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Sequential Multiperiod Repetitive Control Design With Application to Industrial Wide-Format Printing

Lennart Blanken , Patrick Bevers , Sjirk Koekebakker, and Tom Oomen

Abstract—Repetitive control (RC) enables high control performance for systems subject to periodic disturbances. Many disturbances in mechatronic applications involve the sum of multiple periodic signals, which is not necessarily periodic over a small time interval. The contribution of this article is a design procedure for multiple repetitive controllers, each addressing a single periodic component that achieves high performance with fast convergence in the presence of multiple periodic disturbances. The developed approach involves sequential design of the controllers, and systematically addresses modeling errors and coupling between controllers. An experimental case study on an industrial roll-to-roll printer confirms the benefits of the proposed approach, including superior performance compared to preexisting RC designs.

Index Terms—Mechatronics, motion control, repetitive control.

I. INTRODUCTION

MANY disturbances in mechatronic applications involve the sum of multiple periodic signals. Their summation is not necessarily periodic, e.g., consider a signal with a rational period and one with an irrational period, or may be periodic with the least common multiple of all periods, which can be very large. Relevant examples include industrial roll-to-roll printing systems, which is the case study in the present article. The positioning of paper under the printheads is subject to multiple periodic disturbances induced by various rotating components and gear ratios in the paper transport path. Indeed, mechanical gears are often designed with coprime teeth numbers to ensure uniform wear, leading to very high common period lengths.

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Repetitive control (RC) can significantly improve the control performance of systems that are subject to dominantly periodic disturbances, see, e.g., [1], [2]. RC enables perfect asymptotic rejection of periodic signals through the internal model principle [3]. However, a disadvantage of RC is the increased sensitivity to nonperiodic disturbances [4]–[6] due to Bode's sensitivity integral [7], which directly deteriorates control performance. This adverse effect can be reduced through high-order RC, see, e.g., [8]–[10], yet the achievable performance is fundamentally limited for the considered class of multiperiodic disturbances, i.e., possibly nonperiodic summations of periodic signals.

RC for multiperiodic disturbances can be directly achieved by including multiple internal models, see, e.g., [11]–[13], for developments in this direction. By joint implementation of multiple repetitive controllers, each with the task to reject a single periodic component, perfect rejection of multiperiodic disturbances is potentially enabled. Most design approaches consider parallel configuration of multiple repetitive controllers, see, e.g., [11], [14]–[16]. This potentially enables perfect asymptotic performance, yet often leads to slow convergence due to interaction between controllers. In [12] and [17], multiple repetitive controllers are connected in a cascaded manner to improve convergence behavior. Despite the fact that promising results are reported, these approaches involve independent designs of each controller, and address interaction as uncertainty, i.e., through robustness. This leads to unnecessarily conservative designs, which can lead to severe performance limitations.

Although multiperiodic disturbances occur in many systems, at present, a systematic RC approach that is robust and achieves fast convergence is not yet available. The aim of the present article is to address the interaction between multiple repetitive controllers in a systematic and nonconservative manner; hence, enabling fast convergence.

The contributions of this article are twofold.

- C1) A multiperiod RC approach for fast convergence is developed, based on sequential designs.
- C2) The potential of the developed design technique for rejection of multiperiodic disturbances is validated on an industrial roll-to-roll printer in reproducible experiments.

The sequential approach leads to a systematic design that explicitly accounts for interaction from previously designed controllers. The design can be performed using traditional

Fig. 1. Océ Colorado 1640 wide-format roll-to-roll printing system.

techniques, including loop shaping in the frequency domain. The approach relates to sequential design of multivariable RC in [18, Sec. 5], yet is tailored toward achieving fast convergence in the presence of interacting internal models, which is not covered in [18]. The experimental results confirm that the developed approach enables fast rejection of multiperiodic disturbances, and outperforms preexisting approaches.

The rest of this article is organized as follows. In Section II, the design problem is formulated, and illustrated by a case study of a roll-to-roll printer. In Section III, the considered multiperiod repetitive controller structure is introduced. The sequential design framework is developed in Section IV, constituting contribution C1. The experimental validation on the considered case study is presented in Section V, forming contribution C2. Finally, Section VI concludes this article.

Notation: All systems are discrete time, single-input–single-output, and linear time invariant. The results can be directly generalized to multi-input–multioutput systems. The set of real-rational functions bounded on the unit circle and analytic for $|z| > 1$ is denoted $\mathcal{RH}_\infty$. The imaginary unit is denoted ι , i.e., $\iota^2 = -1$. The least common multiple of two integers a, b is denoted $\text{lcm}(a, b)$. The complex indeterminate z is often omitted when it is clear from the context.

II. PROBLEM FORMULATION

In this section, the control problem is formulated. In Section II-A, the considered case study is introduced. The RC setup is presented in Section II-B, and the RC problem is defined in Section II-C.

