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Decentralized Coordination of a Community of Electricity Prosumers via Distributed MILP

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Abstract—In this paper, the decentralized energy management of a community of prosumers is studied. The coordination of the prosumers participating in the community is cast as a mixed-integer linear program (MILP) with local and global coupling constraints. A dual decomposition method with a tightening of the coupling constraints is applied to solve the problem in a decentralized way. The resulting tightened problem provides feasibility guarantees for the obtained solutions. A receding horizon approach that exploits the structure of the problem is proposed to divide the problem in subproblems and solve them sequentially. The proposed approach ensures that the tightened problem is feasible under realistic conditions, whereas the existing methods may result in infeasible problems. Moreover, it may reduce the necessary constraint tightening compared to existing methods. Two setups are evaluated for the organizational structure of the community, one with a community coordinator and one without. Moreover, three different structures of the communication graph are analyzed for the setup without a coordinator. The simulation results demonstrate that the proposed method results in feasible problems that can be solved due to the reduced constraint tightening when other methods are not feasible. In the case of feasible problems, the optimality gap is comparable to the existing methods and even smaller for communities with a smaller number of participating prosumers.

Index Terms—Community of prosumers, Decomposition methods, Distributed optimization, Mixed Integer Linear Programming (MILP), Receding Horizon.

I. INTRODUCTION

A CONSIDERABLE rate of adoption at the consumer level of advanced technologies such as distributed energy resources (DERs), smart meters (SM), and home energy management systems (HEMS), has enabled electricity end-users to become prosumers [1]. The increasing share of prosumers has initiated developments in electricity markets to enable their direct market participation [2]. Thus, novel market structures, such as prosumer-centric, decentralized, bottom-up market models are emerging [3].

In recent years, a number of prosumer-centric market models, commonly referred to as peer-to-peer (P2P) models, have been proposed. These include fully decentralized models, community models, as well as hybrid models designs that combine the former two [2], [4]. Community market models

represent organized groups of prosumers that collectively share some common interests [2]. This concept is of interest because it can be easily integrated into the existing electricity markets, considering local energy markets, neighborhood initiatives, and microgrids [2], [3]. Coordination of the interactions internally within the community and externally with the power system is necessary [4].

In recent literature, several methods are proposed to model the coordination of communities of prosumers. Earlier and more recent studies provide detailed prosumers' optimization models but solve the coordination in a centralized way [5], [6]. However, centralized models may be inappropriate due to a lack of compliance with privacy requirements. Moreover, they are likely to face scalability issues as indicated in [7].

In this regard, distributed approaches are of interest. Within the field of mathematical optimization, decomposition methods that support distributed computation are used. The most prominent technique is the Alternating Direction Method of Multipliers (ADMM), which is implemented in [8]–[10]. Despite its benefits such as relatively fast convergence rate and suitability for parallel implementation, its use is restricted to convex problems due to convergence guarantees. Moreover, even for some non-separable convex problems, the convergence is questionable unless some additional conditions are fulfilled [11]. Approaches that utilize game theory, as in [7] and [12] are also limited to convex models. However, non-convex problem formulations such as Mixed Integer Linear Programming (MILP) can offer a wider and less restrictive modeling framework. MILP problem formulations can include explicit on-off control signals for appliances and expressions of operational constraints. Nevertheless, the use of decomposition methods for MILP to solve the community coordination problem has not been presented in the existing literature.

For the community coordination, a coordinator can manage internal interactions [13], [14], or represent an intermediary with other agents in the power system [15], or do both [8]–[10]. A few studies present energy management of a community without a coordinator, especially when external coupling constraints from the power system are concerned. In [16], a fully distributed energy exchange setup is presented but under the assumption of convexity. In [17], an inner-outer iterative approach is proposed, but no coupling grid constraints are considered. In [18], a setup with and without coordinator is presented, but preference for local exchanges is enforced through price-discrimination towards utility prices, which may not always be a valid assumption.

In the existing studies, a MILP modeling framework

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that incorporates coupling constraints regarding the power exchange of the community and can be solved in a decentralized way has not been presented. Moreover, when considering coupling constraints, the common approach in the literature is to enforce them through a community coordinator. Hence, the combination of a decentralized MILP model with coupling constraints and additionally, coordination without a community coordinator has not been considered in the latest studies. The increasing number of energy communities that are being established, have different goals and organization structure. Hence, it is important to evaluate different possibilities for the structure of their organization, such as the whether a central coordinator will be present or not. Moreover, as an entity participating in the power market, they will likely be subject to constraints imposed by the network operators. In the same time, privacy of the participating households should be protected. Finally, communication requirements for information sharing should also be taken into consideration.

In this paper, the coordination of a community of prosumers that collectively optimize their energy consumption and production is formulated as a MILP problem with coupling constraints and is solved through a decomposition method in a decentralized way. The proposed decomposition method is inspired by the recent methods that solve MILP through decomposition using Lagrangian duality while ensuring the feasibility of the obtained solutions proposed in [19]–[21]. The feasibility of solutions is ensured through the tightening of the coupling constraints. However, for time-coupled optimization problems, existing methods may generate too restrictive constraint tightening and even result in infeasible problems that cannot be solved. To address this problem, another method to calculate the necessary constraint tightening based on receding horizon (RH) approach is proposed in this paper. In this manner, the original problem is divided into smaller sections using a receding horizon approach. This method results in less restrictive tightening that allows for wider applicability of the used decomposition methods, while being in line with maintaining solution feasibility.

Two different setups are presented for the energy management in the community. In the first, limited communication with a community coordinator as a separate agent is considered. In the second, the community coordination is performed solely through communication between the participating prosumers, without a coordinator being present.

The contributions of this paper can be summarized as follows:

- Prosumer coordination in the context of an electricity community is formulated as a MILP problem with coupling constraints. The proposed formulation can be decomposed and solved in a distributed way, considering privacy and communication requirements.
- Two different distributed optimization implementations under a MILP framework with coupling constraints are presented, one with a coordinator and one without. Moreover, for the latter implementation, three different communication network structures are tested.
- A receding horizon based method is proposed in order to divide the original problem to ensure problem feasibility

and improve constraint tightening.

The paper is organized as follows: The mathematical background for the decomposition of MILP and the proposed approach are presented in Section II. The mathematical formulation of the primal and dual optimization problem under study is detailed in Section III. The optimization setups and the algorithms are given in Section IV. The results of a case study are analyzed and discussed in Section V. Finally, Section VI concludes the paper.

