Analysis of chaos in fluidization using electrical capacitance tomography

F T Künn†, J C Schouten†, R F Mudde‡, C M van den Bleek† and B Scarlett†
† Delft University of Technology (DUT), Faculty of Chemical Engineering and
Materials Science, Department of Chemical Process Technology, Julianalaan 136,
NL-2628 BL Delft, The Netherlands
‡ Delft University of Technology (DUT), Faculty of Applied Physics, Kramers
Laboratorium voor Fysische Technologie, Prins Bernardlaan 6, NL-2628 BW Delft,
The Netherlands

Received 28 July 1995, in final form 18 September 1995, accepted for publication
27 October 1995

Abstract. Fluidized beds display a degree of structure which it is vital to
understand in any design and scale-up of equipment. This paper is on the
employment of chaos theory to describe this structure. The measuring technique of
electrical capacitance tomography is used to determine the local porosity of a
gas–solids fluidized bed and to monitor its fluctuation with time. These raw data
are translated into invariants characteristic of the structure mentioned, such as the
Kolmogorov entropy. Initial measurements have shown a strong variation in these
chaotic invariants with bed position and operational parameters. First results are
compared with earlier data from pressure measurements.

Nomenclature

\[
\begin{align*}
D_T & \quad \text{Inner bed diameter (m)} \\
H_S & \quad \text{Settled bed height (m)} \\
K_p & \quad \text{Kolmogorov entropy from pressure data} \\
P & \quad \text{Pressure (Pa)} \\
K_v & \quad \text{Kolmogorov entropy from voidage (bits s}^{-1}) \\
L_{ax} & \quad \text{Axial electrode length (m)} \\
U_O & \quad \text{Superficial gas velocity (m s}^{-1}) \\
U_{mf} & \quad \text{Minimum fluidization velocity (m s}^{-1}) \\
\epsilon(x, y) & \quad \text{Dielectric distribution (Fm)} \\
\phi(x, y) & \quad \text{Electrical potential distribution (V)} \\
\end{align*}
\]

1. Introduction

Up to now, the dynamics in fluidized bed reactors have not been fully understood. The fact that fluidized beds are nonlinear systems and may exhibit a strongly chaotic behaviour makes it difficult for them to be both scaled up and controlled during operation.

Conventional scale-up rules are based on steady state operations such as keeping constant a set of dimensionless numbers like the Reynolds number, the ratio of the solids to fluid density and geometric properties. Since these procedures do not take into account the systems’ dynamics, large plants in industry often fail in comparison with laboratory test set-ups.

To find a better approach to this problem, research is now being done towards the scale-up of the fluidization dynamics of gas–solids fluidized beds. At Delft University of Technology (DUT), an extensive project is worked on with the goal of representing the nonlinear dynamics by chaotic invariants. Therefore, probes were immersed into fluidized beds to record time series of pressure data [1]. These data then served to quantify the chaotic behaviour of the fluidized beds. Techniques were developed in the framework of the project to reconstruct the system attractor in phase space and to calculate the chaotic invariants

- correlation dimension [2], being a measure of the number of degrees of freedom and quantifies the system’s complexity and
- Kolmogorov entropy [3], which measures the time dependency and quantifies the short-term predictability.

Time series of pressure had been measured in bubbling and slugging beds consisting of 560 µm polystyrene particles. The Kolmogorov entropy calculated from these data could be related to the gas velocity, column diameter and bed height (see equation (1)) [4]. Equation (1) was fitted to data measured in columns ranging from 10 to 38.4 cm internal diameter (ID) and has to be evaluated in SI units:

\[
K_p = 10.7 \left( \frac{U_0 - U_{mf}}{U_{mf}} \right)^{0.4} \frac{D_T^{1.2}}{H_S^{0.5}}.
\]

