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Vehicle Energy Management with Ecodriving: A Sequential Quadratic Programming Approach with Dual Decomposition

Z. Khalik, G.P. Padilla, T.C.J. Romijn, M.C.F. Donkers

Abstract—In this paper, we propose to solve the ecodriving problem using a Sequential Quadratic Programming (SQP) algorithm. We formulate the ecodriving problem as a discrete-time (possibly nonconvex) nonlinear optimal control problem, and form convex SQP subproblems by using a linearized objective function with Thikhonov regularization. We will further show that the SQP algorithm can be embedded in a distributed optimization approach, allowing it to be used for Complete Vehicle Energy Management (CVEM), incorporating optimal control of the vehicle's auxiliary systems, in combination with ecodriving. We consider two case studies for the ecodriving problem. The first case study concerns the optimal control of a full electric vehicle, which has one control input and two states and is solved with the SQP algorithm. The second case study lays a foundation for CVEM with ecodriving, where we solve an energy management problem with ecodriving for a series-hybrid electric vehicle, using the aforementioned SQP algorithm and dual decomposition.

I. INTRODUCTION

Hybrid electric vehicles offer the potential to reduce the fuel consumption of a vehicle by adding an electric motor with a high-voltage battery to the powertrain. This allows braking energy to be recuperated and allows the combustion engine to work at a more efficient operating point. In energy management, supervisory control is used to determine the optimal power flow between the electric machine and the combustion engine. A recent trend is to extend the energy management problem to incorporate more auxiliary devices, such as a refrigerated semitrailer or a climate control system [1], engine thermal management [2], battery ageing [3], etc., which is referred to as Complete Vehicle Energy Management (CVEM). The rationale behind this is that the auxiliaries also consume a considerable amount of energy. However, in all these vehicle energy management problems, the vehicle speed is assumed to be given, while most of the power generated by the powertrain is used for propelling the vehicle. Therefore, optimizing the speed of a vehicle over a certain trajectory, thereby allowing for an optimal conversion of potential energy from the road profile into kinetic energy of the vehicle, can lead to a considerable energy consumption reduction. In this paper, we refer to this latter problem as the ecodriving problem.

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Zuan Khalik, Paul Padilla, Constantijn Romijn and Tijs Donkers are with the Department of Electrical Engineering of Eindhoven University of Technology, Netherlands. E-mail: {z.khalik, g.p.padilla.cazar, t.c.j.romijn, m.c.f.donkers}@tue.nl

In vehicle energy management, global optimal solutions are typically achieved using Dynamic Programming (DP) [4], see e.g., [5]. However, DP has the inherent disadvantage that the computational burden increases with the number of states. Optimization methods based on the Pontryagin's Maximum Principle (PMP), see, e.g., [6] can handle computational complexity of multi-state energy management problems. In PMP, the problem is reduced to solving a two-point boundary value problem, which can be difficult to solve in the presence of state constraints. Static optimization methods guarantee a global optimal solution for convex approximations of the energy management problems, e.g., [7]. To increase scalability of the static optimization problem to allow for a large number of auxiliary systems, distributed optimization approaches have been proposed in [1] for complete vehicle energy management. A disadvantage of the convex optimization approach to CVEM of [1] is that it requires the powertrain components to be described by (convex) quadratic models. This renders the distributed optimization approach not usable for the ecodriving problem, as the longitudinal vehicle dynamics are nonlinear. However, a particular extension of the distributed optimization approach of [1] towards a nonlinear battery ageing model has been made in [3], showing its ability to handle nonlinear models as well.

Some of the above mentioned optimization methods have been applied to the ecodriving problem, e.g. PMP in [8], DP in [9], and static optimization in [10]. The approaches based on PMP and DP suffer from their inherent difficulties, i.e., incorporating state constraints in PMP and more dynamics in DP, while the approach based on (convex) static optimization [10] applies an Euler discretization to a continuous-time optimal control problem. As the latter problem is formulated as a second-order cone program, some of the components can only be modeled using (piecewise) linear functions.

In this paper, we will propose to solve the nonlinear (and possibly nonconvex) ecodriving problem using a static optimization approach. To handle the nonlinearity and non-convexity of the resulting optimal control problem, we employ a Sequential Quadratic programming (SQP) algorithm [11], in which the Hessians are approximated to accelerate convergence. Furthermore, we will show an additional advantage of the presented SQP algorithm, as it can be embedded in the distributed optimization approach of [1]. This allows it to be used for CVEM, incorporating optimal control of the vehicle auxiliary systems, in combination with ecodriving. We will benchmark our solution strategy on a full electric vehicle optimal control problem presented in [8], which solves the problem using PMP, and solve the problem

using SQP. We show the advantage of SQP over the PMP approach used in [8], as state constraints can be easily added in our proposed approach. Furthermore, we demonstrate the combined ecodriving problem with the powersplit control of a series hybrid powertrain, as was also considered in [10].

