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Dynamic Equivalencing of Distribution Networks with Dispersed Generation Using Krylov Methods

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Abstract—Nowadays the number of dispersed generators (DG) is growing rapidly. This change will greatly influence the power system dynamics. A distribution network, where DG are connected to the grid, cannot be considered as passive anymore. So in future it will not be possible to use simple equivalents of distribution networks for power system dynamic modeling as it was done before. In dynamic studies the whole power system cannot be represented in a detailed manner because of huge system dimension. Therefore special techniques have to be applied for aggregation and order-reduction of distribution networks with DG. In this paper brief review of existing techniques has been given, and after that dynamic reduction using Krylov methods is performed for a distribution network consisting of DG of different types.

Index Terms—distributed generation, aggregation, model reduction, Krylov methods.

I. INTRODUCTION

LARGE-SCALE implementation of dispersed or distributed generators (DG) leads to a gradual transition from the current ‘vertically-operated power system’, which is supported mainly by several big centralized generators, into a future ‘horizontally-operated power system’, having also a large number of small to medium-size distributed generators [1].

The main feature of DG concept is that such generators are connected to power system at distribution network level. Therefore this network cannot be considered as passive anymore since the penetration of DG becomes more and more significant. The higher the amount of DG is, the greater is the influence on power system dynamics. In the future it will be not possible anymore to neglect dynamics introduced by generators connected at the distribution level. At the same time, detailed modeling of the whole power system is not practical due to computational and time constraints associated to the large number of active elements. Finding reduced-order dynamic equivalents of distribution networks with DG will become more and more important.

II. STATE OF THE ART OF POWER SYSTEM DYNAMIC REDUCTION

Although research in the area of dynamic reduction dates back to the 1970s, experience on this subject is limited and reductions often are done using heuristic methods [2].

Usually, a power system is partitioned into two areas during dynamic reduction:

- the study area, which contains the variables of interest, and

therefore it is represented in a detailed manner;

- the external area, which is important only as far as it influences the analysis of study area, and typically it can be represented in reduced form by a linear system.

The dynamic reduction of power system model can be divided in three stages: coherency identification, generator aggregation, and network reduction.

At the stage of *coherency identification* coherent generators are identified and grouped together. Usually, this identification procedure is simplified by using the following assumptions:

- non-generator dynamics may be ignored;
- classical generator models may be used;
- linearized system preserves well the trends of coherency.

Under these assumptions, the system model can be represented either in state-space form or by a transfer function matrix. Such forms are not only convenient for analysis, but also are required by most of the control design methods.

Three coherency identification methods are most widely used:

1. *Weak Links* [3]. In this method, the coherency is determined by analyzing the coupling of generators in the state matrix (system is represented in state-space form). A group of generators is identified as coherent if the coupling coefficients among them are high.
2. *Two Time Scale* [4]. This method is based on the concept that a slow oscillation is caused by two groups of strongly coherent generators interconnected through weak ties. Computationally, the two oscillating generator groups can be easily identified using the eigenvector associated with the mode of oscillation.
3. *Linear Time Simulation* [5]. This is the classical method to identify coherent generators. The time domain response of the system is solved for a specified fault in the power system, and the rotor angles of generators are compared. Those generators with rotor angles swinging together are identified as coherent.

The second step of dynamic reduction is the *generator aggregation*. Generally, the generator aggregation can take one of two forms:

- *Classical Aggregation*. Generators in a coherent group are represented by an equivalent classical generator model. The equivalent inertia is the sum of the inertia of all generators in the group, and the equivalent transient reactance is obtained by paralleling the transient reactances of all generators in the group.

- *Detailed Aggregation.* If some or all generators in a coherent group have similar control systems, they can be aggregated to a detailed generator model with an equivalent exciter, stabilizer, and governor (as, for example, in [6]). The parameters of the equivalent models are obtained using a combination of two approaches: a least square fit of the frequency responses to determine the linear characteristics, and an evaluation of the time-domain constraints to set nonlinear characteristics.

The last stage of dynamic equivalencing is the *network reduction*. The equivalent generators are inserted in to the system and the generators in the associated coherent groups are removed. The network is modified to maintain the balanced steady-state powerflow conditions.

As can be seen from the description above, most of the reduction techniques, which are referred as classical, are heuristic (experience-based) or semi-heuristic. These methods have been developed for transmission systems with relatively small number of large synchronous generators. However, DG are numerous and are not limited only to synchronous generators: many of them are based on induction machines, they may use power electronics and have quite different control systems, which are rather difficult for aggregation. Some of them even do not have rotating parts (for example, fuel cells and solar panels). Term ‘coherency’ widely adopted by traditional reduction methods becomes less meaningful since induction generators do not have synchronizing torques and power electronics interfaces can almost completely separate generators from the external network (in dynamical sense). Obviously, these methods might give a very rough approximation in case of dynamic analysis of complex power systems with DG. Therefore strictly mathematical techniques that originate from systems theory have to be applied in order to retain necessary precision during dynamic reduction and, at the same time, reduce simulation time.

