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A Port-Hamiltonian Approach to Complete Vehicle Energy Management: A Battery Electric Vehicle Case Study

G.P. Padilla, J.C. Flores Paredes, M.C.F. Donkers

Abstract—In this paper, we present a modelling approach to vehicle energy management based on Port-Hamiltonian systems representations. We consider a network of interconnected port-Hamiltonian systems that describes the powertrain components and auxiliaries in the vehicle. This description is suitable to obtain a systematic approach to formulate a decomposable optimal control problem for Complete Vehicle Energy Management. A physically insightful cost function that describes the total energy consumption of the vehicle is proposed in terms of internal energy and losses of each system connected to the network. Taking advantage of the modularity of the proposed formulation, we present a distributed optimization algorithm to find solutions to the energy management problem. To illustrate this modelling methodology, we consider a case study in which the energy consumption of a battery electric vehicle is optimized.

I. INTRODUCTION

Electrification of vehicles, and particularly the deployment of fully electric vehicles, is considered a way to mitigate CO₂ emissions related to the transport sector, which was responsible for 14% of the global CO₂ emissions in 2014 [1], with 95% of the transportation energy based on fossil fuels, estimated in 2016 [2]. However, electric vehicles are known to have a limited range and a time-consuming charging process, leading to range anxiety, which is the user's concern to be unable to reach the destination with the current battery charge [3]. This concern has a negative impact for the introduction of electric vehicles in the market [4], [5].

In order to mitigate range anxiety, it is possible to use Energy Management Strategies (EMS) [6], [7], which aim to minimize energy consumption during operation, therefore extending the driving range. These strategies can often be implemented as software in the vehicle, which implies limited cost for production. A specific branch of EMS that considers the influence of all auxiliary subsystems in the vehicle is referred as Complete Vehicle Energy Management (CVEM) [8]. Specifically, CVEM strategies are solutions to optimal control problems (OCP) that aim to minimize the power consumption of the vehicle subject to the dynamical behavior of interconnected subsystems in a network, e.g., the internal combustion engine, the battery, the electric motor, and the climate system. CVEM results in an efficient operation of the vehicle.

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In the current literature, the modelling and the OCP formulation are mainly obtained by applying power-based approaches [9]–[12]. The main idea of these approaches is to describe the behavior of the interconnected subsystems in a network, in terms of power interactions. In other words, each subsystem is represented by an energy buffer connected to a power converter. The buffer stores energy and it is modeled as a linear dynamical linear system, which often resembles an integrator. The power converter represents energy consumption and is modeled as a static non-linear function, normally approximated to a quadratic function from data measurements.

However, power-based approaches are difficult to use when the optimization variables cannot be easily described in terms of power. For instance, in [13], [14] attempts to unify CVEM and eco-driving (finding energy optimal velocity profiles) are presented. In this case, velocity becomes a decision variable and describing it in terms of power is a non-trivial task. Other disadvantage of power-based approaches is that it is difficult to constrain physical variables that describe power in the model. For instance, imposing constraints on the battery power do not necessarily imply that the solution of the OCP is physically realizable. Power in the battery is described as the product of voltage and current, thus an optimal solution that satisfies power constraints not necessarily satisfies the bounds on voltage and current. Additionally, power-based approaches do not always allow for an intuitive formulation of decomposable OCP, which are useful when distributed optimization techniques are applied to approximate solutions. For instance, in [10] the concept of 'sum of losses' is introduced to formulate a separable OCP for CVEM. Unfortunately, this concept is not always simple to apply.

In this paper, we propose an alternative modelling framework based on interconnected port-Hamiltonian (pH) systems [15], [16] aiming to overcome the previously discussed limitations of power-based approaches. In this modelling approach, each subsystem is represented as a dissipative dynamical system that inherently describes energy losses and changes of internal energy in the subsystem. The main contributions of this work are twofold. First, we propose to use pH representations to model networks for CVEM applications, thus unifying the power-based concepts of energy buffers and power converters in a single dynamical model, i.e., in terms of internal energy and losses. Second, a systematic approach is proposed to formulate a decomposable OCP for CVEM, which is useful when distributed static optimization techniques are considered to solve the CVEM OPC.