II. ECODRIVING PROBLEM FORMULATION

In this section, we formulate the ecodriving problem and we will propose a discrete-time formulation of the problem. The ecodriving problem is defined as minimizing traction power over a certain trajectory:

$$\min_{v(t), s(t), u(t)} \int_{t_0}^{t_f} P_{\text{trac}}(v(t), u(t)) dt, \quad (1a)$$

where v is speed, u is the mechanical force, t_0 and t_f are the initial and the final time, respectively. In this paper, we assume

$$P_{\text{trac}}(v, u) = \frac{1}{2}\gamma_2 u^2 + \gamma_1 uv + \frac{1}{2}\gamma_0 v^2, \quad (1b)$$

where γ_2 , γ_1 and γ_0 are (nonnegative) parameters. The model (1b) is a reasonable model for electric motors, because v^2 is related to friction losses and u^2 is related to Ohmic losses. The objective function in (1a) is minimized subject to (1b) and the longitudinal dynamics of the vehicle, given by

$$\begin{cases} \dot{v}(t) = \sigma_u u(t) - \sigma_v v(t)^2 - \sigma_r - g \sin(\alpha(s(t))), \\ \dot{s}(t) = v(t), \end{cases} \quad (1c)$$

for $t \in [t_0, t_f]$, where s denotes the distance traveled, and subject to the lower and upper bounds on v and u , i.e.,

$$\underline{v}(t) \leq v(t) \leq \bar{v}(t), \quad \underline{u}(t) \leq u(t) \leq \bar{u}(t), \quad (1d)$$

for $t \in [t_0, t_f]$, and given $v(t_0)$, $s(t_0)$, $v(t_f)$ and $s(t_f)$. In (1c), α is the road slope, which depends on the distance s , $\sigma_r = gc_r$, $\sigma_u = \frac{1}{m}$ and $\sigma_v = \frac{1}{2m} c_d \rho_a A_f$. The coefficients m , g , c_d , ρ_a , A_f and c_r are the vehicle's mass, gravitational acceleration, aerodynamic drag coefficient, air density, frontal drag area and rolling resistance coefficient, respectively.

To arrive at a finite-dimensional optimization problem, we discretize (1) using forward Euler discretization, and arrive at a discrete-time nonlinear optimal control problem, subject to state dynamics, and state and input constraints:

$$\min_{x_k, u_k} \sum_{k \in \mathcal{K}} \frac{1}{2} \begin{bmatrix} x_k \\ u_k \end{bmatrix}^\top H_k \begin{bmatrix} x_k \\ u_k \end{bmatrix} + F_k^\top \begin{bmatrix} x_k \\ u_k \end{bmatrix}, \quad (2a)$$

$$x_{k+1} = f(x_k, u_k), \quad (2b)$$

$$\underline{x}_k \leq x_k \leq \bar{x}_k, \quad \underline{u}_k \leq u_k \leq \bar{u}_k, \quad (2c)$$

for $k \in \mathcal{K} = \{0, 1, \dots, K-1\}$, where $K = \frac{t_f - t_0}{\tau}$ is the optimization horizon with x_0 , x_K given, and $\tau > 0$ is the step size, which is chosen such that $\tau t_f \in \mathbb{N}$. To arrive at a discrete-time approximation of (1), we choose

$$x_k = \begin{bmatrix} v_k \\ s_k \end{bmatrix}, \quad H_k = \tau \begin{bmatrix} \gamma_0 & 0 & \gamma_1 \\ 0 & 0 & 0 \\ \gamma_1 & 0 & \gamma_2 \end{bmatrix}, \quad F_k = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (3a)$$

$$f(x_k, u_k) = \begin{bmatrix} v_k + \tau(\sigma_u u_k - \sigma_v v_k^2 - \sigma_r - g \sin(\alpha(s_k))) \\ s_k + \tau v_k \end{bmatrix} \quad (3b)$$

An electric machine, represented by (1b), typically has a close to linear input-output behavior, such that $\gamma_1 \approx 1$, $\gamma_0 \ll 1$ and $\gamma_2 \ll 1$, as the power losses are generally small. This might cause (2) with (3) to be a non-convex optimization problem, as H_k is not positive definite. As a result, the method presented in this paper will only give a local minimum, which might not be the global minimum. Understanding convexity of the ecodriving problem is a topic of current research.