Most of the strictly mathematical methods of model reduction focus on linear systems, which in many cases provide accurate descriptions of physical systems. There are techniques based on directly identifying and preserving certain modes of interest (modal model reduction) or based on the singular value decomposition (SVD), for instance, balanced truncation, focusing on the observability and controllability properties of the system. Another family of model reduction techniques is based on Krylov methods. Its main objects of interest are the leading coefficients of a power series expansion of the reduced system transfer function around a user-defined point that have to match those of the original system transfer function [7].

All methods mentioned above have certain advantages and disadvantages (they are summarized in Table 1). Generally, SVD-based methods are hardly applicable to systems of large dimensions since singular values of unreduced system have to be computed and such a computation might be cumbersome for systems of large order. Krylov methods are iterative in nature and can be used for systems with large number of states. On the other hand, Krylov methods do not provide the bounds for the error of approximation while SVD-based methods do

so. Therefore the selection of the reduction method has to be performed in accordance with the desired goals and taking into account the structure of the given system. In this paper a Krylov method (the Arnoldi variant) has been chosen for the model reduction since studied system is rather large.

Table 1. Comparison of SVD-based and Krylov methods

SVD-based methods		Krylov methods	
Advantages	Disadvantages	Advantages	Disadvantages
Error is bounded	Not applicable for large systems	Applicable for large systems	Error is not bounded
Stability is preserved	Requires calculation of singular values for original system	Numerically efficient	Stability is not preserved

Krylov-based model reduction has been used for studies of slow electromechanical processes in transmission network with large synchronous generators (for example, see [7]).

In this paper, the Krylov method is applied for a distribution network with generators of different type and size, and it is done in the context of electromagnetic and fast electromechanical transients.

III. BASICS OF KRYLOV METHODS

The algorithm of a Krylov-based model reduction is shown schematically on Figure 1. Below is the explanation, which clarifies the figure.

Consider a linear dynamical system in state-space form:

$$\frac{dx}{dt} = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) + Du(t)$$

where x are states of the system, u – its inputs, y – its outputs.

The transfer function describing the system (1) can be written as:

$$H(s) = C(sI - A)^{-1}B + D \quad (2)$$

The transfer function of the original system can be expanded in series around certain point s_0 :

$$H(s) = \eta_0(s_0) + \eta_1(s_0) \cdot \frac{(s-s_0)}{1!} + \dots + \eta_i(s_0) \cdot \frac{(s-s_0)^i}{i!} + \dots$$

where η_i are called the moments of the system and are defined as:

$$\eta_i(s_0) = (-1)^i \cdot \left. \frac{d^i H(s)}{ds^i} \right|_{s=s_0}, \quad i \geq 0 \quad (3)$$

Series expansion can be done around different points. When the expansion is around infinity ($s_0 = \infty$), the moments are

called Markov parameters and the resulting model reduction problem is known as *partial realization*.

Then the moments can be derived from (2) and (3) as:

$$\eta_0(\infty) = D, \quad \eta_i(\infty) = CA^{i-1}B, \quad i > 0 \quad (4)$$

If the moments are chosen at zero ($s_0 = 0$), the corresponding model reduction problem is known as *Pade approximation* and the moments are equal to:

$$\eta_0(0) = D + CA^{-1}B, \quad \eta_i(0) = CA^{-(i+1)}B \quad (5)$$

Figure 1. Krylov-based model reduction

In the general case $s_0 \in \mathbb{C}$ (s_0 is arbitrary complex number), the reduction problem is known as *rational interpolation*. The moments can be written as:

$$\eta_0(s_0) = D + C(s_0 I - A)^{-1} B \quad (6)$$

$$\eta_i(s_0) = C(s_0 I - A)^{-(i+1)} B, \quad i > 0$$

There exist two algorithms for the solution of the model reduction problem: *Lanczos* and *Arnoldi* algorithms. The algorithms are dependent on the point s_0 , around which the series expansion is done.

The Lanczos algorithm has the advantage of being simple, efficient, and very fast but it suffers from numerical stability [8]. It uses A, B, C matrices of the original system as inputs and produces the projection matrices V_t and W_t as outputs.

The Arnoldi algorithm is not so fast but far more stable. It uses A, B matrices of original system as inputs and produces the projection matrix V_t as output. Since we are aiming at fast phenomena the expansion point is selected to be equal to infinity. The Arnoldi algorithm for partial realization ($s_0 = \infty$) is shown below:

```
function [V_t] = BlockArnoldi(A,B,k)
m = size(B,2)
q = k/m
[V_1, R] = QR(B)
for j = 1:(q-1)
    W = A V_j
    for i = 1:j
        H = V_i^T W
        W = W - V_i H
    end
    [V_{j+1}, H] = QR(W)
    V_t = [V_1 V_2 ... V_q]
```

QR stands for QR matrix decomposition.

