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**Influence of geometry on acoustic end-corrections of slits
in Microslit Absorbers**

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Abstract

The acoustic behavior of individual slits within microslit absorbers (MSAs) is investigated to explore the influence of porosity, edge geometry, slit position and plate thickness. MSAs are plates with arrays of slit-shaped perforations, with the height of the order of the acoustic viscous boundary layer thickness, for optimized viscous dissipation. The slit behaves as confined in a rectangular channel and the flow within the slit is assumed as locally incompressible. The viscous dissipation and the inertia are quantified by the resistive and the inertial end-corrections. These are estimated by using analytical results and numerical solutions of the Linearized Navier-Stokes equations. Expressions for the end-corrections are provided as functions of the ratio of the slit height to the viscous boundary layer thickness (Shear number) and the porosity. The inertial end-correction is sensitive to the far-field behavior of the flow and strongly depends on the porosity, unlike for circular perforations. The resistive end-correction is dominated by the edge geometry of the perforation. The relative position of the slit with respect to the wall of the channel is important for distances of the order of the slit height. The plate thickness does not have a significant effect on the end-corrections.

I. INTRODUCTION

Microslit absorbers and plates (MSAs, MSPs) have been proposed by Maa¹⁶ as sound absorbers at low frequencies, providing light-weight and compact solutions to substitute conventional materials, such as absorptive foams and porous structures. In simple MSAs the plate, consisting of an array of slit-like perforations, is mounted with a shallow or sub-partitioned backing cavity. Alternative designs of MSAs have been recently reported in the literature^{12, 19, 21, 25}. MSAs have several advantages with respect to micro-perforated plates (MPPs) with circular perforations. Using slits one can easily obtain a relatively large porosity, resulting in a higher Helmholtz resonance frequency, when needed. For equal porosity, a single slit replaces a large number of circular perforations. Furthermore, a slit can be used to delimit flexible structures whose vibration can contribute to the sound absorption^{21, 25}. Compared to the literature for circular perforations, fewer publications investigate the acoustic properties of slit-like perforations. Maa¹⁶ states that no theory is available to predict inertial end-correction. He assumes the same viscous dissipation as for circular perforations. In the work of Maa¹⁶, radiation to free space is assumed. The inertial end-correction model fails. This failure is solved when taking the confinement into account. Ingard⁹ obtained a solution for high Shear numbers, assuming a uniform flow in the slit and matching the resulting rigid piston oscillation model to a modal expansion of the flow in the confinement channel. Correct expressions for the inertial end-corrections, without typos, are presented by Jaouen *et al.*¹⁰.

19 The same model is used by Vigran²⁴. Another model, based on an incompressible potential
 20 flow with a thin boundary layer, is proposed by Morse and Ingard¹⁷ for an abrupt transition
 21 with sharp square edges. This model yields both inertial and resistive end-corrections in the
 22 limit of high Shear numbers. For a slit in an infinitely thin plate, the same approach does
 23 predict an inertial end-correction. However, the singularity of the potential flow at the edge
 24 of an infinitely thin plate results in a divergence of the resistive end-correction. Morse and
 25 Ingard¹⁷ propose as an ad-hoc solution to introduce a finite plate thickness. The divergence
 26 of the resistive end-correction due to the singularity at the edge suggests that the viscous
 27 dissipation is a local effect, strongly influenced by the edge geometry. One concludes that
 28 there is a lack of a complete model to describe the acoustic behavior of slits. For instance,
 29 both Ruiz *et al.*²² and Cobo *et al.*² state that all the models proposed in literature do not
 30 fit experimental absorption curves of MSPs. Therefore, the goal of the present work is to
 31 complement the theoretical knowledge concerning the acoustical properties of microslits. In
 32 particular two effects appear to be ignored in the literature for slits: the influence of the
 33 position of the slit and the influence of the edge shape. For a circular perforation, Temiz
 34 *et al.*²³ observed that chamfering the edges reduces the effective plate thickness t_{eff} by a
 35 length of the order of the total length of the chamfers. A non-symmetric position of the slit
 36 can be found when the periodicity of the array is not perfect or in case of sub-partitioned
 37 cavity. In the present work, a combination of analytical models and numerical solution of the

incompressible Linearized-Navier Stokes equations is proposed. In Sec. II, analytical models are developed using a 2D configuration. In Sec. III, the numerical models and solutions of the incompressible Linearized Navier-Stokes equations (LNSE) are proposed using Comsol³ v5.5. In Sec. IV, analytical and numerical results are compared. Findings are summarized in Sec. V.

II. THEORY

A. Definition of the problem

Microslit plates (MSPs) are plates with arrays of slit-like perforations with height b in the sub-millimeter range and width $w \gg b$. The plate thickness t_p is of the order of magnitude of the slit height. The acoustic properties of MSPs are defined by the porosity $\Phi = b/a$, with a the distance between neighboring slits. The hydrodynamical interaction between the perforations in the array can be described by considering a single slit of height b , confined within a channel of height a of rectangular cross-section aw given by the distance between neighboring slits and the lateral width of the slit. At the open front side of the MSA, the confinement channel represents the hydrodynamic interaction between neighboring slits.

56 The confinement channel on the cavity side is resulting from physical walls in case of a
sub-partitioned cavity or is due to hydro-dynamical interactions.

Figure 1: On the left, frontal view of the microslit plate with slit width w . In the middle, lateral view of the microslit plate of thickness t_p with back cavity. On the right, a single slit of height b with confinement channel of height a due to hydrodynamic interactions.

57

58 As illustrated in Fig. 1, for a periodic array of slits, the confinement channel is placed
59 symmetrically with respect to the slit. Assuming a long slit ($w \gg b$) implies that one can
60 consider a two dimensional (2D) acoustical flow through the slit. As the slit forms the neck
61 of a Helmholtz resonator with a portion of the back cavity as volume, the flow within the
62 slit can be considered as locally incompressible up to the first resonance frequency of the
63 resonator, $\omega_H = c\sqrt{\Phi/(d_c t_{eff})}$, with c the speed of sound, d_c the back cavity depth and
64 t_{eff} the effective neck length. In the audio range, the square of the Helmholtz number is
65 small, *i.e.* $He^2 = \left(\frac{\omega b}{c}\right)^2 < 10^{-1}$. Thermal effects in the slit are neglected. The discussion is