III. SQP APPROACH TO ECODRIVING

In this section, we will present a Sequential Quadratic Programming (SQP) algorithm to solve the nonlinear optimal control problem (2). SQP aims at solving a nonlinear optimization problem by sequentially solving linearly constrained quadratic programs (LCQP), which are formed, e.g., by approximating the objective function with a quadratic equation and linearizing the constraints. In particular, we will solve (2) by recursively solving the SQP subproblem

$$\begin{aligned} \{x_k^{i+1}, u_k^{i+1}\}_{k \in \mathcal{K}} = \\ \operatorname{argmin}_{x_k, u_k} \sum_{k \in \mathcal{K}} \frac{1}{2} \begin{bmatrix} x_k - x_k^i \\ u_k - u_k^i \end{bmatrix}^\top R_k \begin{bmatrix} x_k - x_k^i \\ u_k - u_k^i \end{bmatrix} + (H_k \begin{bmatrix} x_k^i \\ u_k^i \end{bmatrix} + F_k)^\top \begin{bmatrix} x_k \\ u_k \end{bmatrix}, \end{aligned} \quad (4a)$$

subject to linearized state dynamics,

$$x_{k+1} = f(x_k^i, u_k^i) + \nabla f(x_k^i, u_k^i) \begin{bmatrix} x_k - x_k^i \\ u_k - u_k^i \end{bmatrix}, \quad (4b)$$

and state constraints and input constraints,

$$\underline{x}_k \leq x_k \leq \bar{x}_k, \quad \underline{u}_k \leq u_k \leq \bar{u}_k, \quad (4c)$$

for all $k \in \mathcal{K}$ and $i \in \mathbb{N}$, and for given x_0 , x_K as well as some suitably chosen $\{x_k^0, u_k^0\}_{k \in \mathcal{K}}$, such that a feasible solution exists for the next iteration of the SQP subproblem (4). We have derived the SQP subproblem (4) by linearizing the state equations (2b), and linearizing the objective function (2a) and adding an $R_k \succeq 0$ to ensure a convex objective function in (4a), which can be regarded as a proximal or Thikonov regularization.

The matrix R_k can be chosen to warrant the SQP subproblem (4) is strictly convex in its decision variables and converges. We note that by choosing $R_k = H_k$, the SQP objective function (4a) becomes exactly the original objective function (2a). Still, for any $R_k \neq H_k$, the First Order Necessary Conditions for optimality (FONC) [12] for (4) are identical to the FONC for (2) if the SQP problem has converged, i.e., $x_k^{i+1} = x_k^i$ and $u_k^{i+1} = u_k^i$ for all $k \in \mathcal{K}$. We will employ a so-called merit function to monitor convergence [12] and to decide if the algorithm has converged.

We remark that the state variables may be eliminated in the SQP problem (4), by rewriting the linearized state equations (4b) in a prediction form, where the state variables are given by a set of prediction matrices and the inputs, i.e.

$$\mathbf{x} = \begin{bmatrix} x_0 \\ \vdots \\ x_K \end{bmatrix} = \Phi(\mathbf{x}^i, \mathbf{u}^i) + \Gamma(\mathbf{x}^i, \mathbf{u}^i) \mathbf{u}, \quad (5)$$

Fig. 1: Topology

where $\mathbf{u} = [u_0 \cdots u_{K-1}]^\top$ and Φ, Γ are matrices chosen so that (5) represents (4b). By substituting (5) into the objective function (4a) and state constraints (4c), as is often done in model-predictive control, we can arrive at an SQP subproblem with only the input variables as decision variables, and the state variables can be updated with (5).

IV. ENERGY MANAGEMENT WITH ECODRIVING

As a case study for CVEM with ecdriiving, we consider a series-hybrid electric vehicle, consisting of an Electric Motor (EM), Engine-Generator Unit (EGU) and a High-Voltage Battery (HVB). The topology is shown in Fig. 1, in which y_{egu} and u_{egu} denote the ICE's fuel and mechanical power, respectively, y_{hvb} and u_{hvb} the battery's electrical and stored chemical power, respectively, y_{br} is an artificial braking power exerted at the interconnection of the subsystems, u_{em} and y_{em} the EM's mechanical force and electrical power, respectively, x_{hvb} denotes the battery state of energy, and $x_{em} = [v \ s]^\top$, denotes the state vector with speed v and distance traveled s of the vehicle. In CVEM, the main goal is to minimize the fuel consumption