The remainder of this paper is organized as follows. In Section II, we briefly introduce the pH modeling theoretical background and use these concepts to model CVEM networks. In Section III, we take advantage of the pH modelling approach to formulate decomposable OCP for CVEM. The pH methodology for CVEM is applied to a case study of a battery electric vehicle presented in Section IV. Finally, the conclusions are drawn in Section V.

II. A PORT-HAMILTONIAN MODELLING APPROACH FOR CVEM

The main objective of this section is to present the pH modeling framework for energy management applications. Specifically, we adapt the pH framework to the holistic philosophy of CVEM presented in [8]. To achieve this, we briefly discuss the main concepts of pH representations. Later, the main features of networks of interconnected subsystems for CVEM are translated into a compatible description with pH models.

A. Port-Hamiltonian Representation

From a modeling perspective, a pH representation is a port-based modelling approach that considers a pair of conjugated variables in each port, whose product represents power. This has been depicted in Fig. 1, where a generic graphical representation of a two-port pH system is presented. Although in literature, there is a large number of possible pH representations, in this paper we will only focus on linear input-state-output models with a bidirect feedthrough term.

Consider a linear subsystem $m \in \mathcal{M} := \{1, \dots, M\}$, where M is the total number of interconnected subsystems in the network. The internal energy of the m subsystem is expressed as a quadratic function given by

$$\mathcal{H}_m(x_m) = \frac{1}{2} x_m^\top Q_m x_m, \quad (1)$$

where $x_m \in \mathbb{R}^n$ represents the state vector and $Q_m \in \mathbb{R}^{n \times n}$ is a symmetric positive semi-definite matrix, which is called energy matrix. This energy function is used in an input-state-output with feedthrough term pH representation [16], [15, §4] of the m subsystem, and is given by

$$\dot{x}_m = (J_m - R_m) Q_m x_m + B_m u_m \quad (2a)$$

$$y_m = (B_m + 2P_m)^\top Q_m x_m + (M_m + S_m) u_m \quad (2b)$$

where $u_m \in \mathbb{R}^2$ and $y_m \in \mathbb{R}^2$ are the system input and output, respectively, given by

$$u_m = \begin{bmatrix} u_m^{\text{in}} \\ u_m^{\text{out}} \end{bmatrix} \quad \text{and} \quad y_m = \begin{bmatrix} y_m^{\text{in}} \\ y_m^{\text{out}} \end{bmatrix}. \quad (3)$$

In (2), $J_m \in \mathbb{R}^{n \times n}$ is a skew symmetric matrix known as the interconnection matrix, and $R_m \in \mathbb{R}^{n \times n}$ is a positive semidefinite matrix known as dissipation matrix.

Fig. 1: Port-Hamiltonian subsystem Σ_m

The port-Hamiltonian model (2) satisfies the following power balance equation [15, §4]

$$\frac{\partial \mathcal{H}_m}{\partial t}(x_m) = - \begin{bmatrix} x_m \\ u_m \end{bmatrix}^\top L_m \begin{bmatrix} x_m \\ u_m \end{bmatrix} + y_m^\top u_m. \quad (4)$$

in which

$$L_m := \begin{bmatrix} Q_m R_m Q_m & Q_m P_m \\ P_m^\top Q_m & S_m \end{bmatrix}. \quad (5)$$

In case that $L_m \geq 0$, passivity of the system is guaranteed [17, §7.1]. Furthermore, an energy balance can be described by reorganizing and integrating both sides of (4) over the interval $[t_0, t_f]$, hence obtaining

$$\Delta E_m = E_m^{\text{in}} + E_m^{\text{out}} = \Delta \mathcal{H}_m + \mathcal{L}_m \quad (6)$$

with

$$\Delta E_m = \int_{t_0}^{t_f} y_m^\top u_m \, dt = \underbrace{\int_{t_0}^{t_f} y_m^{\text{in}} u_m^{\text{in}} \, dt + \int_{t_0}^{t_f} y_m^{\text{out}} u_m^{\text{out}} \, dt}_{E_m^{\text{in}} + E_m^{\text{out}}} \quad (7a)$$