A. Case Study: Positioning Errors in a Roll-to-Roll Printer

Fig. 1 depicts the considered Océ Colorado 1640 printer, which can print on roll-based media, including paper and vinyl, up to 1.6 m wide. The printing operation is illustrated in Fig. 2. The printheads, which contain many closely spaced nozzles that jet ink onto the medium, are located in the carriage. The carriage performs lateral passes over the stationary medium. After printing, the ink is cured by UV light. In between passes, the paper is transported in the y -direction by the medium positioning roll. For high-quality printing, accurate alignment of consecutive print-passes is required.

Fig. 2. Top view of printing operation in a prototype roll-to-roll printer. The carriage performs passes in the x -direction over the paper. In between passes, the paper is stepwise transported in the y -direction using the medium positioning roll. This paper transport induces MPEs, resulting in misalignment of consecutively printed passes. The aim is to measure the MPEs using a scanner mounted on the carriage, and compensate them by controlling the gantry position in the y -direction during printing using multiperiod RC.

Fig. 3. Measured MPEs, consisting of multiple periodic components. The periodic components correlate with various rolls in the system, e.g., the dominant period of 31.4 passes (frequency 0.032 1/print-pass) corresponds to one rotation of the medium positioning roll. (a) Measured medium positioning errors. (b) Power spectral density of measured medium positioning errors.

The alignment of print-passes is deteriorated by medium positioning errors (MPEs), which are induced during stepwise medium transportation. These MPEs originate from multiple rotating components in the system, e.g., due to eccentricities in rolls that transport the paper. A measured MPE is depicted in Fig. 3. Its power spectrum reveals multiple periodic contributions, relating to several rotating components. For example, the dominant contribution of period 31.4 print-passes corresponds to one rotation of the medium positioning roll.

Fig. 4. Control configuration with repetitive controller R .

For high-quality printing, the compensation of the multiperiodic MPEs is required. This is enabled by a scanner in the carriage, see Fig. 2. The MPE measurements result from an image processing algorithm that analyzes each scan *a posteriori*. The main idea is to use the MPE measurements as a reference for the gantry beam, and actively control the gantry beam in the y -direction during lateral passes of the carriage. In this article, an RC framework is developed to achieve the required printing accuracy.

B. Repetitive Control Setup

Consider the control scheme in Fig. 4, consisting of plant $G(z)$, stabilizing feedback controller $C(z)$, and repetitive controller $R(z)$ to be designed, connected in a cascaded configuration. Furthermore, $r(k)$ is the medium position that is to be tracked by gantry position $y(k)$, and $G(z)$ is the gantry dynamics. Note that $r(k)$ represents a fictitious reference; indeed, the MPE $e(k)$ is directly measured and fed into $R(z)$. The signal d denotes disturbances.

The control objective is to reject exogenous disturbance

$$r(k) = \sum_{i=1}^n r_i(k) \quad (1)$$

where $r_i(k + N_i) = r_i(k)$ and $N_i \in \mathbb{N}$ are the periods. That is, minimize the tracking error $e = r - y$, given by

$$e = r - Sd - Tu = r - Sd - TRe \quad (2)$$

$$= (1 + TR)^{-1} (r - Sd) = S_R(r - Sd) \quad (3)$$

with sensitivity function $S = (1 + GC)^{-1} \in \mathcal{RH}_\infty$, complementary sensitivity function $T = (1 + GC)^{-1}GC \in \mathcal{RH}_\infty$, and modifying sensitivity function $S_R = (1 + TR)^{-1}$.

Based on the internal model principle [3], RC enables asymptotic rejection of periodic disturbances by including a memory loop in the feedback loop, see, e.g., [1]. For multiperiodic disturbances (1), a single-period RC may be designed based on the least common multiple of all periods, i.e., $\text{lcm}(N_1, N_2, \dots, N_n)$, if it exists. This can lead to slow convergence, especially if the least common multiple is unacceptably large, e.g., consider Fig. 3(b).

Remark 1: The employed cascaded controller configuration (see Fig. 4) with input to C given by $u - y = Re - y$, is in contrast with plug-in RC implementations that are commonly used, see, e.g., [1], [2]. In plug-in RC implementations, the RC output $u = Re$ is added to the tracking error e , i.e., the input to C is given by $e + u = e + Re$. The cascaded structure is preferred here in view of the considered case study—the reference r , which represents the medium position that is to

be tracked, becomes available after a large time delay due to the required image processing. Hence, it cannot be fed into C in real time, as in conventional plug-in RC structures. In the cascaded configuration, the image processing delay can be incorporated into the delay of repetitive controller R . The design problem of R is equivalent; in both cases, the problem is the design of $S_R = (I + TR)^{-1}$, see, e.g., [2, Sec. 3]. This implies that all results developed in this article are directly applicable to plug-in RC configurations.