$$\sum_{k \in \mathcal{K}} \tau y_{egu,k}, \quad (6a)$$

subject to the dynamics and input-output behavior of the converters in Fig 1, and the power balance at the interconnection of the subsystems, i.e.,

$$y_{em,k} - u_{egu,k} - y_{hvb,k} = y_{br} \leq 0, \quad (6b)$$

for all $k \in \mathcal{K}$. We use constraint (6b) and the energy balance constraint of the HVB, i.e.,

$$\sum_{k \in \mathcal{K}} \tau u_{hvb} = x_{hvb,K} - x_{hvb,0} = 0 \quad (6c)$$

to rewrite (6a) as a 'sum of losses', i.e.,

$$\tau \sum_{k \in \mathcal{K}} y_{egu,k} - u_{egu,k} + u_{hvb,k} - y_{hvb,k} + y_{em,k} - y_{br,k}. \quad (6d)$$

By substituting (6b) and (6c) into (6d), we retrieve the original fuel consumption (6a). In this section, we solve the CVEM problem by formulating it as a convex SQP problem and then apply dual decomposition as presented in [1].

A. Optimal Control Problem

The objective is to minimize fuel consumption, for which we use the equivalent fuel consumption (6d), which may be written as

$$\min_{u_{m,k}, y_{m,k}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} a_m u_{m,k} + b_m y_{m,k} \quad (7a)$$

where $a_m, b_m \in \mathbb{R}$, and $u_{m,k} \in \mathbb{R}$ and $y_{m,k} \in \mathbb{R}$ are the (scalar) inputs and outputs of the converter in subsystem $m \in \mathcal{M} = \{\text{em}, \text{egu}, \text{hvb}\}$ at time instant $k \in \mathcal{K}$, respectively. The optimization problem (7a) is to be solved subject to an equality constraint describing the quadratic input-output behavior of each converter, i.e.,

$$y_{m,k} = \frac{1}{2} \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix}^\top H_m \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + F_m^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + e_m \quad (7b)$$

for all $k \in \mathcal{K}$, $m \in \mathcal{M}$, and subject to the system dynamics of the states in subsystem $m \in \mathcal{M}$, i.e.,

$$x_{m,k+1} = f_m(x_{m,k}, u_{m,k}) \quad (7c)$$

for all $k \in \mathcal{K}$ where the initial state $x_{m,0}$ and final state $x_{m,K}$ of the energy storage device are assumed to be given, and the input $u_{m,k}$ is subject to linear inequality constraints,

$$\underline{u}_{m,k} \leq u_{m,k} \leq \bar{u}_{m,k}, \quad (7d)$$

for all $k \in \mathcal{K}$, $m \in \mathcal{M}$ and the state $x_{m,k}$ is subject to linear inequality constraints, i.e.,

$$\underline{x}_{m,k} \leq x_{m,k} \leq \bar{x}_{m,k}, \quad (7e)$$

for all $k \in \mathcal{K}$ and $m \in \mathcal{M}$, and finally, subject to a linear inequality constraint describing the interconnection of the subsystems given by (6b), which we write as

$$\sum_{m \in \mathcal{M}} c_m u_{m,k} + d_m y_{m,k} = y_{br,k} \leq 0, \quad (7f)$$

for all $k \in \mathcal{K}$, where $c_m, d_m \in \mathbb{R}$. Note that we have left out the term $-y_{br,k}$ in (7a), as $y_{br,k}$ for all $k \in \mathcal{K}$ is already given by the inequality (7f), which renders $y_{br,k}$ as an implicit decision variable. To have that (7a) is equivalent to minimizing the fuel consumption, we choose $a_{egu} = b_{hvb} = -\tau$, $a_{hvb} = b_{egu} = b_{em} = \tau$, $a_{em} = 0$, and to have that (7f) corresponds to the interconnection of the subsystems, we choose $c_{egu} = d_{hvb} = -1$, $d_{em} = 1$, $c_{hvb} = c_{em} = d_{egu} = 0$. Finally, we assume in this paper that the coefficients for the input-output behavior of the converters in (7b) are given by

$$H_{em} = \begin{bmatrix} \gamma_{em0} & 0 & \gamma_{em1} \\ 0 & 0 & 0 \\ \gamma_{em1} & 0 & \gamma_{em2} \end{bmatrix}, \quad F_{em} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad e_{em} = 0, \\ H_j = \begin{bmatrix} 0 & 0 \\ 0 & \gamma_{j2} \end{bmatrix}, \quad F_j = \begin{bmatrix} 0 \\ \gamma_{j1} \end{bmatrix}, \quad e_j = \gamma_{j0}, \quad (8)$$