$$\Delta \mathcal{H}_m = \mathcal{H}_m(x_m(t_f)) - \mathcal{H}_m(x_m(t_0)) \quad (7b)$$

$$\mathcal{L}_m = \int_{t_0}^{t_f} \begin{bmatrix} x_m \\ u_m \end{bmatrix}^\top L_m \begin{bmatrix} x_m \\ u_m \end{bmatrix} \, dt \quad (7c)$$

where ΔE_m represents the net supplied energy to the m subsystem, $\Delta \mathcal{H}_m$ is the change on internal energy and $\mathcal{L}_m$ describes the total energy losses. Moreover, let us define the set of source subsystems

$$\mathcal{S} := \{m \in \mathcal{M} \mid E_m^{\text{in}} = 0\}, \quad (8a)$$

the set of consumers

$$\mathcal{C} := \mathcal{M} \setminus \mathcal{S}, \quad (8b)$$

and the set of terminal subsystems

$$\mathcal{T} := \{m \in \mathcal{M} \mid E_m^{\text{out}} = 0\}. \quad (8c)$$

A physical example of a source subsystem, which only has an output port, is chemical energy stored in a battery. On the other hand, mechanical brake is a physical example of a terminal system, which has only an input port.

B. Port-based Network Topology

Interconnections of pH systems have been studied in [18], where a methodology has been proposed to describe networks of pH systems with the final goal to perform stability analysis. Although in this paper, we consider some ideas from the work presented in [18], we will deviate from this approach to propose a specialized description of network topologies that capture specific features found in CVEM applications.

A tree structure port-based network topology is presented in Fig. 2, where each subsystem $m \in \mathcal{M}$ is depicted by a gray square. The interconnection between subsystems takes place through junctions $j \in \mathcal{J} := \{1, \dots, J\}$, where $J \in \mathbb{N}$ is the total number junctions in the network. These junctions

Fig. 2: Port-based topology for interconnected port-Hamiltonian systems

are power preserving interconnections that are described in terms of the conjugated input output variables as follows

$$\sum_{m \in \mathcal{M}} a_{j,m} u_m^{\text{in}} y_m^{\text{in}} + b_{j,m} u_m^{\text{out}} y_m^{\text{out}} = 0, \quad \text{for all } j \in \mathcal{J}, \quad (9)$$

where $a_{j,m} = 1$ if the input port of subsystem m is connected to junction j , and $a_{j,m} = 0$ otherwise. Similarly, $b_{j,m} = 1$ if the output port of subsystem m is connected to junction j , and $b_{j,m} = 0$ otherwise.

In CVEM networks, there is a type of junction that can be decomposed into an additive node and an equality node. A physical interpretation of these type of nodes is observed in electrical interconnections between subsystems, e.g., Kirchhoff's law of currents can be seen as an additive node, while Kirchhoff's law of voltages is an equality node. Let us define the set of all the decomposable junctions in the network $\mathcal{J}_n := \{1, \dots, J_n\}$, where $J_n \leq J$ is the total number of these type of junctions in the network. Thus, the power preserving equation (9) can be rewritten as

$$\sum_{m \in \mathcal{M}} [a_{j,m} \ 0] u_m + [0 \ b_{j,m}] y_m = 0, \quad (10a)$$

$$[a_{j,m} \ 0] y_m - [0 \ b_{j,m}] u_m = 0 \text{ for all } m \in \mathcal{M}, \quad (10b)$$

for all $j \in \mathcal{J}_n$. Note that (10a) and (10b) are referred as additive node and an equality nodes, respectively.

Another type of power preserving interconnections observed in CVEM networks are power junctions. The power junction artifact introduced in this framework obeys the necessity to resemble the functionality of DC-DC converters in CVEM networks. It is well known that DC-DC converters can be represented as non-linear pH systems, e.g., see [19]. In fact, from steady-state approximations of the models proposed in [19], under the assumption that the DC-DC converter is lossless, it is possible to prove that DC-DC converters satisfy (9). In order to keep consistent notation, we rewrite b(9) as

$$\sum_{m \in \mathcal{M}} y_m^T \begin{bmatrix} a_{j,m} & 0 \\ 0 & b_{j,m} \end{bmatrix} u_m = 0, \quad (11)$$

for all $j \in \mathcal{J}_p$; where $\mathcal{J}_p := \{J_n + 1, \dots, J\}$ is the set of all power junctions in the network.

