Sparse virtual array synthesis for MIMO radar imaging systems

Citation for published version (APA):

DOI:
10.1049/mia2.12129

Document status and date:
Published: 09/09/2021

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.
• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
Sparse virtual array synthesis for MIMO radar imaging systems

Rabia Z. Syeda | Martijn C. van Beurden | A. Bart Smolders

Department of Electrical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

Correspondence
Email: r.z.syeda@tue.nl

Funding Information
NWO TTW (Netherlands Organization for Scientific Research), Grant/Award Number: 13922

Abstract
Multiple-input-multiple-output (MIMO) radar imaging systems require a high angular resolution to realize a high detectability level of the target directly related to the aperture size of the virtual array. Since in MIMO radars, the virtual array topology depends on the transmit and receive array topologies, two methods are proposed for the synthesis of sparse virtual arrays by designing the sparse transmit and receive arrays using a two-step synthesis procedure (TSP) for antenna array sparsity via convex optimization. Both methods are formulated in detail, and it is shown that adjustments in formulations with respect to physical constraints in design specifications are feasible. In the first method, both the sparse transmit and the sparse receive array are synthesized using the TSP independently of each other. In the second method, the sparse receive array is synthesized based on the transmit array topology or vice versa using the method of least squares. With the help of numerical examples, it is shown that a sparse virtual array has better performance in terms of beamwidth and side lobe level compared with the traditional dense virtual array with the same number of transmit and receive array elements. It is also shown that the proposed design methods lead to broadband performance.

1 | INTRODUCTION

Radar imaging systems are multi-antenna systems used for target localization in one or more dimensions providing high system resolution. These systems cover a wide range of commercial applications, such as security, medicine, and automotive, that require high resolution for unambiguous detection of targets [1-4]. Radar imaging systems have traditionally used phased-array for multi-antenna systems [5]. The angular resolution of these imaging radars depends on the effective aperture of the phased array, which means that for a high angular resolution, a large aperture, and thus a large number of array elements for both transmit and receive channels, is required to satisfy the Nyquist sampling criterion [6]. Further, in order to avoid the appearance of grating lobes while scanning, a dense array configuration is commonly chosen. This results in expensive phased-array systems especially at mm-wave frequencies, where because of the shorter wavelength, the array element size and interelement distance are small. To realize high angular resolution with such a solution, a complex antenna system is required [6].

Recently, radar imaging systems have largely employed the concept of multiple-input-multiple-output (MIMO) radar formally introduced by E. Fishler et. al. in [7] and further elaborated in [8-12]. With the use of multiple transmit and receive channels in MIMO mode, a higher angular resolution can be achieved compared with that of traditional phased-array radars [12]. A MIMO radar is different from a phased-array radar in the sense that each transmitter transmits an orthogonal waveform in time, frequency, or phase instead of a coherent signal. Thus, each of the transmitted signals, after reflection from targets, is received by each receiver. This concept results in the formation of a virtual array that is larger in aperture and hence has potentially a higher angular resolution. Like a phased array, a dense virtual array configuration in MIMO radar is desirable for scanning without the appearance of grating lobes [13]. However, the MIMO radar concept is increasingly being used for single-chip mm-wave radar sensors either with
antenna-on-chip or with antenna-in-package; some examples of small-scale radar platforms are a 77 GHz module [14] and a single-chip 79 GHz FMCW radar sensor [15, 16]. To use these sensors in high-resolution imaging applications, the sensors are required to be placed in a cascaded (master/slave) system [17], where a dense configuration is physically not feasible because of size, thermal, or cost limitations. In such cases, it is unavoidable to use a sparse virtual array configuration with desirable array patterns and to consider the aforementioned limitations. In addition, a sparse configuration results in the reduction of mutual coupling between antenna elements, which is a critical performance indicator when it comes to these radar sensors [18].

In the literature, sparse virtual arrays have been proposed and evaluated such as in [19, 20] and in [21] to design a sparse transmit and receive array. The design method for these arrays is based on either traditional synthesis methods or trial and error. More recently, a genetic algorithm was used to determine the positions of the transmit and receive array elements by setting the ambiguity function of the virtual array as the fitness function [22, 23]. The limitation of genetic algorithms is that the fitness function is evaluated repeatedly, and as a result, for a large number of transmit and receive array elements, the optimization problem would become highly computationally expensive to solve. In addition, in [23] the issue of an unambiguous field-of-view (FoV) while scanning has been addressed, but the maximum achieved side lobe level (SLL) is around −6 dB for the virtual array pattern, which is considered too high for many applications. Overall, a systematic approach is lacking to design sparse transmit and receive array configurations individually to obtain a desired virtual array performance. The optimization of the transmit and receive array individually allows more design freedom in terms of practical limitations for physical arrays, such as the size of the array and distance between elements, while controlling the side lobe level for each array individually.

We propose two synthesis methods to determine the topology of sparse transmit and receive arrays whereby they are synthesized independently of each other or one depends on the topology of the other. For the synthesis of each sparse transmit and receive array, we use the concept of antenna array sparsity, which is known as a technique to reduce the number of array elements while satisfying the array performance requirements [24–27]. For its simplicity, efficiency, and ease of use, we use the iterative convex optimization procedure. The use of convex optimization for antenna array pattern synthesis was proposed in [26] and subsequently used in [27, 28] to obtain maximally sparse arrays by using iterative weighted $\ell_1$-norm minimization. This method allows the design of a sparse array with optimal amplitude weights of the antenna array elements that are non-uniform. The non-uniform amplitude weights of array elements are a stringent practical limitation for almost all applications. This problem can be solved by combining this method with a second step of iterative convex optimization, but this time for the antenna array element positions, as proposed in [29]. This leads to a two-step synthesis procedure (TSP) for antenna array sparsity, which is also formulated and explained. Moreover, a sparse antenna array configuration usually results in the appearance of grating lobes in the FoV while scanning, and we solve this by constructing the antenna array topology using the TSP at the maximum scan angle determined by the system requirements. Using TSP at maximum scan angle for synthesis of transmit and receive array (either independently or jointly) allows us to relax the formulation parameters (such as beamwidth and side lobe level) in each array synthesis procedure.