II. METHODOLOGY

A. Dual decomposition method for MILP with constraint tightening

The optimization problem studied in this paper can be written as a MILP and can be formulated as P .

$$\begin{aligned} \min_x \quad & \sum_{i=1}^n c_i^T x_i \quad (P) \\ \text{subject to:} \quad & \sum_{i=1}^n A_i x_i \leq b \\ & x_i \in X_i, i = 1, \dots, n \end{aligned}$$

where $x_i \in \mathbb{R}^{m_i}$ is the decision variable vector of dimension m_i and $c_i \in \mathbb{R}^{m_i}$ is the corresponding cost vector of all agents $i \in I = 1, \dots, n$. The coupling constraints are expressed through the matrices $A_i \in \mathbb{R}^p \times \mathbb{R}^{m_i}$ resulting in the coupling constraints matrix $A \in \mathbb{R}^p \times \mathbb{R}^{n \cdot m_i}$. The resource vector $b \in \mathbb{R}^p$ is of size p . The sets $X_i = \{x_i \in \mathbb{R}^{r_i} \times \mathbb{Z}^{z_i} : D_i x_i \leq d_i\}$ define the local, compact, mixed-integer polyhedral constraint sets. Each agent i has r_i continuous decision variables and z_i discrete variables, summing to m_i variables in total per agent.

These problems can be solved by using the Lagrangian dual problem D , through an iterative process, such as the subgradient method. Decomposition techniques can be applied to the dual problem, but the candidate solutions may not necessarily satisfy the coupling constraints, so feasibility for the obtained solutions cannot be guaranteed [19], [20].

$$\max_{\lambda \geq 0} -\lambda^T(b) + \sum_{i=1}^n \min_{x_i \in X_i} (c_i^T + \lambda^T A_i) x_i \quad (D)$$

In [19], a method for recovering feasible primal solutions for problem P using the dual of a modified primal problem is presented. The modification lies in tightening the coupling constraint by an appropriate amount ρ . The method is enabled by exploring the relation between the solutions of the convex linear problem P_{LP} , derived from P by using the convex hull of the local constraint sets $\text{conv}(X_i)$ and the solutions recovered from their coinciding dual problem D . The relevant proof can be found in [19].

The tightened version of P , denoted as P_ρ , is defined as:

$$\begin{aligned} \min_x \quad & \sum_{i=1}^n c_i^T x_i \quad (P_\rho) \\ \text{subject to:} \quad & \sum_{i=1}^n A_i x_i \leq b - \rho \\ & x_i \in X_i, i = 1, \dots, n \end{aligned}$$

The corresponding dual of P_ρ is D_ρ .

$$\max_{\lambda \geq 0} -\lambda^T(b - \rho) + \sum_{i=1}^n \min_{x_i \in X_i} (c_i^T + \lambda^T A_i)x_i \quad (D_\rho)$$

The contraction vector that ensures the feasibility of the primal solutions obtained from the dual problem $x(\lambda^*)$ for the original problem P is expressed by (1):

$$[\rho]_j = p \cdot \max_{i \in I} (\max_{x_i \in X_i} [A_i]_j x_i - \min_{x_i \in X_i} [A_i]_j x_i) \quad (1)$$

where j denotes the row of the coupling constraint matrix A and the element of the contraction vector $\rho \in \mathbb{R}^p$. The index i denotes the columns of the coupling constraint matrix related to agent i . The size of the contraction is influenced by the number of coupling constraints, i.e. the number of rows p in the matrix A .

The contraction can be determined a priori, by solving the optimization problems in (1). This is possible as the optimization problems in (1), optimize over the local constraint sets X_i for all agents in I , finding the maximum and minimum feasible solutions, which result in the maximum possible difference between these two values. Moreover, for some problems, the contraction can be determined by exploring the structure of the problem, without solving (1), as demonstrated in [19].

B. Conservatism reduction in constraint tightening

Several cases for which the applied contraction can be reduced are proposed in [19]. If the coupling constraint matrix A has linearly dependent constraints, it can be reduced by substituting the length of the contraction vector p with the rank of the coupling constraints matrix $\text{rank}(A)$ as in (2).

$$[\rho']_j = \text{rank}(A) \cdot \max_{i \in I} (\max_{x_i \in X_i} [A_i]_j x_i - \min_{x_i \in X_i} [A_i]_j x_i) \quad (2)$$

Furthermore, sufficient tightening can be provided by considering the sum of the $\text{rank}(A)$ largest contributions to the coupling constraint as in (3)

$$[\rho'']_j = \sum_{r \in \text{rank}(A)} \max_{i \in I} (\max_{x_i \in X_i} [A_i]_j x_i - \min_{x_i \in X_i} [A_i]_j x_i) \quad (3)$$

A significant contribution to the reduction of the level of conservatism in the contraction calculation is proposed in [20]. Rather than calculating the contraction a priori, over the entire local constraint sets $X_i \forall i \in I$, the contraction is calculated in an adaptive and iterative manner, considering the tentative primal solutions obtained at each iteration k . The contraction $[\rho(k)]_j$ at a iteration k is given in (4).

$$[\rho(k)]_j = p \cdot \max_{i \in I} (\max_{r \leq k} [A_i]_j x_{i,(r)} - \min_{r \leq k} [A_i]_j x_{i,(r)}) \quad (4)$$

An adaptive tightening of the coupling constraint following (4) has finite-time feasibility compared to the asymptotic feasibility achieved by using (1), as well as better performance of the primal objective function. Additionally, a less conservative contraction is applicable to a larger class of optimization problems [20]. The methods for contraction reduction proposed in [19] and given in (2) and (3) are also applicable to (4), resulting in the sequences $[\rho'_{(k)}]_j$ and $[\rho''_{(k)}]_j$.

C. Proposed approach

For a set of problems in which the coupling constraints are across different time intervals, the proposed methods for

reducing the conservatism in the contraction may not be sufficient. The contraction calculated using (1), (4) or their alternative forms, is applied to the coupling constraints for each time interval. In this way, the coupling constraints are tightened with the largest contraction based not only the specific time interval, but the entire time horizon. However, for the problem of residential prosumption of electricity, each time interval of the day has different a feasible region expressed through the local constraint sets. This corresponds to different possible solutions for the coupling constraints, across the considered time horizon. Therefore, it is not necessary to tighten each coupling constraint per time interval with a contraction based on the entire time horizon. This may result in either infeasible problems that cannot be solved with the existing decomposition methods with contraction, or feasible problems whose tightening is larger than necessary.