III. OPTIMAL CONTROL PROBLEM FOR pH CVEM

In this section, we first provide a general formulation for a continuous-time optimal control problem (OCP) for the pH CVEM modelling approach described in the previous section. Subsequently, we propose a physically insightful cost function that enables decomposability of the proposed pH OCP for CVEM. Finally, we present a discrete-time OCP for pH CVEM.

The main objective of CVEM is to minimize the total energy consumed by a network of interconnected subsystems over a fixed time interval, subject to the dynamical behavior of each subsystem and bounds on the respective inputs, outputs and states. From a mathematical perspective, this goal can be described as an OCP. Hence, a general formulation of the pH OCP for CVEM is given by

$$\min_{\{x_m, u_m, y_m\}_{m \in \mathcal{M}}} \mathcal{J}(\{x_m, u_m, y_m\}_{m \in \mathcal{M}}) \quad (12a)$$

subject to: (2), (10), (11),

$$\underline{x}_m \leq x_m \leq \bar{x}_m, \quad (12b)$$

$$\underline{u}_m \leq u_m \leq \bar{u}_m, \quad (12c)$$

$$\underline{y}_m \leq y_m \leq \bar{y}_m, \quad (12d)$$

for all $m \in \mathcal{M}$ and all $t \in [t_0, t_f]$. Note that (12a) represents a general cost function that aims to describe the total energy consumed by the subsystems in the network. We propose a physically insightful and decomposable cost function for the OCP (12) below.

A. Power Consumption in pH CVEM

The physical interpretation provided by pH models in terms of internal energy and energy losses of the subsystems can be exploited to formulate a sensible description of the energy consumed in the network. In particular, we could aim to maximize the internal energy in each source subsystem, which implies minimizing consumption, i.e., see (7b). This can be described by (12) if the cost function is defined as

$$\mathcal{J} = \sum_{m \in \mathcal{S}} -\Delta \mathcal{H}_m. \quad (13)$$

Note that the minus sign in (13) is necessary because (12) is defined as a minimization problem.

Interestingly, by decomposing the cost function (13) it is possible to obtain an equivalent expression that can be interpreted as minimizing the aggregated internal energy of all the consumers and the energy losses of all the subsystems. In order to see this, let us recall that power preserving interconnections are considered for all the junctions $j \in \mathcal{J}$ in the network. Therefore, by integrating (9) for a given time interval we obtain the following energy balance

$$\sum_{m \in \mathcal{M}} a_{j,m} E_m^{\text{in}} + b_{j,m} E_m^{\text{out}} = 0, \quad (14)$$

for all $j \in \mathcal{J}$. Hence, it is possible to follow a recursive procedure where we substitute (6) and (14) into (13) to obtain

the partially expanded function

$$\begin{aligned}\mathcal{J} &= \sum_{m \in \mathcal{S}} \mathcal{L}_m - E_m^{\text{out}} \\ &= \sum_{m \in \mathcal{S}} \left(\mathcal{L}_m + b_{m,j} \sum_{n \in \mathcal{C}} a_{n,j} E_n^{\text{in}} \right) \\ &= \sum_{m \in \mathcal{S}} \left(\mathcal{L}_m + b_{m,j} \sum_{n \in \mathcal{C}} a_{n,j} (\Delta \mathcal{H}_n + \mathcal{L}_n - E_n^{\text{out}}) \right). \quad (15)\end{aligned}$$

The recursive expansion of this cost function ends with terminal subsystems in each branch of the tree network. Hence, the complete expansion of (13) is given by

$$\mathcal{J} = \sum_{m \in \mathcal{M}} \mathcal{L}_m + \sum_{n \in \mathcal{C}} \Delta \mathcal{H}_n, \quad (16)$$

which implies that maximization of internal energy is equivalent to minimizing the losses of all subsystems and the internal energy of only the consumers in the network.

These results not only show the importance of pH formulations to obtain physically insightful interpretations of cost functions, but also open the door to obtain possible equivalent cost functions that could be beneficial for specific solution methods used to solve the CVEM OCP (12), e.g., [10], [12].