Finally, we use this TSP in two methods for the synthesis of a sparse virtual array topology. In the first method, both the sparse transmit and the sparse receive arrays are synthesized individually using the TSP and are used to obtain the sparse virtual array topology. In the second method, the transmit array is synthesized using TSP, while the sparse receive array is obtained using the method of least squares (MLS) [50] to determine the desired virtual array topology. We show with specific test cases that by using a sparse configuration of the virtual array, an improved performance can be achieved compared with the equivalent dense virtual array while using the same number of transmit and receive array elements. For the sake of simplicity, both methods are presented for the case of linear arrays. However, both methods can be extended to the planar case for an imaging radar. We are focussing on the optimization of the virtual array performance for MIMO radar systems that ultimately translates into improved signal-to-noise ratio of the obtained information of the target and environment. This will lead to improved imaging performance of the radar systems.

This paper is organized as follows. In Section 2, the virtual array concept in MIMO radars is briefly reviewed in the context of array radiation patterns along with the relations among the transmit, receive, and virtual array element positions. In addition, two known cases of a dense virtual array configuration are explained and compared. In Section 3, two methods are proposed that will be used to determine the element positions of the sparse virtual array and in Section 4, numerical examples are presented to show the performance of the sparse virtual arrays obtained with the proposed methods. These examples are then validated with full-wave solver in Section 5, and the strengths and weaknesses of these methods are explained in Section 6. Finally, Section 7 presents the conclusions.

## 2 | CONVENTIONAL MIMO VIRTUAL ARRAY CONCEPT AND ITS LIMITATIONS

Consider a linear transmit array of $N_t$ isotropic elements with element position vector $\mathbf{d}_t$ and a receive array of $N_r$ isotropic elements with element position vector $\mathbf{d}_r$. The array patterns of
the transmit and receive arrays are given by the following expressions:

\[ \text{AF}_t(\theta) = \sum_{n=1}^{N_t} \sum_{n_1=1}^{N_v} \omega_{n,n_1} \exp(-jkd_{n,n_1}(\sin\theta - \sin\theta_{\text{scan}})), \]

\[ \text{AF}_r(\theta) = \sum_{n=1}^{N_r} \sum_{n_1=1}^{N_v} \omega_{n,n_1} \exp(-jkd_{n,n_1}(\sin\theta - \sin\theta_{\text{scan}})), \]

where \( k = 2\pi/\lambda_0 \), \( \lambda_0 \) is the free-space wavelength, \( \omega_{n,n_1} \) and \( \omega_{n,n_1} \) are the real-valued amplitude weights of the \( n \)th array element in the transmit and receive array, respectively, and \( \theta_{\text{scan}} \) is the scan angle. For the sake of simplicity, and without the loss of generality, we assume that both the transmit and the receive array have the same array orientation axis. According to the concept of MIMO radar arrays, if the transmitted signals from each transmit antenna are orthogonal, then each receive antenna receives different reflected signals from a target. This results in the concept of the virtual array with number of elements \( N_v \) whose element positions are all the possible combinations of the transmit and receive antenna positions. In addition, the array pattern of the virtual array is the Kronecker product of the array pattern of the transmit and receive array and is given by

\[ \text{AF}_v(\theta) = \sum_{n=1}^{N_t} \sum_{n_1=1}^{N_v} \omega_{n,n_1} \exp(-jkd_{n,n_1}(\sin\theta - \sin\theta_{\text{scan}})), \]

where \( d_{n,n_1} \) is the \( n \)th element of the vector \( d_v \) that contains the virtual array element positions, which is given by the equation

\[ d_v = \text{vect}(d_{1,M}^T + J_{1,N_v}d_v^T), \]

where \( J_{1,M} \) is a column vector of all those of dimension \( M \), and \( \text{vect}(C) \) is the operator for the vectorization of matrix \( C \). An illustration of the physical transmit and receive arrays and virtual array is shown in Figure 1.

The definition of the MIMO virtual array suggests that its element positions and array pattern depend on the topology of the physical transmit and receive array. The virtual array is so called because it is not a physically real array but an equivalent array whose focused array beam pattern is obtained by receiving the target reflections of each transmitted signal, by each receiver, hence resulting in \( N_t \) times \( N_r \) reflections of the target that may or may not be unique. In the case of linear arrays, the choice of interelement distance and the mutual orientation of the axes of the arrays for both transmit and receive can allow for different configurations of the virtual array, see [31–34]. We consider two cases where the transmit and receive array topologies result in a dense configuration of the virtual array. The first case uses a dense transmit and a dense receive array while the second case uses a regular sparse transmit array and a dense receive array. Although both cases result in a dense configuration of the virtual array, for a better use of terminology in the rest of the paper, we name the first case 'dense virtual array' and second case 'conventional virtual array', because the latter is most commonly used in practical MIMO radar designs, such as in [14, 31].

### 2.1 Case I: Dense virtual array

When the transmit and receive arrays both use a dense uniform interelement distance of \( d = \lambda_0/2 \), the virtual array is also dense with a total number of elements \( N_v = N_tN_r \). However, not all of the virtual elements have a unique element position. The number of non-redundant elements is given by \( N_v = [(N_t(N_r - 1))/2], \) and therefore the length of the virtual array is given by \( L_v = (\langle N_t(N_r - 1)/2 \rangle - 1)d \). An example in the case of a three-element transmit and four-element receive array is shown in Figure 2 with corresponding element positions and array patterns.