To ensure that the contraction will result in feasible problems, a sequential solution of the problem using a receding horizon approach as in model predictive control (MPC) problems is proposed. Instead of solving the problem for the entire scheduling horizon $t \in T$, only a section of the vector, indicated as prediction horizon $t \in T^{PH}$ is consecutively solved. The set T^{PH} is a dynamic set which is a subset of T and is denoted by $[t_s^{PH}, t_e^{PH}]$. Thus, the tightening of the coupling constraints is calculated considering only the rank of the coupling constraint matrix $[A]^{PH}$ for the corresponding section of resource vector $b^{PH} \subseteq b$.

The retrieved solutions for $t \in T^{PH}$ are fixed for a given control horizon T^{CH} , where $T^{CH} \subseteq T^{PH}$ and the optimization proceeds to the next prediction horizon until all time intervals in the scheduling horizon T are solved. The decision variables related to the time intervals after the current prediction horizon ($t \in T | t \geq t_e^{PH}$) are fixed to 0. A graphical representation of the process is given in Fig. 1

As the objective of the methods proposed in [19], [20] is to guarantee the feasibility of the retrieved solutions rather than optimality, the extension to a receding horizon approach is in line with these methods. The primal solutions recovered within each prediction horizon are thus guaranteed to be feasible. MPC has been previously used for energy optimization in local energy markets [8] and microgrids [22] to exploit new information. However, in this case, all relevant information is known from the beginning of the optimization. Rolling across the prediction horizons is performed solely to sectionalize the problem and reduce the horizon for which the contraction is calculated.

III. MATHEMATICAL FORMULATION

A. Primal problem

In this section, the mathematical formulation of the community market problem as MILP, adapted from [5], is outlined. The symbols used in this section are given in Table I.

1) *Objective function:* The objective of the optimization problem is to collectively minimize the total electricity procurement cost that the community of agents is exchanging with the grid and is expressed by (5).

TABLE I
NOMENCLATURE

Sets and indices	
$h(H)$	Index (set) of households.
$h_p(N^{PH})$	Index (set) of prediction horizons.
$t(T)$	Index (set) of time intervals.
t_s^{PH}, t_e^{PH}	Start/End index of prediction horizon interval.
T^{CH}	Set of time intervals in control horizon.
T^{PH}	Dynamic set of time intervals in prediction horizon.
T^{SH}	Dynamic set of time intervals in scheduling horizon.
X_h	Local constraint set of household h .
Parameters	
c_t^b	Buying price of electricity in period t [\$/kWh].
c_t^s	Selling price of electricity in period t [\$/kWh].
$E_h^{EV,com}$	SOE of EV at end-of-morning for household h [kWh].
$E_h^{EV,eod}$	SOE of EV at end-of-day for household h [kWh].
$E_{h,t}^{EV}$	SOE trajectory of EV for household h [kWh].
$P_{h,t}^{in,fl}$	Inflexible demand for household h in period t [kW].
$P_h^{ESS,R}$	Charging rate of ESS for household h [kW].
$P_h^{EV,R}$	Charging rate of EV for household h [kW].
P_h^{lim}	Power limit of the household h [kW].
$P_{h,t}^{PV}$	PV generated power for household h in period t [kW].
P_{TR}^L	Export power limit of the transformer [kW].
P_{TR}^U	Import power limit of the transformer [kW].
$SOE_h^{ESS,i}$	Initial SOE of ESS for household h [kWh].
SOE_h^{EV,i_m}	Initial morning SOE of EV for household h [kWh].
SOE_h^{EV,i_a}	Initial afternoon SOE of EV for household h [kWh].
$\overline{SOE}_h^{ESS}$	Maximum SOE of ESS for household h [kWh].
$\underline{SOE}_h^{ESS}$	Minimum SOE of ESS for household h [kWh].
$\overline{SOE}_h^{EV}$	Maximum SOE of EV for household h [kWh].
$\underline{SOE}_h^{EV}$	Minimum SOE of EV for household h [kWh].
T_h^a	Arrival time of EV for household h .
T_h^d	Departure time of EV for household h .
$w_{j,(k)}^h$	Weight assigned to household j by h at iteration k .
Δt	Time interval duration [h].
$\eta_h^{ESS,ch}$	ESS charging efficiency for household h .
$\eta_h^{EV,ch}$	EV charging efficiency for household h .
$\eta_h^{ESS,dis}$	ESS discharging efficiency for household h .
$\eta_h^{EV,dis}$	EV discharging efficiency for household h .
$\sigma_{h,t}^b$	Permutation for buying price for household h in period t [\$/kWh].
$\sigma_{h,t}^s$	Permutation for selling price for household h in period t [\$/kWh].
Decision variables	
$P_{h,t}^b$	Power bought by household h in period t [kW].
$P_{h,t}^s$	Power sold by household h in period t [kW].
$P_{h,t}^{ESS,ch}$	ESS charging power for household h in period t [kW].
$P_{h,t}^{ESS,dis}$	ESS discharging power for household h in period t [kW].
$P_{h,t}^{ESS,s}$	Portion of the ESS discharging power exported for
	selling by household h in period t [kW].
$P_{h,t}^{ESS,u}$	Portion of the ESS discharging power self consumed by household h in period t [kW].
$P_{h,t}^{EV,ch}$	EV charging power for household h in period t [kW].
$P_{h,t}^{EV,dis}$	EV discharging power for household h in period t [kW].
$P_{h,t}^{EV,s}$	Portion of the EV discharging power exported for selling by household h in period t [kW].
$P_{h,t}^{EV,u}$	Portion of the EV discharging power self consumed by household h in period t [kW].
$P_{h,t}^{PV,s}$	Portion of the PV power exported for selling by household h in period t [kW].
$P_{h,t}^{PV,u}$	Portion of the PV power self consumed by household h in period t [kW].
P_{TR,ρ^L}^n	Net power of the community in period t [kW].
P_{TR,ρ^L}^L	Contracted export limit of the transformer [kW].
P_{TR,ρ^U}^U	Contracted import limit of the transformer [kW].
$\bar{s}_{h,(k)}^L$	Max contribution of household h to export limit constraint at iteration k [kW].
$\underline{s}_{h,(k)}^L$	Min contribution of household h to export limit constraint at iteration k [kW].
$\bar{s}_{h,(k)}^U$	Max contribution of household h to import limit constraint at iteration k [kW].
$\underline{s}_{h,(k)}^U$	Min contribution of household h to import limit constraint at iteration k [kW].
$SOE_{h,t}^{ESS}$	SOE of ESS for household h in period t [kWh].
$SOE_{h,t}^{EV}$	SOE of EV for household h in period t [kWh].
$u_{h,t}^{ESS}$	Binary variable; 1 if the ESS is charging in period t , else 0.
$u_{h,t}^{EV}$	Binary variable; 1 if the EV is charging in period t , else 0.
$u_{h,t}$	Binary variable; 1 if the household is importing power in period t , else 0.
$\alpha(k)$	Step size at iteration k .
$\lambda_{(k)}^L$	Dual variable for export constraint at iteration k [\$/].
$\lambda_{(k)}^U$	Dual variable for import constraint at iteration k [\$/].
$\lambda_{h,(k)}^{L,est}$	Estimated dual variable for export coupling constraint by household h at iteration k [\$/].
$\lambda_{h,(k)}^{U,est}$	Estimated dual variable for import coupling constraint by household h at iteration k [\$/].
$\rho_{(k)}^L$	Contraction for export constraint at iteration k [kW].
$\rho_{(k)}^U$	Contraction for import constraint at iteration k [kW].
$\rho_{h,(k)}^L$	Contraction of household h for export constraint at iteration k [kW].
$\rho_{h,(k)}^U$	Contraction of household h for import constraint at iteration k [kW].
$\rho_{h,(k)}^{L,est}$	Estimated contraction for household h for export constraint at iteration k [kW].
$\rho_{h,(k)}^{U,est}$	Estimated contraction for household h for import constraint at iteration k [kW].

