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Avalanches in a Bose-Einstein Condensate

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Collisional avalanches are identified to be responsible for an 8-fold increase of the initial loss rate of a large ^{87}Rb condensate. We show that the collisional opacity of an ultracold gas exhibits a critical value. When exceeded, losses due to inelastic collisions are substantially enhanced. Under these circumstances, reaching the hydrodynamic regime in conventional Bose-Einstein condensation experiments is highly questionable.

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One of the current goals in the field of Bose-Einstein condensation (BEC) is the production of a condensate in the collisionally opaque or hydrodynamic regime, where the mean free path of an atom is much less than the size of the sample. This would offer the opportunity to study striking phenomena such as quantum depletion or dynamical local thermal equilibrium. In this context, one possible approach is to increase the interaction among the atoms by means of Feshbach resonances [1]. It has been observed, however, that in their vicinity the large cross section for elastic collisions is accompanied by a dramatic increase of atom losses [2,3]. Hence, it seems advantageous to follow a different route by producing large and dense condensates.

In this Letter, we conclude that the collisionally opaque regime can hardly be reached in alkali BEC experiments. We identify an intrinsic decay process that severely limits the average column density $\langle nl \rangle$ of condensates at values achieved in present BEC experiments. It is based on collisional avalanches that are triggered by inelastic collisions between condensate atoms. A considerable part of the energy released in these initiatory collisions is distributed among trapped atoms resulting in a dramatic enhancement of the total loss from the condensate. In analogy to the critical mass needed for a nuclear explosion, we define a critical value of the collisional opacity $\langle nl \rangle \sigma_s$, with $\sigma_s = 8\pi a^2$ the s-wave cross section for like atoms and a the scattering length. The *critical opacity* equals 0.693, corresponding to a collision probability of 0.5. Related scenarios have been discussed in Refs. [4,5], but were assumed to play a minor role in the experimentally relevant region. However, we present strong experimental evidence that the anomalous decay of our ^{87}Rb condensate is caused by collisional avalanches. This is supported by the good agreement of a simple model with the data.

The crucial point for the occurrence of an avalanche is whether the products of a one-, two-, or three-body decay process have a substantial probability

$$p(E) = 1 - \exp[-\langle nl \rangle \sigma(E)] \qquad (1)$$

of undergoing secondary collisions before leaving the trap [6], with $\sigma(E)$ the total cross section at kinetic energy E. The collision probability varies significantly with temperature and is usually highest in the s-wave scattering regime.

Here, the differential cross section is isotropic in the center-of-mass system; in the laboratory system the two atoms fly apart at an angle of $\pi/2$ on average. The energy of the projectile is on average equally distributed among the two colliding atoms. This implies that each collision results in *two* new atoms that both can continue their collisional havoc in the trap until they leave the condensate (Fig. 1). If the probability for collisions is higher than 0.5, the average number of colliding atoms increases with every step of the collisional chain which now becomes self-sustaining.

To calculate the total loss from the condensate, we start from the well-known loss rates $\dot{N}_i = -K_i N \langle n^{i-1} \rangle$, with $i = 1, 2, 3$ associated with one-, two-, and three-body decay processes with rate constant K_i, respectively. Here, N is the number of atoms in the gas with the density distribution $n(\vec{r})$. Depending on the energy of the decay products, typically a few or even no further collisions are needed to generate an atom with an energy $E_{i,s}$ whose next collision

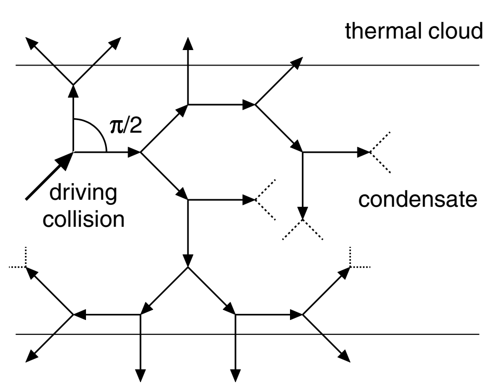

FIG. 1. Sketch of a collisional avalanche in a homogeneous condensate with a radius equal to twice the mean free path $(n\sigma_s)^{-1}$, illustrating the enhancement of the loss rate.

