LONGITUDINAL DISPERSION IN OBLONG AERATED SYSTEMS

S P P OTTENGRAF
Laboratory for Physical Technology, Eindhoven University of Technology, P O Box 513, Eindhoven The Netherlands

(Received 12 March 1979 accepted 23 May 1979)

Abstract—A scale model study to the flow and mixing phenomena has been carried out in oblong aerated basins, where a transversal circulating flow of the liquid is introduced by dispersing air along one side of the basin. A semi-empirical correlation of dimensionless numbers has been developed, which is considered to give a reasonable prediction of the rate of longitudinal mixing. Experiments carried out in commercial basins have shown to be in good agreement with the presented model.

1 INTRODUCTION

In biological sewage treatment the solved and colloidal pollutions are eliminated by micro-organisms, which are agglomerated in activated sludge flocs. To provide the organisms with oxygen, air can be dispersed in the aqueous suspension, or aeration is carried out with rotating brushes or surface paddles. As a result of these aeration processes, circulating flows, turbulence and mixing are introduced in the aeration basin [1-4]. The mixing phenomena are necessary in order to keep the activated sludge in suspension, to distribute it throughout the reactor-volume and for promoting the physical mass transfer processes. As a consequence, also longitudinal mixing is introduced in the system.

The advantages of a strong longitudinal mixing in a continuously operating waste water treatment plant are (1) the dilution effect on toxic materials or slugs of degradable organics in the influent stream and (2) the provision for a more uniform physical environment for the biological culture.

On the other hand, longitudinal mixing is generally detrimental to the degree of conversion of a continuously operating process. Aeration basins are usually of the order of 3.5-4.5 m deep, 4.0-9.5 m wide and 30-100 m long. Generally, air bubbles are introduced into the liquid by means of perforated plates (e.g., the Inka system) or by porous tubes (e.g., the Brandol system). These diffusers generally are located along one side of the basin. Residence time distribution measurements have shown a considerable amount of back mixing [3, 4].

The longitudinal mixing rate depends on the degree of conversion of a continuously operating process. Aeration basins are usually of the order of 3.5-4.5 m deep, 4.0-9.5 m wide and 30-100 m long. Generally, air bubbles are introduced into the liquid by means of perforated plates (e.g., the Inka system) or by porous tubes (e.g., the Brandol system). These diffusers generally are located along one side of the basin. Residence time distribution measurements have shown a considerable amount of back mixing [3, 4].

The objective of this work is a further analysis of the mixing mechanism and the circulating flow. A semi-empirical correlation has been developed, which is considered reasonable to predict the rate of longitudinal mixing as a function of the extrinsic parameters of the system.

2 FLOW PHENOMENA AND MIXING IN THE SCALE MODELS

2.1 Experimental set-up

A scale model investigation has been set up for the analysis of flow and mixing phenomena. These models consisted of oblong basins with a rectangular cross-section, and at a linear scale of 1:20, 1:10 and 1:5 with respect to the investigated commercial basins. Air bubbles were continuously dispersed in water by means of a horizontal tube, provided with a single queue of equidistant perforations. This distance was varied from 0.5 cm to 2.0 cm, the diameter of the perforations being varied between 0.3 and 10 mm. The aeration tube was situated along one side of the basin, thus introducing an overall circulating flow in a cross-section of the basin by the lift of the air bubbles (see Fig 1).

Since the longitudinal rate of mixing and the mixing mechanism were shown to be independent of the hydraulic load to the basin, all experiments were carried out without liquid feed. The rate of longitudinal mixing was measured by injecting a pulse of a sodium chloride solution at one end of the basin and determining the electrical conductivity as a function of time at the other end. The longitudinal dispersion coefficient was calculated by comparing the measured response with the theoretical one [14] (see Fig 2).

\[
\frac{V_c}{\delta} = \frac{1}{\sqrt{(\pi F_0)}} \sum_{n=1}^{\infty} \exp \left\{ -\frac{(1 - 2n)^2}{4F_0} \right\}
\]

The influence on the circulating flow and the longitudinal rate of mixing of the following extrinsic parameters has been investigated: the superficial gas velocity \(v_o\), the height of submergence \(H^*\) of the aeration tube and the geometrical parameters of the basin.

2.2 The transversal circulating flow

By the lift of the air bubbles an overall transversal circulating flow is introduced in the basin. The streamline profile of this flow has been visualised in the (1:10)-basin by suspending polystyrene particles (\(\rho_d = 1002 \text{ kg/m}^3\)) in the liquid (see photographs 1a and 1b).
The polystyrene particles, having a diameter range of 120 up to 200 μm, were illuminated by intensive light sources located behind a narrow split perpendicular to the longitudinal axis of the basin. This resulted in an illuminated cross-section, in which the movement of the particles was observed from a direction perpendicular to it. By measuring the trajectories of the illuminated particles during exposure time, local fluid velocities could also be determined from the photographs (see Figs 3(a)–3(b)).