It was argued in [4] that this type of correlation could be used in a chaos-based scale-up procedure to ensure the same degree of chaotic dynamics in terms of Kolmogorov entropy in fluidized beds of different size (tube or column diameter $D_T$). It must be pointed out that the pressure signals measured are global, i.e. they originate from all over the bed and even events below the gas distributor.
plate may contribute to the signal measured. However, they are dominated by events at the bed surface. In addition, the dependency of the Kolmogorov entropy on operational parameters was investigated theoretically. The use of bubble size correlations from the literature led to a relationship quite similar to equation (1) which fitted measured pressure data [4]. The theoretical relationship is based on the assumption that the rate of loss of information, i.e. Kolmogorov entropy, is proportional to the number of bubbles that erupt at the bed surface and the bubble impact which is given by the ratio of bubble size to bed diameter. As an additional measurement tool the electrical capacitance tomography (ECT) technique, which produces cross-sectional voidage profiles of the fluidization column, has been installed on a 28.4 cm ID fluidized bed. The comparison of the Kolmogorov entropy calculated from the local voidage data to the results from the pressure measurements compiled in equation (1) and values expected from the theoretical investigation is to validate the earlier studies. For the validation of the theoretical dependency bubble size and number of bubble eruptions per unit of time have to be measured. This would enable verification of the relations drawn from the literature, on which the model correlation is based.

Moreover, it has to be pointed out that one single tomography measurement yields information on the overall cross section. This enables radial and angular dependences of the chaotic invariants to be examined. First, a description of the measuring technique will be given.

2. Electrical capacitance tomography (ECT)

This measuring technique uses the measurement of electrical capacitances in different locations and directions of the cross section of a three-dimensional object of nonconductive components. The result is the dielectric or permittivity distribution over the cross section. This distribution represents the material distribution when only two components are present.

The ECT set-up consists of three main parts (see figure 1):

(i) the capacitance sensor,

(ii) the sensor electronics controlled by a personal computer,

(iii) the image reconstruction computer.

2.1. Capacitance sensor

The sensor is set up by mounting 12 electrodes around the periphery of the duct to be imaged. This array enables the measurement of the capacitances of all combinations of two of the electrodes [5]. 12 electrodes give 66 independent measurements per image which depend on the unknown material distribution. A grounded outer shield is wrapped around the whole system. The cross section of the sensor is shown in figure 2.

Axial shields are mounted above and below the electrodes. Depending on their design there are two types of sensors (see figure 3):

The UMIST-type sensor. This sensor uses large sheet metals wrapped around the pipe above and below the ring of electrodes. Both shields are kept at zero potential during the measurement. In the literature the axial length of either axial guard is recommended to be at least the pipe diameter [6]. Image reconstruction algorithms assume the electrostatic field during the measurement to be two-dimensional, i.e. changes in axial direction are neglected. However, in this type of sensor the field lines of the electrostatic field bend in axial direction. This introduces the so-called 2D/3D-error.

The METC-type sensor. This sensor type had first been used by the Morgantown Energy Technology Center (METC), USA [7]. For the field lines to be as parallel as possible to the sensor plane, i.e. to alleviate the 2D/3D-error, the axial shields are cut into segments being as large in the circumferential direction as the electrodes. Applying the same potential to the axial shields as to the electrodes yields a better approach to the assumption of dealing with a two-dimensional electrostatic field during the measurement.

Initially, the UMIST–type sensor was used. It worked well on a small scale test sensor of 10 cm ID but not on the larger 28.4 cm ID bed. There was an influence
caused by the 2D/3D-effect, i.e. the ratio between the axial electrode length $L_{ax}$ and the pipe diameter $D_T$ had a strong influence on the electrostatic field and thus on the capacitances measured.

To verify this effect, finite element method (FEM) calculations were performed at Delft University of Technology (DUT) to get a measure of the influence of the 2D/3D-effect in the two different sensor designs. The commercial software package SEPRAN [8] was used to perform simulations with two-dimensional models representing one half of an axial-radial section through the pipe centre as depicted in figure 4.

The simulation results are the equipotential lines shown in figure 5. They represent the electrostatic field during the measurement. When drawing field lines perpendicular to the equipotential lines one can see that in the UMIST-type sensor the field lines bend in axial direction whereas in the METC-type sensor the field lines are straight and perpendicular to the sensor axis. The latter sensor produces larger capacitance values and complies with the assumption of dealing with a two-dimensional field. The results presented in the following section were obtained using the segmented METC-type sensor.

2.2. Sensor electronics

The second part, the sensor electronics, applies the electrodes with potentials and detects the material response. Various basic electrical circuits for measuring low capacitance values are described in the literature. For this project, 12-channel sensor electronics based on the so-called charge transfer (charge/discharge) principle [5] were purchased from the Process Tomography Group, Department of Electrical Engineering and Electronics, University of Manchester, Institute of Science and Technology (UMIST), UK. As this device was designed for use on the UMIST-type sensor it was modified to drive the segmented axial guard rings with the potentials of the corresponding electrodes.

