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A physical relationship between consumption and displacement speed for premixed flames with finite thickness

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Abstract

The characterization of premixed flames by a flame speed has been a subject that has occupied much interest in the literature in many systematic studies on combustion phenomena. Consumption and displacement speeds are two such flame speeds that are understood to describe the flame dynamics under the effect of flame curvature, flow non-uniformities, Lewis number and turbulence effects along with heat transfer with flame holders and cold walls. As such, much work has been done in the past where either one of these two speeds has been employed along with a linear sensitivity coefficient (Markstein length) for describing different sensitivities to stretch effects. However, despite recent attempts using the asymptotic theory, the relationship between these two quantities has only been clarified in a limited manner for flames of finite thickness. In this study, we use flame stretch theory that takes into account changes of stretch, curvature, heat transfer and Lewis number effects throughout the pre-heat zone and its integral effect on the flame reaction zone. A sound mathematical and physical basis is provided for understanding the two speeds that is valid for weak as well as strong stretch effects. Understanding from theory is further demonstrated by analysing several example 1D stretched flames along with a 2D bluff body flame near extinction.

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Keywords: Consumption speed; Displacement speed; Integral effect; Extinction; Strong flame stretch

1. Introduction

Analysing premixed flames under the influence of different effects such as flame curvature, flow non-uniformities, Lewis number, turbulence and heat transfer is fundamentally important for advancing our understanding of combustion science

and developing new models and devices for fuel-flexible combustion. Two flame speed definitions are often used to characterize these changes effectively, namely the consumption and the displacement speeds [1–4]. In order to illustrate the difference between these two definitions, an illustration of a premixed flame subjected to stretch is shown in Fig. 1 with streamlines, flame normals, reaction progress variable and color contour plot for fuel consumption rate ω_f marking the flame reaction zone for a steady flame. It can be observed that due

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Fig. 1. An illustration of a stretched flame subjected to a non-uniform flow-field. Colored contours show the fuel consumption rate, solid magenta lines are the streamlines and dashed grey lines mark flame normal paths. Black lines show iso-levels of a reaction progress variable Y and s is the flame path coordinate.

to the divergence of the flow field, a reduction in mass flux reaching the reaction zone from the unburnt to the burnt side of the flame takes place. This mass flux can be described as follows for a premixed flame:

$$m = \rho(\mathbf{V}_f - \mathbf{v}) \cdot \mathbf{n} = \rho S_d, \quad (1)$$

where m is the mass burning rate, ρ is the density, $\mathbf{v}$ is the local gas velocity, $\mathbf{n}$ is the flame normal vector, S_d is the local flame displacement speed and $\mathbf{V}_f$ is the absolute speed of the flame front relative to the laboratory frame [1]. For taking into account the flow acceleration in the flame zone, S_d can be weighted with density change inside the flame front represented by $S_{D,Y}$ in Fig. 1, where subscript Y represents an iso-level of a reaction progress variable inside the flame structure (defined later in Section 2). Density weighted displacement speed can then be compared with a reference value of a corresponding 1D flat flame S_L . The displacement speed is not uniquely defined due to flame thickness effects but is usually defined at a reference position in the flame structure. The choice of this position is crucial for flame dynamics since the displacement speed may change drastically inside the flame front. It has been shown that $S_{D,Y}$ taken at the inner reaction layer of the flame characterizes the flame's response to distortions and perturbations in a sound manner due to the consumption of fuel at this location [5]. All the changes due to preferential diffusion, Lewis number and convection-diffusion misalignment in the flame structure can be captured accurately closer to the reaction zone. In other words, what happens at the reaction zone determines the flame speed. Similar conclusions have been drawn by arguing that taking $S_{D,Y}$ closer to the burnt gas collapses the value of the Markstein length and results in better predictions using linear models [6].

The consumption speed on the other hand, is not directly related to a flame-flow kinematic bal-

ance, but describes the rate at which fuel (for lean mixtures) or oxidizer (for rich mixtures) is consumed. The consumption speed S_c for lean mixtures is usually defined as [1]

$$S_c = -\frac{1}{\rho(Y_{f,u} - Y_{f,b})} \int_{s_u}^{s_b} \omega_f ds, \quad (2)$$

where $Y_{f,u}$, $Y_{f,b}$ are fuel mass fractions at the unburnt and burnt side, respectively and s is a spatial coordinate along a flamelet path shown in Fig. 1. Displacement and consumption speeds are known to be equal for 1D flat adiabatic unstretched flames where distortions to the flame structure not present. However, for stretched flames, $S_{D,Y}$ and S_c are two different quantities and as such have attracted interest from researchers for finding a relationship between these two speeds. Recent studies using asymptotic analysis and the hydrodynamic theory [7] have provided insights into the nature of the variation of $S_{D,Y}$ within the flame structure in relation to S_c . The authors of Ref. [7] concluded that both $S_{D,Y}$ and S_c are asymptotic concepts and they did not consider the integral nature of flow-flame misalignment, preferential diffusion and enthalpy changes inside the pre-heat zone that determines the flame burning rate at the inner layer.