B. Discretization

The use of static optimization techniques to approximate solutions of (12) in a finite-time horizon requires the discretization of this OCP. It is important to remark that we are aware that there are no conclusive results about preservation of the discrete-time energy balance equation under interconnection of discrete-time pH systems. Although promising results can be found in literature, e.g., [20], [21], we will neglect this issue in this work.

To arrive at a discrete-time OCP, (12) is discretized at times $t_k = k\delta_t + t_0$, $k \in \mathcal{K} = \{0, \dots, K-1\}$, with time step $\delta_t = \frac{t_f - t_0}{K}$ for some $K \in \mathbb{N}$ using the forward Euler discretization method. This leads to

$$\min_{\{x_{m,k}, u_{m,k}, y_{m,k}\}_{m \in \mathcal{M}, k \in \mathcal{K}}} \sum_{m \in \mathcal{M}} e_m \Delta \hat{\mathcal{H}}_m + \hat{\mathcal{L}}_m \quad (17a)$$

subject to the discrete-time dynamical model

$$x_{m,k+1} = (I + \delta_t (J_m - R_m) Q_m) x_{m,k} + \delta_t B_m u_{m,k}, \quad (17b)$$

$$y_{m,k} = (B_m + 2P_m)^\top Q_m x_{m,k} + (M_m + S_m) u_{m,k}, \quad (17c)$$

discrete-time versions of (10), (11), and $\{x_{m,k}, u_{m,k}, y_{m,k}\} \in \Omega_m$ for all $m \in \mathcal{M}$ and $k \in \mathcal{K}$, where

$$\begin{aligned}\Omega_m &:= \{x_{m,k} \in \mathbb{R}^n, y_{m,k} \in \mathbb{R}^2, u_{m,k} \in \mathbb{R}^2\} \\ &\quad (12b), (12c) \text{ and } (12d) \text{ hold}. \quad (17d)\end{aligned}$$

Note that (17a) is an equivalent formulation of the cost function (16), where $e_m = 1$ if $m \in \mathcal{C}$ and $e_m = 0$ otherwise. This indicates that the cost function (17a) penalizes changes of internal energy only for the set of energy consumers. The

discretized versions of internal energy and losses are given respectively by

$$\Delta \hat{\mathcal{H}}_m = \frac{1}{2} x_{m,K}^\top Q_m x_{m,K} - \frac{1}{2} x_{m,0}^\top Q_m x_{m,0}, \quad (18a)$$

$$\hat{\mathcal{L}}_m = \delta_t \sum_{k \in \mathcal{K}} \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix}^\top L_m \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix}. \quad (18b)$$

The advantage of the pH framework exposed in this paper lays in the possibility to formulate decomposable problems by expanding the cost function in terms of internal energy and losses of the subsystem in the network. Therefore, distributed static optimization techniques can be easily adapted to find solutions of the discrete OCP (17), e.g., [10], [12].

IV. CASE STUDY: BATTERY ELECTRIC VEHICLE

The pH CVEM framework described in Section II and the associated OCP of Section III are illustrated in a case study presented in this section. We consider a Battery Electric Vehicle (BEV), whose topology is depicted in Fig. 3. A high-voltage battery (HVB) is connected via a DC-DC converter to an electric machine (EM). The mechanical part in this driveline yields a power balance between the requested propulsion power and the mechanical braking power. Even though this configuration is simple, it allows to illustrate the proposed modelling approach for CVEM and the solution method.

In the remainder of this section, we provide details for modelling the topology presented in Fig. 3 and formulate an OCP for CVEM. Later, we discuss the connections between power-based and pH OCP formulations highlighting the advantages of the approach proposed in this paper. Finally, simulations results for this case study are presented and analyzed.

A. Modelling

In the topology presented in Fig. 3, it is possible to observe the physical conjugated variables that describe the power preserving interconnection between subsystems. We will use these variables to define the input-output ports for each subsystem. Additionally, the pH models for the subsystems will be obtained from first-principle models and described in terms of the pH representation given in Section II-A.

