.18: Vertical force ($F_z(z_{ss}, i)$) with current through coil for optimized model (2D BEM).
In Figure 5.18 the vertical force as function of a vertical displacement of the moving magnets due to a current through the coil is shown. The required current through the coil was \( i = 573 \) Atorns. The motor constant \( (K_z) \) is found by (4.5): \( K_z(z_{ss} = 0) = \frac{F_z,\text{coil}}{i} = \frac{104.3 - 83.2}{573} = 0.0368 \) N/Atorns. For obtaining the steepness, a reference to (4.21) is made. Inserting the numerical values of \( f = 0.65 \), \( A_{\text{coil}} = 2 \cdot h_{\text{coil}} \cdot b_{\text{coil}} = 108 \cdot 10^{-6} \) m\(^2\), \( K_z = 0.0368 \) N/Atorns, \( \sigma_e = 5.0 \cdot 10^7 \) S/m and \( V_{\text{coil}} = 2.00 \cdot 10^{-5} \) m\(^3\) \( \Rightarrow S_z = 25.7 \) N²/W (at \( T = 293 \) K). For generating a force of \( F_z = 20 \) N a dissipation of \( P = \frac{F_z^2}{S_z} = 15.6 \) W occurs for the boundary element model. The dissipation according to the model, used for the optimization is \( P = 15.1 \) W (see Section 5.3.1). Table 5.7 summarizes the results from both models.

<table>
<thead>
<tr>
<th>Opt. Model</th>
<th>( F_z,\text{static} )</th>
<th>( m_{\text{mover}} )</th>
<th>( m_{\text{stator}} )</th>
<th>( k_z,\text{max} )</th>
<th>( P^4 )</th>
<th>( V_{\text{bound}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>83.7</td>
<td>0.25</td>
<td>0.22</td>
<td>12.3</td>
<td>15.1</td>
<td>0.000172</td>
</tr>
<tr>
<td>2D BEM</td>
<td>83.2</td>
<td>0.25</td>
<td>0.22</td>
<td>18.0</td>
<td>15.6</td>
<td>0.000172</td>
</tr>
</tbody>
</table>

Table 5.7: Comparison of analytical model with 2D BEM model.

As was concluded in Section 4.5.3, the analytical model and the boundary element model are quite similar. The proto-type measurement results are well predicted by the models. Therefore, the need for validation of the previous results by another proto-type is not considered necessary at this moment.

One of the most remarkable results of the optimization process is the vanished outer stator magnet ring. Omitting this ring leads to an even more simple design. The other dimensions of the initial design (proto type) require small changes (see Table 5.5). A discussion on the impact of these different changes was given in Section 5.3.2. Table 5.8 lists the static force, masses, dissipation and bounding volume of the initial design of the gravity compensator and the optimized design, calculated by the 2D boundary element solver. The optimized design has an extremely low vertical stiffness (93% lower) in comparison to the initial design, which was developed under time pressure in an industrial environment. The dissipation is also considerably lower: 55%. The bounding volume is considerably smaller as well, due to the vanished

\(^4\text{at} \ F_z,\text{dynamic} = 20 \ \text{N.}\)
outer stator magnet ring (27%). The mover mass of the optimized model is larger: the moving magnets' volume is larger (25%). The specified value for the static force (85 N), was slightly better achieved by the initial design (99.2%) than the optimized design (97.8%). The increased mover mass is acceptable if the specifications in Table 5.1 are considered.

<table>
<thead>
<tr>
<th>Model</th>
<th>$F_{z,\text{static}}$</th>
<th>$m_{\text{mover}}$</th>
<th>$m_{\text{stator}}$</th>
<th>$k_{z,\text{max}}$</th>
<th>$P^\text{b}$</th>
<th>$V_{\text{bound}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>84.4</td>
<td>0.20</td>
<td>0.24</td>
<td>250</td>
<td>34.8</td>
<td>0.000236</td>
</tr>
<tr>
<td>Optimized</td>
<td>83.2</td>
<td>0.25</td>
<td>0.22</td>
<td>18</td>
<td>15.6</td>
<td>0.000172</td>
</tr>
</tbody>
</table>

Table 5.8: Comparison of initial and optimized design (2D BEM).

From the weight factor sensitivity analysis in Paragraph 5.4 is known that if the mover mass weight factor ($\frac{w_2}{S_2}$) is increased, the effect on the mover mass is considerable (see Table 5.6). If a design with lower mover mass is desirable, the optimization process with increased mover mass weight factor should be performed.

\footnote{at $F_{z,\text{dynamic}} = 20 \text{ N}$.}
Chapter 6

Conclusions and recommendations

6.1 Conclusions

Highly accurate production systems, which have to perform with nanometer accuracy, require actuators with limited parasitic phenomena, such as cogging, reluctance forces, magnetic stiffness, damping and hysteresis. Unknown or badly predictable disturbance forces result in a loss of nanometer accuracy. The mass of the moving parts should be kept to a minimum for two reasons. First, to limit the acceleration forces, the related mechanical deformations of the moving structure, and the related dissipation, and second, to achieve a high servo bandwidth, to increase the capability of correcting for disturbance forces. Mechanical deformations, caused by time-dependent thermal gradients, affect the stability of highly accurate measuring systems as well, resulting in loss of accuracy.

The previously mentioned implications put severe demands on the specifications, as system’s mass, dissipation, stiffness, damping and volume. The initial design of the magneto dynamic gravity compensator (Chapter 4) meets the required specifications. This design has proven to be an alternative for the current systems in wafer scanners, based on pressurized air and high-precision expensive components. Important benefits are the system’s reduced complexity, reliability, costs and
last but not least the possibility of application in a high-vacuum environment. The principle and design configuration are the subject of patent application [15], [17]. The numerical and analytical models show a strong similarity, which validates the replacement of the permanent magnets by a linear current density. The behavior of the proto-type, illustrated by the measurement results, was predicted by these models, but differences were observed. The differences in static force(s), motor constant, and stiffness, as elaborated in Paragraph 4.5, can be attributed mainly to tolerances in magnetization, geometric tolerances, material properties ($\mu_r > 1$), and inaccuracy of the measuring equipment.