C. Problem Formulation

The problem addressed in this article is the design of a repetitive controller R with respect to the following requirements.

R1) High asymptotic performance in the presence of multiperiodic disturbances, i.e., small error (3) after convergence.

R2) Robust and fast convergence of error (3) in the presence of modeling errors and interaction between controllers.

As is argued in Section I, preexisting approaches fail to satisfy both requirements, due to seemingly required robustness to modeling errors and interaction between controllers.

In this article, a sequential design approach is developed for multiple repetitive controllers that meet R1 and R2. The interaction between controllers and modeling errors are addressed in a nonconservative manner through the sequential design and the use of inexpensive and accurate frequency response function (FRF) measurements [19].

In the next section, the multiperiod RC structure is introduced. In Section IV, the sequential design technique is developed. The experimental validation is presented in Section V.

III. MULTIPERIOD REPETITIVE CONTROL STRUCTURE

In this section, the multiperiod RC structure is introduced. Multiple repetitive controllers are implemented in a cascaded structure, see also [12], [17], which potentially improves convergence speed of the closed-loop system (R2) compared to parallel interconnections, see, e.g., [11], [15]. This is demonstrated in examples, including slow convergence of parallel implementations. The considered cascaded structure facilitates nonconservative sequential design (R1) in Section IV, i.e., contribution C1.

The multiperiod repetitive controller R , see Fig. 5(a), is parametrized as

$$R = \sum_{i=1}^n \left(R_i \prod_{j=1}^{i-1} (1 + \hat{T}R_j) \right) \quad (4)$$

with $\hat{T}$ a parametric model of T . Each single-period repetitive controller R_i in Fig. 5(b) is given by

$$R_i = \alpha_i L_i z^{-N_i} Q_i (1 - z^{-N_i} Q_i)^{-1} \quad (5)$$

with learning gains $\alpha_i \in \mathbb{R}$, and filters $L_i(z), Q_i(z)$ such that $z^{-N_i} L_i(z) Q_i(z) \in \mathcal{RH}_\infty$. That is, L_i and Q_i can have finite preview, i.e., be noncausal, provided that R_i is causal, by embedding their finite preview in the delay z^{-N_i} .

Remark 2: The repetitive controller (4) directly allows to include multiple memory loops with equal delay lengths N_i ,

Fig. 5. Structure of multi-period repetitive controller R . (a) Cascaded structure of R . (b) Structure of R_i .

which can be used to increase the robustness to disturbance period variations and nonperiodic disturbances, as in [4], [9], and [10].

The main idea is that each R_i eliminates a part of the error. By filtering through model $\hat{T}$, the predicted residual error is obtained. This is subsequently processed by R_{i+1} . This is illustrated by two examples.

Example 1: Let $n = 2$, and assume $\hat{T} = T$, i.e., perfect model knowledge. Considering Fig. 5(a) and (2), the error signal entering R_2 is given by

$$\begin{aligned} e_2 &= e_1 + \hat{T}u_1 = e + \hat{T}u_1 \\ &= r - Sd - T(u_1 + u_2) + \hat{T}u_1 \\ &= r - Sd - Tu_2. \end{aligned}$$

Hence, the control action u_1 is hidden from R_2 . The input to R_1 is the original error, i.e., $e_1 = e$.

Next, the benefits of the cascaded structure on convergence behavior are demonstrated.

Example 2: Consider a static system $T \in \mathbb{R}$, $T \neq 0$, subject to disturbance $r(k) = \sum_{i=1}^2 r_i(k)$, with $r_1(k+2) = r_1(k)$ and $r_2(k+3) = r_2(k)$, i.e., $N_1 = 2$, $N_2 = 3$, and $d = 0$. A two-period repetitive controller (4) is implemented with $N_1 = 2$, $N_2 = 3$, static $L_i, Q_i \in \mathbb{R}$, and $\alpha_i = 1$, see Fig. 5. Using state-space representations, R_1 and R_2 are given by

$$R_1 \stackrel{s}{=} \left[\begin{array}{cc|c} 0 & Q_1 & 1 \\ 1 & 0 & 0 \\ 0 & L_1 Q_1 & 0 \end{array} \right] \quad R_2 \stackrel{s}{=} \left[\begin{array}{ccc|c} 0 & 0 & Q_2 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & L_2 Q_2 & 0 \end{array} \right].$$

Using (4), the resulting closed-loop system (3) is given by

$$e(k) = \underbrace{\left[\begin{array}{cccc|c} 0 & (1 - TL_1)Q_1 & 0 & 0 & -TL_2Q_2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & (\hat{T} - T)L_1Q_1 & 0 & 0 & (1 - TL_2)Q_2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & TL_1Q_1 & 0 & 0 & TL_2Q_2 \end{array} \right]}_{S_R} r(k). \quad (6)$$

Let $L_i = 1/T$, i.e., perfect model knowledge, and $Q_i \neq 0$ such that S_R is stable. Comparing a cascaded structure with $\hat{T} = T$ to a parallel structure, i.e., $\hat{T} = 0$ in Fig. 5, the following observations are made.