would be in the s-wave regime. The probability for this collisional chain is $p_{i,1} \cdot p_{i,2} \cdots = \tilde{p}_i$ with $p_{i,n} = p(E_{i,n})$. During this process, on average $\tilde{g}_i$ atoms are lost from the condensate without undergoing secondary collisions. Next, an atom with energy $E_{i,s}$ induces an avalanche with a collision probability $p_s = 1 - \exp[-\langle nl \rangle \sigma_s]$ that now is independent of energy. Consequently, atoms with an energy $E_{i,s}/2^k$ in the kth step of an avalanche are generated with a probability of $\tilde{p}_i p_s^k$. Since every collision now results in two projectiles in the next step, the degeneracy of step k is 2^k. The rate at which atoms are lost from this avalanche step is $\tilde{p}_i \dot{N}_i 2^k p_s^k (1 - p_s)$ and the total loss rate from the condensate becomes

$$\dot{\mathcal{N}}_{i,\mathrm{aval}} = \dot{N}_i \left[\tilde{g}_i + \tilde{p}_i (1 - p_s) \sum_{k=0}^{k_i^{\max}} 2^k p_s^k \right], \tag{2}$$

where the sum extends over all relevant avalanche steps.

To determine the cutoff $k_i^{\max}$, note that avalanche-enhanced losses can occur up to a step with an energy on the order of the chemical potential. However, when the energy falls short of the trap depth, E_{trap}, the atoms lost from the cold sample are still trapped. They will repeatedly penetrate the cloud and thus give rise to heating. This will either be compensated by an evaporation of atoms from the trap or will reduce the condensed fraction by increasing the temperature. Both possibilities are not described by Eq. (2). Therefore, we use $k_i^{\max} \le \log_2(E_{i,s}/E_{\mathrm{trap}})$ as a cutoff and, hence, account only for immediate trap losses.

The additional heat-induced depletion caused by trapped avalanche atoms can easily be estimated for the case that the temperature is fixed by the trap depth. Each atom participating in step $(k_i^{\max} + 1)$ of the avalanche will finally dump about the energy $E_{\mathrm{trap}}/2$ into the system. Since any evaporated atom takes the energy E_{trap} with it, about half as many atoms as are produced in the step $(k_i^{\max} + 1)$ of the avalanche have to be evaporated to keep the temperature constant. Hence, the evaporation rate is

$$\dot{\mathcal{N}}_{i,\mathrm{heat}} \simeq (1/2) \tilde{p}_i \dot{N}_i 2^{(k_i^{\max}+1)} p_s^{(k_i^{\max}+1)}. \tag{3}$$

Equation (2) predicts substantially enhanced losses as soon as the critical opacity is exceeded. However, for a given $k_i^{\max}$ there is a second critical value of the opacity above which the loss rate $\dot{\mathcal{N}}_{i,\mathrm{aval}}$ decreases again. Now, with increasing opacity the limited trap depth continuously looses its shielding effect against the products of inelastic collisions, since most avalanches generate trapped particles. In the collisional regime with $p_s \simeq 1$, the energy released in an inelastic process will be entirely dissipated in the system. This results in an explosionlike particle loss according to Eq. (3).