From the photographs it was concluded that the circulating flow generally has its centre in the middle of the cross-section. However, in case the liquid height \( H \) is significantly less than the width \( B \) of the basin, the flow profile is no longer rotationally symmetric at high circulating flow rates (see photograph 1b).

The circulating flow can be characterised by the circulating flow rate and the time of circulation \( \tau_c \). This is defined as the time a fluid element needs to cover a full circulation in a cross-section of the basin. It has been measured by injecting a tracer pulse in the periphery of the circulating flow and determining its concentration as a function of time in the plane of injection (see Fig 4).

If there is only one predominant circulating flow, the circulation time \( \tau_c \) is related to the flow rate \( q_c \) per unit length of the basin according to

\[
\frac{B}{H} \tau_c = \frac{\dot{V}}{q_c}
\]

The flow rate \( q_c \) can also be compared with the flow rate \( Q_c \) calculated from the measured velocity profiles (see Fig 5). From Fig 5 it can be concluded that the flow rate \( q_c \) calculated from the circulation time is generally somewhat higher than \( Q_c \).

From the photographs it will be clear that this is due to the fact that the effective area for circulation is somewhat smaller than the cross-sectional area \( B \times H \). However, it may be concluded that the transversal circulating flow rate can reasonably well be estimated from the circulation time \( \tau_c \), which can be measured very easily.
photograph ia streamline profile of the transversal circulating flow
\( (H = B = 40 \text{ cm}, H^* = 30 \text{ cm}, v_0 = 0.30 \text{ cm/s}, \tau_c = 47 \text{ s}) \)

2.3 Longitudinal dispersion coefficient

Tracer experiments have shown that the RTD of the investigated basins can best be described by the stagewise backflow model [3, 5]. In the further analysis it will nevertheless be started from the continuous backflow model. From its more simple

photograph ib streamline profile of the transversal circulating flow
\( (H = 30 \text{ cm}, B = 40 \text{ cm}, H^* = 29 \text{ cm}, v_0 = 0.19 \text{ cm/s}, \tau_c = 48 \text{ s}) \)
nature this model lends itself better to a physical-mathematical description in modelling the up-scaling factors. Moreover the continuous backflow model and the stagewise backflow model converge at increasing Peclet numbers. The correspondence between the two models has thoroughly been investigated by Roemer and Durbin [6]. The experimentally determined dependence of the longitudinal dispersion coefficient \( E_{ax} \) on the height of submergence \( H^* \) of the aeration device for the (1-10)-model is shown in Fig 6.

From the observed data it appears that

\[ E_{ax}(\cdot)H^*^{2/3} \]

and that for this particular geometry the gas velocity \( v_0 \) hardly influences \( E_{ax} \). (We shall revert to this influence in more detail in section 3.) The dependence of \( E_{ax} \) on \( H^* \) stems from the influence which \( H^* \) has on the circulating flow rate \( q_e \). Figure 7 shows the measured relationship for the (1-10)-model.

The relation between \( E_{ax} \) and \( \tau_e \) is given in Fig 8. In these experiments the circulation time \( \tau_e \) has been changed by varying both the gas velocity \( v_0 \) and the height of submergence \( H^* \).

From the experimental results it can be seen that for a given geometrical size of the cross-section of the basin, the influence of \( v_0 \) and \( H^* \) on the longitudinal dispersion coefficient can integrally be fully described by \( \tau_e \). Furthermore, Fig 8 shows that in the larger basins at low values of the circulation time \( \tau_e \), the longitudinal dispersion coefficient tends to a constant value.
3 TURBULENT DIFFUSION AND ENERGY DISSIPATION

When the circulating flow of the fluid is fully turbulent, mixing processes in the fluid are considered to be governed by $\varepsilon (m^2/s^3)$, the mean rate of energy dissipation per unit mass of the fluid. These diffusion processes take place in the longitudinal as well as in the transversal direction. In the transversal direction the turbulence field is superposed on the mean circulating flow, and therefore mixing in a cross-section of the basin is very rapid. The longitudinal turbulent diffusion is the integral effect from all eddies, which contribute to a transport of mass in this direction. Although the turbulent eddy spectrum extends to a very small scale, the majority of the transport of mass is done by the larger eddies [8]. At the small eddy scale, referred to as the Kolmogorov microscale or "inner scale," velocity fluctuations are smoothed out by viscosity effects. To support turbulent motion, energy has to be supplied continuously to the system to make up for these viscous losses. Via a wide range of eddies with decreasing length scales this energy is transferred to be eventually dissipated into heat. Let the eddies which contribute to the longitudinal diffusive transport have a size $\lambda$ and a velocity $v_\lambda$, then the rate of transfer of energy of these eddies to smaller eddies is proportional to $v_\lambda^3 / \lambda$.