- 1) In a cascaded structure, the five poles of the resulting system S_R are all located in the origin, irrespective of Q_i . Indeed in (6), $(\hat{T} - T) = 0$ and $(1 - TL_2) = 0$. Hence, convergence of $e(k)$ is achieved after $k = 5$ samples for arbitrary disturbance $r(k)$ with period $\text{lcm}(N_1, N_2)$, i.e., $e(k + \text{lcm}(2, 3)) = e(k)$ for $k \geq 5$. If $Q_i = 1$, then zero error is obtained, i.e., $e(k) = 0$ for $k \geq 5$.
- 2) In a parallel structure, i.e., $\hat{T} = 0$ in (6), the poles of S_R are evenly spaced on the circle with radius $\sqrt[3]{|Q_1 Q_2|}$, centered at the origin. Hence, slow convergence may be obtained, irrespective of the perfect model knowledge, i.e., $L_i = 1/T$, $\forall i$. Also, $Q_i = 1$ results in an unstable system S_R with all poles on the unit circle.

The example illustrates that parallel RC structures can lead to slow convergence, whereas fast convergence can be achieved through a cascaded structure. In the next section, a systematic design approach is developed for cascaded multiperiod RC that explicitly accounts for modeling errors $\hat{T} \neq T$ and the resulting remaining interaction between controllers.

IV. SEQUENTIAL DESIGN APPROACH FOR MULTIPERIOD REPETITIVE CONTROL

In this section, a sequential design procedure is developed for cascaded multiperiod RC, constituting contribution C1 of this article. Nonconservative stability conditions are developed in Section IV-A, which form the basis for the developed design procedure in Section IV-B. Stability is guaranteed in the presence of modeling errors $\hat{T} \neq T$, and can be verified on a frequency-by-frequency basis, e.g., by use of FRF measurements, see, e.g., [19]. Consequently, this enables performance improvements compared to [12] and [17]. The developed results are related to sequential design techniques for multivariable control, see, e.g., [20], [21] for feedback control and [18] for applications to RC, yet have different implications for controller design. Specifically, multiperiodic and high-order internal models need to be accounted for in every sequential design step, which is enabled through the use of FRF measurements.

A. Stability Analysis

In this subsection, nonconservative stability conditions are developed for the closed-loop system (3) with multiperiod RC in (4). The following result is fundamental to all derivations.

Lemma 1: Consider closed-loop (3) with R as in (4). Then

$$S_R = (1 + TR)^{-1} = \prod_{i=1}^n (1 + T_i^{\text{eq}} R_i)^{-1} \quad (7)$$

where T_i^{eq} denotes the equivalent plant in loop i , given by

$$T_i^{\text{eq}} = \begin{cases} T & \text{if } i = 1 \\ (1 + T_{i-1}^{\text{eq}} R_{i-1})^{-1} T_{i-1}^{\text{eq}} (1 + \hat{T} R_{i-1}) & \text{if } i > 1 \end{cases} \quad (8)$$

Proof of Lemma 1: Equation (7) is proven by induction. For $n = 1$, it directly follows that $T_1^{\text{eq}} = T$. For $n > 1$, the modifying sensitivity is rewritten by substitution of (4) as

$$\begin{aligned}
 S_R &= (1 + TR)^{-1} = \left(1 + \underbrace{T}_{T_1^{\text{eq}}} \sum_{i=1}^n \left(R_i \prod_{j=1}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= \left(1 + T_1^{\text{eq}}R_1 + T_1^{\text{eq}} \sum_{i=2}^n \left(R_i \prod_{j=1}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= (1 + T_1^{\text{eq}}R_1)^{-1} \\
 &\quad \cdot \left(1 + (1 + T_1^{\text{eq}}R_1)^{-1} T_1^{\text{eq}} \sum_{i=2}^n \left(R_i \prod_{j=1}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= (1 + T_1^{\text{eq}}R_1)^{-1} \\
 &\quad \cdot \left(1 + \underbrace{(1 + T_1^{\text{eq}}R_1)^{-1} T_1^{\text{eq}} (1 + \hat{T}R_1)}_{T_2^{\text{eq}}} \right. \\
 &\quad \cdot \left. \sum_{i=2}^n \left(R_i \prod_{j=2}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= (1 + T_1^{\text{eq}}R_1)^{-1} \\
 &\quad \cdot \left(1 + T_2^{\text{eq}}R_2 + T_2^{\text{eq}}(1 + \hat{T}R_2) \sum_{i=3}^n \left(R_i \prod_{j=3}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= \prod_{i=1}^2 (1 + T_i^{\text{eq}}R_i)^{-1} \\
 &\quad \cdot \left(1 + \underbrace{(1 + T_2^{\text{eq}}R_2)^{-1} T_2^{\text{eq}} (1 + \hat{T}R_2)}_{T_3^{\text{eq}}} \right. \\
 &\quad \cdot \left. \sum_{i=3}^n \left(R_i \prod_{j=3}^{i-1} (1 + \hat{T}R_j) \right) \right)^{-1} \\
 &= \prod_{i=1}^n (1 + T_i^{\text{eq}}R_i)^{-1}.
 \end{aligned}$$