2.3. Image reconstruction algorithms

An image reconstruction algorithm to calculate dielectric distributions corresponding to every set of capacitances measured was implemented. In contrast to the non-electrical tomographic techniques operating with, e.g., radiation, there are no measuring signals passing along straight lines. In any capacitance measurement the whole of the region within the boundary, i.e. the outer shield, contributes to the values measured. How much all the cross sectional regions contribute to the measurement depends on the still unknown dielectric distribution. This effect, usually referred to as the soft-field effect, requires a very sophisticated reconstruction.

Algorithms found in literature are based on models assuming the problem to be both two-dimensional and static. The reconstruction procedure is usually divided into the linear forward problem and the nonlinear inverse problem.

2.3.1. Forward problem. The forward problem represents the calculation of the capacitance values for a given dielectric distribution, $\epsilon(x, y)$. The linear partial differential equation describing electro-static fields, Gauss’s law,

$$\nabla(\epsilon(x, y)\nabla\phi(x, y)) = 0$$

(2)

has to be solved within the Dirichlet boundaries† to obtain information on the system. The forward problem can be solved numerically, e.g. using a finite element method (FEM) software package.

2.3.2. Inverse problem. In the second step, the inverse problem has to be solved with the aim of obtaining a dielectric distribution $\epsilon(x, y)$ which theoretically yields capacitance values equal to the measured ones. At present the linear back projection algorithm (LBP) [9] is the only algorithm available for imaging fluidization flow patterns.

† Dirichlet boundary conditions denote that (parts of) the boundary are applied with a constant potential.
The LBP is merely a linear approach to the nonlinear inverse problem. This approach does not correct for the so-called soft-field error making the voidage data qualitative. The calculated voidages exhibit over- and under-shooting, i.e. voidage values larger than 100% and smaller than 0%, respectively. Xie et al [9] suggested to correct for this effect using threshold functions.

The only quantitative algorithms seem to be iterative algorithms that solve the forward problem after every iteration and minimize the difference between the measured capacitances and simulated values from the forward solutions. This procedure should work in theory. In practice, however, it does not converge to the right solution for any set of capacitance data. Even perfect and noise free data from simulations do not yield the right dielectric solution in any case. When using noisy data small imperfections of the measurement data would be amplified to large errors in the dielectric or permittivity distribution. At DUT, work is being performed to investigate and overcome this phenomenon.

3. Measurement results

In a previous study, experimental time series of pressure fluctuations had been used to estimate the Kolmogorov entropy in columns ranging from 10 to 38.4 cm ID [4]. The results presented in the following were obtained from voidage measurements using a 28.4 cm ID gas–solids fluidized bed. In order to be able to compare these results to the earlier pressure measurements the experimental conditions had not been altered.

The capacitance sensor was mounted to the fluidized bed in such a way that the sensor plane was situated 35 cm above the porous gas distributor plate. The bed was filled with 560 µm polystyrene particles up to the sensor plane, i.e. the settled bed level was 35 cm as well (see figure 6). The superficial gas velocity ranged from 2.6 to 7.8 times the minimum fluidization velocity. With these settings measurements of 60 000 frames were done at a data capture speed of 100 frames per second.

As described earlier, the capacitance data can be deconvoluted into cross sectional voidage distributions using the LBP algorithm with and without thresholding. For chaos analysis it is sufficient to reconstruct time series of several image pixels. The influence of the threshold functions on the reconstructed voidage values was found to be strongly dependent on the radial position of the image pixel in the cross section. Figure 7 shows reconstructed time series of pixels near the pipe wall and the pipe centre. One realizes that the threshold functions defined in [9] have a large impact on the voidage data in the pipe centre whereas data from pixels close to the wall are influenced only slightly.

The time series served as input for the software package RRCHAOS [10] which calculates numbers to quantify the time series in terms of nonlinear chaotic dynamics. In this paper, we show the radial dependencies of the Kolmogorov
entropy as well as of statistical invariants such as peak–peak distance, average absolute deviation and cycle frequency.