Earlier attempts to understand the effects of flame stretch and curvature on the flame speed resulted in considerable knowledge about the sensitivity of different effects on the flame speed [8–11]. Most of these studies culminated in determining sensitivity coefficients called Markstein lengths for stretch, curvature and even different coefficients for S_c and $S_{D,Y}$. These relations are based on the asymptotic theory that assumes the flame thickness to vanish and takes the flame as a discontinuity between unburnt and burnt side and results are only applicable for weak stretch effects. A lot of consideration needs to be taken for calculating different Markstein lengths depending on the flame configuration used and collapsing of data. Following the ideas of integral analysis of premixed flames by Sung and Law [12], de Goeij and ten Thije Boonkkamp [13] introduced a flame stretch theory that accounts for complete changes in the pre-heat zone due to convection and diffusion. This theory relates the displacement speed to the distortions from ideal flame behaviour. They have shown that the displacement speed at the burnt side of the flame front $S_D = \frac{\rho_b}{\rho_u} S_d$ of stretched flames is related to that of the 1D flat unstretched, adiabatic flame S_L by [14]:

$$S_D = S_L(\psi_b)(1 - Ka_f). \quad (3)$$

Here, $\psi_b = (Z_{1,b}, \dots, Z_{N_e,b} - 1, h_b)$ represents the state at the burnt side of the flame in terms of element mass fractions Z_j for $j = 1, \dots, N_e - 1$ in the mixture, and the enthalpy h . N_e is number of elements in the mixture. Eq. (3) states that the displacement speed at the burnt side S_D of a stretched

flame can be described by the burning velocity of a stretchless flame S_L with enthalpy and composition of the burnt side of the stretched flame, and stretch effects indicated by the Karlovitz integral Ka_f , discussed later in this paper. Direct stretch effects arise from a change in displacement speed as a result of the mismatch between flow and flame normal fluxes. Eq. (3) holds for weak as well as strong stretch rates.

Our motivation in this study is to show that S_c and $S_{D,Y}$ are interlinked but different quantities separated by the integral effect of stretch, both mathematically and physically. Our approach differs from the recent work done in Ref. [7] by the usage of exact equations in the flamelet coordinate system, arriving at an exact expression, providing physical arguments for differences between $S_{D,Y}$ and S_c and testing the theory for weak as well as strong stretch rates. The flame location at the inner layer of the flame is also shown to have a physical basis for characterizing flame dynamics in the case of flames near extinction. To the best of our knowledge, this is the first time that a quantitative relation is provided between S_c and $S_{D,Y}$ apart from linear models. The structure of this paper is as follows; first using flamelet equations, a relationship is derived between S_c and $S_{D,Y}$ that holds for weak as well as strong stretch. The relation is then tested for 1D stretched flames where changes from enthalpy and elemental mass fractions are not present. Then 1D and 2D flames with Lewis number effects are analyzed in the presence of flame curvature and flow strain.

2. Displacement and consumption speed from flame stretch theory

Flamelet equations with distortions from 1D flat ideal behaviour are derived by de Goey and ten Thijs Boonkamp [13,14] along a flame coordinate s shown in Fig. 1. Equations of mass conservation and fuel mass fraction Y_f are given by:

$$\frac{\partial}{\partial s}(\sigma m) = -\sigma \rho K, \quad (4a)$$

$$\frac{\partial}{\partial s}(\sigma m Y_f) - \frac{1}{Le_f} \frac{\partial}{\partial s} \left(\sigma \frac{\lambda}{c_p} \frac{\partial Y_f}{\partial s} \right) - \sigma \omega_f = -\sigma \rho K Y_f, \quad (4b)$$