### 2.2 Case II: Conventional virtual array

As discussed in [6], when the receive array is dense with uniform interelement distance \( d \) and the transmit array is regular sparse such that the interelement distance is \( N_r \), the resulting virtual array will be dense with \( N_v = N_tN_r \) elements. All of the virtual elements have unique element positions and therefore the length of the virtual array is given by \( L_v = \langle N_t(N_r - 1)/2 \rangle d \). An example of a three-element transmit and four-element receive array is shown in Figure 3 with corresponding element positions and array patterns.

---

1. The authors use the following definitions of dense, regular sparse and irregular sparse array in this paper. Dense array: when the inter-element distance is regular and less than or equal to \( \lambda_0/2 \). Regular sparse array: when inter-element distance is regular and greater than \( \lambda_0/2 \). Irregular sparse array: when inter-element distance is irregular and different for all elements.
2.3 Comparison of Cases I and II

Table 1 summarizes the comparison of the two cases in terms of the number of elements and the length of the arrays, where $L_t$ and $L_r$ are the physical lengths of the transmit and receive arrays. By comparing the dense and conventional virtual array cases from Figures 2 and 3 and Table 1, a few important observations can be made. It can be seen that for the same number of transmit and receive elements, the beamwidth of the virtual array in Case I is considerably wider than that of Case II. However, the SLL of the virtual array in Case I is more than 20 dB lower than that in Case II. This is because in Case I, half of the virtual array element positions are redundant and overlap with the other elements, thus resulting in a virtual amplitude tapering. Thus, the dense virtual array design does not take full advantage of the MIMO concept in terms of beamwidth and the conventional virtual array suffers from a high SLL, which can lead to unwanted target detection. Moreover, in the conventional virtual array design, the physical length of the transmit array is $N_r$ times larger than that of the transmit array in Case I. Because transmit and receive arrays are the physical arrays in MIMO radars, Case II leads to a large transmit array, which induces higher costs and more complications in design and realization, for example proper mechanical support. Case II is therefore only practically viable when the number of transmit and receive channels is small, such as in single radar chips in [14, 15], but for a large number of channels, such as for cascaded radar chip designs in for example [17] and for antennas-on-chip MIMO radars in for example [21], it is not a suitable solution.

Therefore, a third case for the design of virtual arrays is proposed here in which both transmit and receive arrays are irregular sparse. This results in an irregular sparse virtual array configuration that can outperform both Case I and Case II, and the best of both cases can be achieved, that is, narrow beamwidth and low SLL. Because it is well known that a large sparse array with an on-average interelement distance larger than $\lambda_0/2$ can result in the appearance of grating lobes in the FoV while scanning, this problem must be addressed in the design process. In the following section, we present optimization techniques that can be used to determine sparse configurations for both transmit and receive arrays, such that for the virtual array no grating lobes appear while scanning in the FoV.

FIGURE 2 Case I: Dense virtual array configuration example of three-element transmit (Tx) and four-element receive (Rx) array

FIGURE 3 Case II: Conventional virtual array configuration example of three-element transmit (Tx) and four-element receive (Rx) array
3 | OPTIMIZATION TECHNIQUES FOR VIRTUAL ARRAY SYNTHESIS

We propose two techniques for the synthesis of sparse transmit and receive arrays for MIMO radar. The synthesis result for such sparse arrays should provide a better performance of the corresponding sparse virtual array with scanning as compared with the dense and conventional virtual array cases explained in Section 2. In the first method, the sparse configurations of the transmit and receive arrays are synthesized independently from each other. In the second method the transmit array is synthesized depending on the obtained receive array configuration from the first method. Both methods use the TSP using (reweighted) iterative convex optimization. The TSP will be formulated first, and subsequently each method is described in detail in this section.

3.1 | Two-step synthesis procedure formulation

The TSP for antenna array sparsity via convex optimization with constraints on beamwidth, side lobe level, and minimum element spacing is developed and described here. The goal is to achieve a narrower beamwidth and lower SLL as compared with a dense array. The synthesis procedure follows two steps; the first step is an iterative \( \ell_1 \)-norm minimization via convex optimization for antenna sparsity proposed in [27] and the second step is an iterative algorithm for minimizing the SLL proposed in [29], with a constraint on the minimum element spacing. This TSP is then used to determine the sparse configuration of the transmit and receive array of a MIMO radar with desired beamwidth or SLL. Note that the objective functions and the constraints for the optimization are constructed for one dimension only that is along with the direction of the axis of the array (the x-direction in Figure 1). Here, the procedure is illustrated in more detail only for the transmit array. However, a similar procedure can be applied to synthesize the corresponding receive array.

Consider a dense seed array, as a starting point, with the number of elements \( N \) large enough to meet the beamwidth specification. A dense array implies a spacing of \( \lambda/2 \) or less, where \( \lambda \) is the free-space wavelength at the maximum frequency of operation. This spacing ensures the absence of grating lobes within the FoV even when scanning [35]. The first step is a reweighted \( \ell_1 \)-norm minimization solved via convex optimization for antenna array sparsity and returns \( N_t \) elements with optimal weights and a side lobe level below a preset threshold, that is, SLL\(_{\text{min}}\).