$$\min \sum_{h \in H} \sum_{t \in T} (P_{h,t}^b \cdot c_t^b - P_{h,t}^s \cdot c_t^s) \cdot \Delta t \quad (5)$$

2) *Coupling constraints*: For a community of HEMS agents that are located in geographical proximity, it is assumed that they are connected to the same distribution transformer. Hence, the rated power of the transformer is considered as the limit of the energy that can be exchanged between the agents and the rest of the distribution network. This is enforced by constraints (6) and (7) that account for the bi-directional flow of energy.

These constraints are inherently disjunctive considering the local constraint sets of the HEMS agents.

$$P_{TR}^L \leq \sum_{h \in H} (P_{h,t}^b - P_{h,t}^s), t \in T^{PH} \quad (6)$$

$$P_{TR}^U \geq \sum_{h \in H} (P_{h,t}^b - P_{h,t}^s), t \in T^{PH} \quad (7)$$

It is possible to consider additional constraints regarding the distribution network in the model, resulting in an increased number of coupling constraints. This may limit the extent

Fig. 1. Receding horizon approach

to which the proposed method is applicable, as it is suitable for models with a larger number of participating agents than coupling constraints.

3) *Local constraints*: Each HEMS agent has a set of local constraints that define the operation of the assets in the household, such as the PV system, the electric vehicle (EV), and the static electric energy storage system (ESS), as well as the inflexible demand.

The constraints related to the power exchange at a household level are expressed by (8)-(11). More specifically, the individual household power balance is given by (8). The power that can be sold is determined by (9). Also, at each time interval, each household can either buy or sell electricity, which is decided by the binary variable $u_{h,t}$ in the expressions for the household limit of power that can be bought (10) or sold (11).

$$P_{h,t}^b + P_{h,t}^{EV,u} + P_{h,t}^{PV,u} + P_{h,t}^{ESS,u} = P_{h,t}^{infl} + P_{h,t}^{EV,ch} + P_{h,t}^{ESS,ch}, \forall h, t \in T^{PH} \quad (8)$$

$$P_{h,t}^s = P_{h,t}^{ESS,s} + P_{h,t}^{EV,s} + P_{h,t}^{PV,s}, \forall h, t \in T^{PH} \quad (9)$$

$$P_{h,t}^b \leq u_{h,t} \cdot P_h^{lim}, \forall h, t \in T^{PH} \quad (10)$$

$$P_{h,t}^s \leq (1 - u_{h,t}) \cdot P_h^{lim}, \forall h, t \in T^{PH} \quad (11)$$

The constraints that define the operation of the EV are described by equations (12)-(18). Whether the power discharged by the EV will be used in the household or sold is defined in (12). Charging and discharging limits for the EV are enforced by (13) and (14). Constraints (15) and (16) update the SOE of the EV at the initial time of the simulation day or at arrival, respectively, whereas (17) performs the updates for the remaining time intervals. The bounds for the state-of-energy (SOE) of the EV are defined by (18). The required charge level at the time of departure is enforced by (19), and at the end of the simulation period by (20). Finally, the EV battery level is controlled at the end of each control horizon with (21).

$$P_{h,t}^{EV,u} + P_{h,t}^{EV,s} = P_{h,t}^{EV,dis} \cdot \eta_h^{EV,dis}, \forall h, t \in T^{PH} \quad (12)$$

$$0 \leq P_{h,t}^{EV,ch} \leq u_{h,t}^{EV} \cdot P_h^{EV,R}, \quad \forall h, t \in T^{PH} \wedge t \leq T_h^d \vee t \geq T_h^a \quad (13)$$

$$0 \leq P_{h,t}^{EV,dis} \leq (1 - u_{h,t}^{EV}) \cdot P_h^{EV,R}, \quad \forall h, t \in T^{PH} \wedge t \leq T_h^d \vee t \geq T_h^a \quad (14)$$

$$SOE_{h,t}^{EV} = SOE_h^{EV,i_m} + (\eta_h^{EV,ch} \cdot P_{h,t}^{EV,ch} - P_{h,t}^{EV,dis}) \cdot \Delta t, \quad \forall h, t = 0 \wedge t \in T^{PH} \quad (15)$$

$$SOE_{h,t}^{EV} = SOE_h^{EV,i_a} + (\eta_h^{EV,ch} \cdot P_{h,t}^{EV,ch} - P_{h,t}^{EV,dis}) \cdot \Delta t, \quad \forall h, t = T_h^a \wedge t \in T^{PH} \quad (16)$$

$$SOE_{h,t}^{EV} = SOE_{h,t-1}^{EV} + (\eta_h^{EV,ch} \cdot P_{h,t}^{EV,ch} - P_{h,t}^{EV,dis}) \cdot \Delta t, \quad \forall h, t \in (T_h^a, T_h^d] \cap T^{PH} \quad (17)$$

$$SOE_h^{EV} \leq SOE_{h,t}^{EV} \leq \overline{SOE}_h^{EV}, \quad \forall h, t \in T^{PH} \quad (18)$$

$$SOE_{h,t}^{EV} \geq E_h^{EV,eom} \quad \forall h, t = T_h^d \wedge t \in T^{PH} \quad (19)$$

$$SOE_{h,t}^{EV} \geq E_h^{EV,eod} \quad \forall h, t = T \wedge t \in T^{PH} \quad (20)$$

$$SOE_{h,t}^{EV} \geq E_{h,t}^{EV} \quad \forall h, t = t_e^{ch} \wedge t \leq T_h^d \vee t \geq T_h^a \quad (21)$$