In Lemma 1, S_R is reformulated as a product of recursively dependent factors, through equivalent plant T_i^{eq} in (8). Indeed, T_i^{eq} is the transfer function that maps u_i to e_i with all preceding loops $1, \dots, i-1$ closed, including controllers $R_1, \dots, R_{i-1}$.

Remark 3: If $\hat{T} = T$, i.e., perfect model knowledge, then $T_i^{\text{eq}} = T \forall i$. As a result, (7) reduces to $S_R = \prod_{i=1}^n (1 + TR_i)^{-1}$, and the design of R in (4) simplifies to independent designs of each R_i based on T . In this case, standard single-period RC design techniques can be used, see, e.g., [1], [2].

Next, the factorization in Lemma 1 is exploited for stability analysis. The key point is that S_R is stable if each factor $(1 + T_i^{\text{eq}}R_i)^{-1}$ in (7) is stable. Substitution of R_i in (5) and rewriting

Fig. 6. Nyquist contour Γ with inner radius 1, outer radius infinity, and the parallel lines infinitely close to the real axis.

yields

$$\begin{aligned}
 &(1 + T_i^{\text{eq}}R_i)^{-1} \\
 &= (1 + \alpha_i T_i^{\text{eq}} L_i z^{-N_i} Q_i (1 - z^{-N_i} Q_i)^{-1})^{-1} \\
 &= ((1 - z^{-N_i} Q_i + \alpha_i T_i^{\text{eq}} L_i z^{-N_i} Q_i) (1 - z^{-N_i} Q_i)^{-1})^{-1} \\
 &= (1 - z^{-N_i} Q_i) \cdot (1 - (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i)^{-1}. \quad (9)
 \end{aligned}$$

The main results of this section are presented next.

Theorem 1 (Nyquist stability theorem for multiperiod RC based on sequential conditions): Consider closed-loop system (3) with R as in (4). Suppose all poles of S , T , $\hat{T}$, L_i , and Q_i are in the open unit disk, and there are no unstable pole-zero cancellations between S and the factors in (9), $\forall i$. Then, closed-loop system (3) is stable if and only if, for all $i = 1, \dots, n$, the image of $(1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i$:

- 1) does not encircle the point 1;
- 2) and does not pass through the point 1;

as z traverses the Nyquist contour Γ depicted in Fig. 6, see, e.g., [22, ch. 3].

Proof of Theorem 1: Since S and $(1 - z^{-N_i} Q_i)$ are stable by design, and no unstable pole-zero cancellations are present, the closed-loop S_R is stable if and only if each $(1 - H_i)^{-1}$ is stable, $i \in [1, \dots, n]$, with $H_i = (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i$, see (7) and (9). Essentially, each $(1 - H_i)^{-1}$ constitutes a positive feedback loop with loop gain H_i . The Nyquist theorem, see, e.g., [23, Th. 4.9], states that $(1 - H_i)^{-1}$ is stable if and only if the image of $H_i(z)$ encircles the point $1 + 0\iota$ in a counterclockwise direction P times as z traverses Γ , and does not pass through $1 + 0\iota$. Here, P is the number of poles of $1 - H_i(z)$ inside Γ , i.e., unstable poles of $H_i(z)$.

It remains to determine P , i.e., the number of unstable poles of $H_i = (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i$. Since all poles of L_i and Q_i are on the open unit disk, this reduces to the number of unstable poles of T_i^{eq} . Next, it is shown that T_i^{eq} is guaranteed to be stable, i.e., $P = 0$. From (8)

$$T_i^{\text{eq}} = T_{i-1}^{\text{eq}} \frac{1 - (1 - \alpha_i \hat{T} L_{i-1}) z^{-N_{i-1}} Q_{i-1}}{1 - (1 - \alpha_i T_{i-1}^{\text{eq}} L_{i-1}) z^{-N_{i-1}} Q_{i-1}} \quad (10)$$

and $T_1^{\text{eq}} = T$. Observe that T_i^{eq} is stable if T_{i-1}^{eq} is stable, since the numerator of (10) is stable by the assumptions made in Theorem 1, and the roots of the denominator are inside the unit circle by design of preceding $R_1, \dots, R_{i-1}$ according to