limited to the normal incidence of acoustic waves. One can describe the transition between
 the slit and the confinement channel by assuming over the plate thickness t_p an ideal 2D
 parallel flow for a long slit of height b extended over a so-called end-correction length. The
 extrapolation of the linear acoustic pressure in both the slit and the confinement channel is
 used to define the end-corrections. There is a resistive end-correction δ_{res} and an inertial
 end-correction δ_{in} corresponding to the pressure components $Re[\hat{p}]$ and $Im[\hat{p}]$, respectively in
 phase with the volume flow oscillation and in phase with the acceleration of the volume flow
 oscillation $\hat{U}e^{i\omega t}$. The inertial end-correction determines the Helmholtz resonance frequency,
 as shown in Zielinski *et al.*²⁵. Assuming the same geometry on the front and backside of the
 slit, the effective neck length is given by $t_{eff} = t_p + 2\delta_{in}$. The resistive end-correction, takes
 into account the viscous dissipation and influences the quality of the Helmholtz resonance.
 To optimize viscous dissipation, the slit height is chosen to be of the order of magnitude
 of a few times the acoustical viscous boundary layer thickness $\delta_v = \sqrt{2\nu/\omega}$, where ν is the
 kinematic viscosity of air and $\omega = 2\pi f$, with f the frequency. Hence, for typical applications
 the Shear number $Sh_b = b/\delta_v$ is of order unity. The range $0.05 < Sh_b < 20$ is considered.
 As the plate thickness and end-corrections in MSPs are both typically of the order of the
 slit height, it is important to obtain an accurate prediction of end-corrections to design the
 absorbers.

B. Parallel flow

An analytical model for the flow in a long slit of height b is used as a reference to define the end-corrections and to define low and high Shear number limits. It is also used to assess the accuracy of the numerical solution of the incompressible Linearized Navier-Stokes equations. At low Helmholtz numbers ($He^2 \ll 1$), in absence of main flow, the acoustic field is described by the equation of continuity

$$\nabla \cdot \vec{v} = 0 \quad (1)$$

and the linearized equation of motion

$$\rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla p + \eta \nabla^2 \vec{v}, \quad (2)$$

where $\vec{v}$ is the velocity, p is the pressure fluctuation, ρ_0 density of the air assumed to be uniform and constant and η is the dynamic viscosity. In a long thin slit of height b , width $w \gg b$ and length $t_p \gg b$, for $0 < x < t_p$ and $-b/2 < y < b/2$, the flow can be approximated by a parallel flow $\vec{v} = (u(y, t), 0, 0)$. The continuity equation implies, in a parallel flow, that $\frac{\partial u}{\partial x} = 0$. Hence, the derivative with respect to x of the x -component of the equation of motion implies that $\frac{\partial^2 p}{\partial x^2} = 0$, *i.e.* the pressure is given by a linear function of the x -coordinate. The y - and z -components of the equation of motion reduce to $\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0$. This results in a uniform pressure in a cross-section of the slit. Consequently one has that

101 $\frac{\partial p}{\partial x} = \Delta p / t_p$ with $\Delta p = p(t_p, t) - p(0, t)$. For a harmonic oscillation $\Delta p = \Delta \hat{p} e^{i\omega t}$ the flow
 102 profile satisfying the no-slip boundary condition $(u, v) = (0, 0)$ on the slit walls $y = \pm b/2$ is

$$u(x, t) = \hat{u} e^{i\omega t} = -\frac{i}{\rho_0 \omega} \frac{\Delta \hat{p}}{t_p} \left[1 - \frac{\cosh \frac{(1+i)}{\delta_v} y}{\cosh \frac{(1+i)}{2\delta_v} b} \right] e^{i\omega t}. \quad (3)$$

The cross-sectional averaged amplitude of the velocity $\langle \hat{u} \rangle$ is

$$\langle \hat{u} \rangle = \frac{1}{b} \int_{-b/2}^{b/2} \hat{u} dy = -\frac{i}{\rho_0 \omega} \frac{\Delta \hat{p}}{t_p} \cdot \left[1 - \frac{2}{(1+i)Sh} \tanh \frac{(1+i)}{2} Sh \right] e^{i\omega t}, \quad (4)$$

103 with $Sh_b = b/\delta_v$, the Shear number. The slit impedance Z_b is defined as in Morse and
 104 Ingard¹⁷:

$$Z_b = \frac{\Delta \hat{p}}{wb \langle \hat{u} \rangle}. \quad (5)$$

105 At low Shear numbers $Sh_b < 1$, one can use the approximation

$$Z_b \approx \frac{12\eta t_p}{(wb)b^2} + i\frac{6}{5}\rho_0\omega t_p. \quad (6)$$

106 One recognizes in the real part of Z_b the resistance corresponding to a parabolic flow (quasi-
 107 steady flow approximation). At high Shear numbers $Sh_b \gg 1$, one has

$$Z_b \approx \frac{\rho_0\omega t_p}{(wb)Sh_b} + i\frac{\rho_0\omega t_p}{(wb)} \left(1 + \frac{1}{Sh_b} \right). \quad (7)$$

108 The first part of the imaginary part corresponds to the inertia of a uniform flow, which is
 109 a factor 6/10 lower than that of a parabolic flow (see Eq. 6). The time-averaged viscous

110 dissipation $\bar{P}_W$ in the slit is given by Morse and Ingard¹⁷:

$$\bar{P}_W = \frac{1}{2} Re[Z_b] | \langle \hat{u} \rangle |^2 (wb)^2. \quad (8)$$

111 For $Sh_b \gg 1$ using Eq. 7 one has

$$\bar{P}_W = \frac{1}{2} \rho_0 \omega \delta_v | \langle \hat{u} \rangle |^2 2wt_p. \quad (9)$$

112 This thin boundary layer approximation is used in Sec. II E for channels with non-uniform
 113 height. In this limit, the flow in the boundary layer is quasi-parallel along the wall. There-
 114 fore, one can use the dissipation per unit surface found in Eq. 9 when replacing $| \langle \hat{u} \rangle |^2$ by
 115 the amplitude of the tangential velocity amplitude $|\hat{u}_{tan}|$ prevailing just outside the viscous
 116 boundary layer. Integration over the surface yields the total dissipation. This tangential
 117 velocity corresponds to that of a frictionless potential flow. Alternative derivations of this
 118 thin boundary layer equation are provided in literature^{1, 14, 17, 20}. As explained by Morse and
 119 Ingard¹⁷, this approximation fails for infinitely thin orifice plates. While Morse and Ingard¹⁷
 120 suggest that the approximation is valid for sharp square edges, the numerical integration of
 121 the Linearized Navier-Stokes equations will allow to verify this assumption.