The set $\mathcal{M}$ is the set of interconnected subsystems for the BEV considered in this case. For the sake of readability, we use $\mathcal{M} = \{vs, ec, em, br, req\}$ instead of a enumerated set to denote battery voltage source, battery electric circuit, electric machine, mechanical brake and power request, respectively. Note that *vs* is a source subsystem, while *br* and *req* are terminal subsystems. Similarly, $\mathcal{J}_n = \{1\}$ represents the mechanical node in the topology, while $\mathcal{J}_p = \{2\}$ is the power junction of the network, which corresponds to the DC-DC converter.

1) *High-Voltage Battery*: We consider an equivalent circuit model with constant open circuit voltage V_{oc} and internal resistance R_{hvb} in this paper. The voltage and current in the terminals are denoted as V_{hvb} and I_{hvb} , respectively. The pH formulation of the HVB is constituted of subsystems *vs* and

Fig. 3: CVEM port-based configuration for a battery electric vehicle.

TABLE I: High-Voltage Battery (HVB) parameters

Variable	Value	Definition	Observations
V_{oc}	658	open circuit voltage	[V]
R_{hvb}	0.33	series resistance	[Ω]
q_{hvb}	180	capacity of the battery	[Ah]
I_{hvb}	-800	minimum current	[A]
$\bar{I}_{hvb}$	800	maximum current	[A]
$\underline{V}_{hvb}$	400	minimum voltage	[V]
$\bar{V}_{hvb}$	1000	maximum voltage	[V]

TABLE II: Driveline parameters

Variable	Value	Definition	Observations
R_e	1	equivalent resistance	[Ω]
L	2	equivalent inductance	[H]
I_e	99.68	equivalent axle inertia	[kg m ²]
β	1.1e-03	friction coefficient (rotational)	[Nm s/rad]
κ_τ	4.2175	torque constant	[Nm/A]
κ_V	4.27	back-electromotive constant	[Vs/rad]
r_w	0.5715	wheel radius	[m]
g_r	6.1	gear ratio	[-]
$\underline{I}_{em}$	-600	minimum current	[A]
$\bar{I}_{em}$	600	maximum current	[A]
$\underline{V}_{em}$	-1000	minimum voltage	[V]
$\bar{V}_{em}$	1000	maximum voltage	[V]

ec , which represent an ideal lossless voltage source and the internal resistance of the battery, respectively. The input and output ports and the states of the aforementioned subsystems are defined as

$$u_{vs} = \begin{bmatrix} 0 \\ -I_{hvb} \end{bmatrix}, \quad y_{vs} = \begin{bmatrix} 0 \\ V_{oc} \end{bmatrix}, \quad (19a)$$

$$u_{ec} = \begin{bmatrix} V_{oc} \\ I_{hvb} \end{bmatrix}, \quad x_{ec} = \text{SoC}, \quad y_{ec} = \begin{bmatrix} I_{hvb} \\ -V_{hvb} \end{bmatrix}, \quad (19b)$$

where SoC denotes the HVB state-of-charge. Note that these definitions indicate that the interconnection between the voltage source vs and the electric circuit ec is power preserving, i.e., $y_{vs}^{\text{out}} u_{vs}^{\text{out}} + y_{ec}^{\text{in}} u_{ec}^{\text{in}} = 0$. Therefore, the pH models of the subsystems in the HVB are given by

$$\begin{aligned} Q_{ec} &= J_{ec} = R_{ec} = 0, \quad P_{ec} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad B_{ec} = \begin{bmatrix} 0 & -\frac{1}{q_{hvb}} \end{bmatrix} \\ M_{ec} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad S_{ec} = \begin{bmatrix} 0 & 0 \\ 0 & R_{hvb} \end{bmatrix}, \end{aligned} \quad (19c)$$

where q_{hvb} denotes the HVB capacity. Numerical values for the parameters associated to the HVB are given in Table I.

