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Coupled Spectral Element Method and Harmonic Model for Magnetic Modelling of Linear Synchronous Machines

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Abstract—In this paper a hybrid approach is proposed which couples the semi-analytical solution of Laplace equation with the numerical spectral element method (SEM) formulation. This approach comes to decouple the conforming mesh which is used to independently mesh the moving and stationary part of an electrical machine (EM). A particular example is selected for this work which further takes the advantage from semi-analytical methods developed in literature. A permanent magnet synchronous linear machine (PMSLM) is considered, where both the airgap and the magnet array are semi-analytically solved, the iron core part and the winding are modelled with the high order SEM. The obtained numerical results are contrasted with the outcome from finite element method (FEM) revealing the advantages of newly developed approach.

Index Terms—Spectral Element Method, Finite Element Method, Electrical Machines, Hybrid method.

I. INTRODUCTION

The design of EMs rely largely on so-called semi-analytical (SA) models. The main motivation for their development is the reduced computation time and reasonable accuracy of key performance indicators of EM [1], [2]. One of the most important approaches of SA models is to use the solution of the Poisson equation to compute the field quantities in the most essential regions for electromechanical energy conversion of EM, such as airgap or slots, by employing a so-called subdomain problem [3], [4]. There have been numerous attempts to extend the subdomain technique to capture other aspects of EMs such as saturation of magnetic materials, or incorporate more detailed features of EM complex geometry. The alternative concepts that can incorporate those levels of complexity are pointing at a hybrid approach by coupling the subdomain technique with Finite Element Models [5], [6] or networks of lumped parameters [7]–[9] which would model the parts of the machine containing complex geometries or/and magnetic saturation [10]. On the numerical methods side, in the attempts to avoid high mesh density in the high aspect regions of EM, high order methods have been proposed such as SEM [11], based on high order Legendre basis functions, and Isogeometric Analysis (IgA) [12], [13]. These methods employ relatively large quadrilateral mesh elements, which are also suggested in [5], which lead to the challenge of modelling sliding regions to account for the rotation or translation of machine parts. From IgA perspective, the coupling between

stationary and moving parts is implemented in [14].

These hybrid couplings enable the modelling of the rotor or translator at an arbitrary position without changing the mesh or the configuration of subdomain definition, i.e. avoiding the meshing procedures for each simulation step. An additional advantage is that the solution in the airgap is obtained as Fourier coefficients, thus facilitating the post-processing routines [15]. Although the hybrid coupling provides advantages, it is also subject to the increase of the computational burden compared to SAs which employ a purely subdomain approach. The resulting system of equations, although a lot smaller, loses its sparsity resulting in the increase of computational burden.

This paper introduces a hybrid coupling between the SA method with SEM to efficiently model the geometry of a linear machine. The employed SA solution and the coupling with spectral element formulation is presented. Although not shown in this paper, this concept can be applied for nonlinear problems as well, as it is shown in [13]. To expose the effectiveness of the newly proposed approach, despite of its high susceptibility to increased computational complexity, a convergence experiment is performed where the new results are contrasted with the results from FEM. The paper is concluded with discussions on the convergence test and reflection on the relevance of the proposed approach.

II. MODELLING

The hybrid modelling approaches are attractive thanks to their potential to take advantage from different families of modelling methods. In addition to the capability to adopt independent meshing techniques for stationary and moving part of a electromechanical systems as proposed in [14], other authors replace the entire airgap regions with the solution of a Laplace equation [5], [7]. With the approach proposed in this paper, similar coupling is implemented to the later mentioned works, i.e. decoupling the moving and stationary parts meshed with spectral element (SE) by a SA region, as shown in Fig. 1b. However, a particular example is treated in more detail, where both, the airgap region and the magnet array is modelled semi-analytically as shown in Fig. 1a.

Fig. 1. The case studies for the hybrid modelling. (a) Only the mover is meshed with SE elements, the airgap and stator is modelled with Fourier regions. (b) The mover and the stator are meshed with SE whereas the airgap is modelled with a Fourier region.

