

## Measurements of the turbulent energy dissipation rate

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## Measurements of the turbulent energy dissipation rate

B. R. Pearson<sup>a)</sup>

*School of Mechanical, Materials, Manufacturing Engineering and Management, University of Nottingham, Nottingham NG7 2RD, United Kingdom*

P.-Å. Krogstad

*Department of Applied Mechanics, Thermodynamics and Fluid Dynamics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*

W. van de Water

*Fluid Dynamics Laboratory, Department of Applied Physics, University of Technology, Eindhoven, 5600MB Eindhoven, The Netherlands*

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The one-dimensional surrogate for the dimensionless energy dissipation rate  $C_\epsilon$  is measured in shear flows over a range of the Taylor microscale Reynolds number  $R_\lambda$ ,  $70 \leq R_\lambda \leq 1217$ . We recommend that  $C_\epsilon$  should be defined with respect to an energy length scale derived from the turbulent energy spectrum. For  $R_\lambda \geq 300$ , a value of  $C_\epsilon \approx 0.5$  appears to be a good universal approximation for flow regions free of strong mean shear. The present results for  $C_\epsilon$  support a key assumption of turbulence—the mean turbulent energy dissipation rate is finite in the limit of zero viscosity.

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The mechanism of the dissipation of turbulent energy is probably one of the most fundamental aspects of turbulence. In 1922, Richardson<sup>1</sup> proposed his phenomenological forward cascade model, whereby the rate of turbulent energy dissipation is determined by the rate in which the large-scale eddies pass energy downward to the small-scale eddies. A key consequence of the forward energy cascade in strong turbulence is that the nondimensional mean energy dissipation rate  $C_\epsilon$  is independent of viscosity.<sup>2,3</sup> Thus,  $C_\epsilon$  defined as

$$C_\epsilon = \langle \epsilon \rangle L / \langle u^2 \rangle^{3/2}, \quad (1)$$

(here,  $\langle \epsilon \rangle$  is the mean energy dissipation rate per unit mass,  $L$  and  $u$  are characteristic large length and velocity scales, respectively) should be independent of the Reynolds number and be of order unity.<sup>4</sup> The order of magnitude assumption has been rigorously demonstrated by Lohse.<sup>5</sup> He derived, starting with a mean-field theoretical result for the Navier–Stokes equations and ignoring the possibility of inertial range intermittency effects,  $C_\epsilon$  as a function of the Reynolds number. In summary, Lohse showed the asymptotic form for  $C_\epsilon$  to be

$$C_\epsilon \approx (a/C)^{3/2}, \quad (2)$$

here,  $C$  is the Kolmogorov constant for the second-order structure function  $D(r) = C \langle \epsilon \rangle^{2/3} r^{2/3}$ , and  $a$  is estimated at  $r = L$ , as  $D(r = L) = a \langle u^2 \rangle$ . The latest direct numerical simulations (DNS) results for forced isotropic box turbulence<sup>6</sup> suggest  $a \approx 1.25$  and  $C \approx 2.05$  giving  $C_\epsilon \approx 0.48$ . Note that,

irrespective of the structure function scaling deviating slightly from 2/3 due to small-scale intermittency corrections, the magnitude of  $C_\epsilon$  is  $\sim O(1)$ .

The first experimental evidence for the satisfaction of a one-dimensional equivalent for relation (1), in quasi-homogeneous grid turbulence, was given by Batchelor.<sup>3</sup> Saffman<sup>7</sup> remarked that the evidence was not quite convincing and the possibility of a dependence on viscosity could not be ruled out. Sreenivasan<sup>4</sup> provided an update on the experimental grid turbulence evidence to support relation (1), over the Taylor microscale Reynolds number  $R_\lambda$  [ $R_\lambda \equiv \langle u^2 \rangle^{1/2} \lambda / \nu$ , where  $\lambda$  is the Taylor microscale  $= \langle u^2 \rangle^{1/2} / \langle (\partial u / \partial x)^2 \rangle^{1/2}$ ,  $x$  is the streamwise direction and  $u$  is the streamwise velocity fluctuation] range of  $50 \leq R_\lambda \leq 500$ . Roshko<sup>8</sup> noted that there were simpler ways of acquiring direct experimental evidence for the constancy of  $C_\epsilon$  than relation (1), e.g., the “universal” spreading rate of axisymmetric jets, observed under many different types of laboratory conditions and over considerable Reynolds number ranges was considered by him to be proof enough. Later, Sreenivasan<sup>9</sup> showed that  $C_\epsilon \sim O(1)$  was well obeyed for a number of nonhomogeneous “canonical” turbulent flows and for an ensemble<sup>10</sup> of DNS of forced and decaying isotropic box turbulence.