After applying the Lanczos or the Arnoldi algorithm the state-space matrices of the reduced system $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ can be found. In case of using the Lanczos algorithm the matrices are equal to:

$$\begin{aligned} \tilde{A} &= W_t^T A V_t, & \tilde{B} &= W_t^T B \\ \tilde{C} &= C V_t, & \tilde{D} &= D \end{aligned} \quad (7)$$

In case of the Arnoldi algorithm:

$$\begin{aligned} \tilde{A} &= V_t^T A V_t, & \tilde{B} &= V_t^T B \\ \tilde{C} &= C V_t, & \tilde{D} &= D \end{aligned} \quad (8)$$

Generally, the problem of model reduction using Krylov methods can be formulated as follows. For a given original

system of large dimension (1), find a reduced system of order k (where k is significantly less than the order of the original system n):

$$\begin{aligned} \frac{d\tilde{x}}{dt} &= \tilde{A}\tilde{x}(t) + \tilde{B}u(t) \\ y(t) &= \tilde{C}\tilde{x}(t) + \tilde{D}u(t) \end{aligned} \quad (9)$$

such that the moments of original and reduced systems are equal up to the k -th term of series expansion (for Arnoldi algorithm) or $2k$ -th term (for Lanczos algorithm):

$$\begin{aligned} \eta_j &= \tilde{\eta}_j, \quad j = 1, 2, \dots, k \quad (\text{Arnoldi}) \\ j &= 1, 2, \dots, 2k \quad (\text{Lanczos}) \end{aligned} \quad (10)$$

That is why Krylov methods sometimes are referred as moment matching methods.

In the end, there are two important remarks that have to be made:

1. The reduction can be done using only state-space matrices (i.e. it is not necessary to explicitly compute the moments).
2. The reduction does not modify inputs and outputs of the original system.

IV. DYNAMIC MODELING OF DISTRIBUTION NETWORK WITH DISPERSED GENERATION

The Lelystad 10 kV distribution network is shown on Figure 2. Six feeders with a total load of 10.8 MW are represented in details, while other feeders are modeled in the form of an aggregated load of 18 MW peak power consumption. The total installed capacity of DG units is 10.7 MW.

The DG in the test network is represented by:

- three squirrel cage induction generator wind turbines

(SQIG, SQIG1, SQIG2 on Figure 2) with rated power of 1.5, 0.66 and 0.66 MW respectively;

- 1.5 MW doubly-fed induction generator wind turbine (DFIG);
- 0.25 MW split shaft microturbine (μ Turbine);
- two CHP plants consisting of six 0.25 MW microturbine units each (Plant1, Plant2);
- 3.125 MVA diesel generator (Diesel).

A detailed description of models of a squirrel cage and doubly-fed induction generator wind turbines can be found in [9] and [10]. The mechanical part of the wind turbine is represented by a steady-state relationship between the wind speed and the mechanical power of the turbine. In this simplified relationship the dynamic stall, wind shear and tower shadow effects are neglected.

Power electronic converters used in the rotor circuit of the DFIG are represented by algebraic blocks with averaged complex modulation indices (valid if we neglect transients due to power electronics switchings). In this situation all dynamics of the power electronics will be in its controller behavior. Such assumption is applicable for transient stability studies. It can be used also because we perform model reduction for networks that are at significant distance from the disturbance.

The diesel generator model includes the synchronous machine, excitation and the governor control (for a detailed model description see [11]).

The split shaft microturbine is similar to a combustion gas turbine. It involves an air compression section, a recuperator, a burner or combustor, and a power turbine driving an induction generator through a gearbox, which is directly connected to the grid. Since short-term behavior (in the range of couple of seconds) is the main interest of this study, the recuperator and the

Figure 2. Lelystad distribution network

heat exchanger are not included in the model. The model parameters and its detailed description can be found in [12].

All generators are connected to the distribution network through transformers (not shown on Figure 2). The loads are represented by constant impedances. The grid is modeled as a voltage source.

The distribution network is connected to the transmission grid by means of two 47 MVA 150/10 kV transformers. A distant three-phase short-circuit is applied at the primary side of the transformers with a certain fault impedance resulting in a 50% voltage dip at the 10 kV substation. Duration of the fault is 0.25 s. It occurs at $t = 0.1$ s and is cleared at $t = 0.35$ s. Since the electrical distance from the fault to the generators in the distribution network is significant, it is considered that all dynamic elements of the distribution network can be represented without considerable error by linear models.

