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Citation for published version (APA):

Wu, J., Tijsseling, A. S., Sun, Y. D., & Yin, Z. Y. (2020). In-plane wave propagation analysis of fluid-filled L-Shape pipe with multiple supports by using impedance synthesis method. *International Journal of Pressure Vessels and Piping*, 188, [104234]. <https://doi.org/10.1016/j.ijpvp.2020.104234>

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DOI:
[10.1016/j.ijpvp.2020.104234](https://doi.org/10.1016/j.ijpvp.2020.104234)

Document status and date:
Published: 01/12/2020

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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In-plane wave propagation analysis of fluid-filled L-Shape pipe with multiple supports by using impedance synthesis method

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ARTICLE INFO

Keywords:

Fluid-structure interaction(FSI)
Pipe system
Propagation band gap
Periodic support
Wave transfer
Impedance synthesis method

ABSTRACT

In order to reduce the vibration of planar pipe systems assembled by multiple segments, bends and supports, an impedance synthesis method, which is based on the Timoshenko beam theory, is used to analyze vibration and wave propagation characteristics. Due to junction coupling at unanchored bends, there exists interesting hydraulic and structural wave transmission and reflection. By comparing the natural frequencies and dynamic responses with results available in the literature and with results of the finite-element method, the present calculation method is validated. Based on the relation between the wave number and the propagation band gap, parametric studies of a pipe system with periodic supports are presented. In addition, novel experiments are carried out to validate the present method and to investigate the dynamic response of a pipe system with periodic supports. The results of this paper may provide guidance for vibration reduction of supported pipe systems with elbows.

1. Introduction

Fluid-filled pipe systems are widely used in industrial and civil engineering, such as hydraulic power stations, petroleum installations, nuclear plants and ship building. Thus, the phenomenon of fluid-structure interaction(FSI) in pipe systems has received more and more attention [1–6]. A pipe system consists of many straight reaches connected by elbows and tees. The elbows not only change the fluid flow direction, but – if they are not fully anchored –also the vibration and wave propagation along the pipes.

Tijsseling [7] presented a historical review of sixteen laboratory experiments on single-elbow pipe systems, including eight frequency-domain and eight time-domain measurements. Valentin [8] derived the in-plane vibration governing equation of such a bend system and analyzed the elastic wave reflection and transmission characteristics of it. Tentarelli [9, 10] considered the curvature factor and obtained the out-of-plane vibration governing equation of a bend system. Lesmez [11] analyzed the modes of a pipe system with U-bend by using the transfer matrix method. The literature shows that it is necessary to use a global transfer matrix to couple different types of geometrical discontinuities. Everstine [12] proposed two finite-element (FEM) procedures to predict the dynamic response of a three-dimensional fluid-filled elastic piping system. Xu [13] computed the

natural frequency of a fluid-filled elbow pipe by software ADINA. Li et al. [14] analyzed the influence of pipe-wall thickness on natural frequencies by the transfer matrix method.

Koo and Park [15] investigated the vibration band gap properties of a single straight pipe system with periodic supports by employing a wave approach. The effect of different parameters of periodic support were studied and validated by experiment. Yu et al. [16,17] and Shen et al. [18] applied the phononic crystal theory to the vibration reduction of single pipes. Two greatly different materials were used to build the A-B-A-B type structure of the pipe wall.

From the above, we can draw the conclusion that much attention has been paid to the vibration characteristics of straight pipes. Very little literature has studied the vibration wave propagation characteristics of bend pipe systems with multiple supports. In this paper, the vibration calculation model of a planar pipe-bend-pipe system is established by using the impedance synthesis method(ISM). The results of the present method are validated by comparing the calculated natural frequencies with experimental data in dissertation [19] and the dynamic response with FEM results. Vibration band gap characteristics of a periodically supported pipe-bend-pipe system are studied and an experiment is carried out to study structural wave propagation further.

