Modeling of ironless permanent magnet planar motor structures

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Modeling Ironless Permanent-Magnet Planar Actuator Structures

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This paper describes an analytical model that includes end effects for ironless synchronous permanent-magnet planar actuators. Because of its flexibility, the model can be used to predict the performance of various permanent-magnet array and coil array topologies and commutation schemes. Moreover, since control currents have to be nonsinusoidal, it allows analysis of the motor performance without specifying a commutation scheme by directly dealing with the motor-coupling matrix that links the coil currents to the forces or accelerations acting on the translator.

Index Terms—Analytical modeling, control, end effects, finite-element modeling, Lorentz force, motor coupling, permanent magnets, planar actuator, virtual work force.

I. INTRODUCTION

SIX degree-of-freedom (DOF) synchronous permanent-magnet planar actuators with active magnetic bearings as shown in Fig. 1 are currently being developed to replace the existing multidimensional positioning stages, which often consist of cascaded long and short stroke linear actuators that are supported by mechanical bearings. A planar actuator can combine long- and short-stroke movements in one actuator, thus simplifying the mechanical design of the system. In addition, it allows for operation in high vacuum [1]–[4].

The design of a planar actuator is a complex three-dimensional (3-D) problem, especially when a topology is used with stationary coils and moving magnets, where end effects play a significant role. Finite-element models can be used to evaluate motor performance, but these calculations are time consuming. Existing analytical models often do not include end effects of the magnet or coil arrays [1], [2].

In this paper, a model is described that allows fast and flexible computation of the performance of planar actuator structures by combining analytical solutions of the Maxwell equations with numerical techniques. The model is very useful for design and optimization purposes of planar actuators, e.g., comparing different magnet array and coil array topologies. The model predictions are validated with finite-element-method (FEM) calculations.

II. WORKING PRINCIPLE OF THE ACTUATOR

Most of the planar actuators [1]–[3], [5], [6] are based mainly on repulsive forces between permanent magnets and coils. When coils are used in the translator in combination with a permanent-magnet stator, a cable slab is necessary to provide power and cooling to the moving coils, thus limiting the flexibility of the system [5], [6]. In addition, the cable slab is a source of disturbances such as friction and vibrations that reduces the accuracy of the mover. Therefore, a system with moving magnets and stationary coils is more attractive for high-accuracy applications since the cable slab can be omitted.

Fig. 1. Planar actuator with moving magnets and stationary coils.

The commutation scheme that determines the current in each coil depending on the position of the mover with respect to the stator is very different for a planar actuator than for regular rotating or linear synchronous machines, since all six DOFs must be controlled. A standard sinusoidal commutation scheme using Park-transformation is common in rotating or linear synchronous machines. If applied to a planar actuator, the scheme provides position independent forces on the moving magnet array. However, the torques do vary with the relative position of the mover with respect to the stator [1], [4]. Therefore, each coil or group of coils will have to be energized individually with nonsinusoidal currents to compensate for the torque disturbance [4].

III. FORCE CALCULATION

Since nonsinusoidal currents are necessary to control the six DOFs of the planar actuator, it is desirable to analyze the planar actuator without specifying the commutation scheme in advance. To investigate this, the calculation of the forces on the actuator is analyzed first.

There are three common methods for calculating forces in electromechanical systems, i.e., Maxwell stress, virtual work, and Lorentz force [7]–[9]. In finite-element packages, forces are usually calculated using Maxwell stress or the virtual work method, since these methods can calculate forces accurately between both current carrying objects (e.g., coils, Lorentz force) and noncurrent carrying objects (e.g., iron, reluctance force).

The Lorentz force method can only be used to calculate forces on current carrying objects in free space and is therefore limited in its use. However, the Lorentz force method is computationally much faster for force calculations than Maxwell stress (integral) and especially the virtual work method (differential) so

the Lorentz force method is therefore preferable. To compare Lorentz force and virtual work force values, the force between a filamentary coil and a permanent-magnet array is calculated analytically using both methods.