2) *Electric Machine*: The HVB is connected to an EM through an ideal DC-DC converter represented by junction $\mathcal{J}_p$. The pH formulation for the EM is defined using

$$u_{em} = \begin{bmatrix} V_{em} \\ F_{em} \end{bmatrix}, \quad x_{em} = \begin{bmatrix} LI_{em} \\ \frac{g_r}{r_w} I_m v_{em} \end{bmatrix}, \quad y_{em} = \begin{bmatrix} I_{em} \\ -v_{em} \end{bmatrix}, \quad (20a)$$

where V_{em} , I_{em} denotes the voltage and current in the EM, respectively; F_{em} is the propulsion force and v_{em} the longitudinal velocity. Additionally, the positive constants L , I_m , r_w and g_r denote the electric inductance, moment of inertia, wheel radius and total gear ratio, respectively. These definitions allow the pH model of the EM to be defined using

$$Q_{em} = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{I_m} \end{bmatrix}, \quad J_{em} = \begin{bmatrix} 0 & \frac{\kappa_V + \kappa_\tau}{2} \\ -\frac{\kappa_V + \kappa_\tau}{2} & 0 \end{bmatrix}, \quad (20b)$$

$$R_{em} = \begin{bmatrix} R_e & \frac{\kappa_V - \kappa_\tau}{2} \\ \frac{\kappa_V - \kappa_\tau}{2} & \beta \end{bmatrix}, \quad B_{em} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{r_w}{g_r} \end{bmatrix}, \quad (20c)$$

$$P_{em} = M_{em} = S_{em} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (20d)$$

where R_e , κ_V , κ_τ and β are positive constants that denote the equivalent electric resistance, back-electromotive constant, torque constant and damping coefficient, respectively. In Table II, numerical values of the driveline parameters used for this case study are presented.

3) *Mechanical Brake*: The function of this subsystem is to dissipate all the power provided to it. It is considered as a terminal subsystem and its ports are defined as

$$u_{br} = \begin{bmatrix} v_{br} \\ 0 \end{bmatrix}, \quad y_{br} = \begin{bmatrix} F_{br} \\ 0 \end{bmatrix}, \quad (21)$$

where $F_{br} \geq 0$ is the mechanical braking force and v_{br} denotes the longitudinal velocity of the vehicle. Note that a complete description of the subsystem is not necessary because we are only interested in its consumption. Thus, the terms that describe internal energy and losses for the mechanical brake subsystem in the cost function (16) can be replaced by the total energy consumed by the subsystem, i.e.,

$$\Delta \mathcal{H}_{br} + \mathcal{L}_{br} = \int_{t_0}^{t_f} y_{brk}^\top u_{br} dt. \quad (22)$$

4) *Power Request*: This subsystem describes the power required by the vehicle to travel a certain period of time with a given longitudinal velocity profile v_{req} and traction force F_{req} . It is considered a terminal subsystem, whose ports are defined as

$$u_{req} = \begin{bmatrix} v_{req} \\ 0 \end{bmatrix}, y_{req} = \begin{bmatrix} F_{req} \\ 0 \end{bmatrix}. \quad (23)$$

The requested force and velocity profiles used in this case study are shown in Fig. 4. In this case, a complete description of the subsystem is not provided, because its behavior is already given. However, to preserve consistency in the cost function formulation (16), the terms that describe internal energy and losses are replaced by the total power consumed by the subsystem, i.e.,

$$\Delta \mathcal{H}_{req} + \mathcal{L}_{req} = \int_{t_0}^{t_f} y_{req}^\top u_{req} dt. \quad (24)$$

5) *Interconnections*: The set $\mathcal{J} = \mathcal{J}_n \cup \mathcal{J}_p = \{1, 2\}$ contains the two junctions in network topology depicted in Fig. 3. This junctions are described by

$$b_{em,1} = a_{br,1} = a_{req,1} = 1, \quad (25a)$$

$$b_{ec,2} = a_{em,2} = 1, \quad (25b)$$

and zero for the remaining coefficients.

B. Optimal Control Problem

After describing pH representations for all the subsystems and its interconnections, the procedure to formulate a CVEM OCP for this case study is reduced to a simple substitution. In particular, a continuous-time CVEM OCP for this case study is obtained by substituting (19)-(25) into (12) and (16). Similarly, substituting (19)-(25) into (17) and (18) generates a discrete-time OCP.