<table>
<thead>
<tr>
<th>Case</th>
<th>( N_t )</th>
<th>( L_2 )</th>
<th>( L_4 )</th>
<th>( L_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \left\lfloor \frac{N(N-1)}{2} \right\rfloor )</td>
<td>( \left\lfloor \frac{N(N-1)}{2} \right\rfloor - d )</td>
<td>( (N_t - 1)d )</td>
<td>( (N_t - 1)d )</td>
</tr>
<tr>
<td>II</td>
<td>( N_t N_r )</td>
<td>( (N_t N_r - 1)d )</td>
<td>( (N_t - 1)N_r d )</td>
<td>( (N_t - 1)N_r d )</td>
</tr>
</tbody>
</table>

Because the use of non-uniform weights for the transmit array elements is undesirable in most applications, the optimal weights of the \( N_t \) elements are reset to 1. This results in an increase in the SLL of the sparse array that is then used as an input to the second step to further reduce the SLL by repositioning the array elements. The first step is illustrated below:

**STEP I**: \( \min_{\mathbf{w}_t} \| \mathbf{Z}_t \mathbf{w}_t \|_{\ell_1} \)

subject to

\[
\begin{align*}
\text{AF}(\theta_{\text{scan}}) & = 1 \\
\text{AF}(\theta_i) & \leq \text{SLL}_{\text{min}}, \theta_i \notin [\theta_{\text{scan}} \pm \theta_{\text{FN}}]
\end{align*}
\]

where \( \mathbf{Z}_t = 1/|\mathbf{w}_{t-1} + \epsilon | \) is a diagonal matrix updated on the \( t \)-th iteration, \( \epsilon \) is a small positive number and determines the efficiency of the algorithm, and \( \mathbf{w}_t \) is the vector that contains the amplitude weights of the \( N_t \) array elements (see Equation 1). Further, \( \theta_i \) represents the side lobe region, and \( \theta_{\text{FN}} \) is the angle corresponding to the first null in the radiation pattern. In practice, \( \theta_i \) is represented by a set of discrete samples with a spacing of \( 1^\circ \). After each iteration, the solution vector \( \mathbf{w}_t \) contains varying amplitudes and only the non-negligible weight amplitudes can be selected, thus resulting in a sparse transmit array of size \( N_t \) and element positions collected in the vector \( \mathbf{d}_t \). The number of iterations for this step is usually very small; see [27].

In the second step, this sparse array is used and the non-uniform weights of all \( N_t \) elements are set equal to 1. The goal is now to adjust the antenna positions with small displacements \( \mathbf{d}' \), using a second iterative procedure, while maintaining a minimum distance \( d_{\text{min}} \) between the elements of the array. This procedure uses a first-order Taylor-series approximation of the antenna array factor to linearize it with respect to the antenna position displacements \( \mathbf{d}' \), see [29] for more details. The second step is illustrated below:

**STEP II**: \( \min_{\mathbf{d}'} \max_{\theta_i} \left| \text{AF}_i(\theta_i)(1+jk\mathbf{d}'\sin\theta_i) \right| \)

subject to

\[
\begin{align*}
\text{AF}_i(\theta_{\text{scan}}) & = 1 \\
\mathbf{d}' & = \epsilon, \quad \epsilon \ll \lambda/(2\pi) \\
|d_{t,n+1} + d_{r,n+1}'| & - |d_{t,n} + d_{r,n}'| \geq d_{\text{min}}
\end{align*}
\]

where \( n = 1, \ldots, N_r \). \( \mathbf{d}' \) and \( \mathbf{d} \) are both vectors of size \( N_r \). \( \mathbf{d}' \) must be smaller than \( \lambda/2\pi \) for the first-order Taylor series approximation to hold [29]. Note that the value of \( d_{\text{min}} \) is a design choice and depends on the size of the physical antenna.
element. For an example of the performance of the TSP used to synthesize a broadside array, the reader is referred to Section 2 of [36].

### 3.2 | Method I: TSP with maximum scanning

The outcome of the TSP is a sparse array with \( N_v \) elements and positions \( \mathbf{d} \), with its beamwidth equivalent to that of a larger array of \( N \) elements. This procedure can synthesize a sparse array for an arbitrary scan angle. It is a well-known fact that if the sparsity of the array is large, that is the average array element distance is larger than \( \lambda/2 \), grating lobes appear in the FoV when scanning away from broadside [13]. Therefore, to overcome this problem, we propose here that in Equations (5) and (6) by choosing scan angle \( \theta_{\text{scan}} = \pm \theta_{\text{max}} \) where \( \theta_{\text{max}} \) is the maximum scan angle in the FoV, the grating lobes do not appear in the FoV while scanning. However, it should be noted that the sparsity achieved for a specific scan range is usually lower than what can be achieved at broadside.

Furthermore, the above-stated TSP can be used to determine the sparse receive array for a particular maximum scan angle with \( N_v \) elements. Thus, in this Method I, the sparse transmit and receive arrays are designed individually using the TSP at the maximum required scan angle of the MIMO radar and the virtual array positions can be obtained from Equation (4). Because the virtual array pattern is the product of the transmit and receive array pattern, while designing the sparse transmit and receive arrays, a compromise between the SLL of one array (say the transmit array) and the beamwidth of the other array (say the receive array) can be made in order to achieve the maximum possible performance of the virtual array. A flow chart for Method I is illustrated in Figure 4. A numerical example to show the performance of this method and to validate these claims is presented in Section 4.1 and the performance is compared with an equivalent dense virtual array.

### 3.3 | Method II: Virtual array synthesis using the method of least squares

In Section 3.2, the TSP has been used to determine the sparse array configurations of both transmit and receive arrays of a MIMO radar individually, which means that the element positions of each array are independently selected from each other. However, Equation (4) suggests that the element positions of the receive array can be determined from the element positions of the transmit array or vice versa. Therefore, we propose here a second method to determine the virtual array positions using MLS, where the transmit and receive array element positions are determined from each other. In this method, first a sparse transmit array configuration is determined using the TSP at the maximum scan angle, as explained in Section 3.2. These transmit array element positions are then used to determine the receive array element positions using Equation (4) and via convex optimization of a least squares formulation to minimize the difference between the realized and desired virtual array element positions, which is formulated as follows:

Let \( \mathbf{d}_v^t \) be a vector that contains the element positions of a desired virtual array. The values of \( \mathbf{d}_v^t \) can be user defined, determined from Equation (4), and based on the required performance of the MIMO radar system and the practical limitation of transmit and receive arrays such as maximum length and minimum possible distance between elements. In
addition, let \( \mathbf{d}_r \) contain the realized virtual array element positions, depending on the transmit array element positions \( \mathbf{d}_t \) from the TSP and the receive array element positions \( \mathbf{d}_n \), which are the optimization variables. The objective of this optimization is to minimize the \( \ell_2 \)-norm of the difference between the realized \( \mathbf{d}_r \) and desired \( \mathbf{d}_v' \) virtual array positions, with \( \mathbf{d}_v' \) being the vector consisting of optimization variables with length \( N_v \). Constraints can be placed on \( \mathbf{d}_r \), such as the required size of the array and minimum distance between the elements. The optimization procedure along with the constraints is illustrated as follows:

\[
\begin{align*}
\min_{\mathbf{d}_v'} & \quad \| \mathbf{d}_r - \mathbf{d}_v' \|_2 \\
\text{subject to} & \\
& \ AF_v(\theta_{\text{scan}}) = 1 \\
& \ \max(\mathbf{d}_v') \leq L \\
& \ \min(\mathbf{d}_v') \leq -L \\
& \ \min(d_v(n + 1) - d_v(n)) \geq d_{\text{min}}
\end{align*}
\]

where \( n = 1, \ldots, N_v \) and \( L = L_v / 2 \) is the required length of the receive array, which depends on the physical limitation of the MIMO system.

The vector \( \mathbf{d}_v' \) is used to determine how to select the values for this vector \( \mathbf{d}_v' \), the relationship between the distance vector \( \mathbf{d}_v \), the length \( L_v \) and the number \( N_v \) of the virtual array is illustrated in Figure 5. The solid line shows that, for a conventional array design (Case II), there is a unique virtual element position for each virtual array element, which results in a large array aperture and consequently in narrow beamwidth. However, in the dense array design (Case I), almost half of the array elements have redundant element positions, as shown by the dashed line, thus resulting in a smaller aperture and wider beamwidth, as compared with the conventional array design. Although the conventional array design seems like the optimal option for narrow beamwidth, for a large number of transmit and receive array elements it might not be a practically viable solution, as explained in Section 2. Therefore, an array designer can choose any set of desired virtual array positions \( \mathbf{d}_v' \) that lies in between the dashed and solid lines of Case I and Case II in Figure 5.

A flow chart for Method II is illustrated in Figure 6. A numerical example to illustrate the performance of our proposed method is presented in Section 4.2.

### 4 | NUMERICAL EXAMPLES

Now, we investigate several numerical examples to demonstrate the performance of the methods explained in Section 3 and discuss these results in detail. In addition, for each example, a comparison is made with the corresponding dense array to highlight the improvement that can be achieved with sparse configurations. Each antenna element is assumed to have a cosine element pattern and only far-field radiation patterns are calculated. Furthermore, to solve the optimization routines in both methods, CVX in MATLAB has been used, which is a package to specify and solve convex problems [37, 38].
4.1 Method I: TSP with maximum scanning

In order to evaluate the performance of the method formulated in Section 3.2, we start with a linear seed array of \(N = 40\) elements with element spacing of \(0.5\lambda\). In Equation (5), the \(SLL_{\text{min}}\) is set to be \(-20\) dB and in Equation (6), the minimum element distance \(d_{\text{min}}\) is set to be \(0.45\lambda\). These two steps are executed separately for two scan angles, first at broadside, that is, \(\theta_{\text{scan}} = 0^\circ\), and then at the maximum scan angle \(\theta_{\text{scan}} = 40^\circ\). The element positions that come out of the TSP for these two scan angles are shown in Figure 7, and the corresponding normalized radiation patterns for each configuration are shown in Figure 8.

It can be seen from Figure 7 that for each scan angle the outcome of the TSP is different, both in terms of the number of elements and in terms of the element positions. For the broadside direction, the number of elements is reduced to half of that of the original array, and the array element positions are sparse, see the red crosses in Figure 7. Moreover, the scanning behaviour of this sparse configuration is shown in Figure 8 (top), which indicates that scanning up to the maximum scan angle results in the appearance of grating lobes in the FoV. However, by determining the sparse configuration using the TSP at maximum scan angle, as proposed in Section 3, this issue can be resolved. For a maximum scan angle of \(40^\circ\), the number of elements is reduced by one fourth the original array; see the yellow crosses in Figure 7. The scanning behaviour of this configuration, in Figure 8 (bottom) shows that no grating lobes appear in the entire FoV while scanning. Note that this array configuration is asymmetric but because the difference is small, no grating lobes will appear in the radiation pattern at scan angle \(-40^\circ\). This configuration can be used for either the transmit or the receive array, but for the purpose of illustration, we will use this configuration as the receive array with \(N_r = 31\) for a MIMO radar array design.

Now, for the transmit array of a MIMO radar, we use the same procedure to obtain a sparse configuration from the TSP at \(\theta_{\text{scan}} = 40^\circ\), with \(SLL_{\text{min}} = -15\) dB and \(d_{\text{min}} = 0.45\lambda\), which results in \(N_t = 20\) for the element positions shown in Figure 9.

---

**Figure 9** Optimized transmit positions for \(\theta_{\text{scan}} = 0^\circ\) and \(\theta_{\text{scan}} = 40^\circ\) with \(SLL_{\text{min}} = -15\) dB obtained via Method 1.

**Figure 10** MIMO virtual array patterns for \(\theta_{\text{scan}} = 0^\circ\) for the sparse configuration and the equivalent dense array obtained via Method 1. Inset: close-up of the main beam at broadside showing an improvement in beamwidth.
Subsequently, we determine the virtual array element positions using Equation (4). The virtual array thus has $20 \times 31 = 620$ elements with irregular positions and the corresponding virtual array radiation pattern is shown in Figure 10 for broadside. For comparison, the radiation pattern of the equivalent dense virtual array, with transmit and receive array elements with half-wavelength spacing and uniform amplitude, is also shown. The sparse configuration exhibits approximately a $1.2^\circ$ improvement in beamwidth (i.e. 20%) and a 7 dB improvement in SLL compared with the equivalent dense array (array with the same number of elements but with half-wavelength spacing).