While the order of magnitude of  $C_\epsilon$  is found to be unity, its precise value is thought to depend on the forcing scheme in DNS<sup>10</sup> and on the macroscale for the case of experiments.<sup>4,9–12</sup> To date, the experimental evidence for the independence of  $C_\epsilon$  from  $R_\lambda$  can be considered scant and the issue unresolved.<sup>13</sup> The aim of the present work is to test the constancy of one-dimensional surrogates for relation (1) in shear flows over a  $R_\lambda$  range which is considerably larger than that in Refs. 4, 9–12. Such information will be a useful

<sup>a)</sup> Author to whom correspondence should be addressed. Telephone: +44 (0)115 95 13789; fax: +44 (0)115 95 13800; electronic mail: bruce.pearson@nottingham.ac.uk

addition to the results already obtained in quasi-homogeneous grid turbulence<sup>3,4</sup> and forced/decaying isotropic DNS of box turbulence.<sup>10</sup>

The majority of data is acquired in a simple inexpensive geometry, which we call a NORMAN grid, that “stirs” vigorously on large scales. The geometry is composed of a perforated plate superimposed over a bi-plane grid of square rods. Further details of the geometry and the resulting flow will be described elsewhere and only a brief description of the experimental setup is given here. In order to span a large  $R_\lambda$  range, two wind tunnels are used. The first grid, hereafter  $N1$ , is located in a blow-down wind tunnel<sup>11</sup> of test section dimensions  $35 \times 35 \text{ cm}^2$  and 2 m length. The second grid, hereafter  $N2$ , is located in a recirculating wind tunnel with a test section of  $2.7 \times 1.8 \text{ m}^2$  cross section and length 11 m. For  $N1$ , the central three rows of the original bi-plane grid (mesh size  $M = 50 \text{ mm}$ , original solidity  $\sigma = 33\%$ ) have alternate meshes blocked (final  $\sigma = 46\%$ ). For  $N2$  ( $M = 240 \text{ mm}$ ) the original  $\sigma = 28\%$  and the final  $\sigma = 42\%$ . As well as the NORMAN grid geometries, normal plate wake data and centerline pipe measurements are re-evaluated here and details can be found in Refs. 11 and 12, respectively. Also, measurements are made on the centerline of a wake formed behind a circular disk of 40 mm diameter in the same facility as that for  $N1$ , the normal plate wake and the circular cylinder wake. For the disk flow the measurement station is located at  $x/d \approx 45$ . For all flows, signals of  $u$  are acquired, for the most part, on the mean shear profile centerline. For  $N2$ , data are also obtained slightly off the centerline at a transverse distance of one mesh height. All data are acquired using the constant temperature anemometry (CTA) hot-wire technique with a single-wire probe made of  $1.27 \text{ }\mu\text{m}$  diameter Wollaston (Pt-10% Rh) wire. The instantaneous bridge voltage is buck-and-gained and the amplified signals are low-pass filtered  $f_{lp}$  with the sampling frequency  $f_s$  always at least twice  $f_{lp}$ . The resulting signal is recorded with 12-bit resolution and for the  $N1$  data reduced velocities are saved with 13-bit resolution. Throughout this work, time differences  $\tau$  and frequencies  $f$  are converted to streamwise distance ( $\equiv \tau U$ ) and one-dimensional longitudinal wave number  $k_1$  ( $\equiv 2\pi f/U$ ), respectively, using Taylor’s hypothesis. The mean dissipation rate  $\langle \epsilon \rangle$  is estimated assuming isotropy of the velocity derivatives, i.e.,  $\langle \epsilon \rangle \equiv \epsilon_{\text{iso}} = 15\nu \langle (\partial u / \partial x)^2 \rangle$ . We estimate  $\langle (\partial u / \partial x)^2 \rangle$  from  $\phi_u(k_1)$  [the one-dimensional energy spectrum of  $u$  such that  $\langle u^2 \rangle = \int_0^\infty \phi_u(k_1) dk_1$  and  $\langle (\partial u / \partial x)^2 \rangle = \int_0^\infty k_1^2 \phi_u(k_1) dk_1$ ]. We have chosen not to correct for the decrease in wire resolution that is associated with an increase in  $R_\lambda$ , since all methods known to us rely on an assumed distribution for the three-dimensional energy spectrum. For most of the data, the worst wire resolution is  $\approx 2\eta$  where  $\eta$  is the dissipative length scale  $\equiv (\nu^3 / \epsilon_{\text{iso}})^{1/4}$ . For  $N1$ , the worst wire resolution is  $\approx 4\eta$ . Finally, we also consider the moderately high  $R_\lambda$  data obtained in “active” grid flows (see Refs. 13–16 for further experimental details).