122

123 C. Impedance and end-corrections

124

In this subsection a formal definition of impedance and end-corrections is provided. Consider the transition from a slit of height b to a channel of height $a > b$. In an ideal (reference) configuration the transition from the slit to the channel is abrupt: the flow can be described as a piece-wise parallel flow. In the actual flow, the transition from the slit to the channel is smooth. However, far from the transition one can observe the linear change in pressure, corresponding to a parallel flow in a slit of height b and in a confinement channel of slit of height a . This far field can be extrapolated at each side of the transition towards the plate surface at $x = 0$. The complex difference in pressure $\Delta\hat{p}_t$ obtained across the transition by this extrapolation divided by volume flux amplitude $\hat{U} = \langle \hat{u} \rangle bw$ is defined as the transition impedance Z_t . The inertial end-correction δ_{in} and the resistive end-correction δ_{res} are defined by:

$$\delta_{in} = \frac{Im[Z_t]}{Im\left[\frac{dZ_b}{dt_p}\right]}, \quad (10)$$

$$\delta_{res} = \frac{Re[Z_t]}{Re\left[\frac{dZ_b}{dt_p}\right]}. \quad (11)$$

The resistive end-correction δ_{res} is in principle different from the inertial end-correction δ_{in} . In Fig. 2 several values of the inertial and resistive end-corrections obtained from the literature for perforations with sharp edges are shown as function of the inverse of the plate porosity $1/\Phi = a/b$. Results for circular perforations are also displayed. A critical discussion of these data is provided by Kergomand and Garcia¹¹.

Figure 2: Comparison of end-corrections for sharp-edged slit ($L_{ref} = b$) and circular perforation ($L_{ref} = d_p$) from the literature. Inertial end-corrections δ_{in} for slits (MSPs): $--$ high Sh_b number limit of Morse and Ingard¹⁷, $-. -$ Modal expansion of Ingard⁹, ... Thin boundary layer of Morse and Ingard¹⁷. $-$ Slit Resistive end-correction δ_{res} from Morse and Ingard¹⁷. $---$ Inertial end-correction for circular perforations from Fok⁴. Resistive end-correction for circular perforations: * from Temiz *et al.*²³ and $-$ from Naderyan *et al.*¹⁸.

The reference length L_{ref} , in Fig. 2 refers either to the height b for slits or to the perforation diameter d_p . It can be noted that the various results at high Sh_b numbers for the inertial end-corrections for sharp-edged slits are in close agreement. Note that the inertial end-correction for a slit in an infinitely thin plate is quite close to that for a square edge transition from a slit to a higher channel. This indicates that at high Shear numbers the plate thickness has a minor effect on the inertial end-correction. For a slit at low porosity $\Phi = b/a$ it appears that the inertial end-correction is proportional to $\ln(1/\Phi)$. Hence, it becomes infinitely large for vanishing porosity. This divergence can be removed when taking

into account the influence of the flow compressibility¹³. For a circular perforation the finite limit value¹⁵ $\delta_{in,\Phi \rightarrow 0} = 0.4108d_p$ is found for vanishing small porosities. For the resistive end-correction of slits, Morse and Ingard¹⁷ obtained an analytical result for $Sh_b \gg 1$. The resistive end-correction increases with decreasing porosity but reaching an asymptote for $\Phi \rightarrow 0$. For circular perforation, resistive and inertial end-corrections are of the same order of magnitude. It should be noted that for relevant porosities all end-corrections are of the order of L_{ref} (either b or d_p).

D. Modal expansion

In this subsection, the frictionless modal expansion proposed by Ingard⁹ is used to derive an expression for the inertial end-correction. Given an arbitrary velocity profile at the end of the slit, it is possible to derive the inertial end-correction by matching to acoustic modes in the confinement channel. Outgoing plane wave and evanescent transversal modes are considered. Kergomand and Garcia¹¹ discuss the convergence of the modal expansion. When using the rigid piston approximation in the slit the number of modes used in the channel should be of the order of the inverse of the porosity²⁰ $1/\Phi = a/b$. An expression of the inertial end-correction for low the Sh_b number is obtained by assuming a parabolic flow (see Sec. II B) at the end of the slit. This is used as input for the frictionless modal

expansion of the acoustic pressure in the channel. One finds:

$$\delta_{in} = \frac{5}{6} \sum_{n=1}^{\infty} \frac{3}{2n\pi} \left(\frac{a}{n\pi b} \right)^3 \left\{ 4 \cos^2(n\pi) \cdot \left[\cos\left(\frac{n\pi b}{a}\right) - \frac{a}{bn\pi} \sin\left(\frac{n\pi b}{a}\right) \right] \sin\left(\frac{n\pi b}{a}\right) \right\}. \quad (12)$$

A number of modes of the order of $N_m = 3(a/b)$ is sufficient to reach a reasonable accuracy.

For the asymmetric case, the influence of the position of the slit with respect to the wall is investigated. In Fig. 3, the transition from an asymmetric slit to a channel is displayed.

Figure 3: Geometry of the asymmetric slit of height $b = b_1 + b_2$ emerging in a channel of height $a = a_1 + a_2$.

The slit height is $b = b_1 + b_2$, the channel height is $a = a_1 + a_2$. The geometry is chosen such that the $(a/b) = (a_1/b_1) = (a_2/b_2)$. In the limit case of a slit sharing the flat wall with the channel, one has $a_2 = 0$ or $a_1 = 0$. The vertical positions of the slit edges (at $x = 0$) are

$y_1 = a_1(1 - b/a)$ and $y_2 = a - a_2(1 - b/a)$. Assuming at the end of the slit a uniform acoustic velocity amplitude and expanding the amplitude of the pressure in frictionless modes in the channel one finds:

$$\delta_{in} = \sum_{n=1}^{\infty} \frac{2}{n\pi} \left\{ \frac{\left[\sin\left(\frac{n\pi y}{a}\right) \right]_{y_1}^{y_2}}{\frac{n\pi b}{a}} \right\}. \quad (13)$$

In the symmetric case $a_1 = a_2$ one finds the result of Ingard⁹, where $n = 2m$. The influence of the position of the slit on the inertial end-correction is discussed in Sec. IV C.

E. Thin boundary layer approximation

For the high Sh_b range, the viscosity effects are concentrated in a thin boundary layer at the wall and do not impact the main, potential flow. The incompressible potential flow theory combined with the thin boundary layer approximation proposed by Morse and Ingard¹⁷ can be used. A generalization of this model is presented by Berggren *et al.*¹. In the present work, this approximation is used to investigate the end-corrections for smooth edges and asymmetric slit sharing the flat wall with the confinement channel. It is also used to explore the effect of viscous friction along the confinement channel walls. The smooth edge geometry is obtained using the conformal transformation introduced by Henrici⁶ (Appendix A for details). An analytical solution is proposed for a smooth transition, providing a generalization of the results of Morse and Ingard¹⁷ for sharp edges.

Figure 4: Henrici's transformation of half the channel with smooth transition from the slit to the channel in the physical plane $z = x + iy$ to the ζ -plane. Coordinates of the points: $A(-\infty; (a+b)/2)$, $B(-d; (a-b)/2)$, $C(0, 0)$, $D(\infty; 0)$.