The virtual work principle is based on the conservation of energy [12]. The electrical energy input in a coil (W_e) is equal to the dissipated energy (W_{diss}), the magnetic energy stored in the system (W_{fld}), and the mechanical output energy W_{mech}

$$W_e = W_{\text{diss}} + W_{\text{fld}} + W_{\text{mech}}. \quad (1)$$

When a conservative system is considered subject to an infinitesimal (virtual) displacement dx , this can be rewritten to

$$d\Lambda_m i = \frac{id\Lambda_c}{2} + f_{\text{fld}} dx \quad (2)$$

where i is the current in the coil, Λ_m is the flux linkage of the coil due to the magnetic field of the magnet array, Λ_c is the flux linkage of the coil due to the current flowing through the coil, and f_{fld} is the force applied by the magnetic field on the coil. Rewriting (2) and generalizing for three dimensions yields

$$\mathbf{F} = i\nabla\Lambda_m - \frac{i}{2}\nabla\Lambda_c \quad (3)$$

where ∇ is the vector differential operator. Under the assumption that the coil behavior is linear ($\Lambda_c = Li$) and the self-inductance L does not depend on the position of the coil with respect to the magnet array, the force equation reduces to

$$\mathbf{F} = i\nabla\Lambda_m. \quad (4)$$

These assumptions are reasonable when the current density in the coil is not too high and since there is no iron core, the self-inductance does not change when the magnets move. The effect of the magnets having a μ_r slightly higher than unity (for NdFeB magnets $1.0 \leq \mu_r \leq 1.1$, [15]) on the self-inductance is neglected.

The Lorentz force is calculated according to

$$\mathbf{F} = \int_V \mathbf{J} \times \mathbf{B} dV \quad (5)$$

where $\mathbf{F}$ is the force acting on the coil, $\mathbf{J}$ is the current density in the coil, and $\mathbf{B}$ is the magnetic flux density generated by the magnet array. For a wire filament $d\mathbf{l}$, this reduces to

$$d\mathbf{F} = i d\mathbf{l} \times \mathbf{B}. \quad (6)$$

Consider a rectangular coil with dimensions $2a \times 2b$ located in the local reference frame (x_c, y_c, z_c) in x - y plane as shown in Fig. 2. Above the coil is a magnet array with reference frame x_m, y_m, z_m and pole pitch τ . The distance r_{cm} between the two reference frames is (x_{cm}, y_{cm}, z_{cm}) , so $x_m = x_c + x_{cm}, y_m = y_c + y_{cm}, z_m = z_c + z_{cm}$.

Fig. 2. Magnet array with filamentary coil in local Cartesian coordinate system.

The magnetic field underneath the array can be written as an infinite sum of spatial harmonics [1], [2]:

$$\mathbf{B} = \sum_{n=1}^{\infty} B_n \exp\left(\frac{-2n\pi(z_c + z_{cm})}{\tau}\right) \times \begin{bmatrix} \sin\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \cos\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \\ \sin\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \cos\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \\ \cos\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \cos\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \end{bmatrix}. \quad (7)$$

The divergence of the magnetic flux density should be zero, i.e.,

$$\begin{aligned} \nabla \cdot \mathbf{B} &= \sum_{n=1}^{\infty} B_n \frac{n\pi}{\tau} \exp\left(\frac{-2n\pi(z_c + z_{cm})}{\tau}\right) \\ &\times \left\{ \cos\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \cos\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \right. \\ &+ \cos\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \cos\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \\ &\left. - 2 \cos\left(\frac{n\pi(x_c + x_{cm})}{\tau}\right) \cos\left(\frac{n\pi(y_c + y_{cm})}{\tau}\right) \right\} \\ &= 0. \end{aligned}$$

Now the coil flux linkage due to the magnet array is calculated:

$$\begin{aligned} \Lambda_m &= \int_{-a}^a \int_{-b}^b B_z dx_c dy_c \Big|_{z_c=0} \\ &= \sum_{n=1}^{\infty} B_n \frac{4\tau^2}{n^2\pi^2} \exp\left(\frac{-2nz_{cm}\pi}{\tau}\right) \sin\left(\frac{an\pi}{\tau}\right) \\ &\times \cos\left(\frac{n\pi x_{cm}}{\tau}\right) \sin\left(\frac{bn\pi}{\tau}\right) \cos\left(\frac{n\pi y_{cm}}{\tau}\right). \end{aligned} \quad (8)$$