The validated analytical model is used in an optimization procedure. In this procedure, the dimensions of the design’s basic configuration (as presented in Paragraph 4.2) are varied within sensible intervals. A weight function is used to express the quality of each possible configuration in a numerical value, where the so-called weight factors express the mutual importance of the specifications (such as static force, stiffness, and dissipation). This strategy leads to an optimal design linked to the chosen set of weight factors: changing the weight factors results in another optimal design configuration.

The optimization is done by making use of analytical expressions. The properties, such as static force, mover and stator mass, stiffness, dissipation, and bounding volume, are calculated for each configuration in a relatively short time. A large number of configurations is investigated. If this number of configurations has to be investigated by boundary element model calculations (or finite element model calculations), the calculation time will increase significantly. The optimal design, according to the analytical expressions, is verified by a 2D boundary element model of the optimal design in Paragraph 5.5. The results are very similar.

One of the most remarkable results of the optimization process is the vanished outer stator magnet ring. Omitting this ring leads to an even more simple design. In Paragraph 5.5, a comparison between the initial design and optimized design was made. The optimized design has an extremely low vertical stiffness (93% lower) in comparison with the initial design. The dissipation is also considerably lower: 55%. The
6.2. RECOMMENDATIONS

The maximum allowable damping between the stator and the mover is specified in Paragraph 4.1. However, the damping is not involved in the analytical model and in the optimization procedure. An analytical expression for the damping can be derived, as was done for example for the stiffness. Therefore, all housings (see Figure 4.31 and Figure 4.32) need to be incorporated in the model. Since the optimization is a time-consuming process, the number of variations is kept to the minimum. Any variation of the housing’s dimensions and materials in the optimization process, results in a dramatic increase of calculation steps and time.

For the same reason (required calculation steps and time for the optimization procedure), an equal thickness of the inner and outer moving magnet ring \( b_{mm} \) is chosen for all configurations. This still resulted in 386,790 configurations which had to be evaluated. Since we know that the outer (stator) magnet ring can be omitted, its dimensions \( R_{oo}, R_{oi} \) will not appear in the optimization procedure anymore. So, one variation parameter could be used to vary the thickness of the outer moving magnet ring independently from the thickness of the inner moving magnet ring. This might result in a better optimal design.

Variations of the motor constant for vertical displacements of the moving magnets were not considered in the optimization process. Position-dependent variations of the motor constant are experienced as a current-dependent stiffness, as is mentioned in Section 2.1.3. The total stiffness between the stator and mover is the sum of the static stiffness (as is
derived from the models and proto-type results) and the current dependent stiffness. From Figure 4.9, it was derived that the motor constant varies only 1.3% for the initial design. The total vertical stiffness can be calculated from the same figure (where the dynamic force $F_{z,dynamic} = 20$ N), making use of (4.4): at $z_{ss} = 1$ mm $\Rightarrow k_{z,dynamic} = 700$ N/m. The same can be done for the optimized model (Figure 5.18): at $z = 1$ mm $\Rightarrow k_{z,dynamic} = 700$ N/m. The analytical model should be extended with this motor constant variation for a more accurate result.

The magneto dynamic gravity compensator consists of strong permanent magnets. If permeable materials or magnetic sources (coils or other permanent magnets) are moving relatively close to these magnets, forces will occur. Therefore, the gravity compensators need to be ‘shielded’. The effect of the shielding on the behavior of the gravity compensator needs attention. This was not studied in the scope of this dissertation.

The effect of tolerances (like mechanical tolerances and magnetization strengths and orientations) on the behavior of the gravity compensator is qualitatively discussed in Paragraph 4.5. These effects are not considered in the optimization procedure. A quantification of these effects is recommended. In addition, NdFeB-magnets suffer from aging: magnetic remanence and coercitive field strength decrease with time, leading to a reduced force and motor constant. In [66], measurement results were presented. Their effects on this design are considered to be negligible.
# Appendix A