In Fig. 4, a 2D slit of height b in $x < 0$ and a 2D channel of height $a > b$ in $x > 0$ are shown. The end of the uniform slit (point B in Fig. 4) is at $(x, y) = (-d, a - b)$, with d being the transition length. The uniform confinement channel begins at $x = 0$. The duct can be associated to a region in the complex z -plane by $z = x + iy$, with $i^2 = -1$ and coordinates (x, y) . Using conformal mapping, the flow region in the duct can be mapped into the upper half-plane in the complex ζ -plane. The mapping of the contraction is a modified Schwarz-Christoffel transformation introduced by Henrici^{6,(7)}. The differential form of Henrici's

transformation is

$$\frac{dz}{d\zeta} = \zeta^{-1} [\alpha(\zeta - 1)^{1/2} + \beta(\zeta - G^2)^{1/2}] \cdot (\zeta - G^2)^{-1/2}, \quad (14)$$

where α , β and G are parameters of the transformation depending on the slit and channel heights and on the transition length d . The parameters α and β are functions of the parameter G obtained numerically as the solution of a non-linear equation. Details are discussed in Appendix A. The equation for the sharp square edge transition is recovered for $d = 0$. Using the thin boundary layer approximation one can find the real and imaginary part of the impedance of the transition Z_t and the inertial and resistive end-corrections. Formulas are provided in Appendix B.

A similar approach can be followed for a fully asymmetric slit, presented in Fig. 3. When $a_2 = 0$, the slit and the confinement channel share the flat wall. One has to add the dissipation of the flat wall, shared by the slit and the channel. This will be done by modifying the limits of integration when calculating the total dissipation along the walls (Appendix B). When the confinement channel walls are representing the influence of hydrodynamic interaction, the flow at the channel walls is frictionless. This can also be taken into account by simply modifying the integration limit when integrating to calculate the dissipated power. Details are in Appendix B.

III. NUMERICAL MODEL

A. Uniform channel

Consider a uniform channel of height b and length t_p , with $t_p \gg b$. The x -axis goes from $x = 0$ to $x = t_p$. The y -axis extends between $y = \pm b/2$. As stated in Sec. II A, the low He number approximation is made. The incompressible Linear Navier-Stokes equations for a 2D domain in a dimensionless form in the frequency domain are hereby presented:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \quad (15)$$

$$iu^* = -\frac{\partial p^*}{\partial x^*} + \frac{1}{2Sh_b^2} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right), \quad (16)$$

$$iv^* = -\frac{\partial p^*}{\partial y^*} + \frac{1}{2Sh_b^2} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right), \quad (17)$$

with $x^* = x/b$ and $y^* = y/b$. The dimensionless velocity (u^*, v^*) is $(u/b\omega, v/b\omega)$ and the dimensionless pressure is $p^* = p/(\rho_0(b\omega)^2)$. These equations are solved using *Comsol Multiphysics*. At the inlet ($x^* = 0$) and at the outlet ($x^* = t_p/b$) of the domain the uniform pressure values are imposed: respectively, $p_{inlet}^* = 1$ and $p_{outlet}^* = 0$. At the walls ($y^* = y/b = \pm 1/2$) no-slip boundary conditions, $(u^*, v^*) = (0, 0)$ prevail. An unstructured mesh of quadratic

216 triangular elements is used, with the finest mesh at the walls. The density of elements at
 217 the walls depends on the Sh_b number: the element sizes at the wall are $0.2/Sh_b$ or less, in
 218 order to accurately capture the viscous boundary layer. For a height ratio $a/b = 10$ and
 219 $Sh_b = 20$, in the proximity of the edges the maximum element size is $M_{el}/b = 2 \times 10^{-2}$ and
 220 the minimum is $m_{el}/b = 7 \times 10^{-4}$. The original mesh chosen for the standard calculations
 221 has a total of 13324 total elements, of which 804 are edge elements (at the walls). Several
 222 checks are performed to gain insight into the accuracy of the numerical simulations. Firstly,
 223 the computational domain length t_p is increased to exclude an influence of the boundary
 224 conditions. It appears that the quantity $U_L = Ut_p/t_{ref}$, with U being the flux in a cross-
 225 section of the channel and $t_{ref} = 6 * b$, is constant within a relative deviation of 10^{-5} for
 226 $0.5 < t_p/t_{ref} < 2$. Secondly, a mesh convergence study is performed and shows convergence
 227 to computer accuracy (10^{-13}). For this study, three additional meshes are used: one coarser
 228 and two finer meshes respectively with half, two and four times the basic number of elements
 229 at the wall. To compare the results, the cross-sectional average velocity $\langle \hat{u}^* \rangle = \int_0^{b^*} \hat{u}^* dy^*$
 230 is used. Comsol³ performs the integration element-wise using numeric quadrature of the 4th
 231 order. The cross-sectional average velocity in the channel obtained with the numerical simu-
 232 lations shows a deviation of 10^{-4} from the analytical solution for the parallel flow, discussed
 233 in Sec. II B.

234

B. Change in cross-section with sharp square edges

The set of equations 15-17 is used to study the channel in Fig. 5 presenting at $x^* = 0$ a sharp square edged transition from a uniform height b^* to a uniform height $a^* > b^*$. The channel extends from $x^* = -t_b^*$ to $x^* = t_a^*$, with $t_a^* = 6a/b$ and $t_b^* = t_a^*/2$.

Figure 5: Geometry of a channel with the sudden transition from the slit of height b to the channel of height a .

The symmetry of the problem allows limiting the numerical domain to half the channel. For the inlet segment AF and outlet segment DE constant pressures are imposed, $p_{AF}^* = 1$ and $p_{DE}^* = 0$. At the segments AB and BC the no-slip boundary conditions are applied. At the segment EF (symmetry axis) a slip boundary conditions are implemented: $\partial u^*/\partial y^* = 0$ and $v^* = 0$. The effect of the boundary condition at the lower wall is investigated. When considering a confinement channel due to hydrodynamic interaction, a slip boundary condition is used on the segment CD. Far from the transition located at $x^* = 0$ the acoustic pressure is uniform in the cross-section and the amplitude of the pressure depends linearly

on the position along the duct (parallel flow behavior). Assuming that for $-2a < x^* < -1a$:
 $\hat{p}^*(x) = \hat{A}x^* + \hat{B}$ and for $3a < x^* < 5a$ one has: $\hat{p}^*(x) = \hat{C}x^* + \hat{D}$. The complex constants
can be determined by a linear fit of the pressure data obtained by numerical simulations
for these regions far from the discontinuity. The fit gives a coefficient of determination⁵
 $1 - R^2 = 10^{-6}$. The impedance Z_t of the transition is determined by $Z_t = \frac{\hat{B} - \hat{D}}{\hat{U}^*}$ with $\hat{U}^*$
being the flux calculated in a generic section of the slit far from the discontinuity, defined as
 $\hat{U}^* = w < \hat{u}^* > b$.