The constraints that describe the operation of the stationary ESS are described with equations (22)-(27). The end use of the power discharged by the battery is defined in (22). The charging and discharging limits for the ESS are defined by (23) and (24). The SOE is updated using (25) for the initial time interval and using (26) for the remaining time intervals, while it is restricted by (27).

$$P_{h,t}^{ESS,u} + P_{h,t}^{ESS,s} = P_{h,t}^{ESS,dis} \cdot \eta_{dis}^{ESS}, \forall h, t \in T^{PH} \quad (22)$$

$$0 \leq P_{h,t}^{ESS,ch} \leq u_t^{ESS} \cdot P_h^{ESS,R}, \quad \forall h, t \in T^{PH} \quad (23)$$

$$0 \leq P_{h,t}^{ESS,dis} \leq (1 - u_t^{ESS}) \cdot P_h^{ESS,R}, \quad \forall h, t \in T^{PH} \quad (24)$$

$$SOE_{h,t}^{ESS} = SOE_h^{ESS,i} + (\eta_{ch}^{ESS} \cdot P_{h,t}^{ESS,ch} - P_{h,t}^{ESS,dis}) \cdot \Delta t, \quad \forall h, t = 0 \wedge t \in T^{PH} \quad (25)$$

$$SOE_{h,t}^{ESS} = SOE_{h,t-1}^{ESS} + (\eta_{ch}^{ESS} \cdot P_{h,t}^{ESS,ch} - P_{h,t}^{ESS,dis}) \cdot \Delta t, \quad \forall h, t \in T^{PH} \wedge t > 0 \quad (26)$$

$$SOE_h^{ESS} \leq SOE_{h,t}^{ESS} \leq \overline{SOE}_h^{ESS}, \quad \forall h, t \in T^{PH} \quad (27)$$

Finally, the power generated by the PV is either used or sold as described by (28).

$$P_{h,t}^{PV} = P_{h,t}^{PV,s} + P_{h,t}^{PV,u}, \quad \forall h, t \in T^{PH} \quad (28)$$

B. Dual problem

The dual function of the problem that is defined by equations (5)-(28) is represented by (29)-(30).

$$\max_{\lambda^U, \lambda^L \geq 0} \sum_{h \in H} IP_h + (\lambda_{TR, \rho^L}^L P_{TR, \rho^L}^L - \lambda_{TR, \rho^U}^U P_{TR, \rho^U}^U) \quad (29)$$

$$IP_h = \min_{X_h} \sum_{t \in T} [P_{h,t}^b \cdot (c_t^b + \sigma_{h,t}^b + \lambda_t^U - \lambda_t^L) - P_{h,t}^s \cdot (c_t^s + \sigma_{h,t}^s + \lambda_t^U - \lambda_t^L)], \quad \forall h \quad (30)$$

where $P_{TR, \rho^U}^U = P_{TR}^U - \rho^U$, $t \in T^{PH}$ is the contracted upper limit and $P_{TR, \rho^L}^L = P_{TR}^L + \rho^L$, $\forall t$ is the contracted lower limit. The dual variables λ^U and λ^L are related to the upper and lower transformer power limits respectively. They are recalculated at each iteration step. Small perturbation terms $\sigma_{h,t}^b$ and $\sigma_{h,t}^s$ are added per agent to ensure the necessary uniqueness of the solution of the $P_{LP, \rho}$ and D_ρ as stated in [19].

The inner minimization problem (30) depends on the local constraint set per household agent X_h ((8)-(28)), and the common dual variables λ^U and λ^L . If they are known or fixed, the inner problem can be solved for each agent individually. The outer maximization (29) problem can be solved with an iterative subgradient method. The optimization setups, implemented in this study, that deal with the solving the outer problem are detailed in Section IV.

Fig. 2. Optimization setups

IV. OPTIMIZATION SETUPS

There are different ways in which the information required to solve the outer optimization problem can be exchanged between the agents. Hence, two setups, one with and the other without a community coordinator are described in this section. In this paper, we follow the naming from [20] and [21] for easier comparison between the papers. Hence, the setup with a community coordinator is alternatively referred to as decentralized, whereas the one without a coordinator as distributed. The organizational structure of the setups is presented in Fig. 2. The symbols used in this section are defined in Table I.

A. Common steps and assumptions

The two setups share some assumptions and have common steps. At each iteration, the maximum contribution of each agent to the two coupling constraints is calculated using (31) and (32). The minimum contribution to the coupling constraints is zero. Hence, only the projection on the positive orthant (defined as $[a]_+ = \max\{0, a\}$) is considered in (31) and (32). When this happens for one of the coupling constraints, it means that the constraint is not active and the other coupling constraint is active. This is due to the disjunctive behavior of the two constraints arising from their definition. The contraction is calculated as the difference between the maximum and minimum contribution to the constraint, using (33) and (34), for the upper and lower bound respectively.

$$\bar{s}_{h,(k+1)}^U = \max\{\bar{s}_{h,(k)}^U, [(P_{h,t}^b - P_{h,t}^s)]_+\} \quad (31)$$

$$\bar{s}_{h,(k+1)}^L = \max\{\bar{s}_{h,(k)}^L, [(-P_{h,t}^b + P_{h,t}^s)]_+\} \quad (32)$$

$$\rho_{h,(k+1)}^U = \bar{s}_{h,(k+1)}^U - \underline{s}_{h,(k+1)}^U \quad (33)$$

$$\rho_{h,(k+1)}^L = \bar{s}_{h,(k+1)}^L - \underline{s}_{h,(k+1)}^L \quad (34)$$

The step size sequence $\alpha_{(k)}$ used in the projected subgradient method has to be positive and non-increasing. Moreover, it has to satisfy $\sum_{k=0}^{\infty} \alpha_{(k)} = +\infty$ and $\sum_{k=0}^{\infty} \alpha_{(k)}^2 < +\infty$ [21]. Hence, a sequence of the type $\alpha_{(k)} = \frac{\alpha_1}{(k+1)^{\alpha_2}}$ where $\alpha_1 > 0$ and $\alpha_2 \in (0.5, 1]$ is used in this paper.

