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On the formulation of linear programming problems of the blending type

by

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Abstract
It is shown how a linear quality-blending problem can be expressed as a processing network problem.

In [1] a theoretical basis has been laid for the development of a linear programming solution system for solving processing network problems by fully exploiting their network structure. Here, processing network problems refer to network flow problems which do not consider only pure or generalized flows of materials through arcs of a network, but which do also allow processing of these materials in some of its nodes. The processes are either separation or blending on recipe, i.e. any two components produced by separation have a given mutual ration; also any two components used in the blending process; these ratios constitute the recipe. Blending is an often occurring process in the oil and food industry. Blending on recipe however can be used only when one is interested in a single quality aspect of the blend. In general one is interested in a larger number of qualities. In such a case these qualities are known
for the components (e.g. density and sulfur content) and the blend is required having these qualities between given limits.

If the qualities of component $j$ are expressed by a vector $q$, lower and upper qualities for the blend are specified by the vectors $q$ and $\bar{q}$, and if the available amount of component $j$ is $a_j$, then the blend satisfies the linear system

$$q^T x \leq \sum_j q_j x_j \leq \bar{q}^T x$$

$$\sum_j x_j = x$$

$$0 \leq x_j \leq a_j \quad \forall j.$$

It will be shown now how such a quality blending problem can be expressed as a processing network problem. The possibility of such an expression is already known from Koene [1,p.120] where it has been shown that any linear programming problem can be written as a processing network problem.

For the quality requirements of the blend three possibilities must be considered: qualities for which either only a lower bound, only an upper bound, or for which both a lower and an upper bound has been specified. For simplicity we consider a blend with only three quality requirements, one for each of the above types. We also take only three components. The blending problem is described then by the linear system
\[ q_1 \leq q_{11}x_1 + q_{12}x_2 + q_{13}x_3 \]
\[ q_2 \leq q_{21}x_1 + q_{22}x_2 + q_{23}x_3 \leq \bar{q}_2 \]
\[ q_3 \leq q_{31}x_1 + q_{32}x_2 + q_{33}x_3 \leq \bar{q}_3 \]
\[ x_1 + x_2 + x_3 = x \]
\[ 0 \leq x_1 \leq a_1 \]
\[ 0 \leq x_2 \leq a_2 \]
\[ 0 \leq x_3 \leq a_3 \]

It is easily verified that such a system is also represented by the following processing network problem:

Figure 1. Network processing representation of a quality blending problem; one blend.
Figure 2. Network processing representation of a quality blending problem; two blends
In general, a quality blending problem can be represented by a processing network problem involving a separation process, corresponding to each component, a blending process on recipe corresponding to the requirements for the blend and a special separation process with two separation coefficients both equal to 1 (a "duplicating" process) for each quality for which both a lower and an upper bound is specified for the blend. (In Figure 1, the arrows \(\longrightarrow\) and \(\longrightarrow\) represent slack activities.)

If two or more blends have to be produced from the same set of components, for each blend a similar processing network must be drawn for each blend, linked only by the common component sources. An example is given in Figure 2.