## Symbols and Units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Property</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>magnetic vector potential (vector)</td>
<td>Tm</td>
</tr>
<tr>
<td>( A_c )</td>
<td>current conduction cross section</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{coil} )</td>
<td>cross section of coil</td>
<td>m²</td>
</tr>
<tr>
<td>( b_{coil} )</td>
<td>radial coil dimension</td>
<td>m</td>
</tr>
<tr>
<td>( b_m, b_{mm} )</td>
<td>radial dimension of moving magnets</td>
<td>m</td>
</tr>
<tr>
<td>( b_{mi} )</td>
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<td>m</td>
</tr>
<tr>
<td>( b_{mo} )</td>
<td>radial dimension of outer stator magnet</td>
<td>m</td>
</tr>
<tr>
<td>B</td>
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<td>T</td>
</tr>
<tr>
<td>( B_r )</td>
<td>remanent flux density</td>
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</tr>
<tr>
<td>( B_{sat} )</td>
<td>saturation flux density</td>
<td>T</td>
</tr>
<tr>
<td>( c )</td>
<td>specific heat</td>
<td>J/kg·K</td>
</tr>
<tr>
<td>( C_o )</td>
<td>amplifier output capacitance</td>
<td>F</td>
</tr>
<tr>
<td>( d )</td>
<td>damping</td>
<td>Ns/m</td>
</tr>
<tr>
<td>( d_s )</td>
<td>sensor offset in vertical direction</td>
<td>m</td>
</tr>
<tr>
<td>D</td>
<td>electric field density (vector)</td>
<td>C/m²</td>
</tr>
<tr>
<td>E</td>
<td>electric field strength (vector)</td>
<td>V/m</td>
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<tr>
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<td>scalar field, force density, copper filling factor</td>
<td>- , N/m³</td>
</tr>
<tr>
<td>( f' )</td>
<td>force density (vector)</td>
<td>N/m³</td>
</tr>
<tr>
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<td>force, vector field</td>
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</tr>
<tr>
<td>( g_m )</td>
<td>radial gap between stator and moving magnets</td>
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</tr>
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<td>A/m</td>
</tr>
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<td>coercitive field strength (BH)</td>
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<tr>
<td>( H_{c,J} )</td>
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<td>Property</td>
<td>Unit</td>
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<td>$J$</td>
<td>current density (vector)</td>
<td>A/m²</td>
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<td>$J_i$</td>
<td>applied current density (vector)</td>
<td>A/m²</td>
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<td>magnetic polarization (vector)</td>
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<td>linear current density (vector)</td>
<td>A/m</td>
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<td>stiffness</td>
<td>N/m</td>
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<td>$K_r, K_z$</td>
<td>motor constant</td>
<td>N/A</td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
<td>kg</td>
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<tr>
<td>$M$</td>
<td>magnetization (vector)</td>
<td>A/m</td>
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<td>$M_0$</td>
<td>intrinsic magnetization (vector)</td>
<td>A/m</td>
</tr>
<tr>
<td>$N$</td>
<td>number (of turns)</td>
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<td>$p$</td>
<td>power density</td>
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<td>$P, P_h$</td>
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<td>W</td>
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<td>heat flow</td>
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<td>C/m²</td>
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<td>W/m²</td>
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<td>$Q$</td>
<td>charge</td>
<td>C</td>
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<td>thermal resistances in cooling system</td>
<td>K/W</td>
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<td>$r, r'$</td>
<td>position vector $(x, y, z)$</td>
<td>m</td>
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<td>distance</td>
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<td>inner radius of inner moving magnet</td>
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<td>inner radius of outer moving magnet</td>
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<td>$R_o$</td>
<td>amplifier output resistance</td>
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<td>$S$</td>
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<td>m²</td>
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<td>steepness z-actuator</td>
<td>N²/W</td>
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<tr>
<td>$t$</td>
<td>thickness</td>
<td>m</td>
</tr>
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<td>$T$</td>
<td>torque, temperature</td>
<td>Nm, K</td>
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<td>m³</td>
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<td>$V_c$</td>
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<td>m³</td>
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<td>$V_{coil}$</td>
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<td>m³</td>
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<td>magnetic scalar potential</td>
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<td>Symbol</td>
<td>Property</td>
<td>Unit</td>
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<td>--------</td>
<td>----------</td>
<td>------</td>
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<td>$w$</td>
<td>energy density</td>
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<td>weight factor ($i \in {1,2,\ldots}$)</td>
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<td>$W$</td>
<td>weight function, energy</td>
<td>- J</td>
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<td>$x, y, z$</td>
<td>coordinates in Cartesian system</td>
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<td>$Z_o$</td>
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<td>$\varepsilon, \varepsilon_0$</td>
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<td>permeability</td>
<td>H/m</td>
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<td>N/m$^2$</td>
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<td>thermal time constant</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
<td>rad/s</td>
</tr>
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</table>
Appendix B

Definitions

- Actuator: system that is able to convert electric, magnetic and mechanic energy
- Biot-Savart’s law: relation between a current distribution through space and the resulting magnetic field density
- Boundary element method: field calculation based on integral relations (for electric and magnetic fields the integral Maxwell’s equations are used (3.5), (3.6), (3.7) and (3.8))
- Conservative system: system where all losses are isolated from
- Cogging: force related to preferred position of mover with respect to stator in permanent magnet actuators
- Cross talk (force): (unwanted) force interaction between two systems (i.e. long-stroke and short-stroke system)
- Damping: velocity dependent cross talk between two systems
- Dissipation: electric losses resulting into heating
- Feature size: dimension of a basic structure on a wafer
- Long-stroke: system that is able to perform a long-range motion
- Reluctance: force related to preferred position of mover with respect to stator in icon core actuators
• Reticle: original containing the structures for the integrated circuit

• Short-stroke system: highly accurate system that is able to perform a short-range motion

• Stage: system that executes a predefined motion

• Steepness: actuator’s force related to dissipation (squared force per unit dissipation)

• Stiffness: position dependent cross talk between two systems

• Thermal duty cycle: mean current or force of an actuator

• Wafer: material on which the structures for the integrated circuit are printed

• Weight factor: numerical value expressing the penalty for not satisfying a specification

• Weight function: analytical formula expressing the quality of a design related to the degree the specifications are satisfied
Appendix C

Mathematical definitions

C.1 Gradient, divergence and curl

Cartesian coordinates \((x, y, z)\):

\[
\nabla f = a_x \frac{\partial f}{\partial x} + a_y \frac{\partial f}{\partial y} + a_z \frac{\partial f}{\partial z} \tag{C.1}
\]

\[
\nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{C.2}
\]

\[
\nabla \times F = \begin{vmatrix}
    a_x & a_y & a_z \\
    \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
    F_x & F_y & F_z
\end{vmatrix} \tag{C.3}
\]

C.2 Helmholtz’s theorems

C.2.1 Helmholtz’s first Theorem

Any vector field \(F\) that is continuously differentiable in some volume \(V\) can be uniquely determined if its divergence and curl are known throughout the volume and its value is known on the surface \(S\) that bounds the volume:
\[
F(r) = -\nabla \left[ \int_V \frac{\nabla \cdot F(r')}{4\pi |r - r'|} dV' - \int_S \frac{F(r') \cdot a_n}{4\pi |r - r'|} ds' \right] \\
+ \nabla \times \left[ \int_V \frac{\nabla \times F(r')}{4\pi |r - r'|} dV' - \int_S \frac{F(r') \times a_n}{4\pi |r - r'|} ds' \right] \quad (C.4)
\]

C.2.2 Helmholtz’s second Theorem

Any vector field \( F \) that is continuously differentiable in some region can be expressed at every point in the region as the sum of an \textit{irrotational} vector and a \textit{solenoid} vector, as expressed by:

\[
F = \nabla f + \nabla \times A \quad (C.5)
\]

C.2.3 Helmholtz’s third Theorem

If \( \nabla \times F = 0 \) throughout a region, then \( F \) can be represented as:

\[
F = \nabla f \quad (C.6)
\]

throughout the region, where \( f \) is a scalar field. Vectors for which \( \nabla \times F = 0 \) are called \textit{irrotational} vectors (curl equals zero).