A. Spectral Element Method

Just like FEM, the Spectral Element Method is based on the weak formulation of the magnetic vector potential, A_z where the residual is projected on a set of Legendre basis functions β

$$\int_{\Omega_{\xi,\eta}} (\nabla \cdot (c \nabla A_z) - J_z) \beta d\Omega_{\xi,\eta} = 0. \quad (1)$$

Because of the orthogonality exhibited by these bases, the residual vanishes with the exponential rate with respect to the number of bases used. The order of each element can go high, in some cases reaching 100 or even more without significantly impacting the conditioning of the system or jeopardising the accuracy. In this paper, SEM is implemented with numerical integration, following the principles introduced in [16] and applied for EM modelling in [11].

B. Semi-analytical solution

For crucial regions in the machine geometries where the iron is absent, such as the airgap, it is convenient to employ the SA solution of the Poisson equation

$$A_z = g_0(\eta) + \sum_{n=1}^{\infty} a_n(\eta) \sin(\omega_n \xi) + b_n(\eta) \cos(\omega_n \xi), \quad (2)$$

where, for Cartesian coordinate system, g_0 , a_n and b_n are

$$a_n(y) = C_{1n} e^{\omega_n y} + C_{2n} e^{-\omega_n y}, \quad (3)$$

$$b_n(y) = C_{3n} e^{\omega_n y} + C_{4n} e^{-\omega_n y}, \quad (4)$$

$$g_0(y) = A_0 + B_0 y, \quad (5)$$

and, for polar coordinate system

$$a_n(r) = C_{1n} r^{\omega_n} + C_{2n} r^{-\omega_n}, \quad (6)$$

$$b_n(r) = C_{3n} r^{\omega_n} + C_{4n} r^{-\omega_n}, \quad (7)$$

$$g_0(r) = A_0 + B_0 \ln(r), \quad (8)$$

where ω_n represents the spatial frequency of the solution. The functions g_0 , a_n and b_n in (2), are decoupled from the trigonometric functions. It should be noted, that the trigonometric basis functions are periodic along ξ . Periodic basis functions, make the solution from (2) suitable for modelling the magnetic field distributions of EM since they have a periodic structure.

C. The coupling

Both cases presented in Fig. 1 employ an interfacing line that ensures the coupling between solutions in SA region represented by the solution from (2), also known as Fourier modelling (FM), and the weak form expression for SEM. The integration of FM regions with the geometrical structures meshed with SEs will require a set of boundary conditions enforced on the interfaces. In the literature on SA, the boundary conditions for coupling multiple regions are based on the continuity of the first derivatives of the magnetic vector potential (MVP), and MVP itself [3].

To couple spectral elements (SEs) with the SA layer, the continuity of MVP from SEM region is imposed on the boundary of SA region

$$A_{zSEM}|_{\zeta} - A_{zFM}|_{\zeta} = 0, \quad (9)$$

where A_{zSEM} is the magnetic vector potential (MVP) solution obtained from SEM and A_{zSA} is the semi-analytical solution on the interface. The second boundary condition refers to the tangential component of magnetic field strength H_t

$$H_{tSEM}|_{\zeta} - H_{tSA}|_{\zeta} = J_z \zeta, \quad (10)$$

where $J_z \zeta$ is the source located on the interface. The later boundary condition is applied weakly on the boundary of SEM region.

Computation of frequency domain matrices, which expresses the modal formulation on SEM nodes from ζ is straightforward, as shown in the previous section. However, the Fourier coefficients from the unknown solution on ζ are obtained through an inner product operation.