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<https://doi.org/10.1016/j.ijpvp.2020.104234>

Received 24 July 2020; Received in revised form 3 October 2020; Accepted 14 October 2020

Available online 20 October 2020

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Fig. 1. Pipe-bend-pipe model [19].

Table 1
Natural frequencies of empty pipe-bend-pipe system (Hz).

Pipe models		1	2	3	4
Paper [19]	Continuous	28	62	375	496
	Discrete	28	61	362	480
FEM [19]		28	65	387	506
The present		30	61	386	526

2. Governing equations of vibrating pipe system

2.1. Mathematical model for straight pipe

In a three-dimensional local coordinate system, z is oriented along the axial direction of the pipe, x and y are oriented along orthogonal transverse directions. The in-plane and out-of-plane vibration do not affect each other. The axial vibration comprises the pressure wave in the fluid and the longitudinal stress wave in the pipe wall, which are coupled due to the hoop elasticity of the pipe wall.

For a section of straight pipe filled with liquid and with uniform wall material, the frequency-domain equation of coupled longitudinal wave transmission is:

Fig. 3. Calculation model of pipe-bend-pipe system with two supports.

Fig. 2. First four vibration modes of pipe-bend-pipe system.

Fig. 4. Dynamic response at point P2:(a)flexural displacement, (b)longitudinal displacement.

Fig. 5. Vibration wave transfer and transform efficiency:(a)longitudinal wave, (b) flexural wave.

$$\left\{ \begin{array}{l} \frac{\partial p}{\partial z} + \rho_f \frac{\partial^2 u_f}{\partial t^2} = 0 \\ \frac{\partial p}{\partial t} + K^* \frac{\partial^2 u_f}{\partial t \partial z} - 2\mu K^* \frac{\partial^2 u_z}{\partial t \partial z} = 0 \\ \frac{\partial f_z}{\partial z} - \rho_p A_p \frac{\partial^2 u_z}{\partial t^2} = 0 \\ f_z - \mu A_p \left(\frac{R}{h} + \frac{1}{2} \right) p - EA_p \frac{\partial u_z}{\partial z} = 0 \end{array} \right. \quad (1)$$

where ρ_f is the density of water; μ is Poisson's ratio; R and h are the inner radius and thickness of the pipe, respectively; K_f is the liquid bulk modulus of elasticity, which can be written as: $K^* = K_f / (1 + (1 + \mu)[2R(1 - \mu) + h]K_f / Eh)$. Equation (1) considers the effect of Poisson's ratio of the elastic wall material on the sound velocity and pressure in the pipe. The effect of radial inertia of the pipe wall is ignored; this simplification is justified for vibrations below the ring frequency of the pipe [2]. The transverse vibration of the pipe is modeled by Timoshenko beam theory. The effect of the fluid in the pipe on the transverse motion is only by additional mass. The four transverse vibration equations of the pipeline are [1]:

Fig. 6. Periodically supported one-elbow system.

Fig. 7. The transverse displacement amplitude as function of frequency: (a) at supports S1 (top curve) to S7 (bottom curve), (b) at support S7 together with $\text{Re}(\kappa)$.

$$\begin{cases} \frac{\partial f_y}{\partial z} - (\rho_p A_p + \rho_f A_f) \frac{\partial^2 u_y}{\partial t^2} = 0 \\ f_y - \zeta G A_p \left(\frac{\partial u_y}{\partial z} - \phi_x \right) = 0 \\ \frac{\partial m_x}{\partial z} + f_y - (\rho_p I_p + \rho_f I_f) \frac{\partial^2 \phi_x}{\partial t^2} = 0 \\ m_x - E I_p \frac{\partial \phi_x}{\partial z} = 0 \end{cases} \quad (2)$$

where $\rho_p E$, and G are the density, Young's modulus, and shear modulus, respectively; A_p is the cross-sectional area of the pipe; A_f is the cross-sectional area of the fluid in the pipe; ζ is the Timoshenko shear coefficient; I_p is the area moment of the pipe. The transverse vibration of a

straight pipe is a mix of bending, shearing and inertia. Equations (1) and (2) constitute the in-plane "eight-equation" model of fluid-structure interaction (FSI) in liquid-filled elastic straight pipes coupled at bends and tees.