Theorem 1. Hence, stability of $T_1^{\text{eq}} = T$ implies stability of T_i^{eq} $\forall i$. As a result, the closed loop is stable iff, $\forall i$, the image of H_i makes no encirclements of the point $1 + 0\iota$. ■

Theorem 2 (Small-gain stability for multiperiod RC based on sequential conditions): Let the assumptions of Theorem 1 hold, and assume $(1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i$ is strictly proper for all i . Then, the closed-loop system (3) with R as in (4) is stable if, for all $i = 1, \dots, n$

$$|(1 - \alpha_i T_i^{\text{eq}}(e^{j\omega}) L_i(e^{j\omega})) Q_i(e^{j\omega})| < 1 \quad \forall \omega \in [0, \pi]. \quad (11)$$

Proof of Theorem 2: Let $H_i = (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i$, and consider Theorem 1 with Nyquist contour Γ in Fig. 6. Given that H_i is strictly proper, which is the case for sufficiently large N_i , only the part of Γ around the unit circle needs to be evaluated. This is proven next. First, note that $\lim_{|z| \rightarrow \infty} H_i(z) = 0$, i.e., the part of Γ at infinity maps to 0. Second, by satisfaction of (11), it follows that $|H_i(-1)| = |H_i(e^{j\pi})| < 1$, and hence, the parallel branches along the real axis of Γ do not influence the stability test, see, e.g., [22, p. 86]. Finally, since $|z^{-N_i}| = 1$ for z on the unit circle, (11) is obtained. ■

The key point is that the stability of $S_R = (1 + TR)^{-1}$ is reformulated in Theorems 1 and 2 as sets of sequential nonconservative conditions on R_i . The results are nonconservative in terms of modeling errors $\hat{T} \neq T$ and the interaction between loops; the conditions are based on equivalent plants T_i^{eq} , which directly include true system T and all interaction from preceding loops, see (8). That is, filters L_i , Q_i , and gains α_i can be explicitly designed for modeling errors $\hat{T} \neq T$. This is the topic of the next subsection.

Remark 4: In the single-period case, i.e., $n = 1$, Theorems 1 and 2 recover well known conditions for stability of standard RC systems, see, e.g., [2, Ths. 1, 2].

Remark 5: A parallel RC structure is recovered as a special case by setting $\hat{T} = 0$ in (4), i.e., $R = \sum_{i=1}^n R_i$. Hence, Theorems 1 and 2 also provide stability conditions for parallel multiperiod RC with $T_i^{\text{eq}} = (1 + T_{i-1}^{\text{eq}} R_{i-1})^{-1} T_{i-1}^{\text{eq}}$. In this case, T_i^{eq} may significantly deviate from T , which may severely complicate the design of L_i and Q_i as is motivated next.

B. Design Approach

Theorems 1 and 2 provide sets of conditions for sequential design of filters L_i, Q_i of cascaded repetitive controllers R_i . Based on these conditions, well known synthesis algorithms from literature may be used to design filters L_i, Q_i , see, e.g., [2], which include approximate inversion techniques for L_i , (see, e.g., [24], [25] for overviews) and zero-phase filters for Q_i . In the present article, the following particular design choices are advocated in view of robustness to interaction between controllers R_i and modeling errors $\hat{T} \neq T$, i.e., requirements R1 and R2, which consequently lead to Procedure 1.

- 1) Robust stability in the presence of modeling errors $\hat{T} \neq T$ can be directly verified using FRF measurements, see, e.g., [19]. In particular, $T_i^{\text{eq}}(e^{j\omega})$ can be constructed based on FRF measurements of $T(e^{j\omega})$, enabling a direct evaluation of robust stability in Theorems 1 and 2. Interaction between controllers is directly taken into account by the

sequential design, i.e., in $T_i^{\text{eq}}(e^{j\omega})$. Robust stability with respect to plant uncertainty can be directly evaluated by inclusion of confidence intervals of FRF measurements, which are typically readily available for FRF estimates that result from experimental data [19].