where λ , c_p and Le_f are the mixture conductivity, specific heat and fuel Lewis number. K is the (local) mass based stretch rate defined as $K = \frac{1}{M} \frac{DM}{Dt}$ with M being the mass contained inside a control volume in the flame of finite thickness and $\frac{D}{Dt}$ being the substantial derivative [13]. In this study, K is calculated using $\rho K = -\nabla \cdot (m\mathbf{m})$. κ is the local curvature of a small flame segment σ by $\kappa = -\nabla \cdot \mathbf{n} = -\frac{1}{\sigma} \frac{\partial \sigma}{\partial s}$. These equations take into account all local distortions from 1D behaviour on the R.H.S. of the

equations. Integrating Eqs. (4a) and (4b) from the unburnt to the burnt side gives

$$(\sigma m)_b - (\sigma m)_u = - \int_{s_u}^{s_b} \sigma \rho K ds, \quad (5)$$

$$\begin{aligned} (\sigma m Y_f)_b - (\sigma m Y_f)_u - \int_{s_u}^{s_b} \frac{1}{Le_f} \frac{\partial}{\partial s} \left(\sigma \frac{\lambda}{c_p} \frac{\partial Y_f}{\partial s} \right) ds \\ - \int_{s_u}^{s_b} \sigma \omega_f ds = - \int_{s_u}^{s_b} \sigma \rho K Y_f ds. \end{aligned} \quad (6)$$

Using the assumption that diffusive fluxes vanish at the unburnt and burnt sides, the diffusive term drops out. This results in

$$\begin{aligned} (\sigma m Y_f)_b - (\sigma m Y_f)_u - \int_{s_u}^{s_b} \sigma \omega_f ds \\ = - \int_{s_u}^{s_b} \sigma \rho K Y_f ds. \end{aligned} \quad (7)$$

Further simplifying Eq. (7) by dividing both sides by $(Y_{f,b} - Y_{f,u})$ and substituting $(\sigma m)_u$ from Eq. (5) along with definition of m from Eq. (1) gives

$$\begin{aligned} \sigma_b S_d \rho_b - \frac{1}{(Y_{f,b} - Y_{f,u})} \int_{s_u}^{s_b} \sigma \omega_f ds \\ = - \frac{1}{(Y_{f,b} - Y_{f,u})} \int_{s_u}^{s_b} \sigma \rho K (Y_f - Y_{f,u}) ds. \end{aligned} \quad (8)$$

Further dividing Eq. (8) with $\rho_u S_L \sigma_b$ and defining a curvature corrected consumption speed S_C can lead to a relationship between S_D and S_C . The curvature corrected consumption speed S_C is defined as:

$$S_C = - \frac{1}{\sigma_b \rho_u (Y_{f,b} - Y_{f,u})} \int_{s_u}^{s_b} \sigma \omega_f ds. \quad (9)$$

Eq. (9) is similar in concept to the one defined by Bell et al. [15] for turbulent flames and for flames with no curvature $S_C = S_c$. Using S_C and the definition of a scaled reaction progress variable $Y = \frac{Y_f - Y_{f,u}}{Y_{f,b} - Y_{f,u}}$ the following relation can be derived for the density averaged displacement speed $S_D = \frac{\rho_b}{\rho_u} S_d$ as

$$\frac{S_D}{S_L} = \frac{S_C}{S_L} - Ka_f. \quad (10)$$

Here the Karlovitz integral Ka_f is the non-dimensional stretch rate integrated over the flamelet structure given by:

$$Ka_f = \frac{1}{\sigma_b S_L \rho_u} \int_{s_u}^{s_b} \sigma Y K \rho ds. \quad (11)$$

Eq. (10) gives the mathematical relationship between the displacement speed S_D at the burnt side and the consumption speed. It states that the two speeds only differ by the integral effect of stretch rate Ka_f . It is to be noted that no major assumption

has been made into the derivation of Eq. (10) regarding stretch and as such this relationship is valid for weak as well as strong stretch rates. The only assumptions that have been made here are (1) Diffusive fluxes vanish at the unburnt and burnt sides and (2) reaction zone is thin compared to the flame thickness. S_C is independent of the position or iso-level of Y but S_D can be evaluated at different iso-levels of the progress variable Y with its corresponding integrating limit for the Karlovitz integral. For this purpose, Eq. (10) is modified for calculation of S_D at different locations inside the flame structure by integrating Eq. (4a) from burnt side to an arbitrary location s_Y in the flame zone and subtracting it from Eq. (8) resulting in