The problem is graphically represented in Figure 2, where the interface line is segmented in M one dimensional SEs. Each element can have an arbitrary length and degree of Legendre polynomial, i.e. an arbitrary number of Legendre Gauss-Lobatto (LGL) nodes. The solution, on ζ is approximated by Legendre polynomials in each element, hence in a piecewise manner, as illustrated in Figure 2. In a similar way, the solution

Fig. 2. Visual representation for projection of a piecewise nodal function on Fourier series.

can be approximated by a sum of trigonometric basis functions along the entire ζ

$$f(\zeta) \approx \sum_{n=1}^Q a_n \sin(\omega_n \zeta) + b_n \cos(\omega_n \zeta) + g_0. \quad (11)$$

Finding the trigonometric coefficients, involves projection of the function on ζ onto trigonometric functions with help of the inner product, i.e. through the Fourier transform:

$$g_0 = \frac{1}{2\tau} \int_0^{2\tau} f(\zeta) d\zeta, \quad (12)$$

$$a_n = \frac{1}{\tau} \int_0^{2\tau} f(\zeta) \sin(\omega_n \zeta) d\zeta, \quad (13)$$

$$b_n = \frac{1}{\tau} \int_0^{2\tau} f(\zeta) \cos(\omega_n \zeta) d\zeta. \quad (14)$$

The approximation of $f(\zeta)$ function from SEM side is nodal piecewise, therefore a Lagrangian polynomial representation is adopted for the definition of $f(\zeta)$. The projection takes place on the entire domain as a sum of integrals

$$g_0 = \frac{1}{2\tau} \sum_{k=1}^M \sum_{j=0}^N f_j \underbrace{\int_{\theta_{k-1}}^{\theta_k} l_j(\xi) d\zeta}_{I_{g0,jk}}, \quad (15)$$

$$a_n = \frac{1}{\tau} \sum_{k=1}^M \sum_{j=0}^N f_j \underbrace{\int_{\theta_{k-1}}^{\theta_k} \sin(\omega_n \zeta) l_j(\xi) d\zeta}_{I_{a_n,jk}}, \quad (16)$$

$$b_n = \frac{1}{\tau} \sum_{k=1}^M \sum_{j=0}^N f_j \underbrace{\int_{\theta_{k-1}}^{\theta_k} \cos(\omega_n \zeta) l_j(\xi) d\zeta}_{I_{b_n,jk}}. \quad (17)$$

The integrals from (15), (16) and (17), cannot be solved analytically. Therefore, the LGL quadratures are used to approximate the integrals. Also, it can be seen, that the nodal values of the function f_j on ζ are independent from the integration, allowing to solve it a priori. The Fourier coefficients of the approximated function $f_j(\zeta_j)$ are computed by using the transformation matrices resulting from each of the integral (15), (16) and (17)

$$g_0 = \underbrace{\begin{bmatrix} I_{g00}^1 & I_{g01}^1 & \cdots & I_{g0N_M}^M \end{bmatrix}}_{\mathbf{T}_{g_0}} \begin{bmatrix} f_0 \\ f_1 \\ \vdots \\ f_{N_{tot}} \end{bmatrix}, \quad (18)$$

$$\vec{a} = \underbrace{\begin{bmatrix} I_{a1,0}^1 & \cdots & I_{a1,N_M}^M \\ \vdots & \ddots & \vdots \\ I_{aH,0}^1 & \cdots & I_{aH,N_M}^M \end{bmatrix}}_{\mathbf{T}_{\vec{a}}} \begin{bmatrix} f_0 \\ \vdots \\ f_{N_{tot}} \end{bmatrix}, \quad (19)$$

$$\vec{b} = \underbrace{\begin{bmatrix} I_{b1,0}^1 & \cdots & I_{b1,N_M}^M \\ \vdots & \ddots & \vdots \\ I_{bH,0}^1 & \cdots & I_{bH,N_M}^M \end{bmatrix}}_{\mathbf{T}_{\vec{b}}} \begin{bmatrix} f_0 \\ \vdots \\ f_{N_{tot}} \end{bmatrix}, \quad (20)$$

where I is the result of the integrals, the superscript is the index of each element and the subscript $N_{tot} = \sum_{k=1}^M N_k + 1$ is the total number of nodes on ζ . The matrices which are multiplied with the nodal function on ζ are the transformation matrices which will be denoted as $\mathbf{T}_{g_0}$, $\mathbf{T}_{\vec{b}}$ and $\mathbf{T}_{\vec{a}}$ for future reference.