Then, the forces in x , y , and z direction can be calculated depending on the position of the magnet with respect to the coil, according to the virtual work method and (4)

$$\mathbf{F} = i\nabla_{cm}\Lambda_m \quad (10)$$

$$= i \sum_{n=1}^{\infty} B_n \frac{-4\tau}{n\pi} \exp\left(\frac{-2n\pi z_{cm}}{\tau}\right) \times \begin{bmatrix} \sin\left(\frac{an\pi}{\tau}\right) \sin\left(\frac{n\pi x_{cm}}{\tau}\right) \sin\left(\frac{bn\pi}{\tau}\right) \cos\left(\frac{n\pi y_{cm}}{\tau}\right) \\ \sin\left(\frac{an\pi}{\tau}\right) \cos\left(\frac{n\pi x_{cm}}{\tau}\right) \sin\left(\frac{bn\pi}{\tau}\right) \sin\left(\frac{n\pi y_{cm}}{\tau}\right) \\ 2 \sin\left(\frac{bn\pi}{\tau}\right) \cos\left(\frac{n\pi y_{cm}}{\tau}\right) \sin\left(\frac{an\pi}{\tau}\right) \cos\left(\frac{n\pi x_{cm}}{\tau}\right) \end{bmatrix}. \quad (11)$$

From (6), the Lorentz force on a filamentary coil is calculated according to

$$\begin{aligned} \mathbf{F} = & i \int_{-a}^a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \mathbf{B} dx_c \Big|_{y_c=-b}^{z_c=0} \\ & + i \int_{-b}^b \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \mathbf{B} dy_c \Big|_{x_c=a}^{z_c=0} \\ & + i \int_{-a}^a \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \times \mathbf{B} dx_c \Big|_{y_c=b}^{z_c=0} \\ & + i \int_{-b}^b \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \mathbf{B} dy_c \Big|_{x_c=-a}^{z_c=0} \end{aligned} \quad (12)$$

which results in exactly the same equations for the force in each direction as the virtual work method. Since any coil can be represented as a bundle of filamentary coils, the Lorentz force method can be used to calculate the forces on the coils as well as the forces on the magnet array using Newton's third law.

IV. ANALYTICAL MODEL

In this section, the fast analytical model is described using the Lorentz force method.

A. Permanent-Magnet Model

To calculate the magnetic flux density $\mathbf{B}$ underneath the magnet array, each magnet in the magnet array is assumed to be ideal with a relative permeability $\mu_r = 1$ and is modeled as an equivalent magnetic charge. The magnetic charge model is derived from the Maxwell equations using the magnetic scalar potential ψ :

$$\nabla \cdot \mathbf{B} = 0 \quad (13)$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}(\mathbf{H}) \quad (14)$$

$$\mathbf{H} = -\nabla \psi \quad (15)$$

where $\mathbf{M}$ is the magnetization of the magnet and $\mathbf{H}$ the magnetic field strength. Substituting (14) and (15) into (13) results in

$$\nabla \cdot \mathbf{B} = \mu_0 \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0 \quad (16)$$

$$= \mu_0 (-\nabla^2 \psi + \nabla \cdot \mathbf{M}) = 0 \quad (17)$$

$$\nabla^2 \psi = \nabla \cdot \mathbf{M}. \quad (18)$$

The vector potential $\mathbf{C}$ is introduced:

$$\psi = \nabla \cdot \mathbf{C} \quad (19)$$

and by substituting (19) into (18), a Poisson equation is obtained

$$\nabla^2 \mathbf{C} = \mathbf{M}. \quad (20)$$

The solution of the Poisson equation is

$$\mathbf{C} = -\frac{1}{4\pi} \int_V \frac{\mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV(\mathbf{r}') \quad (21)$$

where $\mathbf{r}'$ is the source point and $\mathbf{r}$ the point of evaluation. Substituting the solution into (19) yields

$$\psi = -\frac{1}{4\pi} \nabla \cdot \int_V \frac{\mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV(\mathbf{r}') \quad (22)$$

$$= -\frac{1}{4\pi} \int_V \frac{\nabla' \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV(\mathbf{r}') + \frac{1}{4\pi} \int_S \frac{\mathbf{M}(\mathbf{r}') \cdot \mathbf{n}}{|\mathbf{r} - \mathbf{r}'|} dS(\mathbf{r}'). \quad (23)$$

Finally, the expression for the magnetic flux density outside the magnet volume is obtained:

$$\mathbf{B} = \mu_0 \mathbf{H} = -\mu_0 \nabla \psi \quad (24)$$

$$= \frac{\mu_0}{4\pi} \int_V (-\nabla' \cdot \mathbf{M}) \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV + \frac{\mu_0}{4\pi} \int_S (\mathbf{M} \cdot \mathbf{n}) \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dS \quad (25)$$

where $-\nabla' \cdot \mathbf{M}$ represents a magnetic volume charge distribution $\rho(\mathbf{r}')$ and $\mathbf{M} \cdot \mathbf{n}$ the magnetic surface charge $\sigma(\mathbf{r}')$. When the magnetization of the magnet is assumed uniform $\nabla' \cdot \mathbf{M} = 0$, the magnetic field outside the magnet can be represented by the magnetic surface charge $\sigma(\mathbf{r}')$ on the boundary surface of the magnet only. The surface integral has an analytical solution for rectangular magnets [13].