Equation (1) is transformed into one fourth-order differential equation in terms of F_z :

$$\frac{d^4 F_z}{dz^4} + (\sigma + \tau + \gamma) \frac{d^2 F_z}{dz^2} + \sigma \tau F_z = 0 \quad (3)$$

where

$$\begin{aligned} \sigma &= \omega^2 / c_p^2, \quad \tau = \omega^2 / c_f^2, \quad \gamma = 2\mu^2 b \sigma / d, \\ b &= r / h, \quad d = \rho_p / \rho_f, \quad c_f^2 = K^* / \rho_f, \quad c_p^2 = E / \rho_p. \end{aligned}$$

Fig. 8. Sensitivity to parameters of wave number.

Suppose that the solution of Eq. (3) has the following form:

$$F_z(z) = \bar{A}e^{\lambda z} \quad (4)$$

where $\bar{A}$ is a constant. Substituting Eq. (4) into Eq. (3) gives the characteristic equation:

$$\lambda^4 + (\sigma + \tau + \gamma)\lambda^2 + \sigma\tau = 0 \quad (5)$$

The solution of this equation for λ is two pairs of conjugate imaginary numbers, $\pm k_1 j$ and $\pm k_2 j$, with

$$k_1^2 = \sqrt{(\sigma + \tau + \gamma) - (\sigma + \tau + \gamma)(\sigma + \tau + \gamma)^2 - 4\sigma\tau} / 2$$

$$k_2^2 = \sqrt{(\sigma + \tau + \gamma) + (\sigma + \tau + \gamma)(\sigma + \tau + \gamma)^2 - 4\sigma\tau} / 2 \quad (6)$$

So, the solution $F_z(z)$ of Eq. (3) is the linear combination:

$$F_z(z) = \bar{A}_1 e^{jk_1 z} + \bar{A}_2 e^{-jk_1 z} + \bar{A}_3 e^{jk_2 z} + \bar{A}_4 e^{-jk_2 z} \quad (7)$$

By substituting Eq. (7) into Eq. (1), we get the solutions $U_z(z)$, $P(z)$ and $V_f(z)$, in this order. The complete solution is written in matrix form as

$$\begin{Bmatrix} U_z(z) \\ P(z) \\ V_f(z) \\ F_z(z) \end{Bmatrix} = \begin{bmatrix} B_1 e^{-k_1 z} & -B_1 e^{k_1 z} & B_2 e^{-k_2 z} & -B_2 e^{k_2 z} \\ B_3 e^{-k_1 z} & B_3 e^{k_1 z} & B_4 e^{-k_2 z} & B_4 e^{k_2 z} \\ -B_5 e^{-k_1 z} & B_5 e^{k_1 z} & -B_6 e^{-k_2 z} & B_6 e^{k_2 z} \\ e^{k_1 z} & e^{-k_1 z} & e^{k_2 z} & e^{-k_2 z} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \quad (8)$$

where A_1, A_2, A_3 and A_4 are constants of integration, and

$$B_1 = \frac{k_1}{A_p E \sigma}, \quad B_2 = \frac{k_2}{A_p E \sigma}, \quad B_3 = \frac{\sigma - k_1^2}{A_p \mu b \sigma}, \quad B_4 = \frac{\sigma - k_2^2}{A_p \mu b \sigma}, \quad B_5 = \frac{(\sigma - k_1^2)k_1}{A_p \mu b K^* \tau \sigma}, \quad B_6 = \frac{(\sigma - k_2^2)k_2}{A_p \mu b K^* \tau \sigma}$$