For a porosity $\Phi = b/a = 1/10$ at $Sh_b = 20$, numerical simulations show that the effect
of the boundary condition at the lower wall of the channel is negligible. This is in line with
confirming that the dissipation is mainly concentrated inside the slit and around the edges.

In the assumption of locally incompressible flow, the volume flux along the duct axis
is constant. This is verified numerically with a maximum relative deviation of 10^{-4} . The
coefficients $\hat{A}$ and $\hat{C}$ of the linear fittings of $\hat{p}^*$ can be compared to the theoretical values
of the $\Delta\hat{p}^*/t^*$ for the parallel flow in a long channel, respectively of height b and a . The
discrepancy is in the order of 10^{-4} . Apparently, the accuracy in the calculation of the volume
flux is the limiting factor for the global accuracy of the numerical model.

IV. RESULTS

A. Symmetrical slit with sharp square edges

4.1.1 Inertial end-correction at low and high Sh_b number

In this subsection, the inertial end-correction for a sharp square edged transition derived from the numerical simulations are compared with the analytical solutions proposed in Sec. II. For low Shear numbers, the inertial end-correction can be calculated using the oscillating parabolic flow approximation. For high Shear numbers the modal expansion of Ingard⁸ and the thin boundary layer approximation of Morse and Ingard¹⁷ are used. The inertial end-correction calculated with the parabolic flow approximation is about twice (5/3) the value for uniform flow. In Fig. 6 the comparison between the numerical, the modal expansions and thin boundary layer approximation, are shown as function of the inverse of the porosity Φ . The numerical results are obtained for a $Sh_b = 0.05$ and for $Sh_b = 20$. It appears that the parabolic flow approximation captures well the behavior of the inertial end-correction for $Sh_b = 0.05$, whereas the rigid piston and thin boundary layer models are in good agreement with the result for $Sh_b = 20$.

4.1.2 End-corrections at Low Sh_b number

For $Sh_b < 0.6$, the dimensionless inertial end-correction δ_{in}/b and the resistive end-correction δ_{res}/b are functions of the porosity and, to a much lesser degree, of the Sh_b number. The

Figure 6: Comparison of inertial end-correction δ_{in} as functions of $1/\Phi$: — Parabolic flow approximation, — Uniform flow approximation⁹. — High the Sh_b number limit¹⁷. Stars refer to the results of numerical calculations for * $Sh_b = 0.05$ and * $Sh_b = 20$.

dependency of the end-corrections on Sh_b is therefore neglected. The dependency of δ_{res}/b on both porosity and the Sh_b number is negligible. The following fits are proposed:

$$\frac{\delta_{in,fit}}{b} = -2.17 + 2.18 * \left(\frac{1}{\Phi} \right)^{0.13}, \quad (18)$$

$$\frac{\delta_{res,fit}}{b} = 0.425, \quad (19)$$

for $Sh_b < 0.6$ and $3 < 1/\Phi < 30$. The coefficient of determination⁵ $1 - R^2$ for δ_{in}/b is 0.997.

The choice of the fit for δ_{res}/b results is a maximum underestimation of the actual value of 2.5% (see Fig. 7). The negligible effect of the porosity of δ_{res}/b indicates again that the dissipation is a local effect at the sharp edges.

Figure 7: Behavior of a) δ_{in}/b and b) δ_{res}/b as function of the Sh_b number for several porosities for $Sh_b < 0.6$: $\bullet$ $1/\Phi = 3$, $\bullet$ $1/\Phi = 5$, $\bullet$ $1/\Phi = 10$, $\bullet$ $1/\Phi = 15$, $\bullet$ $1/\Phi = 20$, $\bullet$ $1/\Phi = 30$. Solid lines refer to the fit, asterisk refer to numerical data.

4.1.3 End-corrections at high Sh_b number

In the region $0.6 < Sh_b < 20$ the deviations of δ_{in} and δ_{res} from the high Sh_b limits $\delta_{in,ref}$ and $\delta_{res,ref}$, predicted by Morse and Ingard¹⁷, have been obtained (see Appendix B). Proposed fits of the numerical results are:

$$\frac{\delta_{in}}{\delta_{in,ref}} - 1 = \frac{C_1}{C_2 + Sh_b}, \quad (20)$$

$$\frac{\delta_{res}}{\delta_{res,ref}} - 1 = \frac{C_3}{Sh_b * (C_4 + Sh_b)}, \quad (21)$$

$$\text{with } C_i = D_{i,1} + D_{i,2} \cdot (\Phi). \quad (22)$$

From Eq. 22 appears that the coefficients C_i are linear functions of the porosity. Table 1 provides the values of the coefficients $D_{i,j}$.

In Fig. 8a and 8b the fits are compared with numerical simulation data. Both the inertial

Table 1: Values of the coefficients for the fitting in the range $0.6 < Sh_b < 20$.

	C_1	C_2	C_3	C_4
First coefficient $D_{i,1}$	0.52	1.27	5.19	1.69
Second coefficient $D_{i,2}$	9.34	7.45	28.74	3.97

and resistive dimensionless end-corrections show a dependency on the porosity that becomes less important for decreasing porosity. This behavior is more noticeable for $\delta_{res}/\delta_{res,ref}$. In

Figure 8: Result of the fits in the range $0.6 < Sh_b < 20$ for a) $\delta_{in}/\delta_{in,ref}$ and b) $\delta_{res}/\delta_{res,ref}$ as function of the Sh_b number for several $1/\Phi = a/b$: — $1/\Phi = 3$, — $1/\Phi = 5$, — $1/\Phi = 10$, — $1/\Phi = 15$, — $1/\Phi = 20$, — $1/\Phi = 30$. Solid lines refer to the fit, asterisk refer to numerical data..

296

297 Fig. 9a the linear approximations of the coefficients C_1 and C_2 for the inertial end correction
 298 are compared with the actual values. In Fig. 9b the results for C_3 and C_4 for the resistance

are presented. The average adjusted coefficients of determination⁵ $1 - R^2$ are 0.987 for the

Figure 9: Comparison of the coefficients C_i of the fitting of the inertial and resistive end-corrections as function of the porosity Φ in the range $0.6 < Sh_b < 20$. In a) — C_1 and — C_2 . In b) — C_3 and — C_4 . In both, asterisks refer to the numerical data and solid lines are referred to the results of the fitting process.