C.2.4 Helmholtz’s fourth Theorem

If \( \nabla \cdot F = 0 \) throughout a region, then \( F \) can be represented as:

\[
F = \nabla \times A \quad (C.7)
\]

throughout the region, where \( A \) is a vector field. Vectors for which \( \nabla \cdot F = 0 \) are called \textit{solenoid} vectors (divergence equals zero).
C.3 Stokes’ theorem

For any vector field $\mathbf{F}$ that is continuously differentiable in an open surface $S$, bounded by the closed contour $C$ holds:

$$
\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS = \oint_C \mathbf{F} \cdot \mathbf{\tau} dl \quad (C.8)
$$

C.4 Leibnitz’s theorem

For any vector field $\mathbf{F}$ that is continuously differentiable in some region, enclosed by surface $S$, holds:

$$
\frac{d}{dt} \iint_S \mathbf{F} \cdot \mathbf{n} dS = \iint_S \left\{ \frac{\partial \mathbf{F}}{\partial t} + \nabla \times (\mathbf{F} \times \mathbf{v}) + \mathbf{v} \nabla \cdot \mathbf{F} \right\} \cdot \mathbf{n} dS \quad (C.9)
$$

where $\mathbf{v}$ is the local velocity.

C.5 Elliptic integrals

The elliptic integral of the first kind $\mathbf{E}$ is defined as:

$$
\mathbf{E}(k) = \int_{\varphi=0}^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} \cdot d\varphi \quad (C.10)
$$

The elliptic integral of the second kind $\mathbf{K}$ is defined as:

$$
\mathbf{K}(k) = \int_{\varphi=0}^{\pi/2} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}} \quad (C.11)
$$

And finally the integral of the third kind $\Pi(\rho, k)$:

$$
\Pi(\rho, k) = \int_{\varphi=0}^{\pi/2} \frac{d\varphi}{(1 + \rho \sin^2 \varphi) \cdot \sqrt{1 - k^2 \sin^2 \varphi}} \quad (C.12)
$$
APPENDIX C. MATHEMATICAL DEFINITIONS

C.6 Poynting’s theorem

If the dot product of both sides of Maxwell’s curl-$H$ equation (3.1) is taken, we obtain:

$$E \cdot \nabla \times H = E \cdot J + E \cdot \frac{\partial D}{\partial t} \quad (C.13)$$

Knowing that $\nabla \cdot (E \times H) = H \cdot \nabla \times E - E \cdot \nabla \times H$, (C.13) is rearranged as

$$-\nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t} - H \cdot \nabla \times E \quad (C.14)$$

Using Maxwell’s curl-$E$ equation (3.2) yields

$$-\nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t} + H \cdot \frac{\partial B}{\partial t} \quad (C.15)$$

The quantity $-\nabla \cdot (E \times H)$ is known as the Poynting’s vector. Physically it represents the sum of the electric and magnetic power flow to an infinite small volume. The equivalence with (3.37) is directly seen.

C.7 Function transforms

C.7.1 Laplace transform

If $f(t)$ is a function in the time domain the Laplace transform converts the function to the Laplace domain by applying

$$F(s) = \mathcal{L}(f(t)) = \int_{-\infty}^{+\infty} f(\tau) \cdot e^{-st} d\tau \quad (C.16)$$

The Laplace transform of differentiated and integrated functions can easily be derived from (C.16):

$$\mathcal{L}\left(\frac{df(t)}{dt}\right) = s \cdot F(s) = s \cdot \mathcal{L}(f(t)) \quad (C.17)$$

$$\mathcal{L}\left(\int f(\zeta) \cdot d\zeta\right) = \frac{F(s)}{s} = \frac{\mathcal{L}(f(t))}{s} \quad (C.18)$$
C.7. FUNCTION TRANSFORMS

C.7.2 Fourier transform

If $f(t)$ is a function in the time domain the Fourier transform converts the function to the frequency domain by applying

$$ F(\omega) = \int_{-\infty}^{+\infty} f(\tau) \cdot e^{-j\omega \tau} d\tau \quad (C.19) $$
Appendix D

Derivations

D.1 The electric potential

For electrostatics (3.2) is known that $\nabla \times E = 0$. According to Helmholtz’s third theorem (C.6) $E$ can be described as a scalar potential ($V_e$):

$$\nabla \times E = 0 \Rightarrow E = -\nabla V_e \quad (D.1)$$

From (3.4) and (3.10) follows (in regions where no electric polarization $P_e$ is present$^1$):

$$\nabla \cdot D = \rho_v \Rightarrow \nabla \cdot (\varepsilon_0 \cdot E) = \rho_v \quad (D.2)$$

Combining (D.1) and (D.2) results in the Poisson’s equation:

$$\nabla^2 V_e = -\frac{\rho_v}{\varepsilon_0} \quad (D.3)$$

for which the (general) solution

$$V_e = \frac{1}{4\pi\varepsilon_0} \int \int \int \frac{\rho_v(r')}{|r - r'|}dV(r') \quad (D.4)$$

is finally found.

$^1$So-called electrets, materials with polar molecules ($P_e \neq 0$), produce a net electric field even when no polarizing E-field is present. Mind the equivalence with permanent magnets.
D.2 The magnetic vector potential

The existence of a magnetic vector potential was proven by Maxwell’s relation (3.3) in combination with Helmholtz’s fourth theorem (Section C.2). Therefore this quantity can be written as:

\[ \nabla \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B} = \nabla \times \mathbf{A} \quad (D.5) \]

Applying the curl to (3.1) and substituting (3.3) yields

\[ \nabla \times (\nabla \times \mathbf{A}) = \nabla \times \mathbf{B} = \mu_0 \cdot \mathbf{J} \quad (D.6) \]

Expanding the curl-curl operation \((\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A})\) we obtain

\[ \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \cdot \mathbf{J} \quad (D.7) \]

When introducing the vector potential \(\mathbf{A}\) only \(\nabla \times \mathbf{A}\) was specified. As a result, we can choose its divergence to be anything that is convenient. The curl and divergence of a property are independent quantities, since \(\nabla \cdot (\nabla \times \mathbf{A}) = 0\). So \(\nabla \cdot \mathbf{A} = 0\) is selected, yielding

\[ \nabla^2 \mathbf{A} = -\mu_0 \cdot \mathbf{J} \quad (D.8) \]

This equation is the vector form of Poisson’s equation. The equivalence of (D.8) and (D.3) is seen directly. Replacing \(\rho_v\) by \(\mathbf{J}\) and \(\varepsilon_0\) by \(\frac{1}{\mu_0}\) in (D.4) the magnetic vector potential is found:

\[ \mathbf{A} = \frac{\mu_0}{4\pi} \iiint J(r') \frac{dV(r')}{|r - r'|} \quad (D.9) \]