III. THE SYSTEM OF LINEAR EQUATIONS

The matrix components introduced in the previous sections are assembled to form a new global matrix which incorporates the sub matrices of SEM for the elements of both the stator and the moving part of the EM plus the coupling matrices introduced in the previous section. First, the interface elements are identified, the information such as the number of elements, their individual length in the computational domain and the number of LGL nodes is necessary. Secondly, the matrices $\mathbf{T}_f$, $\mathbf{H}_s$ and $\mathbf{H}_f$ are built. And lastly, the computed matrices are assembled with $\mathbf{E}$ creating a new global matrix. The new global matrix is identified as

$$\mathbf{E}_{new} = \begin{bmatrix} \mathbf{E} & -\mathbf{H}_s \\ \mathbf{T}_f & -\mathbf{H}_f \end{bmatrix}. \quad (21)$$

The transformation matrices $\mathbf{T}_{g_0}$, $\mathbf{T}_{\vec{b}}$ and $\mathbf{T}_{\vec{a}}$ are used to transform the nodal values on the interface of spectral elements with the FM region into dc, sine and cosine coefficients, respectively to satisfy the boundary conditions. Together they form the transformation matrix $\mathbf{T}_f$. The Matrix $\mathbf{H}_f$, is a square matrix of $[4Q + 2]^2$ in size which corresponds to the number of unknowns in the FM region. The matrices $\mathbf{T}_f$ and $\mathbf{H}_s$ are populated with more columns and rows with zeros, to match the degrees of freedom (dof) of $\mathbf{E}$. Finally, the matrices $\mathbf{T}_f$ and $\mathbf{H}_s$ will have sizes of $[(4Q + 2) \times (\text{dof}_{SEM})]$ and $[(\text{dof}_{SEM}) \times (4Q + 2)]$, respectively.

IV. NUMERICAL ANALYSIS

The coupling approach between SEM and the SA method introduced in the section II is applied to a linear machine presented in Fig 3. The stator core together with the coils in the slots are modelled with SEM. The SEM mesh is contoured with blue. The air gap and the magnet array are modelled with SA regions described by (2). Preliminary results are shown in Fig. 4. The source code for the model used in this paper is openly available [17]. The back iron is considered to have an infinite permeability, therefore a Neumann boundary condition is imposed at the bottom of the magnet array. The boundary of

Fig. 3. The permanent magnet synchronous linear machine selected as benchmark for modelling of the electrostatic and thermal domains.

Fig. 4. The flux lines distribution for the model which employs SA and SEM regions.

the SA regions is denoted with a dashed black line. The SEM region is interfacing the SA region with nine spectral elements. The SEM mesh is chosen such that it uses least number of elements and at the same time the mesh is conforming. Given that the geometry is quite simple, the elements interfacing the SA region consist of two widths only, the elements meshing the teeth are larger than the one for slots.

A. Refinement

The performance of SA coupling with the numerical SE method is compared against a FEM commercial software named after Altair [18]. In the FEM model of the machine second order triangular elements are used. Furthermore, three parameters are defined to distribute the mesh density across the geometry, and example of this mesh is shown in the left half of the geometry from Fig. 5. The parameters define the lengths of the elements in the airgap, teeth and the back-iron.

Fig. 5. The optimal mesh obtained for both methods. The left side of the figure features the triangular FEM mesh, whereas the right side shows the hybrid discretisation, here Q is the number of harmonics in both regions, airgap and magnet array.

For the p -refinement of SEM, which defines the degree of each spectral element, the refinement along x -axis and y -axis is identified. The x -axis refinement is adjusted to the number of Fourier harmonics such that it satisfies certain accuracy and does not allow for aliasing to occur. It is obvious that the part of geometry close to the back iron is over-refined in the x -direction due to the exploitation of the conforming mesh type. A relaxation of this refinement would lead to significant reduction of dofs in the system without impacting the accuracy of the solution. The y -axis refinement is restricted to two parameters, the elements meshing teeth, slots and the back-iron.