$j \in \{\text{egu}, \text{hvb}\}$, for some (nonnegative) parameters $\gamma_{m2}, \gamma_{m1}, \gamma_{m0}$, $m \in \mathcal{M}$, the state dynamics for the EM are given by (3b), and the state dynamics for the EGU and HVB are given by (7c), with $f_{egu}(x_{egu,k}, u_{egu,k}) = 0$ and $f_{hvb}(x_{hvb,k}, u_{hvb,k}) = x_{hvb,k} - \tau u_{hvb,k}$, respectively.

B. SQP Formulation

To form a convex SQP formulation, we propose to relax the (nonconvex) quadratic input-output behavior of the converters (7b) to a convex quadratic approximation by linearizing (7b) around $[(x_{m,k}^i)^\top (u_{m,k}^i)^\top]^\top$ and adding a convex quadratic part, i.e.,

$$y_{m,k} \approx \frac{1}{2} \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right)^\top R_m \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) \\ + \left(H_m \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} + F_m \right)^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + e_m, \quad (9)$$

for all $k \in \mathcal{K}$ and $m \in \mathcal{M}$, where the matrix $R_m \succeq 0$ is chosen such that (9) is convex. We note that the approximation error disappears when $x_{m,k} \rightarrow x_{m,k}^i$, $u_{m,k} \rightarrow u_{m,k}^i$, and by choosing $R_m = H_m$, we retrieve the original quadratic equation (7b). By substituting (9) into (7a) and (7f) and linearizing the dynamics (7c), we arrive at the convex SQP subproblem

$$\begin{aligned} & \{x_{m,k}^{i+1}, u_{m,k}^{i+1}\}_{k \in \mathcal{K}, m \in \mathcal{M}} = \\ & \underset{x_{m,k}, u_{m,k}}{\operatorname{argmin}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} \frac{1}{2} b_m \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right)^\top R_m \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) \\ & + (b_m H_m \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} + b_m F_m + \begin{bmatrix} 0 \\ a_m \end{bmatrix})^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + b_m e_m, \quad (10a) \end{aligned}$$

subject to the linearized state dynamics, i.e.,

$$\begin{aligned} & x_{m,k+1} - f_m(x_{m,k}^i, u_{m,k}^i) \\ & - \nabla f_m(x_{m,k}^i, u_{m,k}^i) \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) = 0, \quad (10b) \end{aligned}$$

for all $k \in \mathcal{K}$, $m \in \mathcal{M}$, and subject to linear inequality constraints, i.e.,

$$\underline{x}_{m,k} \leq x_{m,k} \leq \bar{x}_{m,k}, \quad \underline{u}_{m,k} \leq u_{m,k} \leq \bar{u}_{m,k} \quad (10c)$$

for all $k \in \mathcal{K}$, $m \in \mathcal{M}$, and further for all $m \in \mathcal{M}$ subject to the convex quadratic inequality constraint specifying the interconnection of the subsystems

$$\begin{aligned} & \sum_{m \in \mathcal{M}} \frac{1}{2} d_m \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right)^\top R_m \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) \\ & + (d_m H_m \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} + d_m F_m + \begin{bmatrix} 0 \\ c_m \end{bmatrix})^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + d_m e_m \leq 0. \quad (10d) \end{aligned}$$

C. Dual Decomposition

Note that (10a) subject to (10b) and (10c) is entirely separable, and the only complicating constraint is (10d), which is the constraint that acts on all components $m \in \mathcal{M}$. Therefore, we propose to decompose (10) via dual decomposition by augmenting the objective function with the constraint (10d), which results in the so-called ‘partial Lagrangian’:

$$\begin{aligned} L(\{x_{m,k}, u_{m,k}, \lambda_k\}) = & \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} \frac{1}{2} \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right)^\top \hat{R}_{m,k} \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) \\ & + (\hat{H}_{m,k} \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} + \hat{F}_{m,k})^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} + \hat{E}_{m,k}, \quad (11a) \end{aligned}$$

in which,

$$\begin{aligned} \hat{R}_{m,k} &= (b_m + d_m \lambda_k) R_m, \\ \hat{H}_{m,k} &= (b_m + d_m \lambda_k) H_m, \\ \hat{F}_{m,k} &= (b_m + d_m \lambda_k) F_m + (a_m + c_m \lambda_k) \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \hat{E}_{m,k} &= (b_m + d_m \lambda_k) e_m, \quad (11b) \end{aligned}$$