This energy is eventually dissipated at a rate $\varepsilon$, which should be equal to the supply rate in the steady state. Hence [10]

$$\varepsilon \sim \frac{v_\lambda^3}{\lambda} \tag{1}$$

![Image 6](image6.png)

Fig 6 The longitudinal dispersion coefficient $E_{ax}$ as a function of the height of submergence $H^*$
With the foregoing premises the turbulent diffusion coefficient is given by [7-9]

\[
E_{ax} = a_1 \tau^{4/3}
\]

(2)

where \(a_1\) is a constant

\(\varepsilon\) can be calculated from the knowledge on the volumetric air flow rate \(Q_\varepsilon\) and the hydrostatic pressure drop \(\Delta P_{\text{hydr}}\) of the bubbles. If liquid stowage in the vicinity of the bubble street is relatively low, \(\Delta P_{\text{hydr}}\) is given by

\[
\Delta P_{\text{hydr}} = \rho_c g H^*
\]

Fig 7 The circulating flow rate \(q_c\) as a function of the height of submergence \(H^*\).

Fig 8 The longitudinal dispersion coefficient \(E_{ax}\) as a function of the circulation time \(\tau_c\).
Longitudinal dispersion in oblong aerated systems

Substituting eqn (3) in eqn (2) yields

\[ E_{ax} = a_1 \left( v_0 \frac{g}{H} \right)^{1/3} \lambda^{4/3} \]  

The experimental dependence of \( E_{ax} \) on \( v_0 \) for several geometrical sizes of the basin is shown in Fig 10. Up to gas velocities of about 0.5 cm/s the relation between \( E_{ax} \) and \( v_0 \) is in accordance with eqn (4).

As pointed out before, a further increase in \( v_0 \) has experimentally shown to suppress the circulating flow rate, as a result of which the longitudinal diffusion coefficient becomes constant. From the experimental results presented in Fig 10 it can furthermore be concluded that the nozzle diameter of the aeration tube—and consequently the bubble diameter—has hardly any influence, even while this diameter has been varied by a factor 3–4. The same conclusion can be drawn from the experiments carried out in the two investigated commercial basins the Inka and the Brandol systems. The nozzle diameter of the aeration devices in both systems differs about a factor 25 [13]. We shall revert to these experiments below.

It may be expected that the influence of the size \( \lambda \) of the eddies on \( E_{ax} \) must be given by a function containing the geometrical parameters. It is evident that this function has to be determined experimentally. Since the turbulent diffusion coefficient is strongly related to the transversal circulating flow, the hydraulic radius \( R_H = \frac{2BH}{2H + B} \) of the cross-section can be expected to play a role in this function.

By means of trial and error it was concluded that the relation

\[ \lambda^{4/3} = R_H \left( \frac{H^*}{H} \right)^{1/3} \left( \frac{B}{H} \right)^n \]

with

\[ n = +1 \text{ for } B \geq H \]
\[ n = -1 \text{ for } B < H \]

described the influence of the geometry of the basin on \( E_{ax} \) in eqn (4) reasonably well.

For the several scale models this function changes in its magnitude about a factor 10, depending upon the basin utilised. Figure 11 shows a plot of \( E_{ax} \) according to eqn (4) as a function of that geometrical factor for the (1 5)-model with the

![Fig 10 The longitudinal dispersion coefficient \( E_{ax} \) as a function of the superficial gas velocity \( v_0 \)]
The parameter $\frac{B}{H}$ ratio as a parameter. Down to values of $\frac{B}{H} = 10$ a decrease in the $\frac{B}{H}$ ratio results in a linear decrease in $E_{ax}$.

However, below this value a further decrease in $\frac{B}{H}$ increases $E_{ax}$. It is surprising to note—and this has been verified in lots of experiments—that the points return reasonably well to the original linear path, if the reciprocal value of the $\frac{B}{H}$ ratio is substituted in the geometrical factor for values of $\frac{B}{H} < 1$.

The correlation thus obtained, which describes the longitudinal diffusion coefficient, is

$$E_{ax} = a_2 \left( \frac{v_0 g H^*}{H} \right)^{1/3} R_H H^{*1/3} \left( \frac{B}{H} \right)^n$$

where $n = +1$ if $B \geq H$
$n = -1$ if $B \leq H$

Equation (5) can be transformed into an equation of dimensionless numbers

$$Pe^* = a_3 \left( \frac{v_0 R_H}{E_{ax}} \right), \quad Fr^* = \left( \frac{gH^*}{v_0^2} \right)$$

A plot of the observed data in the several models is shown in Fig. 12. The semi-empirical correlation seems good, with $a_3 = \text{constant}$ at an average of 20.