The peak–peak distance gives the difference between points in the time series at the 1% and 99% values of its probability density distribution. Thus the peak–peak distance provides a measure of the difference between the maximum and minimum values in the time series. The average absolute deviation of the data points from the average value is a robust estimator of the data’s width around its mean. The average cycle frequency is the reciprocal of the average cycle time. The latter number is defined by the average time needed to complete a full cycle after the first passage of the average of the time series.

Average absolute deviation and average cycle time are measures of the characteristic length and time scale in the time series, respectively. Both invariants are used as such in the reconstruction of the attractor from which the Kolmogorov entropy is estimated (see [2] and [3] for a more detailed description).

The Kolmogorov entropy was calculated along a straight line throughout the centre of the cross section using reconstruction software with and without correction for under- and over-shooting (see figure 8). The limiting values of entropy of 0 and ∞ define the system to be periodic and completely stochastic, respectively.

Both entropy distributions show symmetry around the pipe centre. The entropy distribution calculated with threshold functions has odd peaks and low values in the pipe centre. This effect can be explained by the fact that depending on the radial position of the pixel, the characteristics of the time series are changed by the threshold functions. It was observed that over- and under-shooting rarely occurred near the pipe wall and increased with a decrease of the radial distance (see figure 7). Consequently, the time series change most near the pipe centre. Figure 8 therefore shows an underestimation of the degree of chaos in the pipe centre with respect to the reconstruction without thresholding.

The entropy values calculated without truncation, i.e. allowing the voidage to be negative or larger than 100%, describe the continuous real state of chaos and are comparable to earlier results from pressure fluctuations. Therefore, the following estimations of the Kolmogorov entropy are based on image reconstructions without thresholding.
Additional calculations have been performed along lines turned at an angle of 45° and 90°, respectively, to realize if there was an angular dependency of the Kolmogorov entropy (see figure 9). The entropy distribution calculated under an angle of 0° deviates from those of the other angles. The reason is likely to be a tab used to empty the fluidized bed. It is situated below the place where the entropy differs from the average near-wall value.

There is a strong dependency on fluidization velocity. The higher the gas velocity the more turbulent the hydrodynamics is. This causes an increase of entropy as can be seen from figure 10.

In earlier studies, the Kolmogorov entropy based upon the time series of pressure fluctuations was calculated for a wide range of operating conditions [4]. The comparison of the entropy values from pressure measurements (equation (1)) to the voidage entropies is given in table 1.

Equation (1) is based on measurements in the pipe centre. Therefore, for the comparison the entropy value of the voidage distribution calculated under an angle of 0° is used. One can see that the values of \( K_p \) are about 40% larger than values of \( K_v,centre \). Reasons for the deviation may be:

- The larger the distance of an image pixel from the electrodes the smaller the variation of the capacitances due to a variation of the pixel’s dielectric constant. This means that the image resolution of the ECT system is poorest in the pipe centre where the distance between the pixels and electrodes is at a maximum. This has also been proven by the observation of reconstructed images being blurred most in the centre region of the pipe. Small bubbles in the pipe centre are no longer resolved but appear as some grey shadow in the reconstructed image. Moreover, discontinuities are always reconstructed into gradual changes of the dielectric distribution. Translating this spatial effect into the time dependency of the voidage measured in single image pixels results in a smoothing of the time series, i.e. sudden changes of the pixel’s voidage are also reconstructed into gradual increases or decreases. This effect makes the reconstructed time series more regular and periodic and therefore results in an underestimation of the Kolmogorov entropy.

- The estimation of the Kolmogorov entropy is based upon the average rate at which interpoint distances on the attractor grow beyond a characteristic length scale. The average absolute deviation described earlier was found to be an appropriate measure of this length scale. Only if there is a perfect match between the reconstructed attractors of pressure data and voidage measurements, the average absolute deviation will in both cases be similar, resulting in similar estimates of Kolmogorov entropy. Most likely this will not be the case and lead to a systematic deviation between the entropies estimated from pressure and voidage measurements.

- For an estimation of the Kolmogorov entropy, the number of points per average cycle of the time series used, i.e. the number of data points per average cycle time should be of the order of at least 50, and preferably of the order of 100. However, the voidage time series from the ECT measurements show a number of points per cycle of, roughly speaking, 50 near the pipe wall and about 30 in the centre of the column. The latter is rather low and may therefore lead to a bias in the entropy estimation. In this case a low number of points per cycle, the entropy will generally be over estimated. Consequently, an entropy calculated from time series recorded at a higher sampling rate (say 200 frames per second) might make the deviation from the pressure data even larger.