Reduce the above formula to matrix-vector form: $Y(z) = [B(z)]A$. The longitudinal transfer matrix T^{FP} of a pipe of length L , relating the state vectors Y at both ends of the pipeline (at $z = 0$ and $z = L$), is

$$[T^{FP}] = [B(L)][B(0)]^{-1} \quad (9)$$

$$\text{with } [B(0)] = \begin{bmatrix} 0 & -B_1 & 0 & -B_2 \\ B_3 & 0 & B_4 & 0 \\ 0 & B_5 & 0 & B_6 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$[B(L)] = \begin{bmatrix} B_1 e^{-k_1 L} & -B_1 e^{k_1 L} & B_2 e^{-k_2 L} & -B_2 e^{k_2 L} \\ B_3 e^{k_1 L} & B_3 e^{-k_1 L} & B_4 e^{k_2 L} & B_4 e^{-k_2 L} \\ -B_5 e^{-k_1 L} & B_5 e^{k_1 L} & -B_6 e^{-k_2 L} & B_6 e^{k_2 L} \\ e^{k_1 L} & e^{-k_1 L} & e^{k_2 L} & e^{-k_2 L} \end{bmatrix}$$

Equation (2) is transformed into a fourth-order differential equation for Φ_x :

$$\frac{\partial^4 \Phi_x(z)}{\partial z^4} + (\sigma + \tau) \frac{\partial^2 \Phi_x(z)}{\partial z^2} + (\sigma\tau - \gamma) \Phi_x(z) = 0 \quad (10)$$

where

$$\sigma = \frac{(\rho_p A_p + \rho_f A_f)}{\kappa G A_p} \omega^2, \quad \tau = \frac{(\rho_p I_p + \rho_f I_f)}{E I_p} \omega^2, \quad \gamma = \frac{(\rho_p A_p + \rho_f A_f)}{E I_p} \omega^2. \quad (11)$$

Suppose that the solution of Eq. (10) is of the form:

$$\Phi_x(z) = A e^{\lambda z} \quad (12)$$

By substituting Eq. (12) into Eq. (10), the characteristic equation of transverse vibration is obtained:

$$\lambda^4 + (\sigma + \tau)\lambda^2 + (\sigma\tau - \gamma) = 0 \quad (13)$$

Fig. 9. Dimensions and photo of fluid-filled pipe-bend-pipe system.

The four characteristic roots are

$$\lambda_1 = -k_1, \quad \lambda_2 = k_1, \quad \lambda_3 = -jk_2, \quad \lambda_4 = jk_2 \quad (14)$$

where

$$k_1^2 = \left[\gamma + \frac{1}{4}(\sigma - \tau)^2 \right]^{1/2} + \frac{1}{2}(\sigma + \tau) \quad (15)$$

$$k_2^2 = \left[\gamma + \frac{1}{4}(\sigma - \tau)^2 \right]^{1/2} - \frac{1}{2}(\sigma + \tau)$$

Now, the transfer matrix of lateral vibration of a straight fluid-filled pipe becomes

$$\begin{Bmatrix} U_y(z) \\ \Phi_x(z) \\ M_x(z) \\ F_y(z) \end{Bmatrix} = \begin{bmatrix} B_1 e^{\lambda_1 z} & B_2 e^{\lambda_2 z} & B_3 e^{\lambda_3 z} & B_4 e^{\lambda_4 z} \\ e^{\lambda_1 z} & e^{\lambda_2 z} & e^{\lambda_3 z} & e^{\lambda_4 z} \\ C_1 e^{\lambda_1 z} & C_2 e^{\lambda_2 z} & C_3 e^{\lambda_3 z} & C_4 e^{\lambda_4 z} \\ D_1 e^{\lambda_1 z} & D_2 e^{\lambda_2 z} & D_3 e^{\lambda_3 z} & D_4 e^{\lambda_4 z} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \quad (16)$$

$$\text{with. } B_i = \frac{1}{\lambda_i} \left[1 - \frac{1}{\kappa} (\lambda_i^2 + \tau) \right], \quad C_i = EI_p \lambda_i, \quad D_i = -EI_p (\lambda_i + \tau) \quad (i = 1, 2, 3, 4)$$