D.3 Derivation of Biot-Savart’s law

The magnetic vector potential is used to derive the Biot-Savart’s law, an expression for the magnetic induction \(\mathbf{B}\) as function of steady current sources.

\[ \mathbf{B} = \nabla \times \mathbf{A} = \nabla \times \mathbf{A} = \nabla \times \frac{\mu_0}{4\pi} \iiint J(r') \frac{dV(r')}{|r - r'|} \]
D.3. DERIVATION OF BIOT-SAVART’S LAW

Using the product law \( \nabla \times (f \mathbf{F}) = f \nabla \times \mathbf{F} + (\nabla f) \times \mathbf{F} \) the previous integral expression can be rewritten as:

\[
\mathbf{B} = \frac{\mu_0}{4\pi} \iiint \nabla \times \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, dV(\mathbf{r}')
\]

(D.11)

The first term equals zero because the \( \nabla \) operator only acts on the vector \( \mathbf{r} \) and therefore \( \nabla \times \mathbf{J}(\mathbf{r}') = 0 \). Taking a closer look to the second term, we can express \( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \) in Cartesian coordinates as:

\[
\frac{1}{|\mathbf{r} - \mathbf{r}'|} = \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{-\frac{1}{2}}
\]

(D.12)

Taking the gradient of that function:

\[
\nabla \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -\left( \frac{(x - x') \cdot \mathbf{a}_x + (y - y') \cdot \mathbf{a}_y + (z - z') \cdot \mathbf{a}_z}{\left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{\frac{3}{2}}} \right) = -\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3}
\]

(D.13)

Substituting these results reduces (D.11) to:

\[
\mathbf{B} = \frac{\mu_0}{4\pi} \iiint \frac{-\mathbf{J}(\mathbf{r}') \times \mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \, dV(\mathbf{r}')
\]

(D.14)

and \(- \mathbf{r} - \mathbf{r}' \times \mathbf{J}(\mathbf{r}') = \mathbf{J}(\mathbf{r}') \times [\mathbf{r} - \mathbf{r}']\), so finally:

\[
\mathbf{B} = \frac{\mu_0}{4\pi} \iiint \frac{\mathbf{J}(\mathbf{r}') \times \mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \, dV(\mathbf{r}')
\]

(D.15)

Biot-Savart’s law is derived.
D.4 Derivation of Earnshaw’s theorem in polar coordinates

In Section 3.4 Earnshaw’s theorem was presented as:

$$\nabla \cdot \mathbf{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = 0$$  \hspace{1cm} (D.16)

In Figure D.1 the forces on a (infinite small) body in $x$-direction ($f_x$) and $y$-direction ($f_y$) are shown. These forces are expressed in polar coordinates: $f_{x,r}$, $f_{x,\theta}$, $f_{y,r}$ and $f_{y,\theta}$. Since in the rotationally symmetric case there is no driving force present in the $\theta$-direction (and thus $f_{y,\theta} - f_{x,\theta} = 0$) the following relation exists between $f_x$ and $f_y$:

$$f_y = f_x \cdot \tan \theta$$  \hspace{1cm} (D.17)

The sum of the forces in the radial direction is:

$$f_r = f_x \cdot \cos \theta + f_y \cdot \sin \theta$$  \hspace{1cm} (D.18)

Figure D.1: Coordinate transformation
Inserting (D.17) into (D.18) leads to $f_r = f_x \cos \theta + f_x \frac{\sin^2 \theta}{\cos \theta}$ $\Rightarrow f_x = f_r \cos \theta$ and $f_y = f_r \sin \theta$. An incremental displacement $dr$ is easily transformed into incremental displacements in the x- and y-directions: $dx = dr \cos \theta$ and $dy = dr \sin \theta$. Substituting these results into (D.16) leads to:

$$\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} = \frac{\partial (f_r \cos \theta)}{\partial r \cos \theta} + \frac{\partial (f_r \sin \theta)}{\partial r \sin \theta} + \frac{\partial f_z}{\partial z} = 2 \frac{\partial f_r}{\partial r} + \frac{\partial f_z}{\partial z} = 0$$

(D.19)

D.5 Radially magnetized permanent magnets

In [5] and [31] was derived that permanent magnets can be modelled as a current distribution on the boundary surface of the magnetic material. In Figure D.2 a radially magnetized ring of permanent magnetic

![Figure D.2: Current distribution for radially magnetized permanent magnets.](image-url)
Figure D.3: Current distribution for a radially magnetized permanent magnet.

material, with an intrinsic magnetization of $M_0$, is shown. The magnetization $M_0$ is assumed to be constant in the radial direction (in which case $\nabla \cdot M_0 \neq 0$). From this ring an infinite small fraction is isolated, which has still a magnetization of $M_0$. This fraction can be considered to consist of uniformly magnetized volumes for which the constant current distribution $J_s$, as shown in Figure D.2, is valid. The entire ring is therefore properly replaced by a circular current distribution $J_s$ as shown in Figure D.3, under the assumption that the intrinsic radial magnetization $M_0$ is constant within the volume of the magnet.

\section{D.6 Dissipation of heated systems}

From (3.13) the dissipation is known in a volume $V_c$ with electric conductivity $\sigma_e$ due to a current density $J$:

$$P = \iiint \frac{J \cdot J}{\sigma_e} dV_c$$  \hfill (D.20)

Or, making use of (4.22) and assuming a uniform current density in the volume $V_c$:

$$P = \iiint \rho_e J \cdot J dV_c = \rho_e \cdot J^2 \cdot V_c$$  \hfill (D.21)
From (4.40) is known that the mean temperature increase of a system with thermal resistance \( r \), dissipating a heat \( P \), will be:

\[
\Delta T = r \cdot P \tag{D.22}
\]

or inserting this in (4.22):

\[
\rho_e(P) = \rho_{e,293} \cdot (1 + \alpha \cdot r \cdot P) \tag{D.23}
\]

Substitution into (D.21) results into:

\[
P = \rho_{e,293} \cdot (1 + \alpha \cdot r \cdot P) \cdot J^2 \cdot V_c
= \rho_{e,293} \cdot J^2 \cdot V_c \cdot (1 + \alpha \cdot r \cdot P) \tag{D.24}
\]

which leads finally to:

\[
P(J) = \frac{\rho_{e,293} \cdot J^2 \cdot V_c}{1 - \rho_{e,293} \cdot \alpha \cdot r \cdot J^2 \cdot V_c} \tag{D.25}
\]
Appendix E

Behavior of the power amplifier

In Chapter 2 was described how the actuator’s behavior affects the accuracy of the drive system. However, an actuator is powered by an amplifier. In this appendix is illustrated how the non-ideal behavior of an amplifier affects the accuracy of the drive system.