The present investigation is limited to one-dimensional measurements and suitable surrogates for relation (1). For the mean energy dissipation rate  $\langle \epsilon \rangle$  we use  $\epsilon_{\text{iso}}$ . There are two convenient possibilities for  $L$ , the characteristic length scale of the large-scale motions. The first is  $L_u$ , the stream-

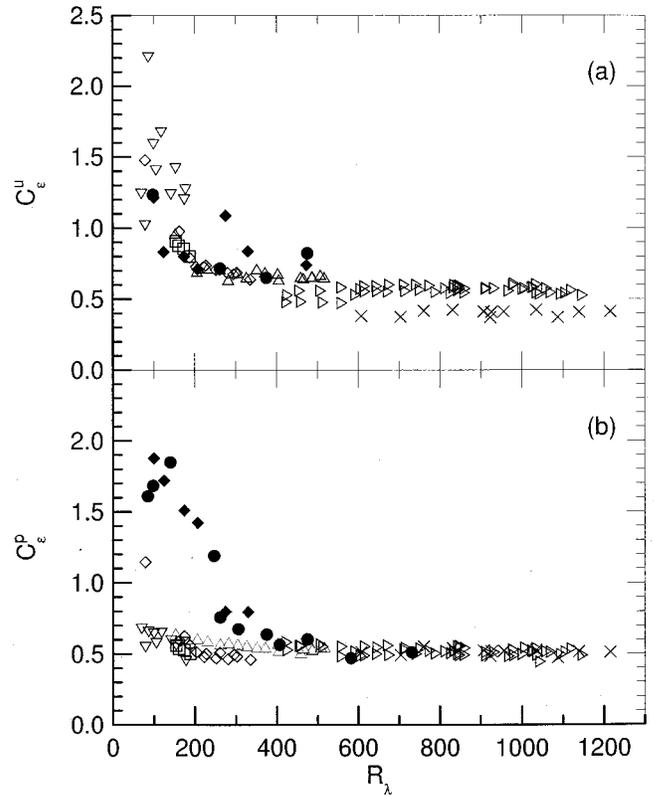


FIG. 1. Normalized dissipation rate for a number of shear flows. Details as found in this work and Refs. 14–16. (a)  $C_\epsilon^u$  [Eq. (3)]; (b)  $C_\epsilon^p$  [Eq. (4)].  $\square$ , circular disk,  $154 \leq R_\lambda \leq 188$ ;  $\nabla$ , pipe,  $70 \leq R_\lambda \leq 178$ ;  $\diamond$ , normal plate,  $79 \leq R_\lambda \leq 335$ ;  $\triangle$ , NORMAN grid,  $174 \leq R_\lambda \leq 516$ ;  $\times$  NORMAN grid (slight mean shear,  $dU/dy \approx dU/dy|_{\text{max}}/2$ ),  $607 \leq R_\lambda \leq 1217$ ;  $\triangleright$ , NORMAN grid (zero mean shear),  $425 \leq R_\lambda \leq 1120$ ;  $\bullet$ , “active” grid Refs. 14, 15,  $100 \leq R_\lambda \leq 731$ ;  $\blacklozenge$ , “active” grid, with  $L_u$  estimated by Ref. 16. For Ref. 14 data, we estimate  $L_p \approx 0.1 \text{ m}$  and for Ref. 15 data we estimate  $L_p \approx 0.225 \text{ m}$ .