The transverse vibration transfer matrix of a pipe of length L is

$$[T^{\circ}] = [B(L)][B(0)]^{-1} \quad (17)$$

Fig. 10. Measured and calculated horizontal acceleration:(a) at point A1,(b) at point A2.

$$\text{where } [B(0)] = \begin{bmatrix} B_1 & B_2 & B_3 & B_4 \\ 1 & 1 & 1 & 1 \\ C_1 & C_2 & C_3 & C_4 \\ D_1 & D_2 & D_3 & D_4 \end{bmatrix},$$

$$[B(L)] = \begin{bmatrix} B_1 e^{\lambda_1 L} & B_2 e^{\lambda_2 L} & B_3 e^{\lambda_3 L} & B_4 e^{\lambda_4 L} \\ e^{\lambda_1 L} & e^{\lambda_2 L} & e^{\lambda_3 L} & e^{\lambda_4 L} \\ C_1 e^{\lambda_1 L} & C_2 e^{\lambda_2 L} & C_3 e^{\lambda_3 L} & C_4 e^{\lambda_4 L} \\ D_1 e^{\lambda_1 L} & D_2 e^{\lambda_2 L} & D_3 e^{\lambda_3 L} & D_4 e^{\lambda_4 L} \end{bmatrix}$$

2.2. Assembling of global pipe system

Large pipe systems can be divided into pipes, bends and branches. The transfer matrix of the straight pipe derived in the previous section is expressed in terms of a local coordinate and must be incorporated in the global coordinate system to establish a model for the entire pipe system. The relationship between the global coordinate system and the local coordinate system of a straight pipe is expressed by

$$\mathbf{q} = \begin{bmatrix} q_z \\ q_y \\ q_x \end{bmatrix} = \begin{bmatrix} \cos(z, \bar{z}) & \cos(z, \bar{y}) & \cos(z, \bar{x}) \\ \cos(y, \bar{z}) & \cos(y, \bar{y}) & \cos(y, \bar{x}) \\ \cos(x, \bar{z}) & \cos(x, \bar{y}) & \cos(x, \bar{x}) \end{bmatrix} \begin{bmatrix} \bar{q}_z \\ \bar{q}_y \\ \bar{q}_x \end{bmatrix} \quad (18)$$

where x, y, z are the local coordinates and $\bar{x}, \bar{y}, \bar{z}$ are the global coordinates; for example $\cos(z, \bar{z})$ is the cosine of the angle between the local coordinate system z and the global coordinate system $\bar{z}$. This coordinate transformation is applicable to displacement, rotation angle,

Fig. 11. Experimental research on bend pipe with periodic supports.

Fig. 12. Measured transverse accelerations at five different positions:(a) Transverse excitation at point P1,(b) Transverse excitation at point P1

shear force and bending moment in equation (2) and displacements, pressure and longitudinal force in Eq. (1). All internal forces in the pipe wall will be redistributed at an elbow, and the fluid pressure will be affected as well if the elbow is allowed to move. Thus, the elbow provides an important coupling mechanism. In general, the theoretical method to deal with the elbow is to obtain the transfer relationship of the state vector of the straight pipe at both ends of the elbow (indicated with superscripts R and L) according to the conditions of ① joint force balance, ② joint displacement continuity, ③ pressure continuity and ④ fluid continuity. Therefore, the transfer matrix of an elbow of angle α can be composed as follows:

$$\begin{bmatrix} P \\ \Phi_x \\ U_y \\ U_z \\ V \\ M_x \\ F_y \\ F_z \end{bmatrix}^R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos \alpha & -\sin \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin \alpha & \cos \alpha - 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \sin \alpha & 0 & 0 & 0 & 0 & 0 & \cos \alpha & -\sin \alpha \\ 1 - \cos \alpha & 0 & 0 & 0 & 0 & 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} P \\ \Phi_x \\ U_y \\ U_z \\ V \\ M_x \\ F_y \\ F_z \end{bmatrix}^L \quad (19)$$