The output impedance of an amplifier \( (Z_o) \) is defined as the sensitivity of the output current \( (i_{out}) \) as function of an applied voltage in series with the output of the amplifier \( (U_o) \). In Figure E.1 the electric diagram for a current power amplifier is shown. It consists of a voltage controlled voltage source \( (V) \). The voltage across the sensing resistor \( (R_{sens}) \), which is proportional to the output current \( (i_{out}) \), is used in a feedback loop \( (H(s)) \). The power amplifier is supplying an actuator, modelled by a resistor \( (R_{actuator}) \), an inductance \( (L_{actuator}) \) and a voltage source \( (U_{emf}) \). The output impedance of current amplifiers is infinite in the ideal case, i.e. any applied voltage in series with the output of the amplifier’s power stage does not affect the output current. In practice however, the output impedance turns out not to be infinite. This has consequences for the dynamic behavior of the application.

To consider the amplifier’s output impedance the current set-point is zero \( (i_{set} = 0) \). From Figure E.1 is then noticed that:

\[
V = -i_{out} \cdot R_{sens} \cdot H(s) \quad (E.1)
\]

\[
Z = r_o + R_{actuator} + s \cdot L_{actuator} \quad (E.2)
\]
Figure E.1: Electric diagram for a current power amplifier.

\[ i_{\text{out}} = \frac{V - U_{\text{emf}}}{Z + R_{\text{sens}}} \]  \hspace{1cm} (E.3)

In general \( R_{\text{sens}} \) and \( r_o \) are small compared to \( Z \). Substitution of (E.1) and (E.2) in (E.3) leads to:

\[ \frac{-U_{\text{emf}}}{i_{\text{out}}} = R_{\text{actuator}} + s \cdot L_{\text{actuator}} + r_o + R_{\text{sens}} \cdot (1 + H(s)) \]
\[ = R_{\text{actuator}} + s \cdot L_{\text{actuator}} + Z_o \]  \hspace{1cm} (E.4)

Knowing that

\[ U_o = U_{\text{emf}} + (R_{\text{actuator}} + s \cdot L_{\text{actuator}}) \cdot i_{\text{out}} \]  \hspace{1cm} (E.5)

and substitution into (E.4) results into:

\[ \frac{-U_o}{i_{\text{out}}} = r_o + R_{\text{sens}} \cdot (1 + H(s)) = Z_o \]  \hspace{1cm} (E.6)

where \( Z_o \) is the output impedance of the amplifier. Since \( r_o \ll R_{\text{sens}} \cdot H(s) \) and \( R_{\text{sens}} \ll R_{\text{sens}} \cdot H(s) \), one yields for the output impedance:

\[ Z_o = R_{\text{sens}} \cdot H(s) \]  \hspace{1cm} (E.7)
Figure E.2: Measured output impedance $Z_o$ of current amplifier.
The output impedance was measured for the amplifier used. In Figure E.2 the measured output impedance of the current amplifier is shown as a function of the frequency. From that figure is concluded that the output impedance is dependent on the frequency. For lower frequencies \((f < 10 \text{ [Hz]})\) the impedance turns out to be real (resistive). At higher frequencies \((f > 10 \text{ [Hz]})\) the impedance is imaginary (capacitive); the decay is \(20 \text{ [dB/decade]}\). In Figure E.3 the equivalent circuit of the non-ideal current amplifier is shown. The output stage is modelled as an ideal current source in parallel with a resistor \((R_o)\) and a capacitor \((C_o)\). In general (and so in the dynamic gravity compensator application) the actuator is modelled as a voltage source \((U_{emf})\), a resistor \((R_{actuator})\) and an inductor \((L_{actuator})\). In case of a Lorentz actuator (as the z-actuator in the dynamic gravity compensator) the voltage source is:

\[
U_{emf} = -\frac{d\lambda}{dt} = -\frac{d\lambda}{dz_{ss}} \cdot \frac{dz_{ss}}{dt} = -K_z \cdot \dot{z}_{ss} \tag{E.8}
\]

where \(\lambda \text{ [Wb]}\) is the coil’s flux linkage with the magnets attached to the short-stroke system, \(K_z \text{ [N/A]}\) is the motor constant in vertical direction and \(\dot{z}_{ss} \text{ [m/s]}\) is the velocity in vertical direction of the moving magnets with respect to the coil.
E.1 Resistive output impedance

From the measurement data, as shown in Figure E.2, was concluded that for lower frequencies the output impedance of the amplifier can be modeled as a resistor. From the same figure one easily obtains \( R_o = 40 \, [\text{k}\Omega] \) \((|Z_o| = R_o \text{ for } f \to 0)\). The actuator’s resistance \((R_{\text{actuator}} = 0.79 \, [\Omega])\) and inductance (typically \( L_{\text{actuator}} = 1 \, [\text{mH}] \)) can be neglected in comparison to this value (see Paragraph 4.4). The induced voltage \((U_{\text{emf}})\) across the resistor of the amplifier results into a current (the current generated by the amplifier is not considered):

\[
i_{\text{resistive}} = \frac{U_{\text{emf}}}{R_o} = -\frac{K_z \cdot \dot{z}_{ss}}{R_o} \quad (E.9)
\]

Since this current is present in the coil of the actuator, this results into a vertical force:

\[
F_z = K_z \cdot i_{\text{resistive}} = -\frac{K_z^2 \cdot \dot{z}_{ss}}{R_o} \quad (E.10)
\]

In (E.10) we recognize that the force is dependent on the velocity of the moving magnets. In Section 2.1.2 this phenomenon was indicated as damping. The damping is according to (2.4):

\[
d_z = \frac{dF_z}{d\dot{z}_{ss}} = \frac{K_z^2}{R_o} \quad (E.11)
\]

In Section 4.4.1 was found for the motor constant \( K_z = 3.06 \, [\text{N/A}] \). Inserting the numbers in (E.11) leads to a damping of \( d = 2.3 \cdot 10^{-4} \, [\text{Ns/m}] \) for low frequencies, which is extremely low compared to the maximum specified damping (see Table 4.1).