Moreover, the scan behaviour of the virtual array is shown in Figure 11 for four different scan angles, that is, $\theta_{\text{scan}} = -40^\circ$, $-20^\circ$, $20^\circ$, and $40^\circ$. This figure shows that grating lobes do not appear in the scan range and that the maximum side lobe level is well below $-30$ dB. Thus, with this synthesis procedure an improvement in beamwidth and SLL can be achieved for a MIMO radar array with a sparse configuration for both the transmit and the receive array, especially for scanning up to the maximum scan angle. It is important to mention here that for some radar applications a SLL higher than $-30$ dB is acceptable. In such cases, the SLL threshold for the transmit and/or receive array can be relaxed, which ultimately results in either more sparsity or more improvement in beamwidth of the virtual array.

4.2 | Method II: virtual array synthesis using the method of least squares

Now, we use the receive array with $N_r = 31$ elements from the TSP as shown in Figure 7 and determine the transmit array element positions $d_t$ with number of elements $N_t = 20$ using the optimization formulation formulated in Section 3.3. In Equation (7), the maximum length of the array is set to be $\pm L = \pm 10\lambda$ and $d_{\text{min}} = 0.5\lambda$. The optimized transmit, receive,
and virtual array element positions are shown in Figure 12, and the corresponding array patterns are shown in Figure 13. In this example, the desired virtual array element position vector \( \mathbf{d}'_n \) is obtained by using Equation (4) for \( d_n(n) = 2n\lambda/2 \) with \( n = -N_x/2, -(N_x - 1)/2, \ldots, N_x/2 \) and \( d_n(n) = n\lambda/2 \) with \( n = 0, 1, \ldots, N_x \). The desired and realized virtual array positions related to the number of virtual array elements are shown in Figure 14.

It can be seen that the synthesized virtual array length is large compared with the dense virtual array and thus results in a narrow beamwidth. Also note that the synthesized transmit array in Figure 12 is nearly symmetric across positive and negative \( x \)-axis, but it is inconsequential because the array pattern is determined by the relative positions of the elements. The comparison of the dense, conventional, and sparse virtual array for broadside scan is shown in Figure 15. In comparison with the equivalent dense virtual array (blue line), the synthesized virtual array (black line) has a 3° improvement in beamwidth (i.e., 41%) and a 7 dB improvement in maximum SLL (indicated by the blue and black horizontal lines). However, in comparison with the conventional array (red line), the synthesized virtual array has a much lower side lobe level (indicated by the red and black horizontal lines). Thus, it is shown that with our sparse configuration synthesis of the virtual array, the best of Cases I and II can be obtained.

For the four scan angles \( \theta_\text{scan} = -40^\circ, -20^\circ, 20^\circ \), and \( 40^\circ \), the virtual array patterns are shown in Figure 16. It is shown that no grating lobes appear while scanning and the maximum SLL remains below \(-20 \) dB for the maximum scan angle of \( \theta_\text{scan} = 40^\circ \). In addition, the side lobes around the main beam are lower as compared with the larger angles, but the maximum value does not exceed \(-20 \) dB, which is acceptable for most radar applications.

It is useful to mention here that this virtual array synthesis procedure can be used to further increase the system resolution by selecting different desired virtual array positions \( \mathbf{d}'_n \) with a larger slope in Figure 5. In addition, the value of \( L' \) can be increased or decreased based on the physical design requirements.

In comparison with the Method I, the SLL performance of this method is slightly deteriorated especially at higher scan angles. However, the beamwidth in case of Method II is improved by 1.8°. This is because of the fact that the transmit array in Method II is a larger sparse array as compared with the transmit array of Method I. Hence, the advantage of Method II is that it allows improved beamwidth that is as close as possible to Case II of MIMO virtual arrays.

**Figure 13** Transmit, receive, and virtual array patterns at broadside for the optimized sparse configurations in Figure 12. TSP, two-step synthesis

**Figure 14** Position versus number of elements: desired, synthesized (realized), and dense comparison

**Figure 15** Multiple-input multiple-output virtual array patterns at broadside for the sparse configurations and the equivalent dense and conventional configuration. The margins for maximum side lobe are shown for each plot
While considering scanning in an array system, another parameter of importance is the scan loss. When the main beam is scanned to a larger angle, the peak gain of the array is reduced. For the purpose of clarity, a vertical line at maximum side lobe level at 40° obtained only from MATLAB synthesis models is added to each graph. The positions used for the transmit and receive arrays is negligible, the transmit and receive arrays are simulated individually [18]. The virtual array patterns are obtained for transmit and receive arrays synthesized with Method I and are shown in Figure 17 for three scan angles.

In addition, the virtual array patterns are obtained for transmit and receive arrays synthesized with Method II and are shown in Figure 18 for three scan angles. It can be seen that in comparison with Figures 11 and 16, the virtual array performance is in good agreement with the results of the synthesis models. However, in particular at the maximum scan angle, the side lobe levels are 3–4 dB higher because of the mutual coupling between the array elements.

For the purpose of clarity, a vertical line at maximum side lobe level at 40° obtained only from MATLAB synthesis models is added to each graph. The positions used for the transmit and receive arrays for wavelength at 64 GHz are given in Table 2.