The transfer matrixes of straight pipe and miter bend can be transferred to impedance matrixes [15]. If the global transfer matrix of a pipe system (without branches) is written as

$$\begin{bmatrix} F_L \\ \vdots \\ W_L \end{bmatrix} = \begin{bmatrix} T_1 & \cdots & T_2 \\ \vdots & \ddots & \vdots \\ T_3 & \cdots & T_4 \end{bmatrix} \begin{bmatrix} F_R \\ \vdots \\ W_R \end{bmatrix} \quad (20)$$

where F_L and F_R represent the forces, moments and pressures at both ends, and W_L and W_R represent the corresponding displacements and rotations, as obtained from Eq. (9), Eq. (17) and Eq. (19), then it can be rewritten in impedance form as

$$\begin{bmatrix} F_L \\ \vdots \\ F_R \end{bmatrix} = \begin{bmatrix} Z_1 & \cdots & Z_2 \\ \vdots & \ddots & \vdots \\ Z_3 & \cdots & Z_4 \end{bmatrix} \begin{bmatrix} W_L \\ \vdots \\ W_R \end{bmatrix} \quad (21)$$

where $Z_1 = T_1 T_3^{-1}$, $Z_2 = T_2 - T_1 T_3^{-1} T_4$, $Z_3 = T_3^{-1}$, $Z_4 = -T_3^{-1} T_4$ are the impedance matrices.

When two sections of pipe share one rigid and mass less joint, the conditions of displacement continuity and force balance must be satisfied. For example, in Fig. 1, point B is shared by straight pipe A-B and bend B-C, so the force and displacement at the right end of A-B must be equal to those at the left end of the bend. The impedance matrix of the pipe-bend-pipe system relates forces to displacements according to

$$\begin{bmatrix} F_A \\ F_B \\ F_C \\ F_D \end{bmatrix} = \begin{bmatrix} Z_1^{AB} & Z_2^{AB} & 0 & 0 \\ Z_3^{AB} & Z_4^{AB} + Z_1^{BC} & Z_2^{BC} & 0 \\ 0 & Z_3^{BC} & Z_4^{BC} + Z_1^{CD} & Z_2^{CD} \\ 0 & 0 & Z_3^{CD} & Z_4^{CD} \end{bmatrix} \begin{bmatrix} W_A \\ W_B \\ W_C \\ W_D \end{bmatrix} \quad (22)$$

If there is a support at point B, the stiffness of this support should be added to the impedance matrix $Z(2,2)$. Actually, every force F includes force and moment, and every displacement W includes displacement and rotation, so that for the pipe-bend-pipe system, the impedance matrix has size 8×8 .

2.3. Natural modes and frequencies

When the determinant of the impedance matrix in Eq. (22) is set to zero, the natural frequencies of the pipe-bend-pipe system can be obtained. In order to verify the present method, the natural frequencies of an empty pipe with 90° elbow are compared with results from thesis [19]. The layout of the system is shown in Fig. 1. The length of the straight pipes is $L = 0.9$ m the radius of curvature of the bend $R = 0.127$ m, Young's modulus $E = 210$ GPa, mass density $\rho_p = 7800$ kg/m³, and Poisson's ratio $\mu = 0.3$. The outside diameter of the pipe is 0.1 m, and the wall thickness is 0.005 m. The boundary at position P1 is fully constrained and the boundary at P2 is fully free.

Table 1 shows the comparison of the natural frequencies presented in Refs. [19] and those obtained with the present method. There were three kinds of data in Ref. [19], including continuous bend model, discrete bend model, and FEM model. It is found that all the results are in acceptable agreement with differences due to different ways of modeling the elbow.

The first four modes of vibration are shown in Fig. 2.

2.4. Dynamic response

Supports are used in pipe systems to keep them in position. Based on Fig. 1, a system with two spring supports and different pipe lengths of 0.9 m and 0.5 m is established in Fig. 3. The supports are located at the middles of the straight pipes, and the transverse stiffness of each support is $k = 10^6$ N/m. Both ends of the empty system are now free. The outer diameter and thickness of the pipes are changed to 80 mm and 4.5 mm, respectively. The FEM model (used for comparison), solving Eq. (1) and Eq. (2), consists of two-node beam elements with a mesh size of 1 mm. The total number elements is 6785 and the mass of the whole FEM model is 56.48 kg.