Interestingly, for this case study it is also possible to find an equivalent OCP using a power-based approach. For instance, let us consider the modelling framework proposed in [10]. In this approach, the source subsystem is represented as a linear system that describes the accumulation of energy, and the conversion of power in all subsystems is defined by a quadratic function of the form

$$P_{out} = \frac{1}{2} \gamma_2 P_{in}^2 + \gamma_1 P_{in} + \gamma_0, \quad (26)$$

Fig. 4: Requested traction force and longitudinal velocity.

where P_{out} , P_{in} represent the power in the input and output ports of the subsystem, respectively. To illustrate the connection between the power-based approach in [10] with the pH framework proposed in this paper, let us consider the EM model described by (2) and (20) in steady state. After some algebraic manipulations and by defining $P_{out} = V_{em} I_{em}$ and $P_{in} = F_{em} V_{em}$, it is possible to obtain

$$\begin{aligned} \gamma_2 &= \frac{R_e r_w^2}{\kappa_V^2 g_r^2 v_{em}^2}, \quad \gamma_1 = \frac{2R_e \beta}{\kappa_V^2} + \frac{\kappa_\tau}{\kappa_V}, \\ \gamma_0 &= \left(\frac{R_e \beta^2}{\kappa_V^2} + \frac{\kappa_\tau \beta}{\kappa_V} \right) \frac{g_r^2}{r_w^2} v_{em}^2. \end{aligned} \quad (27)$$

Hence, considering that the longitudinal velocity v_{em} is given, (26) describes only a static relation between input and output powers. Performing a similar procedure in the rest of the subsystems of this example allows us to obtain an equivalent OCP that has power in the input-output ports of the subsystems as decision variables.

The advantage of this equivalent formulation is that if static optimization methods are used to obtain solutions, the number of decision variables is lower than in the port-based case. However, this framework fails to describe problems of higher complexity. For instance, see [13] where an attempt to unify the optimization of velocity profiles, i.e., eco-driving, and CVEM for the subsystems of a vehicle is presented. Using the pH framework for CVEM presented in this paper, the integration of eco-driving and CVEM is natural. In fact, it is equivalent to include the longitudinal vehicle dynamics as an additional subsystem in the port-based network.

C. Simulation Results

We use the approach presented in [12] to find a solution to the discrete-time OCP obtained from the substitution of (19)-(25) into (17) and (18). In this case, we use a sampling time $\delta_t = 1[s]$ and a horizon of length $K = 990$. The results obtained are compared to a benchmark case where all the mechanical braking power is zero, which means that the energy produced by deceleration of the vehicle is completely stored in the HVB. In Fig. 5, the SoC and the mechanical braking force can be observed as function of time for both the benchmark as well as the optimal solution. The final SoC corresponding to the optimal solution is 0.0717% higher than the benchmark solution, which implies energy savings of 4.896% for the given power request. Interestingly,

Fig. 5: State of charge and braking force.

Fig. 6: Battery voltage.

Fig. 7: Battery current.

the optimal solution still requires some mechanical braking. This clearly shows that the optimal solution selects efficient operational points for the subsystems, instead of only trying to recover as much mechanical energy as possible.

Physical insight about the HVB operation for both cases can be observed in Fig. 6 and Fig. 7, where the behavior of battery voltage and current, is respectively depicted for both solutions. Note that regions where the voltage in the battery terminals increases correspond to negative currents in the HVB. This implies charging the battery, which is also observed in Fig. 5. An advantage of the pH CVEM framework proposed in this paper is that it is possible to give independent treatment to each conjugated variable in the port of the subsystems, i.e., setting independent constraints to each variable. For this study case, the constraints on the voltage and current of the HVB aim to capture the charge acceptance phenomenon in the battery. This is not directly possible in power-based approaches because voltage and current are combined as power in a single decision variable.

V. CONCLUSIONS

We have proposed a systematic modelling approach to describe CVEM networks in terms of interconnected pH systems representations. This is an alternative to power-based approaches for CVEM and has allowed us to unify the concepts of energy storage and losses directly in the dynamical models of the subsystems. We have highlighted the relevance of the pH framework in terms of formulation of a physically insightful decomposable OCP. Moreover, the extra freedom obtained by describing power in terms

of conjugated variables has opened the door to include additional physical phenomena as constraints in the OCP.

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