B. Results

The comparison of the results resulting from both methods is based on the computation of so-called "cogging force" which is the force occurring from the tendency of the magnet flux to align with the teeth when there is no current in the winding, in this way minimizing the energy stored in the magnetic circuit of the machine. This effect causes a great deal of energy displacement resulting from a small displacement and is challenging the conditioning of the linear system, which for this problem cannot be very favourable due to the high difference in permeability values and aspect ratios of the geometry. The result of the cogging force from Fig. 6 is computed with the refinement from Fig. 5. The attempts to obtain better match between the models by increasing the mesh density for both methods did not succeed, at least for the refinement set-up described in the section above. Therefore, a provisional rationale is that the condition number of the linear system for both method does not allow it.

C. Convergence test

A convergence test for both methods is performed using the PMSLM model. The L^2 norm is used to compute the errors between the reference and a particular case i , when a set of

Fig. 6. The results of cogging torque computation for hybrid SEM coupling and FEM for the displacement of one full section.

Fig. 7. Convergence test contrasting the error versus dofs employed for both SEM and hybrid SEM.

Fig. 8. Convergence test contrasting the error versus computation time employed for both SEM and hybrid SEM.

refinement parameters are tested

$$\text{err} = \int_0^{2\tau_p} |F_{x,\text{ref}} - F_{x,\text{ref}}|^2 dx. \quad (22)$$

Given that the error function is periodic, the integral from (22) is approximated using Fourier quadrature. The force is computed for 240 positions implying that an exact integration of the error is ensured up to 239 harmonics. The force in FEM is computed with virtual work principle [19], whereas in the case of hybrid coupling, the advantage is taken of the solution being represented in the airgap in terms of harmonics and the force is computed from the Maxwell Stress Tensor method [20]. As explained in the previous section, the results are greatly affected by the conditioning of the linear systems of both methods. Therefore, a reference result is identified for each method separately. As it is shown in Fig. 6, the simulations are performed for a full periodic section displacement revealing six periodic variations of the cogging force. To generate the reference value of this profile, a mean

value is computed to obtain one cogging period which is repeated six times to span the entire simulation displacement.

Finally, the convergence tests reveal that, as it is expected, the coupling approach yields a similar accuracy compared to FEM with a considerably reduced amount of dof, shown in Fig. 7. This comes at a price that the linear system in the hybrid approach is much less sparse than FEM counterpart affecting the computation time which is reflected in Fig 8, where the pareto-set for the optimal dofs of a given error is plotted versus the computational time. Still, the quasi-plateau that is obtained from SEM results indicates that a model which takes half of the simulation time compared to the most accurate model will still give very good results saving plenty of computational burden.

D. Discussions

The numerical results presented in this paper cannot be generalised. The outcome rather shows that there are cases when one or the other method can be more advantageous. Still, for each method there is room for improvement to make the model faster without affecting the accuracy.

In case of SEM, coupling errors between the SA and SE regions are computed for each harmonic equally, however this turns out to be quite a high penalty resulting in a high refinement which can also be observed in Fig. 5. In reality, higher harmonics will carry significantly lower magnitudes of the solution. Also, the relaxation of SE can be implemented close to the back-iron to reduce the redundant dofs.

The first highlight from the computational test results in Fig. 8 is the vertical pattern of the convergence plot. This indicates that neither the meshing time, which lasts for about four second in every simulation in FEM and non-existing in the hybrid coupling approach, nor the solving time affects the computational burden with the increase of the problem size. It rather shows that the workflow during the simulation process has some tasks which within the problem sizes considered for the problem in this paper takes similar amount of time.

V. CONCLUSION

In this paper a hybrid model has been proposed for modelling the electrical machines. The model incorporates the semi-analytical solution of the Poisson equation and the high-order spectral element method. For the case of a PMSLM, both the magnet array and the airgap has been modelled with the SA approach. Also, the Harmonic nature of the solution in the airgap allowed to compute the force in the airgap using the Maxwell Stress Tensor with a great degree of accuracy. The use of a reduced amount of harmonics in the semi-analytical region and high accurate elements in the SEM region makes the hybrid model competitive compared to the low-order finite element counterpart.

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