Solution to Problem 85-25: Series involving Bessel functions

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and the analogue of (1) is

$$\begin{aligned} \frac{(c; q)_m (c/ab; q)_m}{(c/a; q)_m (c/b; q)_m} {}_3\phi_2 \left[\begin{matrix} q^{-m}, a, b; \\ cq^n, (ab/c)q^{1-m}; \end{matrix} q^{1+n} \right] \\ = \frac{(c; q)_n (c/ab; q)_n}{(c/a; q)_n (c/b; q)_n} {}_3\phi_2 \left[\begin{matrix} q^{-n}, a, b; \\ cq^m, (ab/c)q^{1-n}; \end{matrix} q^{1+m} \right]. \end{aligned}$$

Comment by R. G. BUSCHMAN (University of Wyoming).

If we apply [2, (4.8.3.12)] to the ${}_{p+2}F_{q+1}$ functions and do some simplifications, we can write the sum in the form

$$\begin{aligned} \frac{(\gamma)_n (\gamma - \alpha - \beta)_n}{(\gamma - \alpha)_n (\gamma - \beta)_n} \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma - \alpha)} \int_0^1 x^{\gamma - \alpha - 1} (1 - x)^{\alpha - 1} {}_2F_1 \left(\begin{matrix} -n, \beta; \\ 1 - \gamma + \alpha + \beta - n; \end{matrix} 1 - x \right) \\ \cdot {}_{p+1}F_q \left(\begin{matrix} \gamma - \beta, (a_p); \\ (b_q); \end{matrix} zx \right) dx. \end{aligned}$$

The formula [1, (A.1.2.6)] allows us to rewrite this as

$$\frac{(\gamma)_n (\gamma - \alpha - \beta)_n}{(\gamma - \alpha)_n (\gamma - \beta)_n} F \begin{matrix} 0: p+2; 3 \left(\begin{matrix} - : \gamma - \alpha, \gamma - \beta, (a_p); -n, \beta; \\ \gamma: (b_q); 1 - \gamma + \alpha + \beta - n; \end{matrix} z, 1 \right) \\ 1: q; \end{matrix}$$

REFERENCES

- [1] HAROLD EXTON, *Handbook of Hypergeometric Integrals*, Ellis Horwood Ltd., 1978.
- [2] L. J. SLATER, *Generalized Hypergeometric Functions*, Cambridge University Press, Cambridge-London, 1966.

Also solved by the proposer, who employed his generalization of Saalschütz theorem,

$$\begin{aligned} (\dagger) \quad {}_3F_2 \left[\begin{matrix} a, b, -N; \\ c+n, a+b-c-N+1; \end{matrix} 1 \right] \\ = \frac{(c-a+n)_{N-n} (c-b+n)_{N-n}}{(c-n)_{N-n} (c-a-b)_N} \sum_{k=0}^n \binom{n}{k} \frac{(a)_k (b)_k (c-a-b)_{n-k}}{(c+N)_k}, \end{aligned}$$

and who also obtained the generalization (**) above.

Series Involving Bessel Functions

Problem 85-25, by ELDON HANSEN (Lockheed Missiles & Space Co.).

Assume that a closed form is known for the series

$$S_m(a) = \sum_{k=1}^{\infty} c_k k^{m-a} J_a(kx)$$

where J_a is the Bessel function of the first kind and where the coefficients c_k are independent of a . Sum the series $S_{m+2n}(a)$ ($n=0, 1, 2, \dots$) by expressing it as a finite sum of terms involving $S_m(b)$ for various values of b .

The result can be used to sum the series in Problem 84-18* [1] which was proposed by Henry E. Fettis.

REFERENCE

- [1] HENRY E. FETTIS, *Problem 84-18**, *A summation of Bessel functions*, this Review, 26 (1984), pp. 430-431.

The solutions by O. P. LOSSERS (Eindhoven University of Technology, Eindhoven, the Netherlands) and the proposer were the same and are as follows:

From G. N. Watson, *Theory of Bessel Functions*, 1966, p. 45, equation (1) we borrow the formula

$$2aJ_a(kx) = kx [J_{a-1}(kx) + J_{a+1}(kx)],$$

so that

$$\begin{aligned} S_{m+2}(a) &= \sum_{k=1}^{\infty} c_k k^{m-(a-2)} J_a(kx) \\ &= \sum_{k=1}^{\infty} c_k k^{m-(a-2)} \left[\frac{2(a-1)}{kx} J_{a-1}(kx) - J_{a-2}(kx) \right] \\ &= \frac{2(a-1)}{x} S_m(a-1) - S_m(a-2). \end{aligned}$$

Using the latter formula repeatedly, it follows by induction that

$$S_{m+2n}(a) = \sum_{k=0}^n (-1)^k \left(\frac{x}{2} \right)^{k-n} \binom{n}{k} \frac{\Gamma(a-k)}{\Gamma(a-n)} S_m(a-n-k).$$

Inequality for a Simplex

Problem 85-26, by M. S. KLAMKIN (University of Alberta).

If O , I , R and r denote the circumcenter, incenter, circumradius, and inradius, respectively, of an n -dimensional simplex, prove that

$$R^2 \geq n^2 r^2 + \overline{OI}^2$$

and with equality iff the simplex is regular.

Solution by the proposer.

First we derive a more general inequality. Let F_i denote the content of the $(n-1)$ -dimensional face opposite vertex A_i of a given simplex, $i=0, 1, \dots, n$. If P is an arbitrary point, then by Cauchy's inequality,

$$(1) \quad \sum F_i \sum F_i \overline{PA_i}^2 \geq \left\{ \sum F_i \overline{PA_i} \right\}^2$$