To calculate the magnetic flux density of a magnet array, the magnetic flux density of each magnet can be calculated separately using the charge model. Then, the total magnetic flux density generated by the magnet array is calculated by superposition.

B. Coil Model

The coil is now modeled as a bundle of filamentary wires. Since the current can be taken out of the line integral (12), the force on the magnet only depends on the geometry of the coil, the magnitude and sign of the current and the magnetic flux density of the magnet array. Therefore, each filamentary wire can be modeled as a sequence of vectors $d\mathbf{l}$, making the model very flexible by allowing almost any coil geometry. This is shown in Fig. 3.

By eliminating the current from the line integral, any commutation scheme can be used and the six DOF equivalent of

Fig. 3. Part of coil with vector sequences as filamentary wires.

the motor constant can be calculated for each coil resulting in a matrix that prescribes the force or acceleration per unit current for each coil. End effects are included as well, since the charge model is used to calculate the magnetic flux density underneath the magnet array, instead of spatial harmonics.

C. Model Implementation

The model is implemented using a combination of C and Matlab codes, where a compiled C function is used to improve the calculation time of the magnetic flux density underneath the magnet array. The data is then loaded into the Matlab workspace for the force calculation.

The magnetic flux density is not directly calculated in each coil vector $d\mathbf{l}$ but in a 3-D grid to further save calculation time. The magnet flux density in each coil vector $d\mathbf{l}$ is then estimated by interpolation. The advantage of this approach is that it is not necessary to recalculate the magnetic flux density underneath the magnet array when the position of the coils changes with respect to the magnet array or the current in the coil changes.

V. MODEL VALIDATION

In Fig. 4, a 5×5 N-S magnet array is shown with four coils arranged as a two-phase coil system. The coils in Fig. 4 are moved one pole pitch ($\tau = 30$ mm) from the initial position in both the x and y directions. The dimensions of the coil and magnet array are shown in Table I.

This system is modeled in the finite-element package Maxwell 3D Optimetrics [16] and using the proposed analytical model. The forces are calculated with both models at 16 different positions of the coil array with respect to the magnet array and at four different current magnitudes at each position. All four coils carry the same current.

A. FEM Model

The coil material is copper and the current is uniformly distributed over the cross section of the coil. The magnets are NdFeB with a B_r of 1.23 T and a μ_r of 1.09. First, the nominal project (500 At in each coil, 10 A/mm²) calculation was done with increasing number of elements up to 838 688 elements resulting in a 0.0881% energy error. The current density is

Fig. 4. N-S magnet array with two-phase coil system.

TABLE I
DIMENSIONS OF COIL AND MAGNET ARRAY

Parameter	Symbol	Value	Dimension
Pole pitch	τ	30	[mm]
Magnet distance	d	10	[mm]
Magnet height	mh	8	[mm]
Coil height	ch	5	[mm]
Coil thickness	ct	5	[mm]
Airgap	g	1	[mm]

Fig. 5. Virtual force and Lorentz force versus number of elements.

TABLE II
CALCULATED FORCES IN NOMINAL FEM MODEL

	Lorentz force	Virtual force on coils	Virtual force on magnets
F_x	5.3632 N	5.3626 N	-5.3122 N
F_y	5.3629 N	5.3574 N	-5.3952 N
F_z	7.6328 N	7.6363 N	-7.8640 N

chosen high, since such a planar actuator would have a ceramic heat sink, possibly in combination with water cooling, in an industrial application.

In Fig. 5, the magnitude of the virtual force on the magnet array and the Lorentz force on the coils, which are in theory equal in magnitude but opposite in sign, are shown versus the number of elements in the FEM simulation. The calculated forces in each direction are shown in Table II. The convergence problem in the virtual force on the magnets is mainly due to F_z

Fig. 6. Relative errors of analytical model at -600 At. Fig. 7. Relative errors of analytical model at -200 At. Fig. 8. Relative errors of analytical model at $+200$ At.

because of the small air gap (numerical error). The other force components converge more rapidly.