To apply our model, the column density must be evaluated according to

$$\langle nl \rangle = \int [n(\vec{r})/N] \int [n(\vec{r} + \vec{R})/4\pi R^2] \, d^3R \, d^3r \tag{4}$$

$$= c n_p W_\perp \int_0^\infty dx (1 + x^2)^{-1} (1 + \varepsilon^2 x^2)^{-1/2}, \tag{5}$$

where the second line is the result for the case of a harmonic potential with cylindrical symmetry. Here, $\varepsilon = \omega_\parallel/\omega_\perp$ is the ratio of the trap frequencies and n_p is the peak density of the cold sample. For the parabolic density distribution of the condensate, $W_\perp$ is the half radial width and $c = 5/12$. Because of the scaling $n_p \propto N^{2/5}$ and $W_\perp \propto N^{1/5}$ the column density of a condensate scales as $N^{3/5}$, so that the effect of multiple collisions is quite persistent. For a Gaussian distribution we find $W_\perp = \sigma_\perp$, $c = \sqrt{\pi/8}$, and a scaling according to $\langle nl \rangle \propto N$. In a harmonic potential, the ideal Bose distribution can be represented as a sum of Gaussian distributions and the latter result can thus be used to evaluate the column density close to degeneracy. For a Bose distribution, the opacity scales disproportionate to N, resulting in a faster decline of the avalanche enhancement than in the case of a Thomas-Fermi distribution.

The next step is to identify the energies of the initial decay products. For a background gas collision, $E_{1,1}$ depends on the mass of the impinging particle that is assumed to be Rb in our system. In the case of spin relaxation, $E_{2,1}$ equals either the Zeeman energy or the hyperfine splitting energy. For three-body recombination $E_{3,1}$ has to be derived from the binding energy of the most weakly bound level in which the dimer is predominantly formed. Clearly, the molecule is likely to be deactivated in a subsequent inelastic collision with a condensate atom [7,8]. Deactivating collisions will be a serious problem in highly opaque clouds where atoms with higher energies still have high collision probabilities. In our experiment, however, the collision probability is significantly smaller at typical deactivation energies of 0.1 K than at the binding energy of the molecules in the last bound level. In our analysis we therefore do not account for avalanches triggered by deactivating collisions. The values of all parameters used to calculate the effective losses are listed in Table I. Note that in order to account for the avalanches triggered by the two-body decay, the partial rates associated with the various exit channels are needed since they correspond to different energies released in the process.

Finally, the presence of a diffuse atom cloud in the trapping volume can cause additional losses (see, e.g., [4]). In a steep magnetic trap with a depth of a few mK, such an "Oort" cloud is mainly a consequence of incomplete evaporation at high magnetic fields [9] or low radio frequency (rf) power. In our experiment, the temperature of the diffuse cloud will probably be on the order of 400 μK, corresponding to the measured initial temperature of the magnetically trapped cloud. Even in a rf-shielded trap these atoms will penetrate the condensate giving rise to an additional decay rate according to $1/\tau = n_{\mathrm{Oort}} \sigma_s v_{\mathrm{Oort}}$, with n_{Oort} and v_{Oort} the density and the thermal velocity of the penetrating atoms, respectively. Collisions with Oort atoms will also trigger avalanches, because the collision energy is close to the s-wave regime.

To compare our data with the predictions of the model, the differential and the total scattering cross sections are

TABLE I. Rate constants for the initiatory processes and energies of the subsequent collisions that are necessary to generate the first avalanche atom with energy E_s.

i	Type	Rate constant K_i	$\Delta E_1, \ldots, \Delta E_n = E_s$ $[(3/2)k_B]$
1	Background	$1/(39\ s)$[a]	4 K, 100, 5, 0.5 mK
2	Zeeman[b]	1.4×10^{-18} cm^3/s	0.022 mK
	2 × Zeeman[c]	3.7×10^{-17} cm^3/s	0.045 mK
	Hyperfine[d]	2.2×10^{-16} cm^3/s	109, 5, 0.5 mK
	2 × Hyperfine[e]	1.3×10^{-16} cm^3/s	219, 8, 0.5 mK
3	Recombination[f]	1.8×10^{-29} cm^6/s	0.54 mK[g]