B. Decentralized setup with a coordinator

In the decentralized setup, there is a community coordinator which represents a joint communication point for all households participating in the community. The coordinator

Algorithm 1 RH-DEC

```

1:  $\lambda^U(0), \lambda^L(0) = 0, \forall h$ 
2:  $\bar{s}_h^U(0), \bar{s}_h^L(0) = -\infty, \underline{s}_h^U(0), \underline{s}_h^L(0) = 0, \forall h$ 
3:  $\rho_h^U(0), \rho_h^L(0) = 0, \forall h$ 
4: for  $h_p \in N^{PH}$  do
5:   Update  $X_h, t \leq t_s^{ph}$  if  $t_s^{ph} > 0$ 
6:    $k = 0$ 
7:   repeat
8:     for all  $h \in H$  do
9:        $P_h^b(k+1), P_h^s(k+1) \leftarrow$  using (30)
10:    end for
11:     $\bar{s}_h^U(k+1) \leftarrow$  using (31),  $\bar{s}_h^L(k+1) \leftarrow$  using (32)
12:     $\rho_h^U(k+1) \leftarrow$  using (33),  $\rho_h^L(k+1) \leftarrow$  using (34)
13:     $\rho_h^U(k+1), \rho_h^L(k+1) \leftarrow$  using (35)
14:     $\lambda^U(k+1) \leftarrow$  using (36),  $\lambda^L(k+1) \leftarrow$  using (37)
15:     $k \leftarrow k + 1$ 
16:  until  $k = k^{max}$  or stopping criteria is met
17: end for

```

receives the relevant information from all households, calculates the relevant contraction, updates the dual variables in the subgradient method and communicates this information back to the households for the next iteration (Algorithm 1). The division of the calculation steps between the households and the community coordinator within the iterative method is presented in Fig. 3a.

The contraction is determined using (3) and the rank of the coupling constraint matrix for the current prediction horizon T^{PH} , as in (35).

$$\rho_{(k+1)}^{U(L)} = \sum_{rank(A^{PH})} \max\{\rho_{h,(k+1)}^{U(L)}\} \quad (35)$$

To update the dual variables, a subgradient method with a step $\alpha_{(k)}$ is used. Moreover, a projection on the positive orthant is performed as in (36) and (37).

$$\lambda_{(k+1)}^U = \left[\lambda_{(k)}^U + \alpha_{(k)} \cdot \left(P_t^n - (P_{TR}^U - \rho_{(k+1)}^U) \right) \right]_+ \quad (36)$$

$$\lambda_{(k+1)}^L = \left[\lambda_{(k)}^L + \alpha_{(k)} \cdot \left(-P_t^n + (P_{TR}^L + \rho_{(k+1)}^L) \right) \right]_+ \quad (37)$$

where $P_t^n = \sum_{h \in H} (P_{h,t}^b - P_{h,t}^s), \forall t \in T^{PH}$.

C. Distributed setup without a coordinator

In the distributed setup, a community coordinator is not present. Hence, its function is replaced by steps that are performed by all participating agents. The agents communicate the information that they would share with the coordinator to a subset of the agents set through a connected communication graph (Algorithm 2). The division of the calculation steps between the household and this subset, denoted as neighborhood set, within the iterative method is given in Fig. 3b.

Before the optimization step, each agent consolidates the information received from the agents in its neighborhood set N_h to estimate the contraction and the dual variables used at the given iteration step. Hence, estimates of the dual variables are calculated as a weighted sum of the values received from the other agents, using (38). The estimate of the contraction is calculated by taking the maximum of contractions of the

Algorithm 2 RH-DIS

```

1:  $\lambda_h^U(0), \lambda_h^L(0), \lambda_h^{U,est}(0), \lambda_h^{L,est}(0) = 0, \forall h$ 
2:  $\bar{s}_h^U(0), \bar{s}_h^L(0) = -\infty, \underline{s}_h^U(0), \underline{s}_h^L(0) = 0, \forall h$ 
3:  $\rho_h^U(0), \rho_h^L(0), \rho_h^{U,est}(0), \rho_h^{L,est}(0) = 0, \forall h$ 
4: for  $h_p \in N^{PH}$  do
5:   Update  $X_h, t \leq t_s^{ph}$  if  $t_s^{ph} > 0$ 
6:    $k = 0$ 
7:   repeat
8:     for all  $h \in H$  do
9:        $\lambda_h^{U,est}(k), \lambda_h^{L,est}(k) \leftarrow$  using (38)
10:       $P_h^b(k+1), P_h^s(k+1) \leftarrow$  using (30)
11:       $\rho_h^{U,est}(k), \rho_h^{L,est}(k) \leftarrow$  using (39)
12:       $\bar{s}_h^U(k+1) \leftarrow$  using (31),  $\bar{s}_h^L(k+1) \leftarrow$  using (32)
13:       $\rho_h^U(k+1) \leftarrow$  using (40),  $\rho_h^L(k+1) \leftarrow$  using (41)
14:       $\lambda_h^U(k+1) \leftarrow$  using (42),  $\lambda_h^L(k+1) \leftarrow$  using (43)
15:    end for
16:     $k \leftarrow k + 1$ 
17:  until  $k = k^{max}$  or stopping criteria is met
18: end for

```

neighborhood set and own calculated contraction, as in (39). The necessary assumptions regarding the weight coefficients are as presented in [21].

$$\lambda_{h,(k)}^{U(L),est} = \sum_{j \in N_h} w_{j,(k)}^h \lambda_{j,(k)}^{U(L)} \quad (38)$$

$$\rho_{h,(k)}^{U(L),est} = \max\{\max_{j \in N_h} [\rho_{j,(k)}^{U(L)}], \rho_{h,(k)}^{U(L)}\} \quad (39)$$

If the neighborhood set is not very large, i.e. $\dim(N_h) > \text{rank}(A)$, it is not possible to use (3) to calculate the contraction. Thus, the variant in (2) is used as in (40) and (41), where the contraction per agent is the maximum of the estimated contraction and the agent's own contribution to the coupling constraint.