E.2 Capacitive output impedance

For higher frequencies \((f \gg 10 \, [\text{Hz}])\) the output impedance is modeled as a capacitor. From Figure E.2 the value of the capacitor is obtained, making use of:

\[
Z_o = \frac{R_o}{j \cdot \omega \cdot R_o \cdot C_o + 1} \quad (E.12)
\]
\[ |Z_o| = \frac{1}{\omega C_o} = \frac{1}{2\pi f C_o} \text{ for } \omega \rightarrow \infty. \]

\[ Z_o = 300 \Omega \text{ for } f = 1000 \text{ [Hz]} \text{ and thus } C_o = 530 \text{ [nF]}. \]

The induced voltage across the capacitor of the amplifier results into a current according to:

\[
i_{\text{capacitive}} = C_o \frac{dU_{emf}}{dt} = -C_o \frac{d(K_z \cdot \dot{z}_{ss})}{dt} = -C_o \left( \dot{z}_{ss} \cdot \frac{dK_z}{dt} + K_z \cdot \frac{d\ddot{z}_{ss}}{dt} \right) = -C_o \cdot K_z \cdot \dddot{z}_{ss} \quad (E.13)
\]

Since the motor constant does hardly depend on a displacement, the assumption that \( \frac{dK_z}{dt} = \frac{\partial K_z}{\partial z} \cdot \dot{z}_{ss} = 0 \) is valid. The current \( i_{\text{capacitive}} \) results again in a force on the moving magnets:

\[
F_z = K_z \cdot i_{\text{capacitive}} = -C_o \cdot K_z^2 \cdot \dddot{z}_{ss} \quad (E.14)
\]

From Newton’s law of motion (2.5) is known: \( F_z = m \cdot \dddot{z}_{ss} \). The capacitor leads to a force, which is directed opposite to the acceleration. The actuator has to generate this force. This force can be seen as a virtual increase of moving mass for the short-stroke system. Inserting the numbers again for the application under consideration leads to: \( m = 4.9 \times 10^{-6} \text{ [kg]} \). This is an extremely small effect.

The effects due to the non-ideal output impedance of the amplifier used are negligible for this application.
Bibliography


Summary

Electromechanical actuators form the workhorses within the industry. The demand for increasingly accurate production systems put severe specifications on the actuators, being part of these production systems. In high-end industrial applications the accuracy of the systems is increasing steadily with as background increasingly powerful, precise, efficient and cost-effective actuators. Parasitic phenomena, affecting the actuator’s linearity, as cogging, reluctance forces, stiffness, damping, position and current dependent variations in motor constant and magnetic hysteresis, should be avoided. Moreover, the accuracy of other components, being part of the control loop (sensors, amplifiers and controller), must increase simultaneously.

With these increasingly severe demands in mind, the design of the magnetic gravity compensator was realised. Two main functions are distinguished: on the one hand a static force against gravity must be generated and on the other hand a controllable (dynamic) force must be added to the static force. The cross talk (force interaction) between stator and mover must be restricted to a minimum value to achieve the required nanometer accuracy. Moreover, the dissipation must be kept within limits, since heat affects the thermal stability of the mechanics and the sensor systems. The design had to be realised in a relatively small volume, since the available volume is restricted by the machine’s volume claims. Three models were used for modelling and verification: two numerical boundary element models\(^1\) (two- and three-dimensional) and an analytical model. For the latter, all permanent magnets were modelled as linear current distributions, by which the magnetic induction due to the stator magnets was calculated according to Biot-Savart’s

\(^1\)magnetic field solvers.
law. The force on the moving magnets is found by applying Lorentz’s law and the stiffness between mover and stator is derived from the force. The design’s thermal behavior is determined by two models: a numerical two-dimensional boundary element model and a (quasi analytical) thermal network model. A proto-type was built and forces, stiffness and thermal behavior were measured to verify the results from the numerical and analytical models. The small discrepancy between the measurements and models was investigated and explained.

The analytical model was used in a numerical optimization procedure, in which the dimensions of the design’s geometrical configuration were varied. In total 386790 variations were calculated. This could be done by a powerful processor in a relatively short time: approximately 70 hours. If numerical methods (as boundary element methods or finite element methods) were used, the required calculation time would be much longer (>1000 days). The way each variation meets the specification was expressed numerically, making use of a so-called weight function. The mutual importance of the specifications was determined by the weight factors. The best design was modelled by the boundary element method as well for validating purposes. The results of both models were discussed and compared.
Electromechanische actuatoren vormen de werkpaarden binnen de industrie. De vraag naar steeds nauwkeurigere productiesystemen stelt strenge eisen aan de actuatoren, die deel uitmaken van deze systemen. De nauwkeurigheid van alle high-end industriële toepassingen neemt toe, met als achtergrond krachtigere, nauwkeurigere, efficiënte en goedkopere actuatoren. Positie- en stroomafhankelijke parasitaire fenomenen die het lineaire gedrag van een actuator negatief beïnvloeden, zoals cogging, reluctantiekrachten, stijfrheid, demping, variatie van de motorconstante en magnetische hysterese, dienen zoveel mogelijk vermeden te worden. Verder geldt, dat de nauwkeurigheid van de overige componenten, die zich in de regellus bevinden (sensoren, versterkers and regelaars), gelijktijdig moet toenemen.

De magnetische zwaartekrachtscompensator is het resultaat waarbij deze strenge eisen een rol spelen. De functie van de zwaartekrachtscompensator laat zich opsluiten. Enerzijds moet een statische kracht worden opgewekt, die de zwaartekracht op een massa compenseert. Daarnaast dient er een regelbare kracht opgewekt te kunnen worden. De overspraak tussen stator en bewegend deel (mover) moet tot een minimum worden beperkt. Verder moet de dissipatie voldoende laag blijven, aangezien warmte de thermische stabiliteit van de mechanica en de sensoren beïnvloedt. De beschikbare ruimte voor het ontwerp is beperkt, vanwege de volume claim van de machine. Tijdens het ontwerp zijn drie modellen gebruikt: twee numerieke boundary element modellen² (twee en drie dimensionaal) en een analytisch model. In het analytische model zijn alle permanente magneten gemodelleerd als lineaire stroomverdelingen, waarmee de magnetische inductie ten gevolge

²magnetische berekeningen.