While considering scanning in an array system, another parameter of importance is the scan loss. When the main beam is scanned to a larger angle, the peak gain of the array is

5 | VALIDATION USING FULL-WAVE SOLVER

The two numerical examples of the two proposed methods, presented in Section 4, have been implemented in a full-wave solver (CST Microwave studio). For this purpose, a single-layer patch antenna has been used at 64 GHz, which has been matched in the range of 62–64 GHz, and the transmit and receive arrays of these patch antennas with the configurations obtained from the two proposed methods (Figures 7, 9, and 12) have been implemented in CST. Assuming that the separation between the transmit and receive physical arrays is sufficiently large, such that the coupling between the transmit and receive arrays is negligible, the transmit and receive arrays are simulated individually [18]. The virtual array patterns are obtained for transmit and receive arrays synthesized with Method I and are shown in Figure 17 for three scan angles.

In addition, the virtual array patterns are obtained for transmit and receive arrays synthesized with Method II and are shown in Figure 18 for three scan angles. It can be seen that in comparison with Figures 11 and 16, the virtual array performance is in good agreement with the results of the synthesis models. However, in particular at the maximum scan angle, the side lobe levels are 3–4 dB higher because of the mutual coupling between the array elements.

For the purpose of clarity, a vertical line at maximum side lobe level at 40° obtained only from MATLAB synthesis models is added to each graph. The positions used for the transmit and receive arrays for wavelength at 64 GHz are given in Table 2.

While considering scanning in an array system, another parameter of importance is the scan loss. When the main beam is scanned to a larger angle, the peak gain of the array is

**FIGURE 16** MIMO virtual array patterns at scan angles −40°, −20°, 20°, and 40° for sparse configurations of Method I

**FIGURE 17** Full-wave simulation results of MIMO virtual array patterns at 0°, 20°, and 40° for sparse configurations of Method I
reduced because of the element patterns [13]. In Figure 19, the peak gain over the scan angles for the transmit array and receive array synthesized for both methods are shown. At the maximum scan angle the gain of the main lobe reduces by maximum 3 dB.

5.1 Mutual coupling inclusion in optimization

Although the full-wave simulation validated the two proposed methods for the claim of scanning without appearance of grating lobes in the FoV, it has however raised an issue of mutual coupling between the physical array elements that ultimately increased the side lobe level of the focused virtual array pattern. Therefore it is important to include the effects of mutual coupling while synthesizing the array. In the case of MIMO radars and virtual arrays, the physical arrays are the transmit and receive arrays, and the mutual coupling between their elements can be accounted for by considering the embedded element pattern (EEP) of each antenna element. These EEPs deviate from the cosine antenna patterns that we assumed in Section 4 for the numerical examples of the two methods. In order to include the effects of mutual coupling, instead of using a simple cosine pattern in the synthesis of the transmit and receive arrays, the EEPs of all the elements obtained from the CST simulations are used in STEP II of Equation (6). This allows further adjustment of the positions of the array elements while accounting for mutual coupling and maintaining the minimum distance.

As an illustration, we take the transmit and receive arrays of the numerical example of Method I in Section 4 and the EEPs from the CST simulations. We then perform STEP II of TSP procedure with the simulated EEPs instead of ideal cosine patterns and obtain a new configuration for both transmit and receive arrays. The virtual array patterns after including the mutual coupling are shown in Figure 20, for maximum scan angle 40°. It can be seen that by accounting for the mutual coupling between the array elements while synthesizing the transmit and receive arrays in MATLAB, the simulated virtual array pattern from CST is more closely related to the synthesized pattern from MATLAB.

<table>
<thead>
<tr>
<th>#</th>
<th>Transmit for Method I (mm)</th>
<th>Transmit for Method II (mm)</th>
<th>Receive for Method I/II (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.95</td>
<td>-73.84</td>
<td>-2.89</td>
</tr>
<tr>
<td>2</td>
<td>16.50</td>
<td>-56.96</td>
<td>6.16</td>
</tr>
<tr>
<td>3</td>
<td>21.39</td>
<td>-46.73</td>
<td>11.37</td>
</tr>
<tr>
<td>4</td>
<td>24.03</td>
<td>-38.77</td>
<td>14.02</td>
</tr>
<tr>
<td>5</td>
<td>26.77</td>
<td>-31.98</td>
<td>16.80</td>
</tr>
<tr>
<td>6</td>
<td>29.38</td>
<td>-25.75</td>
<td>19.44</td>
</tr>
<tr>
<td>7</td>
<td>33.78</td>
<td>-19.98</td>
<td>22.33</td>
</tr>
<tr>
<td>8</td>
<td>36.03</td>
<td>-14.64</td>
<td>24.97</td>
</tr>
<tr>
<td>9</td>
<td>38.60</td>
<td>-9.70</td>
<td>27.80</td>
</tr>
<tr>
<td>10</td>
<td>42.04</td>
<td>-4.84</td>
<td>30.50</td>
</tr>
<tr>
<td>11</td>
<td>49.79</td>
<td>0.12</td>
<td>33.36</td>
</tr>
<tr>
<td>12</td>
<td>53.26</td>
<td>4.99</td>
<td>36.31</td>
</tr>
<tr>
<td>13</td>
<td>56.68</td>
<td>9.99</td>
<td>38.86</td>
</tr>
<tr>
<td>14</td>
<td>59.26</td>
<td>15.29</td>
<td>41.17</td>
</tr>
<tr>
<td>15</td>
<td>61.76</td>
<td>20.98</td>
<td>43.56</td>
</tr>
<tr>
<td>16</td>
<td>65.05</td>
<td>27.20</td>
<td>45.73</td>
</tr>
<tr>
<td>17</td>
<td>67.68</td>
<td>34.07</td>
<td>47.85</td>
</tr>
<tr>
<td>18</td>
<td>70.91</td>
<td>42.03</td>
<td>49.97</td>
</tr>
<tr>
<td>19</td>
<td>77.19</td>
<td>52.24</td>
<td>52.44</td>
</tr>
<tr>
<td>20</td>
<td>87.20</td>
<td>69.28</td>
<td>54.82</td>
</tr>
<tr>
<td>21</td>
<td>97.87</td>
<td>86.42</td>
<td>57.21</td>
</tr>
<tr>
<td>22</td>
<td>57.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>59.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>63.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>66.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>69.70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>72.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>75.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>78.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>80.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>83.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1Dimensions of patch antenna are 1.19 × 1.52 mm and dimensions of ground plane are 2.05 × 2.20 mm.
element spacing of the sparse array to exceed the critical value at which it will result in the appearance of grating lobes in the FoV at the maximum frequency of operation [6]. Thus, for any frequency below the maximum it will not result in the appearance of grating lobes. Therefore, the synthesized sparse array is broadband, which was verified by means of simulations. However, the physical effects of bandwidth on the performance of the array, such as distortions in element patterns and variation in element input impedance due to mutual coupling, can be incorporated in the design by co-simulation with a full-wave solver, as presented in [28].