where $\lambda_k \geq 0 \in \mathbb{R}^N$ is a Lagrange multiplier. The partial Lagrange dual function is then given by

$$\begin{aligned} g(\{\lambda_k\}) &= \min_{x_{m,k}, u_{m,k}} L(\{x_{m,k}, u_{m,k}, \lambda_k\}) \\ &= \sum_{m \in \mathcal{M}} g_m(\{\lambda_k\}) + \hat{E}_{m,k}, \quad (12a) \end{aligned}$$

subject to (10b) and (10c), where

$$\begin{aligned} g_m(\{\lambda_k\}) &= \min_{x_{m,k}, u_{m,k}} \sum_{k \in \mathcal{K}} \frac{1}{2} \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right)^\top \hat{R}_{m,k} \left(\begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} - \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} \right) \\ &+ (\hat{H}_{m,k} \begin{bmatrix} x_{m,k}^i \\ u_{m,k}^i \end{bmatrix} + \hat{F}_{m,k})^\top \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix}, \quad (12b) \end{aligned}$$

subject to (10b) and (10c), with x_0 given, is the dual problem for each component. Note that for the components whose dynamics $f_m(x_{m,k}, u_{m,k})$ are linear and $R_m = H_m$ yields a convex dual problem (12b), SQP is not needed to solve the dual problem, as they can be solved as a QP problem. Typically, g_{egu} and g_{hvb} are both QP problems and g_{em} is an SQP problem that can be solved through the approach given in Section III. However, it may still be beneficial to apply regularization to the other components, since it may improve convergence properties. The dual problem is given by

$$\max_{\lambda_k} g(\{\lambda_k\}) = d^*, \quad (13)$$

subject to (10b) and (10c), where d^* is defined as the dual optimal solution. The dual problem (13) gives a lower bound on the primal optimal value p^* of problem (10), i.e.,

$$d^* \leq p^*. \quad (14)$$

The dual problem equals the primal problem, i.e. $d^* = p^*$, if problem (10) is convex and the constraints satisfy Slater’s constraint qualifications [12]. As we have derived a convex SQP subproblem (10) and assume that the Slater’s constraint qualifications are satisfied for this problem formulation, we have that $d^* = p^*$ in our SQP and dual decomposition approach. We maximize the dual problem (13) with a steepest ascend method, i.e.,

$$\lambda_k^{i+1} = \max\{0, \lambda_k^i + \rho_k \left(\sum_{m \in \mathcal{M}} c_m u_{m,k}^{i+1} + d_m y_{m,k}^{i+1} \right)\}, \quad (15)$$

for all $k \in \mathcal{K}$, where $\rho_k \geq 0$ is chosen small enough such that the dual problem converges. In our approach, note that in every iteration, we update both the SQP subproblem (10) as well as the Lagrangian multiplier λ_k for all $k \in \mathcal{K}$. We have found that as long as R_m for all $m \in \mathcal{M}$ are well chosen, this is the fastest way to let the dual problem converge. This concludes the SQP and dual decomposition approach presented in this section, where by formulating (7) as a convex SQP subproblem (10), we could form the dual problem (13), which solves the CVEM problem (7).

V. RESULTS

In this section, we show the results of two simulation studies where the ecodriving problem is present. In the first simulation study, we will use the SQP approach presented in

Fig. 2: Results of the FEV study. The dashed colored lines are the results for which a speed constraint is introduced.

Section III to replicate the results of the ecodriving problem for a Full Electric Vehicle (FEV) detailed in [8]. In the second simulation study, we consider an energy management problem for a Series-Hybrid Electric Vehicle (SHEV), and solve this with the approach presented in Section IV. The results of this simulation are comparable to the results presented in [10], yet not exactly the same, due to slightly different models used.

A. Full Electric Vehicle

The FEV simulation study as presented in [8] has a powertrain consisting of an Electric Motor (EM) and a High-Voltage Battery (HVB). However, in this case study, the HVB is not considered, i.e., it is assumed that the HVB has infinite energy storage and no power limitations, which makes it a simple but representative example for ecodriving. Furthermore, the optimal control problem given in [8] does not consider input and state constraints. This follows from the fact that [8] applies Pontryagin's Maximum Principle (PMP), in which it is difficult to introduce state and input constraints. We solve the ecodriving problem presented in [8] using the discretization approach as presented in Section II, with $\tau = 0.0005$, and SQP, as presented in Section III. We also show that with the SQP approach, adding state and input constraints becomes trivial.