- 2) In view of the design of L_i , equivalent plants T_i^{eq} may closely approximate T , also if $\hat{T} \neq T$. That is, all L_i can be designed based on the inversion of $\hat{T}$, rather than the inversion of T_i^{eq} . Indeed, the inversion of T_i^{eq} is undesired due to high-order terms $z^{-N_1}, \dots, z^{-N_{i-1}}$ from preceding loops. To see that $T_i^{\text{eq}} \approx T$, note from (9) that

$$\begin{aligned} T_{i+1}^{\text{eq}} &= (1 + T_i^{\text{eq}} R_i)^{-1} T_i^{\text{eq}} (1 + \hat{T} R_i) \\ &= \left(\frac{1 - z^{-N_i} Q_i}{1 - (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i} \right) T_i^{\text{eq}} \\ &\quad \cdot \left(\frac{1 - (1 - \alpha_i \hat{T} L_i) z^{-N_i} Q_i}{1 - z^{-N_i} Q_i} \right) \\ &= T_i^{\text{eq}} \frac{1 - (1 - \alpha_i \hat{T} L_i) z^{-N_i} Q_i}{1 - (1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i}. \end{aligned}$$

Hence, at frequencies where $|(1 - \alpha_i T_i^{\text{eq}} L_i) z^{-N_i} Q_i| \ll 1$, see also (11), it holds that $T_{i+1}^{\text{eq}} \approx T_i^{\text{eq}} \approx T$. This is confirmed in Fig. 7.

- 3) The loop-closing order must be selected with care, since the resulting designs and closed-loop system may depend on the loop-closing order. Important considerations include (see also [20] and [21] for related results in sequential feedback design): 1) performance deterioration of previously designed loops due to successive designs and 2) influences of previously designed loops on subsequent designs. This implies that iterative redesign may be needed. Practical guidelines include: 1) to close loops in an increasing order of period lengths N_i for fast convergence and 2) to end with loops for which performance guarantees are desired. The loop-closing order can be altered as $\bar{T} = PTP$, where P is a permutation matrix.

These design choices lead to Procedure 1. In the next section, Procedure 1 is experimentally validated.

V. EXPERIMENTAL VALIDATION

In this section, the sequential design technique for multiperiod RC is validated on a roll-to-roll printer in reproducible experiments, constituting contribution C2 of the present article. It is demonstrated that multiperiod RC enables accurate compensation of MPEs with fast convergence. First, the experimental system is introduced. In Section V-B, the developed RC design technique is applied to the system. The results are presented in Section V-C.

A. Experimental Setup

The gantry beam of a roll-to-roll printer, see Fig. 2, is considered. The system is actuated by a voltage-driven motor, and the output position y [m] is measured using an optical encoder with resolution $0.25 \mu\text{m}$. The system is operated in discrete time with

Procedure 1: Sequential Multiperiod RC Design.

Given FRF measurement of $T(e^{j\omega})$ and parametric model $\hat{T}$, perform the following steps.

- 1) Design $L \equiv L_i$ based on parametric model $\hat{T}$, e.g., by inversion.
- 2) Choose the order in which the controllers are implemented.
- 3) Set the index $i = 1$, and perform the following steps.
 - a) Construct $T_i^{\text{eq}}(e^{j\omega})$ in (8) based on the FRF of $T(e^{j\omega})$.
 - b) Design filter $Q_i(z)$ and gain α_i according to Theorem 1 or Theorem 2, based on the equivalent plant $T_i^{\text{eq}}(e^{j\omega})$.
 - c) Until $i = n$, set $i \rightarrow i + 1$ and return to step 3a.
 - d) If the resulting closed-loop system is unsatisfactory, reset $i = 1$, return to 3a, and redesign each R_i .
- 4) If the resulting closed-loop system after iterations is unsatisfactory, return to step 2 and change the loop-closing order.

Fig. 7. Bode diagram of identified FRF of T (—), approximate parametric model $\hat{T}$ (—) used for L -filter design, and FRF of equivalent plant T_2^{eq} (—) that is used to guarantee stability of sequential multiperiod RC design.

sampling frequency 1 kHz. A stabilizing feedback controller is implemented, yielding a closed-loop bandwidth of 14 Hz. An FRF measurement of the resulting system $T(z)$ is depicted in Fig. 7, together with the low-order parametric model $\hat{T}(z)$ used for design, given by

$$\hat{T}(z) = \frac{0.00357(z + 0.959)(z + 0.762)}{(z - 0.823)(z - 0.872)(z^2 - 1.97z + 0.968)} \cdot \frac{(z^2 - 1.97z + 0.966)(z^2 - 1.78z + 0.952)}{(z^2 - 1.35z + 0.639)(z^2 - 1.36z + 0.895)}.$$

The antiresonance/resonance pairs of T between 20 and 80 Hz are deliberately not modeled by $\hat{T}$, to include the effect of significant modeling errors.

Fig. 8. Disturbance signal $r = r_1 + r_2$ (—), consisting of periodic signals r_1 , r_2 (—, —, respectively) with periods $N_1 = 4500$ and $N_2 = 12000$.

Reproducible experiments are performed to compare all the considered approaches. A two-periodic disturbance $r = r_1 + r_2$ is constructed, see Fig. 8, with period lengths spaced apart to demonstrate potential benefits of the proposed multiperiod RC design. Signals r_1 and r_2 represent a disturbance due to stepwise paper transportation, and an eccentricity of the medium positioning roll, see Fig. 2. Their periods are $N_1 = 4500$ and $N_2 = 12000$ samples, and $\text{lcm}(N_1, N_2) = 8N_1 = 3N_2$. This is in agreement with measured disturbance periods in wide-format roll-to-roll printers, see Fig. 3, for which typical print-pass periods are in the order of several seconds.