It is worthwhile to mention a few general limitations of the realization of sparse transmit and receive arrays in MIMO radars. Firstly, the feeding of antenna elements in a sparse array can be a challenging task because the feed lines from RF channels to the antenna are not of equal length for every element in the array, which can result in a complex design for the feed network, especially for a large array. This is relevant, in particular, in time division multiplexing mode in MIMO radars, because the reception in this mode is simultaneous and requires equal length of the feed lines [21]. Secondly, the phase and amplitude errors caused by the manufacturing tolerances on the element positions and electronics can result in higher SLL and an increase in beamwidth, which should be addressed while realizing the sparse arrays. Last, in target-detection radar imaging systems, the use of the fast Fourier transform (FFT) is desirable for the evaluation of the angle of arrival, which assumes a regular spatial sampling in the antenna arrays, that is, a regular interelement distance. Thus, the FFT is not suitable for irregular sparse arrays. Several non-uniform FFT algorithms have been proposed in the literature, for example [39, 40], and this problem has been efficiently solved especially for the case of MIMO radars in [16].

7 | CONCLUSION AND FUTURE WORK

Two methods for the synthesis of sparse virtual array have been proposed, formulated, and illustrated with numerical examples. Both methods determine the topology of the sparse transmit and receive array that results in a sparse virtual array design. In the first method, both transmit and receive array element positions are independently determined using a TSP for the antenna array sparsity, which are formulated and explained. In the second method, the sparse transmit array element positions are determined using the TSP, while the receive array positions are determined using the MLS and depend on the transmit array element positions. It has been shown with the help of numerical examples that while synthesizing a sparse array via the TSP, if the element positions are determined with the main beam at the maximum scan angle, no grating lobes appear while scanning over the whole FoV. Further, it has been shown that a sparse virtual array can achieve a better performance in terms of beamwidth and side lobe level than possible with the equivalent dense virtual array. An improvement of a maximum of 3° in

FIGURE 19  Peak gain versus the scan angle for receive array of Method I and transmit array of Method I or II

FIGURE 20  MIMO virtual array patterns at 40° for sparse configurations of Method I after including mutual coupling

6 | DISCUSSION OF STRENGTHS AND LIMITATIONS

The methods presented in preceding sections can be used for a wide variety of applications because the preset parameters and formulations of the optimization problems can be adjusted according to the design requirements and physical limitations induced by the application. Therefore, an array designer has the freedom to make several adjustments that still fit the framework of the methods proposed such that desirable results can be achieved. Furthermore, by choosing $\lambda$ to be the wavelength at the maximum frequency of the required band of operation and by selecting $\theta_{max}$ to be the maximum scan angle in the required FoV, we guarantee that no grating lobes appear in the FoV even for the entire frequency band of operation. The reason for this is that, in Equations (5) and (6), the constraints on SLL and minimum distance do not allow the average
beamwidth and a maximum of 7 dB side lobe level is reported in the examples. Furthermore, in the presented formulations of the synthesis methods, physical limitations on the maximum size of the arrays and minimum element spacing can be included, which allows for these methods to be used in a wide variety of applications. In the future, it is valuable to consider the gain of the array for the synthesis of sparse arrays for MIMO radar, because the gain of a linear array also depends on the distance between the array elements and strongly affects the signal-to-noise ratio of these kinds of systems.

ACKNOWLEDGEMENTS

This work is part of research programme HTSM with project number 13922 funded by NWO TTW (Netherlands Organization for Scientific Research).

ORCID

Rabia Z. Syeda [https://orcid.org/0000-0001-9707-8063]

REFERENCES

34. Harter, M., Ziroff, A., Zwick, T.: Three-dimensional radar imaging by
digital beamforming. In: Proceedings of the 8th European Radar Con-
ference (EuRAD), pp. 17–20, Manchester (2011)
USA (2005)
36. Syeda, R.Z. et al.: Sparse MIMO array for improved 3D mm-
waveimaging radar. In: Proceedings of the 17th European Radar Con-
ference (EuRAD), Utrecht (2020)
37. Grant, M., Boyd, S.: CVX: Matlab software for disciplined convex pro-
2020
38. Grant, M., Boyd, S.: Graph implementations for nonsmooth convex
programs. Recent Advances in Learning and Control (a tribute to M.
Vidyasagar). In: Blondel, V., Boyd, S., Kimura, H. (eds.) Lecture Notes in
Control and information Sciences, pp. 95–110. Springer-Verlag, London
(2008)
A529–A547 (2018)
40. Wang, J., Cetinkaya, H., Yarovoy, A.: NUFFT based frequency-
wave number domain focussing under MIMO array configurations. In:
https://doi.org/10.1109/RADAR.2014.7133686

How to cite this article: Syeda, R.Z., van Beurden,
M.C., Smolders, A.B.: Sparse virtual array synthesis for
MIMO radar imaging systems. IET Microw. Antennas