From these plots, it can be concluded that the virtual force is equal to the Lorentz force and that 1×10^5 elements are sufficient to obtain a converged solution for the Lorentz force. Therefore, the predictions of the analytical model are compared to the Lorentz force of the FEM calculations.

B. Analytical Model

The coils are defined as filamentary wires spaced 1 mm apart in the x - y plane. Each wire is divided in points with a corresponding vector $d\mathbf{l}$, which are spaced 1 mm apart. Along the height of the coils, multiple 2-D coil grid layers are spaced 2.5 mm apart. The magnets are represented as magnetic surface charges with $\mu_r = 1$ and $B_r = 1.23$ T. The magnetic flux density is calculated in the space underneath the magnets with a 4 mm resolution in x and y direction. There is no interpolation in the z direction.

C. Results

The force components in each direction predicted by the analytical model and the FEM model are compared and the relative error is calculated for each component

$$F_i \text{ error} = \frac{F_i^{\text{FEM}} - F_i^{\text{analytical}}}{F_i^{\text{FEM}}} 100\%. \quad (26)$$

The errors are plotted versus position of the coil array at four different current magnitudes in Figs. 6, 7, 8, and 9 for -600 , -200 , $+200$, and $+600$ At, respectively. The total calculation

Fig. 9. Relative errors of analytical model at $+600$ At.

Fig. 10. Halbach magnet array with two-phase coil system.

of the FEM model with 200 000 elements was 30 h, and the analytical model took 12 s.

The errors are between -6% and 0% and can be further reduced by increasing the resolution of the magnetic flux density calculation in the analytical model or by eliminating the interpolation altogether. In addition, the resolution of the coil grid points can be increased to further reduce the error variation between -6% and -4% . However this increases calculation time e.g., calculating the forces without interpolation with a 1 mm resolution in x , y and z direction takes 3.5 min instead of 12 s. The offset is purely related to the assumption that $\mu_r = 1$, which has been verified by FEM simulation with ideal permanent magnets. The shape of the relative errors, as shown in Figs. 6–9, remains the same. However, the range changes from -3% to 3% .

VI. APPLICATIONS

The model has been developed to evaluate different designs for planar actuator research at the Eindhoven University of Technology. To illustrate the use of the model, several applications are discussed.

A. Topology Optimization

In every application, the key question is to find the best possible topology to meet the application demands. There are two magnet array topologies that are often used for planar actuators: N-S array and the Halbach array shown in Figs. 4 and 10, respectively. The Halbach array produces more force since it generates a higher magnetic flux density, but the extra Halbach magnets also add weight to the structure so more force does not necessarily mean higher acceleration.

Fig. 11. Mean acceleration for magnet array without load.

Fig. 13. Mean acceleration for magnet array with 1 kg load.

Fig. 12. Minimum acceleration for magnet array without load.

Fig. 14. Minimum acceleration for magnet array with 1 kg load.

For two situations, the acceleration of a planar actuator with both topologies is calculated based on sinusoidal commutation of the coil currents. This commutation scheme is based on the assumption that higher order spatial harmonics of the magnetic flux density underneath the magnet array (7) can be neglected. All dimensions are in Table I, however, the size of the N-S ($\tau-d$) and Halbach (d) magnets is varied to determine the optimal array configuration. To cover the complete workspace of the planar actuator the magnet array is moved half a pole pitch in both the x and y direction in ten increments, resulting in a total of 100 points where the accelerations are calculated for each topology. Total calculation time is about 6 min.

Case 1) Magnets are glued onto a 5-mm-thick aluminum back plate and no additional load.

Case 2) Magnets are glued onto a 5-mm-thick aluminum back plate and 1 kg additional load.

The mean acceleration for the magnet array without load is shown in Fig. 11 and for the magnet array with load in Fig. 13. The minimum acceleration for the magnet array without load and the magnet array with load is shown in Figs. 12 and 14, respectively. The acceleration ripple is the same for both cases and is shown in Fig. 15.

Fig. 15. Acceleration ripple for both magnet arrays.

The minimum acceleration ripple for the Halbach array results in very large Halbach magnets. For the N-S array with the same pole pitch the minimum ripple occurs when the N-S magnets are two third of the pole pitch $(\tau-d)/(\tau) = 0.67$. The minimum ripple is found, when for this topology the distribution of the B -field of the magnet array matches best the commutation

scheme (i.e., coil currents). The minimum acceleration ripple of the Halbach array is smaller than the minimum acceleration ripple of the N-S array.