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van de stator magneten werd berekend door middel van de wet van Biot-Savart. De kracht op de bewegende magneten werd door middel van de wet van Lorentz bepaald en de stijfheid tussen stator en bewegend deel werd berekend uit het krachtsverloop. Voor het thermische gedrag zijn twee modellen gebruikt: een numeriek boundary element model en een (quasi analytisch) thermisch netwerkmodel. Teneinde de modellen te verifiëren is er een proto-type gebouwd waarvan krachten, stijfhenen en het thermische gedrag werden gemeten. De sumiere verschillen tussen de metingen en de modellen zijn onderzocht en verklaard.

Curriculum vitae

Sven Hol was born in Nijmegen (Netherlands) on October 1st in 1971. After having finished his secondary school (Thijcollege in Oldenzaal) in 1990 he studied mechanical engineering at the Twente University (Enschede). His Master of Science thesis, *Design of an Active Controlled Laser cutting head*, was finished in August 1996. Subsequently he worked for a short period at a company in Amersfoort after which he started the Mechatronic Design Program of the Stan Ackermans Institute (Eindhoven University of Technology). The second year’s project, *Design of a planar drive*, was carried out at ASML (Veldhoven), where he started working in the field of electro-mechanics in september 1999. In September 2000 the Ph D project started on a part-time basis, that lead to this dissertation. Supervisors were prof. dr. ir. A.J.A. Vandevenput and prof. dr. ir. J.C. Compter from the Electromechanics and Power Electronics Group at the Electrical Engineering Department of the Eindhoven University of Technology.
Stellingen behorende bij het proefschrift

**DESIGN AND OPTIMIZATION OF A MAGNETIC GRAVITY COMPENSATOR**

doors Sven Antoin Johan Hol
1. Het blijkt mogelijk om een zwaartekrachtscompensatie met extreem lage stijfheid door middel van permanente magneten te realiseren. 

2. Het onderdrukken van verstoringen door het beperken van interacties tussen verschillende machine-onderdelen blijkt effectief voor het halen van nanometernauwkeurigheid.

3. De uitkomsten van de in dit proefschrift afgeleide theoretische modellen worden bevestigd door de metingen aan het proto-type.

4. De magnetische vector potentiaal en de wet van Lorentz blijken nauwkeurige hulpmiddelen voor het bepalen van de onderlinge krachten tussen permanente magneten.

5. De analytische modellen blijken effectief te kunnen worden ingezet bij de zoektocht naar een optimale zwaartekrachtscompensator. Het optimale ontwerp heeft een voor deze toepassing significant beter gedrag dan het initiële ontwerp.

6. Met behulp van de in dit proefschrift afgeleide modellen voor permanente magneten, valt geen uitspraak te doen over de krachtverdeling binnen permanente magneten.

7. Het bepalen van de stroompaden, teneinde de dempingskracht op een zich in een extern magneetveld bewegend lichaam uit te rekenen, is niet triviaal.


9. Bij de toepassing van de wet van Lorentz dient telkenmale de geldigheid te worden aangetoond. Het blindelings aanleren van deze wet, zonder te vermelden welke voorzichtigheid moet worden betracht, is discutabel.

10. Indien wij het aardmagnetisch veld zouden ontberen, zou er geen leven, zoals wij dat kennen, op aarde mogelijk zijn. Verder zou een kompas een waardeloos object zijn.

11. De toename van de rekenkracht en rekensnelheid van micro-processoren heeft tot nu toe geen merkbare vermindering van de tijd, die een gebruiker gemiddeld op een computer moet wachten, opgeleverd.

12. Indien Ludwig van Beethoven een wetenschapper was geweest, zou de unificatie van de fysica reeds twee eeuwen geleden zijn voltrokken.

13. Binnen de ontwikkeling van de muziek is het verval ingezet sinds 1900. In technisch onderzoek worden nog steeds doorbraken gerealiseerd.

14. De toename van de mobiliteit van mensen lijkt te leiden tot een toename van relatieve afstanden.
Propositions belonging to the dissertation

**DESIGN AND OPTIMIZATION OF A MAGNETIC GRAVITY COMPENSATOR**

by Sven Antoin Johan Hol
1. It is possible to realize a gravity compensating system with extreme low stiffness, making use of permanent magnets.

   this dissertation

2. Disturbance suppression by limiting interactions between different machine modules is effective for achieving nanometer accuracy.

   this dissertation

3. The results of the theoretical models, which were derived in this dissertation, are confirmed by the measurements on the prototype.

   this dissertation

4. The magnetic vector potential and Lorentz’s law turn out to be accurate means for determining the mutual force between permanent magnets.

   this dissertation

5. The analytical models can effectively be used for finding the optimal gravity compensator. The optimal design has a significantly better performance, for this application, than the initial design.

   this dissertation

6. The models derived in this dissertation for permanent magnets can not be used for calculating the force distribution within permanent magnets.

   this dissertation

7. Determining the current trajectories for calculating the damping force on a body moving in an external magnetic field is not trivial.

   this dissertation

8. The heat problem usually forms the restrictive factor within the design of electro-mechanical converters. During the design process of machines more attention should be spent on the thermal behavior.

   this dissertation

9. Each time Lorentz’s law is applied, the validity should be proven. Blindly teaching this law, without mentioning which care should be taken, is debatable.

10. If the geomagnetic field would not be present, life on earth, as we know it, would not be possible. More over, a compass would be a useless object.

11. The increase of calculation power and speed of micro-processors did not accomplish a reduction in the time a user has to wait for his computer.

12. If Ludwig van Beethoven had been a scientist, the unification of physics would have been completed two centuries ago.

13. The decline of music development was started in 1900. Breakthroughs in technical research still occur.

14. The increase of people’s mobility seems to lead to an increase of relative distances.