299

300 inertial term and 0.998 for the resistive term. In Fig. 8a and Fig. 8b it appears that both
301 $\delta_{in}/\delta_{in,ref}$ and $\delta_{res}/\delta_{res,ref}$ are converging to the unit value for high Sh_b numbers. For higher
302 Sh_b numbers, some additional calculations are carried out for a typical porosity $1/\Phi = 10$.
303 At $Sh_b = 100$, one has $\delta_{in}/\delta_{in,ref} = 1.0116$ and $\delta_{res}/\delta_{res,ref} = 0.9465$. At $Sh_b = 200$,
304 $\delta_{in}/\delta_{in,ref} = 1.0061$ and $\delta_{res}/\delta_{res,ref} = 0.996$. This confirms the validity of the thin boundary
305 layer approximation for sharp square edges. The effect of the boundary condition (slip or
306 no-slip) on the channel walls is investigated for a typical porosity $1/\Phi = 10$ with $Sh_b = 2$
307 and $Sh_b = 20$. Numerical simulations for $1/\Phi = 10$ show that the introduction of a no-
308 slip boundary condition at the walls of the confinement channel has a negligible effect on

the results. For $Sh_b = 2$, one finds a ratio $\delta_{res,no-slip}/\delta_{res,slip} = 1.032$. For $Sh_b = 20$, $\delta_{res,no-slip}/\delta_{res,slip} = 1.044$. Using the thin boundary layer theory, for high Sh_b one finds $\delta_{res,no-slip}/\delta_{res,slip} = 1.041$, in agreement with numerical results. One expects that this ratio increases for increasing porosity. For an extremely large porosity $1/\Phi = 3$, one finds a ratio $\delta_{res,no-slip}/\delta_{res,slip} = 1.185$. One can conclude that the inertial end-correction is determined by the porosity. The porosity has a modest effect on the resistive end-correction. The negligible effect of the no-slip boundary condition in the channel suggests that, for $\Phi = 0.1$, dissipation is mainly concentrated around the edges.

B. Symmetric slit with smooth edges

Consider a slit of height b with rounded edges of radius r placed symmetrically with respect to a channel of height a . The results of incompressible LNS simulations are compared to the high the Sh_b number approximation for a smooth transition discussed in Sec. II E with the shape determined by the transformation of Henrici⁶. Experimental and numerical data for a circular perforation obtained for a 45° chamfered circular perforation by Temiz *et al.*²³ are also displayed. The reference length L_{ref} is introduced. For the round edges $L_{ref} = r$, the radius of curvature of the rounded edge. For Henrici's transformation, $L_{ref} = d$ is the transition length. For chamfered, $L_{ref} = c_{ch}$ is the chamfer length. It appears that the transition length d well approximates the radius r of an equivalent rounded edge for

$d/b < 1$. In Fig. 10, $\delta_{in,round}/\delta_{in,sharp}$ and $\delta_{res,round}/\delta_{res,sharp}$ are displayed as function of L_{ref}/b . Numerical results for a slit with a height ratio of $a/b = 10$ are shown for: 1) rounded edges at several Sh_b numbers ($Sh_b = 0.2, 2, 20$), 2) chamfered edges for $c_{ch} = 0.5b$ at $Sh_b = 20$, 3) Henrici's geometry for $Sh_b = 20, 200$. The analytical potential solution for smooth edges is validated by the LNSE numerical simulations for Henrici's geometry at high Sh_b number. In Fig. 10a, for the inertial end-correction the analytical solution well approximates the numerical results for a rounded edge. The 2D planar result for the 45° chamfered edge is relatively far from the analytical and numerical results for a smooth transition. In Fig. 10b, for the resistive end-correction the analytical solution provides a good approximation for high Sh_b numbers, both for a round edge and for a chamfered edge. It is interesting to note that the resistive end-correction becomes negative for L_{ref}/b of order unity. For comparison, the influence of chamfer on circular perforations²³ is also displayed in Fig. 10. In Fig. 11a and 11b, $\delta_{in,round}/\delta_{in,sharp}$ and $\delta_{res,round}/\delta_{res,sharp}$, calculated using the high Sh_b number limit, are shown for height ratios a/b relevant in MSPs. The inertial end-correction shows a dependency on a/b that increases with the increase of the ratio L_{ref}/b . The resistive end-correction shows again a much more modest dependency on the porosity, as already observed for sharp edges. Rounded edges and chamfered edges have a similar effect on the end-correction, for a small radius of curvature of the edge compared to the slit height b . The effect of rounded edges on a slit is similar to the effect of a chamfered

edge for circular perforations. In conclusion, it appears that a fair estimation of the edge
 geometry is necessary to obtain meaningful estimations of the end-correction for both slits
 and circular perforations.

Figure 10: Comparison of high the Sh_b number approximation for a smooth tran-
 sition with numerical results for several ratios L_{ref}/b for a) $\delta_{in,round}/\delta_{in,sharp}$ and
 b) $\delta_{res,round}/\delta_{res,sharp}$ for several Sh_b numbers: — Slit with smooth transition, * Slit with
 rounded edges for $Sh_b = 0.2$, + Slit with rounded edges for $Sh_b = 2$, $\times$ Slit with rounded
 edges for $Sh_b = 20$, ∇ Slit with Henrici's transition for $Sh_b = 20$, $\triangle$ Henrici's transition
 for $Sh_b = 200$, $\circ$ Chamfered edge for $Sh_b = 20$, -- Fit of numerical results and $\square$ Experi-
 mental result for circular perforations of Temiz *et al.*²³.

C. Asymmetric slit

Figure 11: behavior of a) $\delta_{in,round}/\delta_{in,sharp}$ and b) $\delta_{res,round}/\delta_{res,sharp}$ as function of the edge rounding L_{ref}/b for several $1/\Phi = a/b$: — $1/\Phi = 3$, — $1/\Phi = 5$, — $1/\Phi = 10$, — $1/\Phi = 15$, — $1/\Phi = 20$, — $1/\Phi = 30$.

353 In this section results for asymmetric slits are discussed. The position of the slit is
354 determined using the distances a_1 and a_2 defined in Fig. 3. The inertial end-correction is
355 calculated for high Sh_b numbers, using the modal expansion method of Ingard⁹ presented
356 in Sec. II D. In the extreme case that $a_2 = 0$, the high Sh_b number limit of Morse and
357 Ingard¹⁷ can be used to calculate both the inertial and the resistive end-corrections. In
358 Fig. 12 the ratio of the inertial end-corrections for the asymmetric case ($\delta_{in,asym}$) and the
359 symmetric case ($\delta_{in,sym}$) is displayed as function of a_2/a_1 for several height ratios a/b , with
360 $a = a_1 + a_2$. The value of a_2/a_1 where the effect of the position has a significant effect
361 decreases with the increase of a/b . It appears that for a slit positioned at the wall ($a_2 = 0$),
362 the inertial end-correction is double the value for the symmetric case, for all the ratios a/b .