Although the Halbach array produces more force, the weight of the additional Halbach magnets results in higher accelerations of the N-S array when there is no additional load on the platform. However, when the platform has an additional load of 1 kg the extra force of the Halbach array is needed to lift the platform and the load, so the Halbach array performs better than the N-S array in this case.

B. Controllability

The commutation scheme, that determines the current in each coil depending on the position of the mover with respect to the stator, cannot be a standard sinusoidal scheme (using Park transformation) in case of a contactless planar actuator. Therefore, it is necessary to investigate the performance of the motor without specifying a commutation scheme. In addition, it is important to determine whether the motor has good controllability within the workspace. To investigate the controllability, the coupling between currents in the coils and accelerations of the mover must be analyzed.

In general, accelerations in an electromechanical motion system [10], [11] can be described as

$$\begin{bmatrix} \ddot{x} & \ddot{y} & \ddot{z} & \ddot{\theta}_x & \ddot{\theta}_y & \ddot{\theta}_z \end{bmatrix}^T = K \mathbf{i} \quad (27)$$

where $\mathbf{i}$ is a $n \times 1$ vector with all coil currents and K is the $6 \times n$ motor coupling matrix linking currents to accelerations. To ensure good controllability, matrix K should have numerical rank 6 and its condition number should be sufficiently low to allow decoupling of the forces and torques (e.g., by using the pseudoinverse $(K^T K)^{-1} K^T$). A well-known technique to test these properties is the singular value decomposition (SVD, [14]). The SVD decomposes K into

$$K = U \Sigma V^T \quad (28)$$

where U and V are unitary and Σ is a diagonal matrix. The values on the diagonal of Σ are called the singular values of K . Since K is real, U and V are orthogonal. The 2-norm of any vector multiplied by an orthogonal matrix remains the same. This means that all the energy conversion from the current vector $\mathbf{i}$ to accelerations is defined by Σ .

The condition number c of a nonsquare matrix is calculated using the singular value decomposition and its value is defined as

$$c = \frac{\max \text{diag}(\Sigma)}{\min \text{diag}(\Sigma)}. \quad (29)$$

In Fig. 16, a planar actuator is shown with 25 coils underneath the 5×5 N-S magnet array. Using the model, the force per unit current is calculated for each coil at different positions to assemble K . The calculation takes about 4 min and results are shown in Fig. 17. Assuming that the model has two significant digits, an upper limit for the condition number is $10^2 = 100$.

The six singular values of K are a measure of how efficiently currents are converted to forces for each DOF. Therefore, the

Fig. 16. Layout of planar actuator with N-S magnet array.

Fig. 17. Condition number of K of a 5×5 N-S array.

Fig. 18. Mean singular value of K of a 5×5 N-S array.

mean value of the singular values is a measure of the motor performance. In Fig. 18, the mean singular value is shown for the N-S magnet array. There is an undesirable drop in performance when the magnet array is moved one pole pitch from its initial position.

If possible, the magnet array should be changed in order to get a more constant behavior. The same simulation is done with a 6×6 magnet array to investigate the effect of the magnet array size on its controllability and performance. The simulation

Fig. 19. Condition number of K of a 6×6 N-S array.

Fig. 20. Mean singular value of K of a 6×6 N-S array.

results are shown in Figs. 19, and 20 for the N-S array condition number and the mean singular value, respectively.

The controllability of the array is now almost constant throughout the workspace of the planar actuator. The performance has a lower maximum than the 5×5 arrays but it is almost constant which is not the case for the 5×5 array. The 6×6 array is therefore preferable over a 5×5 array in combination with this two-phase coil array.

VII. CONCLUSION

The analytical model, that uses the charge model to describe the permanent magnets and filamentary wires to describe the coils, is a fast and reliable tool for use in the design of an ironless permanent-magnet planar actuator with air-core coils.

By using the charge model to calculate the magnetic flux density underneath the magnet array, instead of spatial harmonics, end effects are also included in the model.

The model can predict the performance of various magnet array and coil array topologies. Its flexibility allows for the

testing of different commutation algorithms and the calculation of the motor coupling matrix that links the current in each coil to accelerations or forces on the translator.

Using the motor coupling matrix, the motor controllability and performance can be evaluated without specifying a commutation scheme or control method.

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