[a][17]; [b,c,d,e][11,18]; [f][15,19]; [g][20].

needed. Above a kinetic energy E/k_B of 60 mK we calculate the energy transfer by collisions using a model function for the small-angle differential cross section [10]. For collisions below 60 mK we use the numerical results from a full quantum treatment [11]. For ^{87}Rb in the $|2,2\rangle$ state, the large contribution of a d-wave scattering resonance to the total cross section leads to $\sigma(E) \simeq 4 \times \sigma_s$ at an energy of $E/k_B = 580\ \mu$K in the lab system. This almost coincides with the energy transferred to the third atom in a recombination event (Table I). Hence, a secondary collision of this atom will occur with a probability of 0.99 already when the probability for s-wave collisions among condensate atoms is 0.7. For kinetic energies $E/k_B \leq 1.5$ mK, the total cross section obeys $\sigma(E) \gtrsim \sigma_s$. For simplicity, we use σ_s for calculating avalanches in this energy range. Our model therefore yields a lower bound for the total loss.

The apparatus used to study the condensates has been described previously [12,13]. The experiment is performed with ^{87}Rb atoms in the $|2,2\rangle$ state. A Ioffe-Pritchard magnetic trap with a bias field of 2 G and oscillation frequencies of $\omega_\perp/2\pi = 227$ Hz and $\omega_\parallel/2\pi = 24.5$ Hz is used. The atoms are cooled by rf evaporation and then held in the trap for a variable time interval. During the storage time, the trap depth is set to $E_{\mathrm{trap}}/k_B = 4.4\ \mu$K by means of the rf shield. From the width of the density distribution after expansion the atom number N_C in the condensate is determined. At minimum storage time, we find $N_C = 1.1 \times 10^6$ atoms and $n_p = 6.4 \times 10^{14}$ cm^{-3} and no discernible noncondensed fraction.

The decay curve of the condensate is shown in Fig. 2, revealing that about half the initial number of atoms is lost within the first 100 ms. The dotted line shows the theoretical prediction assuming that losses occur solely due to background gas collisions, spin relaxation and recombination (Table I). The observed loss is 8 times faster than predicted. Moreover, the additional decay is clearly nonexponential and can therefore not result from primary collisions with Oort atoms. Hence, multiple collisions have to be taken into account.

Indeed, with $\langle nl\rangle\sigma_s = 1.4$ the critical opacity is considerably exceeded. To the best of our knowledge, the corresponding s-wave collision probability of $p_s = 0.76$ has not been reached in published work on Rb condensates in the off-resonant scattering regime. This explains why our observations differ from those made in other experiments [14–16]. The dashed line displayed in Fig. 2 has been obtained by numerically integrating the rate equation $\dot{N} = \sum_{i=1}^{3}(\dot{\mathcal{N}}_{i,\mathrm{aval}} + \dot{\mathcal{N}}_{i,\mathrm{heat}})$ that describes avalanche-enhanced losses according to Eqs. (2) and (3), without any adjustable parameter and neglecting the contribution of an Oort cloud. We find good agreement between theory and experiment within the first 200 ms, showing that collisional avalanches triggered by recombination events are responsible for the fast initial decay.

To investigate the role of an Oort cloud, we have performed a similar experiment with an atom cloud at a lower density. Figure 2 shows the decay of a noncondensed cloud with 1×10^7 atoms at a temperature of 1 μK and a peak density of 3.5×10^{14} cm^{-3}. The number of atoms is determined from the total absorption of near-resonant laser light. The trap depth is limited to 10 μK, according to the higher temperature of the sample. Again, the decay is nonexponential and initially about two times faster than