$$\frac{S_{D,Y}}{S_L} = \frac{S_C}{S_L} - Ka_f + Ka_{f,Y}. \quad (12)$$

With the Karlovitz integral $Ka_{f,Y}$ evaluated at a value of Y defined as

$$Ka_{f,Y} = \frac{1}{\sigma_b S_L \rho_u} \int_{s_Y}^{s_b} \sigma K \rho ds, \quad (13)$$

where $s_u < s_Y \leq s_b$ and $S_{D,Y} = \frac{\sigma_Y \rho_Y}{\sigma_b \rho_u} S_d$ are flame coordinate and density weighted displacement speed at a certain level of the progress variable Y . From Eq. (10) or (12) it becomes clear that S_D and S_C only differ from each other by direct stretch effects [5,14]. Here we separate ‘direct’ stretch effects (resulting from misalignment of convective and diffusive fluxes due to flow strain and curvature) given by $Ka_f - Ka_{f,Y}$ and indirect stretch effects. Indirect effects results from Lewis number effects in pre-heat zone changing the local enthalpy and stoichiometry under the impact of stretch along with changes due to heat transfer. S_D and S_C are affected by these indirect effects but their relation to each other is determined by Eqs. (10) and (12) in all cases.

The set of equations Eqs. (10), (12) and (3) describe an integral model which relates S_C and $S_{D,Y}$ at different locations inside the flame structure and can be used for further understanding of these flame characteristic speeds for steady as well as unsteady flames. In this study, however, we only analyze steady flames so that $\mathbf{V}_f = \mathbf{0}$ in Eq. (1). For an ideal 1D flat adiabatic unstretched flame, $S_D = S_C$ as is well-known and this is also evident from our derived result using flame stretch theory. In the next sections we use our derived relations to test the theory for also 1D and 2D flames with distortions from ideal flat flame behaviour.

3. 1D flames

In this section, we numerically test the relationship between displacement and consumption speed for 1D flat stretched (strain only) and 1D tubular (strain + curvature) premixed flames. The 1D flame

solver CHEM1D is used for a mixture of 60 percent CH_4 and 40 percent H_2 by volume with mixture Lewis number $Le = 0.76$ using the DRM19 chemical mechanism [16] used previously in Ref. [17] with the multi-component transport model and the fuel equivalence ratio is set to $\phi = 0.52$. This results in $S_L = 10 \text{ cm s}^{-1}$ and flame temperature $T_b^0 = 1545.8 \text{ K}$. Results for flat stretched flames are presented in Section 3.1 and for curved tubular flame in Section 3.2. The fuel mass fraction is calculated as $Y_f = Y_{\text{CH}_4} + Y_{\text{H}_2}$ and the fuel consumption rate as $\omega_f = \omega_{\text{CH}_4} + \omega_{\text{H}_2}$.

3.1. Flat stretched flames

In this subsection, 1D flat stretched flames [14] are simulated with $Le = 1$ for all species, set artificially in order to focus only on the so called direct stretch effects. Such flames have no curvature $\sigma = 1$ and ρK is taken constant in Eqs. (13) and (11). The consumption speed is not expected to change as there are no changes in enthalpy and elemental mass fractions occurring in the pre-heat zone. The displacement speed $S_{D,Y}$ is expected to change as per Eq. (12) due to the integral effect. $S_{D,Y}$ from numerical simulations is calculated by interpolating Eq. (1) at s_Y . Results are shown in Fig. 2 for S_C and $S_{D,Y}$ at different locations inside the flame structure along with predictions from Eq. (12). First, it is observed that $S_C/S_L = 1$ for all cases with increasing stretch rate and is not affected by the integral effect ($Le = 1$). Second, $S_{D,Y}/S_L$ decreases with K for $Y = 0.4 - 0.9$ in the pre-heat zone and in the inner layer of the flame while it increases for $Y = 0.2$. Third, the prediction from Eq. (12) is in excellent agreement with calculation from the numerical simulation results showing that indeed the flame stretch theory predicts the $S_{D,Y}$ at different locations inside the flame structure in a perfect manner.

Next, Lewis number effects are enabled along with Soret diffusion for capturing the effect of changes in enthalpy and preferential diffusion on S_C for predicting $S_{D,Y}$ from theory. Results are shown in Fig. 3 for the same stretch field and Y iso-levels at which $S_{D,Y}$ is evaluated in Fig. 2. Due to changes in enthalpy and elemental mass fractions resulting from strong preferential diffusion and Lewis number effects, S_C increases with an increase in stretch as observed in Ref. [18] for H_2 -air flames. The displacement speed in the pre-heat zone $S_{D,0.4}$ closely follows the S_C response to stretch, while at further advanced locations inside the flame, the integral of direct stretch increases. This causes $S_{D,0.7}$ and $S_{D,0.9}$ to decrease more as compared to S_C . Again, we observe an excellent agreement between prediction of $S_{D,Y}$ using theory and calculations using numerical simulations. These results give strong evidence on the integral effect of stretch in terms of direct and indirect effects and the physical nature of the flame stretch theory.