The optimal control problem defined in [8] is given by the ecodriving problem (1) as presented in Section II. The parameters used for the FEV simulation study can be found in [8], and the control input is normalized, i.e., $-100 \leq u \leq 100$. To select initial conditions for the input and states, we choose a constant velocity profile $v_k = 0$ m/s for all $k \in \mathcal{K}$, and determine s_k and u_k for all $k \in \mathcal{K}$ from the longitudinal vehicle dynamics (2b).

In Fig. 2, the results of the FEV simulation study are shown using solid lines. In this figure, we can observe that the initial and final state constraint are satisfied. We further note that the speed profile is not symmetrical, due to the road profile; close to where the slope is steepest, speed is maximized, as is expected. The cost as defined by the discrete-time objective function (2a) is $J = 1227247.8$, which differs 0.1% from the cost obtained in [8]. This small difference may be caused due to numerical inaccuracies

TABLE I: SHEV Component Parameters

$\gamma_{em0} = 0.5856$	$\gamma_{egu0} = 19000$	$\gamma_{hvb0} = 0$
$\gamma_{em1} = 1.005$	$\gamma_{egu1} = 2.52$	$\gamma_{hvb1} = 1$
$\gamma_{em2} = 5.052 \times 10^{-4}$	$\gamma_{egu2} = 2 \times 10^{-8}$	$\gamma_{hvb2} = -1.671 \times 10^{-6}$

presented in both this work, as well as in [8], and the finite sampling time. It is interesting to note that although global optimality is guaranteed neither in the approach used in this paper nor in the approach used in [8] (because both approaches only consider necessary conditions for optimality), the solutions obtained are practically identical. As an illustration to the ease of adding state and input constraints, in Fig. 2, the results of the simulation study with a maximum speed constraint of 12 m/s are also shown using dotted lines. We observe that with a cost of $J = 1511796.5$, by adding this constraint, the cost becomes higher than without state constraints, as is expected.

B. Series-Hybrid Electric Vehicle

In this simulation study, we consider the SHEV case study presented in Section IV. We solve the case study in a 'forward' and 'backward' simulation. The terms 'forward' and 'backward' refer to way in which the longitudinal vehicle dynamics differential equations (1c) are treated. In 'backward' simulation, the differential equations are treated as 'quasi-static' equations, i.e., the states v and s are given a priori, and in 'forward' simulation, the differential equations are treated 'dynamically', i.e., the states remain decision variables. We will refer to 'forward' optimization as solving the vehicle energy management problem with ecodriving (7), and refer to 'backward' optimization as solving (7) as an energy management problem without ecodriving, where the vehicle trajectory information, i.e., speed v_k and distance s_k for all $k \in \mathcal{K}$, are given.

We base our simulation study on the work done in [10], in which a convex optimization approach is taken to solve the SHEV case study, where it is formulated as a second-order cone program. Due to the chosen problem formulation, the authors in [10] have opted for a piece-wise linear EM model, linear Engine-Generator Unit (EGU) model and a quadratic HVB model. Furthermore, the authors in [10] define the optimization problem in the space domain, for which the physical interpretation of some parts of their problem formulation is not easy to understand. As we have defined the SHEV case study in the time domain, we avoid this issue. To have somewhat comparable results, we fit the parameters of our quadratic EM model using least-squares. Furthermore, we approximate the linear EGU model with a quadratic EGU model by choosing the quadratic coefficient γ_{egu2} in (8) sufficiently small. The parameters for the components can be found in Table I, in which all the parameters are given in SI units. The vehicle parameters, initial and final conditions, as well as bounds on the states and inputs can be found in [10]. The parameters $R_m, m \in \mathcal{M}$ are chosen such that they yield convex objective functions in the dual problem (12b).

The simulations are done for 1080 s over a distance of 21

Fig. 3: Backward vs forward optimization. The dotted lines represent minimum and maximum state constraints.

km with a step size $\tau = 5$ s, which gives an optimization horizon $K = 216$. The road altitude is given by a sinusoidal profile with an amplitude of 280 m. In ‘backward’ optimization, the speed is given by a constant speed of $v_k = 70$ km/h for all $k \in \mathcal{K}$, and in ‘forward’ optimization the initial and final velocity are constrained to 70 km/h, while over the trajectory the speed is allowed to vary by 70 ± 10 km/h. As initial conditions for the ‘forward’ optimization, we choose the solutions of the ‘backward’ optimization.