V. LINEARIZATION OF DISTRIBUTION NETWORK WITH DG

The first step towards Krylov-based model reduction is linearization since this method deals with linear systems. It has been performed for all nonlinear elements (i.e. for all DG, since other network elements in this study are already modeled in linear form). The operating points (states x_0 , inputs u_0 and outputs y_0) has been defined based on the loadflow results, and the Jacobians of the system at the operating point have been determined:

$$\begin{aligned} A &= \left. \frac{\partial f}{\partial x} \right|_{x_0, u_0}, \quad B = \left. \frac{\partial f}{\partial u} \right|_{x_0, u_0} \\ C &= \left. \frac{\partial g}{\partial x} \right|_{x_0, u_0}, \quad D = \left. \frac{\partial g}{\partial u} \right|_{x_0, u_0} \end{aligned} \quad (11)$$

where the original nonlinear system is represented by

$$\begin{aligned} \frac{dx}{dt} &= f(x(t), u(t)) \\ y(t) &= g(x(t), u(t)) \end{aligned} \quad (12)$$

Afterwards, the linearized state-space equations in incremental form have been obtained:

$$\begin{aligned} \frac{d}{dt}(\Delta x) &= A \cdot \Delta x + B \cdot \Delta u \\ \Delta y &= C \cdot \Delta x + D \cdot \Delta u \end{aligned} \quad (13)$$

where $\Delta x = x(t) - x_0$,

$$\Delta u = u(t) - u_0,$$

$$\Delta y = y(t) - y_0.$$

A three-phase short-circuit was simulated, and the currents on the terminals of the generators and the total current of the feeder were observed. Currents of the nonlinear and the linearized generator models are in good agreement (not shown in the paper due to space limitations). Responses of the nonlinear and the linearized overall system are also matching quite well as it can be seen in Figure 3. Therefore, the linear model adequately represents the dynamics of the system with the nonlinear elements.

The linear state-space model of the whole system has 529 states in total.

Figure 3. Response to the fault of a nonlinear and a linearized system model

VI. DYNAMIC EQUIVALENCING OF A DISTRIBUTION NETWORK WITH DG

One of the features of the chosen Arnoldi-Krylov method is that each iteration adds to the projection matrix a block of the same size as matrix B . Therefore the order of the reduced system is multiple to the number of inputs. In this specific example there are 9 inputs. After the third iteration the reduced system is a 27th order system with the response, which is still reasonably close to the response of the original nonlinear system (see Figure 4).

Figure 4. The total current of the original nonlinear and a reduced 27th order system

Figure 5. The total current of the original nonlinear and a reduced 18th order system

Further reduction to a 18th order system (i.e. using only two iterations of the algorithm) results in a bad agreement between the original and a reduced model. This fact is illustrated in Figure 5.

Therefore, for this study case a 27th order reduced order model appears to be the optimal in the sense of order versus performance.

VII. CONCLUSIONS

This paper illustrates the application of a model reduction using an Arnoldi-Krylov method on a distribution network with various DG. In most of the cases such a task can be hardly performed using traditional methods for aggregation and model reduction of power systems due to quite different dynamic behavior of DG, which is strongly influenced by controllers and power electronics. Therefore advanced mathematical techniques have to be used instead of classical heuristic methods.

Krylov methods are simple, they can be automated and they can provide a significant order reduction while retaining small error with respect to the original model. They are especially profitable when applied to large systems. The task of model reduction (or dynamic equivalencing) is extremely important for transient stability analysis, the coordination of controls, and an adequate representation of the external networks for analysis of dynamic response to disturbances.

VIII. ACKNOWLEDGEMENT

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X. BIOGRAPHIES

Anton Ishchenko was born in Krasnodar, Russia in 1980. He received MSc. in Electrical Engineering from the Kuban State Technological University, Russia in 2002. In 2001 he was honored with Russian Academy of Electrotechnical Sciences Award for outstanding student's paper in the area of "Power and Electrical Engineering". At the same year he received Russian Ministry of Education Award for outstanding student in the area of technical science. In August 2003 he joined the Electrical Power Systems group, Technical University of Eindhoven as PhD student. His major fields of interest are: renewable energy sources, control systems, power system protection and emergency controls, power system transients and stability.

Johanna M.A. Myrzik was born in Darmstadt, Germany in 1966. She received her MSc. in Electrical Engineering from the Darmstadt University of Technology, Germany in 1992. From 1993 to 1995 she worked as a researcher at the Institute for Solar Energy Supply Technology (ISET e.V.) in Kassel, Germany. In 1995 Mrs. Myrzik joined the Kassel University, where she finished her PhD thesis in the field of solar inverter topologies in 2000. Since 2000, Mrs. Myrzik is with the Eindhoven University of Technology, the Netherlands. In 2002, she became an assistant professor in the field of distributed generation. Her fields of interests are: power electronics, renewable energy, distributed generation, electrical power supply.

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