$$\rho_{h,(k+1)}^U = \max\{\rho_{h,(k)}^{U,est}, \dim(T^{PH}) \cdot (\bar{s}_{h,(k+1)}^U - \underline{s}_{h,(k+1)}^U)\} \quad (40)$$

$$\rho_{h,(k+1)}^L = \max\{\rho_{h,(k)}^{L,est}, \dim(T^{PH}) \cdot (\bar{s}_{h,(k+1)}^L - \underline{s}_{h,(k+1)}^L)\} \quad (41)$$

The dual variables are calculated using (42) and (43)

$$\lambda_{h,(k+1)}^U = \left[\lambda_{h,(k)}^U + \alpha_{(k)} \cdot \left(P_{h,t}^n - \frac{P_{TR}^U - \rho_{h,(k+1)}^U}{|H|} \right) \right]_+ \quad (42)$$

$$\lambda_{h,(k+1)}^L = \left[\lambda_{h,(k)}^L + \alpha_{(k)} \cdot \left(-P_{h,t}^n + \frac{P_{TR}^L + \rho_{h,(k+1)}^L}{|H|} \right) \right]_+ \quad (43)$$

where $P_{h,t}^n = P_{h,t}^b - P_{h,t}^s, \forall t \in T^{PH}$.

V. CASE STUDY

A. Input data and case study description

Electricity measurements of 2018 with 15-minute resolution from the Pecan Street Database [23] are used as inputs for the household models. The subset of 25 households from Austin is used, out of which 9 have EVs and 19 have PV. A static ESS is assigned randomly to 12 of the households with PV. The specifications of the Tesla Powerwall [24] are used as inputs

Fig. 3. Flowchart for the two algorithms

for the ESS model. The initial state of energy for each ESS is drawn from a uniform distribution over the interval [40%, 70%] multiplied by the energy capacity of the battery $E^{ESS,cap}$.

The energy demand for each EV, as well as the arrival and departure times are derived from the dataset, when available. Otherwise, the initial states of energy for each EV, both in the morning SOE_h^{EV,i_m} and the afternoon SOE_h^{EV,i_a} , are drawn from a uniform distribution over the interval [30%, 60%] multiplied by the EV battery capacity $E^{EV,cap}$. For the EV departure and arrival times, unimodal beta distribution defined by Mode 1 of the weekday plug-out times and Mode 2 of the weekday plug-in times from the beta mixture model presented in [25] are used for the departure and arrival time respectively.

In order to simulate a larger number of household agents, the data available for the 25 households in the dataset is assigned to other HEMS agents in the following manner. For a given simulation day, the data of the other weekdays of the same week and the following weeks are assigned to the agents, as if it is consumption and production data of the simulation day. The use of this method is restricted to the same month of the simulation day because the electricity data does not vary significantly within a month.

For the electricity prices, the hourly day-ahead prices of 2018 from [26] are adapted. Net-metering is assumed and thus the buying and selling prices for electricity are the same. The simulations are performed for 8th August 2018. This month was selected due to the highest electricity demand during the year according to the dataset. The transformer power rating for each community size was scaled in such a way that the bounds

Fig. 4. Communication graphs (a single household's connections highlighted in blue)

Fig. 5. Comparison of community net power results

are tight enough to avoid trivial solutions and demonstrate that the applied algorithms can provide feasible solutions within a finite number of iterations.

The simulation results of the receding horizon approach proposed in this paper for a setup with coordinator (Algorithm 1) and without (Algorithm 2) are compared to the results obtained using algorithms with the entire time horizon from [20] and [21]. Moreover, all methods are compared to the results of a benchmark, centralized model, which directly solves the primal problem given by (5)-(28).

For the distributed setup without a coordinator, three different types of communication graphs are simulated: complete graph (A) in which each agent is connected to every other agent, Chord connected graph (C) [27] in which agents are connected in a Chord $O(\log N)$ network, and direct neighbors connected graph (N) in which each agent is connected only to two other neighboring agents. The participating households are randomly assigned their location in the communication graph, before the simulation starts. The location of each household agent in the communication graph remains fixed throughout the iterations. The three types of graphs are presented in Fig. 4.

For the step size sequence, $\alpha_2 = 1$, whereas $\alpha_1 = 10^{-5}$ for Algorithm 1 and $\alpha_1 = |H| \cdot 10^{-5}$ for Algorithm 2.

B. Simulation results

1) *Decentralized setup*: The simulations for the receding horizon approach for the decentralized setup with coordinator

are run with prediction horizon length $|T^{PH}| = 40$ and control horizon length $|T^{CH}| = 24$. The results for the optimality gap of the receding horizon (RH-DEC) and the full-time (FT-DEC) approach are given in Table II. The optimality gap is calculated in relation to the centralized solution according to $\Delta J\% = \frac{J_{ALG} - J_{CEN}}{J_{CEN}} \cdot 100$, where J_{ALG} is the cost of the primal recovered solutions from the algorithm used and J_{CEN} is the objective function calculated by the centralized algorithm.

The receding horizon approach results in a smaller optimality gap compared to the full-time approach for a smaller number of participating households. As the number of prosumers in the community grows, the results are improved for the full-time approach, FT-DEC. This can be explained by the results shown in Fig. 5. For $|H| = 100$ the contraction for the full-time approach is larger than the contraction for the receding horizon approach. This results in much tighter bounds which leads to solutions that differ more than those obtained by the centralized algorithm, whereas the solutions obtained by the receding horizon approach follow the solutions of the centralized model closely. On the other hand, for a $|H| = 300$ the contraction in both approaches is relatively small. In this case, full observability of the entire scheduling horizon will generate better decisions compared to the limited information considered in the receding horizon simulation. The average contraction reduction with the proposed method is calculated as an average of the reduction for all time intervals calculated using $\Delta\rho\% = \frac{\rho_{FT} - \rho_{RH}}{\rho_{FT}} \cdot 100$. The reduction for upper bound is in the range of 93.53 - 99.10 %, and 92.48 - 100.00 % for the lower bound.