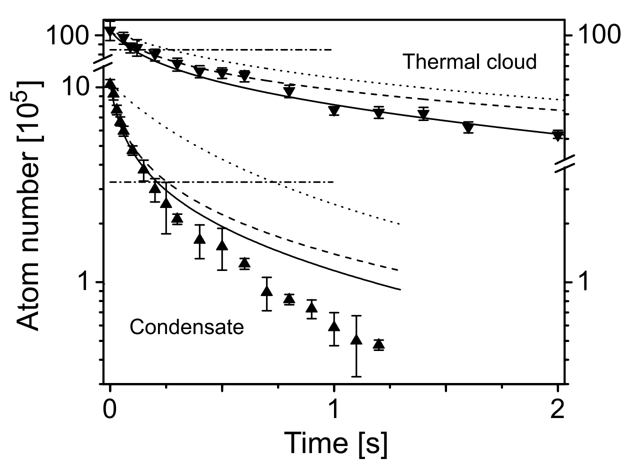

FIG. 2. Decay of the condensate and the thermal cloud. The horizontal line corresponds to the critical opacity. For comparison, the calculated decay due to the initial one-, two-, and three-body loss rates without (dotted line) and with avalanche enhancement (dashed line) are shown. The full line includes the effect of an Oort cloud with avalanche enhancement.

predicted by the primary loss rates (dotted line). At an opacity of 0.9, obtained by assuming an ideal Bose distribution, we already expect a weak avalanche enhancement. This allows us to test our model in a different regime since in a thermal cloud avalanches are less persistent than in a condensate. In addition, the intrinsic two- and three-body decay rates will die out during the observation time whereas the effect of an Oort cloud as a one-body decay will persist. The solid line in Fig. 2 is the prediction of our model where we have included an avalanche-enhanced decay rate caused by an Oort cloud. Good agreement with the data is obtained for $1/\tau = 1/7.8$ s, corresponding to $n_{\rm Oort} = 5 \times 10^8$ cm^{-3} at 400 μK. Such a density is produced by only a few times 10^5 atoms and appears realistic in view of the more than 10^9 atoms that were loaded into the magnetic trap. It is also consistent with the fact that we have no direct experimental evidence for an Oort cloud and that the initial decay is correctly predicted by the model even if the contribution of the Oort cloud is neglected (dashed line).

We can now calculate the extra loss rate of the condensate due to an Oort cloud. Since the two experiments described above are performed under identical conditions, the density of the Oort cloud is essentially unchanged in the two measurements. As can be seen from the solid line in Fig. 2, the small extra contribution from the Oort cloud does not change the predicted initial decay, but slightly improves the agreement between the model and the data for longer times. The small remaining discrepancy can be the result of an additional decay not accounted for in our model. In particular, avalanches will seriously perturb the equilibrium of the condensate by inducing local fluctuations of the mean-field energy [5]. Since the damping rate of excitations can be small compared to the elastic collision rate, we expect that this process introduces a second time scale to the decay that depends on the history of the condensate.

The simultaneous agreement of our model with the two complementary data sets strongly supports the evidence for the occurrence of collisional avalanches in our experiments. Our analysis reveals that the density of a cold gas is severely limited as soon as the s-wave collisional opacity exceeds the critical value of 0.693. It is important to point out that the anomalous initial decay of the condensate is attributed to collisional avalanches almost exclusively triggered by the intrinsic process of recombination and that no free parameters are introduced in the model. We have no evidence for the contribution of an Oort cloud to the fast initial decay observed in our experiments.

We conclude that it will be hard to enter the collisional regime in alkali BEC systems. For ^{87}Rb in the $|2,2\rangle$ state the prospects are even worse due to the large collision cross section of the recombination products. Hydrodynamic conditions might be reached in the longitudinal direction in an extremely prolate geometry, as can be seen from Eq. (5). In the vicinity of Feshbach resonances, collisional deactivation of the highly excited molecules can also cause avalanches [7] which, in turn, might contribute to the fast decay reported in Ref. [2]. This offers a new application for a condensate of, e.g., ground-state helium atoms, where recombination is not possible.

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