When a unit force in Y direction is applied at the position P1, the

flexural and longitudinal dynamic responses at position P2 is calculated by the present method and by the FEM. The two results are in excellent agreement as shown in Fig. 4.

3. Wave propagation characteristics

3.1. Wave transfer properties

Due to junction coupling at bends, Poisson coupling in straight pipes, and the effect of geometric discontinuities, there exists vibration wave type interaction at unrestrained elbows. For example, applying a transverse force at point P1 in Fig. 3, the excitation is of the transverse wave type, but due to the bend it causes both longitudinal and transverse wave types at point P2. We define τ_{in} as the wave transfer efficiency and τ_{out} as the wave transmission efficiency, where P_{total} is the input vibration power flow of excitation, P_{in} is the same vibration wave type at the end of the bend (point P3), and P_{out} is the opposite wave type, as follows:

$$\tau_{in} = \frac{P_{in}}{P_{total}}, \quad \tau_{out} = \frac{P_{out}}{P_{total}} \quad (23)$$

The results of longitudinal and transverse incoming (at point P3) vibration are shown in Fig. 5(a) and (b), respectively. It is found that, both the transfer efficiency and transmission efficiency are frequency dependent, and the longitudinal wave easier transfers to flexural wave than the other way around. The transverse wave type is the main vibration in the pipe-bend-pipe system.

3.2. Vibration band gap properties

The vibration band gap properties due to periodic (i.e., equidistant) supports are investigated. According to the wave propagation approach, the dynamic response of a pipe under harmonic excitation can be written as

$$U = U_0 e^{kx + j\omega t} \quad (24)$$

where ω is the circular frequency, already introduced in Eq. (1) and Eq. (2), U is an arbitrary variable, and the vibration wave number $\kappa = \kappa_{real} + j\kappa_{imag}$ has a real part and an imaginary part. The real part of the wave number accounts for attenuation, provided that it is negative, while the imaginary part stands for propagation. Fig. 6 is a schematic diagram of a periodically supported one-elbow system. The distance between the supports is $\Delta L = 1$ m, the transverse stiffness of each support is $k = 10^6$ N/m, the diameter of the pipes is 80 mm, and the wall thickness is 4.5 mm. The one-elbow system is closed at both ends and filled with liquid of sufficient pressure to avoid cavitation. One pipe is 3 m long with 4 supports and the other pipe is 2 m long with 3 supports; the length and mass of the elbow are ignored. The 7 supports are labeled S1–S7.

The displacement spectra at the seven supports are shown in Fig. 7 (a). There are obvious propagation regions and wave stop regions. By comparing the displacement of support S7 with the real part of the periodic support system's wave number in Fig. 7(b), the corresponding relationship between the wave number and the dynamic response is easily found. Note that $\text{Re}(\kappa)$ is plotted on linear scale, i.e. not on logarithmic scale. When the real part of the wave number is of the order of 1 it presents the wave stop region where the displacement has an obvious stop band. When the real part of the wave number is zero (at the scale plotted in Fig. 7(b)) and the imaginary part of the wave number is neither zero nor π , the displacement has an obvious propagation band.

The sensitivity to parameters is studied in Fig. 8. Fig. 8(a) shows how the distance ΔL between supports changes the wave number. When this length is 1 m, there are two wave stop areas in the range of 1000 Hz, but when it is 2 m, there are four such areas. There are two "bifurcation" frequencies (238 Hz and 890 Hz) that are independent of ΔL . Fig. 8(b) shows the influence of pipe diameter on band gaps. All "bifurcation" frequencies increase with pipe diameter.

The influence of support stiffness is shown in Fig. 8(c). When the stiffness is 10^7 N/m , the first attenuation wave band becomes smaller, whereas for the larger stiffness this band keeps unchanged. When the stiffness increases from 10^9 to 10^{10} N/m , there is basically no change. Apparently, there is a range of stiffness where the band gaps are most sensitive to changes.