In Fig. 3, the simulation results for ‘backward’ and ‘forward’ optimization are given. In the ‘forward’ optimization results, we see that between 0 and 2 km, where the slope becomes negative, the vehicle reaches the lower speed bound. This action allows the available potential energy from the road profile, between 2 and 13 km, to be maximally converted to kinetic energy; the vehicle speed is maximized in this interval. After 13 km, the road gradient becomes positive, and speed is minimized such that the final speed constraint is met. Therefore, we can see that as a result of having the speed as a decision variable, in the ‘forward’ case, the EGU may provide less power over the course of the trajectory, and noticeably less braking power is applied, when it is compared to the ‘backward’ case. Assuming that the EGU consumes diesel fuel with a specific energy of 830 kg/m^3 , the fuel consumption of the ‘backward’ and ‘forward’ simulation cases are 23.41 l/100 km and 22.31 l/100 km, respectively. Thus, by including the ecodriving problem into the vehicle energy management problem, approximately 4.7 % decrease in fuel consumption is achieved. This is comparable to the fuel consumption obtained in [10], which are 24.35 l/100 km and 23.98 l/100 km for the ‘backward’ and ‘forward’ case respectively. This is a 1.54 % decrease in fuel consumption between the ‘backward’ and ‘forward’ case. We may explain this difference in fuel consumption savings largely due to the different models used.

VI. CONCLUSION

In this paper, we have solved the ecodriving problem using a Sequential Quadratic Programming (SQP) algorithm. We have formulated the ecodriving problem as a discrete-time nonlinear optimal control problem. In the SQP algorithm, we have formed convex SQP subproblems by using a linearized objective function with Thikonov regularization. To demonstrate the scalability of the distributed optimization approach presented in [1], we have embedded the SQP algorithm into this distributed optimization approach, which allows it to be used for complete vehicle energy management in combination with ecodriving. We have done so by formulating a vehicle energy management with ecodriving problem as a convex SQP problem and applying dual decomposition to solve the dual problem. We have considered two case studies for the ecodriving problem. In the first case study, we have solved the ecodriving problem for a full electric vehicle using the SQP algorithm and have shown that the algorithm yields a very similar result as the benchmark problem set in [8], and shown that adding state constraints is trivial using the approach proposed in this paper. In the second case study, we have solved an energy management problem with ecodriving for a series-hybrid electric vehicle with the SQP algorithm and dual decomposition. We have shown that using our approach, we have had the opportunity to use more representable powertrain models than in [10], and as a result have the potential to save more fuel by incorporating ecodriving to the energy management problem.

REFERENCES

- [1] T. Romijn, M. Donkers, J. Kessels, and S. Weiland, “A distributed optimization approach for complete vehicle energy management,” *Trans. Control Syst. Technol.*, 2017.
- [2] M. Donkers, J. Van Schijndel, W. Heemels, and F. Willems, “Optimal control for integrated emission management in diesel engines,” *Control Engineering Practice*, 2017.
- [3] Z. Khalik, T. Romijn, M. Donkers, and S. Weiland, “Effects of battery charge acceptance and battery aging in complete vehicle energy management,” *Proc. of the IFAC World Congress*, 2017.
- [4] D. P. Bertsekas, *Dynamic programming and optimal control*. Athena Scientific Belmont, MA, 1995, vol. 1, no. 2.
- [5] L. V. Pérez, G. R. Bossio, D. Moitre, and G. O. García, “Optimization of power management in an hybrid electric vehicle using dynamic programming,” *Math. Comput. Simulation*, 2006.
- [6] T. Pham, J. Kessels, P. van den Bosch, R. Huisman, and R. Nevels, “On-line energy and battery thermal management for hybrid electric heavy-duty truck,” in *Proc. American Control Conf.*, 2013.
- [7] N. Murgovski, L. Johannesson, J. Sjöberg, and B. Egardt, “Component sizing of a plug-in hybrid electric powertrain via convex optimization,” *Mechatronics*, 2012.
- [8] N. Petit and A. Sciarretta, “Optimal drive of electric vehicles using an inversion-based trajectory generation approach,” *IFAC World Congress*, 2011.
- [9] E. Hellström, J. Åslund, and L. Nielsen, “Design of an efficient algorithm for fuel-optimal look-ahead control,” *Control Engineering Practice*, 2010.
- [10] N. Murgovski, L. Johannesson, X. Hu, B. Egardt, and J. Sjöberg, “Convex relaxations in the optimal control of electrified vehicles,” in *Proc. American Control Conference*, 2015.
- [11] P. T. Boggs and J. W. Tolle, “Sequential quadratic programming,” *Acta numerica*, 1995.
- [12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, Cambridge, 2004.