wise integral length scale, computed from the streamwise autocorrelation function  $\rho_{uu}(\tau)$  [ $L_u$  is defined as the area under the corresponding autocorrelation function  $\rho_{uu}(\tau)$  such that  $L_u = U \int_0^{\tau_0} \rho_{uu}(\tau) d\tau$  with time  $\tau_0$  chosen as the first zero-crossing] and a plausible surrogate for relation (1) is

$$C_\epsilon^u = \epsilon_{\text{iso}} L_u / \langle u^2 \rangle^{3/2}. \tag{3}$$

The second possibility for  $L$  is  $L_p$ , the predominant energy scale that follows directly from the spectrum  $\phi_u(k_1)$ . The length scale  $L_p$  is estimated from the wave number  $k_{1,p}$  at which a peak in the compensated spectrum  $k_1 \phi_u(k_1)$  occurs, i.e.,  $L_p = 1/k_{1,p}$ . A second suitable surrogate for relation (1) is

$$C_\epsilon^p = \epsilon_{\text{iso}} L_p / \langle u^2 \rangle^{3/2}. \tag{4}$$

Since the majority of flows investigated in the present work are wake flows it is useful to recall that all wakes form some semblance of a vortex street and the governing parameter of a vortex street is the Strouhal number  $St$ . For the flows considered here, it is simple to show, noting that  $k_{1,p} = 1/L_p$ , that  $St$  can be defined as

$$St = L_u / 2\pi L_p, \tag{5}$$

which, when rearranging relations (3) and (4), gives

$$St = C_\epsilon^u / 2\pi C_\epsilon^p. \quad (6)$$

Relation (6) indicates that  $St$  accounts for the difference between relations (3) and (4). It is interesting to note that Refs. 4, 9–12 observed a distinction between different flows or different large-scale forcing schemes for  $C_\epsilon^u$ . It may very well be that these flows are distinguishable on the basis of the Strouhal number, and that possible differences in  $C_\epsilon^u$  can be reconciled using relation (6). Uncovering possible differences between the result of relations (3) and (4) is, therefore, another aim of the present work.

Figure 1(a) shows  $C_\epsilon^u$  for each flow calculated by relation (3). After a rapid decrease in  $C_\epsilon^u$  for  $R_\lambda \lesssim 300$ ,  $C_\epsilon^u$  tends to a constant value as  $R_\lambda$  increases. However, the apparent magnitude for  $C_\epsilon^u$  is different for each flow and indicates that  $L_u$ , in relation (3), is a sensitive indicator of the particular manner in which each flow is forced at large scales and also may reflect the influence of initial and/or boundary conditions. Figure 1(b) shows  $C_\epsilon^p$ , based on the spectral energy scale  $L_p$ , for all flows. The overall agreement between all data is significant, especially when  $R_\lambda \gtrsim 300$ . Of particular note is the collapse of the  $N1$  data measured at  $dU/dy=0$  and  $dU/dy=dU/dy|_{\max}/2$  and also the collapse of the active grid data with the present data at high  $R_\lambda$ . Figure 1(b) suggests an asymptotic value of  $C_\epsilon^p \approx 0.5$  to be an adequate “universal” approximation. Note that all of the measurements are acquired in shear flows in regions of minimal mean shear. It would be fortuitous, indeed, if the estimate of  $C_\epsilon^p$  in strong mean shear should also be  $\approx 0.5$ . We expect the effect of strong mean shear is to reduce the magnitude of  $C_\epsilon$  and there is some evidence to suggest this is so.<sup>9</sup> In conclusion, the difference between  $C_\epsilon^p$  and  $C_\epsilon^u$ , where  $C_\epsilon^u$  depends on the realization of the flow field, can be reconciled by accounting for the large-scale forcing in different flows with the respective  $St$  number.

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