- Fluidized beds are known to be spatio-temporal systems. As already mentioned earlier, the measurement of a global signal such as pressure is also influenced by events taking place at some distance from the pressure probe, the intensity of their contribution depending on the separation. Therefore, pressure time series does not only contain information on chaotic dynamics at the location of the pressure probe. For this reason the global measurement is expected to yield larger entropy values with respect to the more local tomographic voidage measurement.

All these factors probably contribute to some extent to the difference in the estimation of the Kolmogorov entropy from voidage and pressure data. With respect to the pressure measurements, the poor image resolution of the ECT system in the pipe centre seems to have the largest impact on the result shifted to small entropy values.

Figure 11 shows the radial dependency of the peak–peak distance, i.e. the difference between the minimum and maximum values in the time series. The peak–peak distance is much larger in the centre of the column than at the wall. This is related to the average radial distribution

**Table 1.** Comparison of the Kolmogorov entropy estimated using pressure measurements \( (K_p) \) and voidage measurements \( (K_v) \).

<table>
<thead>
<tr>
<th>( U_0/Umf [-] )</th>
<th>( K_p ) (bits s(^{-1}))</th>
<th>( K_v,centre ) bits/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>3.3</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>3.9</td>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>4.6</td>
<td>21</td>
<td>15</td>
</tr>
<tr>
<td>6.5</td>
<td>25</td>
<td>18</td>
</tr>
<tr>
<td>7.8</td>
<td>27</td>
<td>20</td>
</tr>
</tbody>
</table>

**Figure 10.** Dependency of the Kolmogorov entropy on the fluidization velocity; \( H_B = 35 \) cm.
of voids and bubbles at the axial location of the bed. In general, the bubbles rise preferably throughout the centre part of the column and not along the pipe wall. This implies that low or even zero solids concentrations are more likely towards the pipe centre, irrespective of the gas velocity. Consequently, independent of the gas velocity, the largest variation of the measured voidages is expected in the centre pixels rather than in near wall pixels, which is in agreement with the results in figure 11.

Figure 12 gives the radial dependency of the average absolute deviation. The largest deviations are observed towards the centre of the column which is in agreement with the more intense dynamics expected more in the centre than in the wall region due to the more frequent passage of bubbles. The slight dependency of the average absolute deviation on the gas velocity may be attributed to the passage of larger bubbles rising at the higher gas velocities.

The radial dependency of the average cycle frequency is shown in figure 13. The cycle frequency at the centre of the bed is about 25% to 50% higher than at the wall. This is also in agreement with the picture of a stream of bubbles rising towards the centre of the column rather than to the wall, causing a more frequent variation in voidage at the centre than at the wall. Furthermore, a clear dependency on gas velocity is observed, showing an increase in the average cycle frequency with increasing gas velocity. This effect corresponds with a higher frequency of bubble passages at a higher gas velocity.

4. Conclusions

- A 12-electrode ECT system has been applied to a 28.4 cm ID fluidized bed as an instrument to record voidage data for chaos analysis. Two types of primary capacitance sensors were tested. As predicted by FEM simulations, only the METC-type sensor yielded reliable results on the large scale (28.4 cm ID) fluidized bed.

- The preliminary results using the linear back projection (LBP) image reconstruction algorithm without threshold functions can be related to earlier results from pressure fluctuation measurements. ECT is shown to be a promising technique for characterizing the chaotic fluidization dynamics.

- The first results stimulated measurements to be performed over a wide range of operational parameters to get a profound insight into the chaotic fluidization dynamics. A wide variety of measurements will serve to further compare voidage and pressure measurements and, finally, to enhance the reliability of rules for scale up to large industrial facilities.

- Since the image reconstruction algorithm used (LBP) merely gives qualitative data, a quantitative algorithm using multiple iterations is being developed. If this algorithm reliably converged to the correct solution for any presumed dielectric distribution, the raw capacitance data used in this study would be reconstructed once again for a further comparison with these and earlier pressure results.

References

[8] Segal G SEPRAN manuals Ingenieursbureau SEPRAN, Leidschendam, The Netherlands