B. Repetitive Control Designs

Three repetitive controllers are designed. The multiperiod designs are implemented with memory lengths N_1 and N_2 , i.e., $n = 2$. All filters L_i are constructed through inversion of model $\hat{T}$, i.e., $L_i(z) = 1/\hat{T}(z)$, and the robustness filters Q_i are designed as 50th order zero-phase low-pass FIR filters with cutoff frequencies 35 Hz.

- 1) A *cascaded multiperiod RC* (4) is designed according to Procedure 1 and Theorem 2, see Fig. 9. Due to the cascaded structure, the equivalent plant T_2^{eq} in loop 2 closely matches the true system T , see Fig. 7. As a result, gains $\alpha_i = 1$ are permitted by Theorem 2.
- 2) A *parallel multiperiod RC*, i.e., without $\hat{T}$ in Fig. 5, is designed according to [16, Th. 1] and [11, Th. 1]. Due to the absence of $\hat{T}$ in Fig. 5, the learning gains have to be set to $\alpha_i = 0.5$ to guarantee stability.
- 3) A *single-period RC* is designed according to Theorem 2 with $n = 1$, i.e., $|(1 - \alpha T(e^{j\omega})L(e^{j\omega}))Q(e^{j\omega})| < 1 \forall \omega$. The memory length of the RC is $N = \text{lcm}(N_1, N_2)$.

C. Experimental Results

The results are presented in Figs. 10 and 11. In Fig. 10, the norm $\|e_p\|_2 = \sqrt{\sum_{k=1}^{N_1} |e_p(k)|^2}$ is depicted as a function of repetitions of r_1 , where $e_p(k) = e(k + pN_1)$. The following observations are made.

Fig. 9. Sequential multiperiod RC using Procedure 1. With $L_i = 1/\hat{T}$, first filter Q_1 is designed such that $|(1 - TL_1)Q_1| < 1, \forall \omega$ (—). Second, Q_2 is designed using T_2^{eq} such that $|(1 - T_2^{\text{eq}} L_2)Q_2| < 1, \forall \omega$ (—).

Fig. 10. Results: Error norm as function of periods N_1 . The proposed cascaded multiperiod RC (○) results in fast convergence, whereas parallel multiperiod RC (×) leads to very slow convergence, and single-period RC (+) updates every $\text{lcm}(N_1, N_2) = 8N_1$ samples, indicated by the dotted line. The error obtained without RC is shown as (◇).

Fig. 11. Results: Errors with proposed cascaded multiperiod RC (—), parallel multiperiod RC (—), and single-period RC (—). After convergence, all designs achieve peak error $2 \mu\text{m}$, approaching the encoder resolution of $0.25 \mu\text{m}$. Due to the employed RC configuration, see Fig. 4, the errors equal disturbance r until the RC signals u become nonzero.

- 1) The cascaded multiperiod RC (○) achieves fast convergence. After N_1 samples, a large part of sinusoidal disturbance r_1 is compensated by $R_1(z)$, see Fig. 11. Since the control action of R_1 is approximately hidden from R_2 in the cascaded structure, R_2 is able to reject disturbance r_2 within two periods N_2 .
- 2) The parallel RC (×) leads to very slow convergence. This is due to the absence of $\hat{T}$ in Fig. 5, resulting in interaction between R_1 and R_2 , see also Example 2.
- 3) Single-period RC (+) yields zero control action until $\text{lcm}(N_1, N_2) = 8N_1$ samples, i.e., the control action is updated every $8N_1$ samples. The least common multiple of general multiperiodic disturbances may be much higher, e.g., consider Fig. 3(b), which can lead to unacceptably long delays before learning.
- 4) For all RC designs, the remaining error after convergence has peak magnitude $2 \mu\text{m}$, i.e., disturbance r in Fig. 8 is accurately compensated. Hence, it is expected that the control designs are suitable for the actual compensation of MPEs, such as depicted in Fig. 3.

VI. CONCLUSION

The results in this article allow systematic RC design with fast convergence and accurate tracking for a class of, possibly nonperiodic, signals that consist of a finite summation of periodic components. The developed approach involves sequential design of multiple repetitive controllers in a cascaded interconnection. The cascaded structure reduces interaction between repetitive controllers, and the sequential approach enables to address robustness to modeling errors in a nonconservative manner. Experiments on a wide-format roll-to-roll printer demonstrate accurate compensation of multiperiodic disturbances in medium transport, with superior convergence speed compared to preexisting approaches.

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