2) *Distributed setup*: The simulations for the receding horizon approach for the distributed setup without coordinator (RH-DIS) are run with prediction horizon length $|T^{PH}| = 24$ and control horizon length $|T^{CH}| = 8$ for communities with 100-300 households. For 50 households, the prediction horizon is set to a smaller value $|T^{PH}| = 12$, as larger horizon lengths result in infeasible contracted bounds calculated with (2). The horizon lengths for the distributed setups are smaller compared to the decentralized setups for the same reason of obtaining feasible contracted bounds. Due to the tight original bounds, the full-time distributed algorithm results in infeasible contracted bounds which limits the usage of that algorithm for solving a realistic case. Hence, the results of the receding horizon distributed algorithm are compared to the results of a decentralized algorithm with the same horizon lengths and contraction that is calculated using the variant (2), denoted as RH-DEC-v2.

3) *Setup comparison*: The optimality gap for the simulations compared to the centralized solutions are given in Table III. The results for the different communications graphs are almost identical, with a slightly smaller gap for the RH-DIS-A setup. However, the optimality gap for the decentralized setup with the same horizon lengths results in much larger optimality gaps, especially for the smaller communities, with 50 or 100 prosumers. This can be attributed to the smaller size of the problem and thus the larger influence of the constraint tightening.

The behavior of the feasibility of the upper bound for

Fig. 6. Upper bound feasibility for $H = 200$ for distributed and decentralized setup

a community with 200 households in the different prediction horizons for the different distributed and the decentralized setup is presented in Fig. 6. In this figure, the maximum coupling constraint violation for the upper bound for all intervals in the prediction horizon per iteration using $\|\sum_{h \in H} (P_{h,t}^b - P_{h,t}^s) - P_{TR}^U\|_1, t \in T^{PH}$ is presented. If the values are negative, the obtained solutions are feasible. Only in horizon h_3 , the RH-DEC-v2 and RH-DIS-A violate the bound in the first 10 iterations, after which the solutions are feasible. It can be noted that RH-DIS-N demonstrates the largest variability from the three distributed communication graphs, which is understandable given the slow propagation of information in that type of graph. The RH-DIS-C setup usually follows the pattern of RH-DIS-A and at times with smaller variability. Even the decentralized setup is not exempt from cyclic variability as it can be seen in horizon h_{12} . However, in the decentralized setup because of the existence of a community coordinator, the feasibility of the obtained solutions can be verified on the central, community level. Hence, it is possible to stop the iterative process after the coupling constraints are satisfied for a given number of consecutive iterations. Nevertheless, in the distributed setup, evaluating the

feasibility of the obtained solutions is not straightforward.

4) *Computational properties:* The proposed algorithms were implemented in Python 3.7. and the optimization problems were solved using the commercial solver Gurobi 9 [28]. The simulations were executed on a workstation with 2 3-GHz (24 cores) and 128 GB RAM. At each iteration, the optimization problems of the household agents are independent, so they can be performed in parallel. Process parallelization provides the benefit of increased calculation speed, but at the cost of time required to setup and exchange information between the parallel processes. Thus, as a trade off and for fair comparison, the calculations were parallelized with 25 workers for all setups and community sizes. The total simulation times for the different optimization setups are portrayed in Fig. 7. The total simulation time increases in an approximately linear manner as the number of households increases, since at each iteration, a larger number of computations need to be performed. The rolling horizon models result in longer simulation times than the full time model. Shorter prediction horizons, result in a larger number of prediction horizons for the scheduling period and thus lead to longer simulation times, as seen when comparing RH-DEC

TABLE II
OPTIMALITY GAP FOR DECENTRALIZED SETUP [%]

H agents	$\bar{P}_{TR}$ [kW]	FT-DEC	RH-DEC
50	250	3.03	0.41
100	360	6.30	1.15
150	550	1.53	1.33
200	720	1.35	1.44
250	900	1.29	1.49
300	1110	1.22	1.44

TABLE III
OPTIMALITY GAP FOR DISTRIBUTED SETUP [%]

H agents	RH-DEC-v2	RH-DIS-A	RH-DIS-C	RH-DIS-N
50 *	11.35	11.07	11.07	11.10
100	8.18	2.26	2.26	2.39
150	2.91	2.39	2.40	2.43
200	3.19	2.48	2.49	2.50
250	3.22	2.45	2.47	2.46
300	2.52	2.33	2.33	2.34

* Note: $T^{PH} = 12$ and $T^{CH} = 8$ for $H=50$ and $T^{PH} = 24$ and $T^{CH} = 8$ for the rest

and RH-DEC-v2. The total simulation time is not significantly influenced by the communication graph in the distributed setup, as they all display similar simulation times.

Even though the total simulation times are relatively long, this is to an extent due to the fact that they are run for a predetermined, large number of iterations. This is done to examine the behavior of the algorithm for a longer period of iterations as well as to enable standard base for comparison across models. In practice, a stopping criterion can be used and the iterations can be stopped if the coupling constraints are satisfied for a given number of iterations. This would significantly reduce the simulation time, as for many prediction horizons, the coupling constraints are feasible within a small number of iterations, as it can be observed in Fig. 6.

To further support this observation, the average time per iteration is presented in Fig. 8. The average time grows linearly with the increase of the number of households. There is not a significant difference of the time for difference setups. In fact, the iterations in the full-time horizon model (FT-DEC) take slightly longer than in the receding horizon models, possibly due to the larger problem size compared to the receding horizon models. In consideration of an application in which every household will have a dedicated computing node, it can be concluded that the calculations would be tractable.

VI. CONCLUSION

In this paper, the energy management of a community-based market was studied. The problem of coordinating a community of prosumers that can collaboratively share electricity behind a transformer was modeled as a MILP with coupling constraints. The problem was decomposed and solved using the Lagrangian dual and limited information exchange. The feasibility of the primal obtained solutions was ensured through applying a contraction to the coupling constraints. A receding horizon approach was proposed to reduce the conservatism of the contraction and extend the applicability of the method. Two setups regarding the communication infrastructure were considered. One involved communication between the prosumers and a community coordinator while the

Fig. 7. Total simulation time for different optimization setups

Fig. 8. Average time per iteration for different optimization setups

other did not consider a coordinator and assumed information exchange solely among the participating prosumers. In the latter, three communication graph structures were modeled. The algorithms were implemented in a realistic case study and compared to a centralized model. The receding horizon approach resulted in a significant reduction in the necessary constraint tightening. The proposed approach resulted in solutions with a smaller optimality gap for smaller sized problems and was applicable in a distributed setup whereas the original method was not. Future work may include incorporation of strict equality constraints as well as consider agents with several and varying objective functions, extending the problem to the field of multi-objective optimization.

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