Under normal conditions, where $B \simeq H \simeq H^*$, it can be concluded from the slope of Fig. 12 and eqns (4)–(6), that the macroscale $\lambda$ of the vortices contributing to the longitudinal diffusional transport must be about 0.08 of the linear size of the cross-section. This value corresponds quite well with the values encountered in other turbulent systems, like fully developed turbulent pipe flow [11] and in mechanically agitated systems [12].

The correlation given in eqn (6) holds as long as the transversal circulating flow is not suppressed by the phenomenon mentioned in 2.3. Exceeding the critical gas velocity at which $E_{ax}$ remains about constant, the path of $Pe^*$ is as shown by the dotted lines in Fig. 13 for the (1 10)-model and the commercial Inka basin.

Some observed data are given in Table 1.

In Table 1 the observed data have also been given for a commercial Brando basin, where the aeration devices are located nearly at the bottom. These data are plotted in Fig. 14 (It should be borne
in mind that in this figure the two axes have been expanded by a factor 10.) It can be concluded that in the Brandol system the critical gas velocity is not exceeded and that the semi-empirical relation satisfies quite well.

Accordingly, eqn (6) is considered to give a reasonable prediction of the rate of the longitudinal mixing in oblong basins, where a transversal circulating flow of the liquid is introduced by dispersing air along one side of the basin. However, the determination of the point at which suppression of the circulating flow sets in is a subject for further study.

Acknowledgements—The author would like to express his thanks to Ir J H A M Rubbens, Ir N H Gerardu and Ir G J M Verbruggen for their contribution during the last year of their course of engineering. The many fruitful discussions with Prof Dr K Rietema of the same laboratory are also gratefully acknowledged.

Fig. 12 Plot of the observed data in the several scale models according to eqn (6)

Fig. 13 Plot of the observed data in the (1 10)-basin and the commercial Inka basin exceeding the critical gas velocity (The dotted lines show the expected path at constant $E_{ax}$.)
Fig 14 Plot of the observed data in the commercial Brandol basin according to eqn (6)

### NOTATION
- $a_1, a_2, a_3$: constants
- $B$: breadth of the basin, m
- $E_{ax}$: longitudinal dispersion coefficient, $m^2/s$
- $F_0 = \frac{E_{ax} t}{L^2}$: Fourier number
- $\Gamma r^* = \frac{g H^*}{v_0^2}$: modified Froude number
- $H$: liquid height in the basin, m
- $H^*$: height of submergence of the aeration device, m
- $L$: length of the basin, m
- $n$: exponent either (+1) or (-1)
- $\Delta P_{hyd}$: hydrostatic pressure drop, N/m$^2$
- $Pe^* = \frac{v_0 R_H}{E_{ax}}$: modified Peclet number
- $\dot{q}_c$: circulating flow rate per unit length of the basin, calculated from the circulation time $\tau_c$, $m^2/s$
- $Q_c$: circulating flow rate per unit length of the basin, calculated from the measured velocity profiles, $m^2/s$
- $Q_s$: air flow rate, $m^3/s$
- $R_H$: hydraulic radius of the cross-section, m
- $v_0$: superficial gas velocity, $m/s$
- $\bar{v}_c$: average circulating flow velocity, $m/s$
- $v_{la}$: velocity scale of turbulent eddies, $m/s$
- $V_c$: volume of the continuous phase, $m^3$

### Greek symbols
- $\varepsilon$: rate of energy dissipation per unit mass of fluid, $m^2/s^3$
- $\lambda$: length scale of turbulent eddies, m
- $\rho_c$: density of the continuous phase, kg/m$^3$
- $\rho_d$: density of the dispersed phase, kg/m$^3$
- $\tau_c$: circulation time, s

### REFERENCES
1. Ardern E and Lockett T, J Soc Chem Ind 1914 33 523

| Table 1 |
|----------|----------|----------|----------|
| Basin | (1-10)-model | Inka basin | Brandol basin |
| $L$ (m) | 2.40 | 24.00 | 24.00 |
| $H$ (m) | 0.40 | 3.40 | 3.40 3.05 3.40 |
| $B$ (m) | 0.40 | 4.00 | 4.00 |
| $H^*$ (m) | 0.38 | 0.73 | 3.20 2.85 3.20 |
| $v_0$ (cm/s) | 0.30 1.28 2.47 4.36 | 0.38 0.95 1.25 1.50 | 0.28 0.28 0.55 |
| $E_{ax}$ (cm$^2$/s) | 29 31 38 35 | 234 235 220 232 | 583 854 797 |