Numerical calculations are performed for a slit positioned at the wall and compared to the analytical results. In Fig. 13, for $Sh_b = 20$ the end-corrections for an asymmetric slit ($a_2 = 0$) as function of the height ratio a/b are plotted using the corresponding values (same Sh_b number) for a symmetric slit as a reference. The inertial end-correction is double the value for the symmetric slit. The resistive end correction instead increases for decreasing porosity $\Phi = b/a$. It approaches the asymptotic value of $\delta_{res,asym} = 2.3\delta_{res,sym}$. This asymptotic value reduces for increasing Sh_b approaching the analytical value for very high Sh_b number. Considering the common wall as a mirror, the flow corresponds to that in a slit with double width $2b$ placed symmetrically with respect to a channel of width $2a$. This explains the behavior of the inertial end-correction. For the resistive end-correction, the dissipation occurs in a small region around the edge. This region can be addressed as the dissipation region. When keeping the flow velocity in the slit constant, but doubling the slit and channel width, one increases the dissipation region length by a factor 2. The resulting resistive end-correction doubles. In practice, the end-correction increase is larger than the factor 2 because one has to account for an additional dissipation along the flat wall common to the slit and the channel. While the dissipation is localized around the edge, it is not limited to the edge.

In conclusion, it appears that the influence of the position on the end-corrections cannot be neglected for positions of the slit with respect to the channel of the order of magnitude

Figure 12: Analytical results for the inertial end-correction obtained by means of modal expansion for an asymmetric slit for several $1/\Phi$: — $1/\Phi = 3$, — $1/\Phi = 5$, — $1/\Phi = 10$, — $1/\Phi = 15$, — $1/\Phi = 20$, — $1/\Phi = 30$.

Figure 13: Comparison of the numerical simulations (*) for $Sh_b = 20$ and potential flow theory (—) results as function of a/b for a) $\delta_{in,asym}/\delta_{in,sym}$ and b) $\delta_{res,asym}/\delta_{res,sym}$.

of the slit height.

D. Finite thickness plate with sharp square edges

In Fig. 14a and 14b the deviations of the inertial and resistive end-correction for a finite thickness are compared with the transition between a very long slit and the confinement

channel discussed in the previous sections. In the range of interest, the deviation lays within 10% and 5% accuracy, respectively for the inertial and the resistive end correction. $\delta_{res,plate}$ shows a negligible dependency on t_p/b with respect to the dependency on the Sh_b number. From this study, one can state that for practical purposes the influence of the thickness of the plate on the end-corrections can be neglected.

Figure 14: Deviation of a) $(\delta_{in,plate} - \delta_{in,inf})/\delta_{in,inf}$ and b) $(\delta_{res,plate} - \delta_{res,inf})/\delta_{res,inf}$ from the semi-infinite slit as function of the ratio t/b for: — $Sh_b = 0.2$, — $Sh_b = 2$ and — $Sh_b = 20$.

V. CONCLUSIONS

This study combines analytical and numerical solutions of the incompressible Linearized Navier-Stokes equations to investigate the acoustic behavior of microslit absorbers (MSAs and MSPs). A single slit of height b is studied as confined in a rectangular channel of height a determined by the porosity of the plate $\Phi = b/a$. The flow within the slit is assumed

to be locally incompressible (low He numbers). Thermal effects are neglected. Focus is
 given to the frequency range of application for MSAs and resonant metamaterials. For sharp
 edges, numerical simulations demonstrate that for low Sh_b numbers a parabolic flow ap-
 proximation provides a good approximation of the inertial end-correction, whereas the thin
 boundary layer approximation predicts both the end-corrections at high Sh_b numbers. The
 inertial end-correction is strongly dependent on the porosity, showing a very different behav-
 ior compared to that of circular perforations. A striking result is that the ratio of the resistive
 end-correction and the slit height is weakly dependent on the porosity, independently of Sh_b
 number. This indicated that the viscous friction is a local phenomenon occurring near the
 edges. This is confirmed by the negligible influence of the no-slip boundary condition at the
 walls of the confinement channel, for $\Phi < 0.1$. Final prove is gathered in Sec. IV B where
 the effect of the edge geometry is discussed. The analytical model for a smooth transition
 provides a reasonable prediction for rounded and chamfered edges at high Sh_b numbers. In
 Sec. IV C it is shown that the position of the slit becomes an important effect for distance
 from the wall in the order of the slit height b . For the limit case of a slit sharing the wall
 with the channel, the inertial and resistive end-corrections are both approximately twice the
 values for a symmetrical slit. In Sec. IV D it is shown that, for $t_p > 0.1b$, the effect of the
 plate thickness on the end-corrections is negligible.

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APPENDIX A: TRANSFORMATION OF HENRICI

In this appendix the high Sh_b limit theory is described for the smooth and asymmetric transitions presented in Sec. II E. The duct can be associated to a region in the complex z -plane by $z = x + iy$, with $i^2 = -1$ where (x, y) are the coordinates in the physical plane. Using conformal mapping, the flow region in the duct in the complex z -plane is mapped into the upper half-plane in the complex ζ -plane. The transformation of Henrici⁶ is used to derive the results for a smooth transition from a slit of height b to a channel of height a . The geometry is presented in Fig. 4. The integral form of the transformation proposed by Henrici⁶ is:

$$z = \alpha \left[\ln \frac{1 + \tau}{1 - \tau} - \frac{1}{G} \ln \frac{G + \tau}{G - \tau} \right] + \beta \ln \left[\frac{\zeta}{G^2} \right]^\tau, \quad (23)$$

431 where τ is:

$$\tau = \sqrt{\frac{\zeta - G^2}{\zeta - 1}}. \quad (24)$$

432 The point far downstream of the transition $A((-\infty, a))$ can be mapped into point $A'(\zeta = 0)$,
 433 the start of the transition $B(-d, a - b/2)$ corresponds to $B'(\zeta = 1)$, the end of the transition
 434 $C(0, 0)$ corresponds to $C'(\zeta = G^2)$. The coefficients are related to parameter G by:

$$\alpha = \frac{a - b}{\pi} \left[\frac{G}{G - 1} \right] \quad (25)$$

435 and

$$\beta = \frac{G b - a}{\pi(G - 1)}. \quad (26)$$

436 The parameter G is found by solving the non-linear equation:

$$G = \frac{a}{b} \left[1 + \frac{\pi d}{2 a \ln G} (G - 1) \right]. \quad (27)$$

437 This equation can be solved by successive substitution for $\pi d (2b) < 2$ using $G_0 = a/b$
 438 as initial guess. For $\pi d (2b) > 2$ the successive substitutions should be applied to:

$$G = \exp \left[\frac{\pi d}{2b} \left(\frac{G - 1}{G - \frac{a}{b}} \right) \right], \quad (28)$$

439 using $G_0 = \exp \left(\frac{\pi d}{2b} \right)$. For sharp edges $d = 0$ and $G = a/b$. For an asymmetric slit positioned
 440 at the wall it is necessary to identify the point ζ_0 on the ζ -axes that corresponds to $z_0 = ia$ on
 441 the flat wall in the z -plane. ζ_0 is found by solving numerically the equation $z_0 = z(\zeta_0) = ia$.
 442 This can be done for any value of the transition length d . Here, only the sharp edge ($d = 0$)
 443 is considered for the fully asymmetric slit position ($a_2 = 0$).