Fig. 2. S_C and $S_{D,Y}$ for 1D flat stretched flames with varying stretch rate and $Le = 1$ for all species. $\circ$ symbols represent results from numerical simulations while solid lines show calculations from Eq. (12).

Fig. 3. S_C and $S_{D,Y}$ for 1D flat stretched flames with varying stretch rate. $\circ$ symbols represent results from numerical simulations while solid lines show calculations from Eq. (12).

3.2. Curved stretched flames

In this section we look at tubular premixed flames [19], where flow strain as well flame curvature are present (due to a cylindrical geometry). In more realistic laminar and turbulent flames, both flow strain and flame curvature effects strongly influence the flame. Local extinction may occur causing the flame speed to change drastically at different flame sections. As such, tubular flames are ideal for studying these effects using 1D flames. In Eq. (13) σ is not constant and changes along the flamelet while ρK is kept constant. Stretch rate is varied towards the extinction value resulting in strong stretch effects. The extinction limit for the current settings was found to be 1170 s^{-1} . Results for S_C and $S_{D,Y}$ are shown in Fig. 4 for the same

Y locations as in the previous subsection. Here, we again observe excellent prediction of $S_{D,Y}$ from theory for all the stretch rates in comparison with numerical results although there are minor differences now. The maximum value of Ka_f is 2.25 near the extinction stretch rate and thus represents a strong stretch case. The consumption speed S_C increases with increasing stretch rate as it did for the 1D flat stretched flames. The maximum value of S_C rises to almost 3 times the reference value of S_L near the extinction limit. It is to be noted that near extinction, S_C does not drop but is increasing with stretch rate instead. This results from strong preferential diffusion effects associated for stretched flames with curvature. $S_{D,0.2}$ also increases with stretch rate but at a certain stretch rate, it begins to decrease near the extinction point. Moving further towards the

Fig. 4. S_C and $S_{D,Y}$ for a 1D tubular flames with varying stretch rate towards the extinction limit. $\circ$ symbols represent results from numerical simulations while solid lines show calculations from Eq. (12).

Fig. 5. Scaled heat release rate $\hat{\omega}$, burnt fuel mass fraction $\hat{Y}_{F,b}$ and burnt temperature $\hat{T}$ for 1D tubular flames with varying scaled flame curvature $\hat{\kappa}$.

inner layer of the flame, we can again observe the integral effect of $S_{D,Y}$ due to misalignment in flow-flame fluxes even though the preferential diffusion effects are present. At the inner layer of the flame, i.e., at $Y = 0.9$, the displacement speed $S_{D,0.9}$ increases for a small range of stretch rate initially, and then decreases towards the extinction level resulting in strong bending of the flame speed curve.

In order to understand the observations in the previous paragraph and illustrating why $S_{D,0.9}$ near the inner layer characterizes the flame dynamics, a plot of the scaled heat release rate (with maximum value of the corresponding 1D flat flame) $\hat{\omega}$, scaled residual fuel mass fraction (with residual value at extinction) $\hat{Y}_{F,b}$ and scaled temperature (with adiabatic flame temperature T_b^0) $\hat{T}$ are plotted in Fig. 5 as a function of scaled flame curvature (determined at the inner layer) with flame

thickness of 1D flat flame $\hat{\kappa} = \kappa \delta^0$. It can be observed that with increasing $\hat{\kappa}$, $\hat{\omega}$ increases throughout the stable flame region while $\hat{T}$ first increases and then decreases towards the extinction limit at higher curvature (and stretch rates). This happens as a result of incomplete combustion resulting due to incomplete reaction at the axis of symmetry. This phenomenon is quantified by $\hat{Y}_{F,b}$ which is zero for $\hat{\kappa} < 0.25$ and then increases gradually towards the extinction limit. It can be deduced from these results that S_C follows the behaviour of $\hat{\omega}$ which increases always with increase in stretch (and curvature) induced preferential diffusion effects and does not ‘feel’ the extinction effects. However, $S_{D,Y}$ calculated at the inner layer of the flame, follows the trend of $\hat{T}$ which first rises for weaker stretch rates and then decreases at higher stretch rates due to incomplete combustion. This shows that $S_{D,0.9}$

Fig. 6. Scaled heat release rate $\hat{\omega}$ (colors) along with streamlines (green lines) for near limit flame stabilized behind a bluff body with diameter $D = 8$ mm. Purple lines are 3 flamelet paths.

characterizes flame propagation in a physical way and S_C alone might result in a less informed interpretation of burning and propagation of a flame.