Fig. 8(d) shows the influence of liquid on the vibration band gaps. The liquid adds mass to the system thus lowering the frequencies of vibration. This effect is the strongest at higher frequencies.

4. Experimental research

4.1. Dynamic response

In order to validate the model and method presented in this paper, the results of calculations are compared with the results of experiments. The test system consists of two straight pipes and an elbow, connected by flanges, in the X–Y plane as shown in Fig. 9. The horizontal pipe is closed and rigidly clamped (via a plate) to a massive steel block. The vertical pipe has an open and free end. The total length of the system (including elbow) is 1.377 m with the flanges 0.120 m away from the elbow. The diameter and thickness of the steel pipes are 80 mm and 4.5 mm, respectively. The density, Young's modulus and Poisson's ratio of the pipes are 7900 kg/m^3 , 210 GPa and 0.3, respectively. The elbow is assumed to have the same cross-sectional dimensions and material properties as the adjacent pipes. The length of the elbow is 188.4 mm and its mass is 4.5 kg. The mass of each flange including bolts is estimated at 2.0 kg. The fluid is water with density and acoustic bulk modulus set to 1000 kg/m^3 and 1.95 GPa. Transverse external excitation is applied on top of the vertical pipe (point F1) by an electric shaker. Two horizontal-acceleration sensors (a1 and a2) are attached to the flanges at the points A1 and A2.

The measured and computed horizontal acceleration levels ($\text{ref} \cdot 10^{-6} \text{ m/s}^2$) are compared in Fig. 10. The experimental results agree nicely with the predictions over a wide frequency range. There is some difference near the first resonance frequency (25–30 Hz), noting that the double peak is a typical effect of FSI [2]. There is also error because of undesirable out-of-plane vibration in the experiment.

4.2. Vibration band gaps due to periodic support

Fig. 11(a) shows photos of the periodically supported horizontal test rig and Fig. 11(b) is the schematic layout. In the experiment, the transverse excitation is applied by the electromagnetic exciter at the positions P1 and P2, respectively. The length of the long pipe is 3.00 m and there are 3 supports with spacing of 800 mm. The length of the short pipe is 1.50 m and there are 2 supports with spacing of 500 mm. The pipe system is empty. Pipe support is drawing detail in Fig. 11(a).

The transverse accelerations of five supports (A1–A5) are shown in Fig. 12. The measured spectra are rather complex, partly because the stiffness of the supports varies with frequency and partly because the supports introduce longitudinal-flexural coupling.

5. Conclusion

It has been shown that the impedance matrix synthesis method is an appropriate tool to analyze fluid-structure interaction in pipe systems with elbows. The method has been successfully verified by comparing calculated natural frequencies and dynamic responses with those predicted by alternative approaches. The wave propagation and interaction in a pipe-bend-pipe system has been studied.

Generally speaking, the transverse vibration has lower frequency and larger amplitude than longitudinal vibration, so it is mainly the flexural vibration that should be controlled. Periodic supports were used to

reduce the vibration of pipe systems with one bend. The numerical results show that there exist wave stop bands and propagation bands which are affected by the distance between the supports and the relative thickness of the pipe wall. However, the transverse stiffness of the supports has the largest influence. The research shows that periodic support is a suitable method to reduce the level of pipe vibration in a given frequency window. The results of one new experiment validated the impedance matrix synthesis method and proved that the underlying FSI model is correct. The results of a second new experiment need further investigation.

Author statement

Jiang-hai Wu : Conceptualization, Methodology, Software, Writing-Original draft preparation, Arris S. Tijsseling : Writing- Reviewing and Editing, Supervision, Yu-dong Sun : Investigation , Supervision, Zhi-yong Yin : Experimental data analysis, Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was supported by the Foundation of National Key Laboratory on Ship Vibration & Noise under Grant number JCK-Y2019207C102, People's Republic of China.

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