445 APPENDIX B: THIN BOUNDARY LAYER APPROXIMATION

446 The thin boundary layer method of Morse and Ingard¹⁷ for the transition from a slit of
 447 height b to a channel a with sharp edges is extended to a smooth transition and to a fully
 448 asymmetric slit positioned at the wall ($a_2 = 0$). The inertial and resistive end-corrections
 449 can be found comparing the actual configuration with an ideal configuration. The ideal
 450 reference flow, used to define the end-corrections, has for $x > 0$ a uniform velocity u_a in
 451 the channel of height a and for $x < 0$ a uniform velocity $u_b = (a/b)u_a$. The potential flow
 452 far upstream is obtained by placing a volume source at the origin $\zeta = 0$ (far downstream
 453 the transition) with potential $\varphi = (au_a/\pi) \ln(\zeta)$. The local flow velocity is the vector field
 454 $\vec{v}_{wall} = (u, v) = \nabla\varphi$. The linearized form of the frictionless equation of motion is

$$-\nabla p = \rho_0 \frac{\partial \vec{v}}{\partial t}. \quad (29)$$

455 To compare the actual and the reference configurations two points in the transformed
 456 ζ -plane are necessary. Choosing $\zeta_1 \rightarrow \infty$ and $\zeta_2 = 0$ corresponds to z_1 and z_2 respectively
 457 far upstream and far downstream the transition. Integrating Eq. 29 between $z_1 = (x_1; y_1)$
 458 and $z_2 = (x_2; y_2)$ with $x_1 > 0$ and $x_2 < 0$, one has for a harmonic oscillating acoustic field:

$$i\rho\omega(\varphi_2 - \varphi_1) = p_1 - p_2, \quad (30)$$

459 with $\varphi = \int \vec{v} \cdot d\vec{z}$. If the flow velocity would remain uniform $(u_a, 0)$ for $x > 0$ and jump to

460 $(u_b, 0)$ with $u_b = u_a a/b$ for $x < 0$, we would have:

$$(\varphi_2 - \varphi_1)_{ideal} = u_a \frac{a}{b} x_2 - u_a x_1. \quad (31)$$

461 The inertial end-correction δ_{in} is given by:

$$\delta_{in} = \frac{u_a a}{b \Delta \varphi}, \quad (32)$$

462 Where $\Delta \varphi$ is defined as the difference $(\varphi_2 - \varphi_1)_{actual} - (\varphi_2 - \varphi_1)_{ideal}$. Choosing real values

463 ζ_1 and ζ_2 , so that the values of z_1 and z_2 are far from the origin of the axis, one has:

$$\delta_{in} = \frac{b}{\pi} \ln \left(\frac{\zeta_2}{\zeta_1} \right) - Re(z_2) + \frac{b}{a} Re(z_1) \quad (33)$$

For $\zeta_1 \rightarrow \infty$ and $\zeta_2 \rightarrow 0$ in Eq. 23 and Eq. 24 we can expand at the first order τ and obtain an expression for z_1 and z_2 to substitute in Eq. 33. One arrives at Eq. 34. For $d = 0$ this expression recovers the result of Morse and Ingard¹⁷.

$$\delta_{in}/b = \frac{1}{\pi} \left\{ \frac{(a-b)^2}{2ab} \ln \left(\frac{G+1}{G-1} \right) + \frac{a-b}{b(G-1)} \cdot \left[\frac{Gb+a}{2a} \ln \left(\frac{(1+G)^2}{4G^2} \right) \right] + \ln G \right\}. \quad (34)$$

464 The additional dissipation due to the transition can be derived integrating along the wall
 465 the dissipation per unit surface presented in Sec. II B for the actual and the reference
 466 configuration. It should be noted that the actual configuration and the ideal configuration
 467 should be combined to obtain converging integrals. In terms of potential the velocity at the
 468 wall is:

$$|\widehat{u}_{tan}|^2 = \left| \frac{d\varphi}{dz} \right|^2 = \left| \frac{d\varphi}{d\zeta} \right|^2 \left| \frac{d\zeta}{dz} \right|^2. \quad (35)$$

The power dissipated at the junction compared to an ideal configuration is:

$$2\vec{P} = \frac{1}{\delta_v} \eta w \left[\int_{\zeta_2}^{\zeta_0} \left(\left| \frac{d\varphi}{d\zeta} \right|^2 \frac{d\zeta}{dz} - u_a^2 * \left(\frac{a}{b} \right)^2 \operatorname{Re} \left[\frac{dz}{d\zeta} \right] \right) d\zeta \right. \\ \left. + \int_{\zeta_0}^{\zeta_1} \left(\left| \frac{d\varphi}{d\zeta} \right|^2 \frac{d\zeta}{dz} - u_a^2 \operatorname{Re} \left[\frac{dz}{d\zeta} \right] \right) d\zeta \right], \quad (36)$$

where for a symmetric slit $\zeta_1 \rightarrow \infty$, $\zeta_2 \rightarrow 0$ and ζ_0 corresponds to $z = 0$ and it is found from $\zeta_0 = G^2$. The second integral in Eq. 36 contains the effect of the dissipation in the channel. For a slip boundary condition prevailing in a confinement channel resulting from hydrodynamic interactions, one can take $\zeta_1 \rightarrow \zeta_0$ and calculate the dissipation using only the first integral. The resistive end-correction of the discontinuity can be defined as Morse and Ingard¹⁷:

$$\delta_{res} = \frac{2\vec{P}}{(2awu_a)^2}. \quad (37)$$

Solving the integrals for the symmetric smooth-edged configuration with friction at the channel walls leads to

$$\delta_{res}/b = \frac{(G-1)}{2G(a-b)} \left\{ (G-1) \left[\frac{(G+1)}{\pi(G-1)} \left(\frac{(a-b)^2}{b^2(G+1)} - 1 \right) \cdot \ln \left(\frac{G+1}{G-1} \right) + 1 \right] - \frac{2DG^2}{\pi} \ln(G) \right\}, \quad (38)$$

with $D = \frac{Gb-a}{G(a-b)}$. Again for $d = 0$ one recovers the result of Morse and Ingard¹⁷, with $G = a/b$ and $D = 0$.

For an asymmetric slit, the dissipation of the transition, in this case, is the sum of the dissipation of the edge and the dissipation at the opposite flat wall. The same integrals can

be solved by changing the integration to $\zeta_1 \rightarrow \infty$, $\zeta_2 \rightarrow -\infty$ and ζ_0 can be found solving numerically the equation $z_0 = z(\zeta_0) = ia$.

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