4. A 2D flame example

In this section, we present a 2D lean limit flame stabilized behind a bluff body for the same mixture of 40 percent H_2 and 60 percent CH_4 at $\phi = 0.35$. Figure 6 shows the scaled heat release rate $\hat{\omega}$ (scaled with maximum heat release rate of a 1D flat adiabatic unstretched flame) along with flow streamlines along with a portion of the bluff body colored in grey. This case has been previously presented in Ref. [17] where local stretch rates were compared with extinction stretch rate of corresponding tubular flame and it was found that extinction at the

neck occurs due excessive stretching and incomplete reactions. More details about the settings can be find in Ref. [17] and are omitted here for the sake of brevity. It can be observed that the flame burns much stronger than the respective 1D flat adiabatic flame (which burns extremely weak with $S_L = 1$ cm s^{-1}). Stronger burning results from preferential diffusion effects under the impact of flame curvature and stretch. The flame foot stabilizes inside the recirculation vortex (RZ) and the flame forms a neck downstream of the vortex between $2 < z/D < 3$. In this region incomplete combustion can be observed resulting in $\hat{\omega} \neq 0$ at the axis of symmetry similarly as for tubular flame extinction discussed previously.

Results from numerical simulations and predictions from theory are plotted in Fig. 7 as a function of axial coordinate z along the flame. Consumption speed $S_C < 1$ at the flame leading edge can be observed which further increases towards the region of the flame neck $z/D = 2$ and then drops beyond that. In the neck region, $S_{D,Y}$ starts to decrease from the pre-heat zone towards the inner layer at $Y = 0.9$, due to extinction effects as observed in the case of tubular flames. The theory is able to predict the results from numerical simulations in an excellent manner. Some differences exist due to the limited resolution in the flame zone (25 μm due to computational restrictions). Another interesting flame section is the flame leading edge submerged inside recirculation vortex. Negative convective fluxes exist inside the flame zone resulting in negative displacement speeds. This can be observed in Fig. 7 between $0 < z/D < 0.5$. It is also found that theory predicts this behaviour as well showing that the physics is modelled well by Eq. (12).

Local predictions of $S_{D,Y}$ from theory along three interesting flamelet paths marked in Fig. 6 with purple lines are shown in Fig. 8

Fig. 7. S_C and $S_{D,Y}$ for the bluff body flame near lean limit. $\circ$ symbols represent results from numerical simulations while solid lines show calculation from Eq. (12).

Fig. 8. $S_{D,Y}$ as a function of progress variable Y along three flamelets as shown in Fig. 6. $\circ$ symbols represent results from numerical simulations while solid lines show calculation from Eq. (12).

along with results from numerical simulations. The flamelet at the flame neck and RZ exit location show good agreement between theory and numerical simulations. $S_{D,Y}$ decreases while approaching the flame inner layer. The flamelet inside the RZ where negative $S_{D,Y}$ values are observed shows surprisingly good agreement between theory and numerical simulations between $0.1 < Y < 0.8$ and results in an accurate prediction of $S_{D,Y}$ becoming negative at the inner layer. Thus it can be concluded that the theory is able to predict the complete $S_{D,Y}$ field within the flame zone with knowledge of S_C and $Ka_f - Ka_{f,Y}$ integrals confirming the integral nature of both the flame speeds.

5. Conclusions

A clear cut mathematical and physical relationship has been provided between consumption and displacement speed throughout the flame zone using flame stretch theory. It is shown that both consumption and displacement speeds change due to the integral nature of the flame structure with finite thickness resulting from direct and indirect stretch effects. It is also elaborated why the displacement speed at the inner layer of the flame is representative of flame dynamics by analysing tubular flames near extinction. Theory is also shown to predict displacement speeds for a fairly complex bluff body stabilized 2D flame near the lean limit. Negative displacement speeds occurring inside the recirculation vortex are also predicted. It can be concluded from this study that the finite flame thickness effects are crucial for flame propagation and burning and these effects can be predicted by flame stretch theory.

Declaration of Competing Interest

None.

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