Model order reduction for nonlinear problems in circuit simulation
Verhoeven, A.; Voss, T.; Astrid, P.; ter Maten, E.J.W.; Bechtold, T.

Published: 01/01/2007

Document Version
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 10. Oct. 2018
Model order reduction for nonlinear problems in circuit simulation

A. Verhoeven¹, T. Voss², P. Astrid³, E.J.W. ter Maten¹,4*, and T. Bechtold⁴

1 Eindhoven University of Technology, Den Dolech 2, 5600 MB Eindhoven, The Netherlands
2 University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands
3 Shell Global Solutions International BV, P.O. Box 38000, 1030 BN Amsterdam, The Netherlands
4 NXP Semiconductors, High Tech Campus 37, 5656 AE Eindhoven, The Netherlands

Electrical circuits usually contain nonlinear components. Hence we are interested in MOR methods that can be applied to a system of nonlinear Differential-Algebraic Equations (DAEs). In particular we consider the TPWL (Trajectory PieceWise Linear) and POD (Proper Orthogonal Decomposition) methods. While the first one fully exploits linearity, the last method needs modifications to become efficient in evaluation. We describe a particular technique based on Missing Point Estimation.

Copyright line will be provided by the publisher

1 Introduction

Simulation for nanoelectronics requires that eventually circuit equations can be coupled to electromagnetics, to semiconductor system, are reduced and combined to a weighted global system. The time points for updating the local linearized systems are determined dynamically, based on error control [7]. The technique has been successfully applied to reduce a DAE model of an inverter chain model [1, 6]. The method can also be combined with LSPOD [6].

The same inverter chain model has also been considered in [7] to study Trajectory Piece-Wise Linear (TPWL) [5], combined with "PoorMan’s TBR" [3]. The locally linearized systems, created along a typical time-domain trajectory of the original system, are reduced and combined to a weighted global system. The time points for updating the local linearized systems are determined dynamically, based on error control [7].

2 Diode chain model

We consider the diode chain model shown in Fig. 1 (with the parameters $I_s$, $V_T$, $R$, $C$), described by the following system of DAEs. Here the diode functionality is modelled by the function $g(V_o, V_b)$ and the input function by $U_{in}(t)$.

$$
\begin{align*}
V_1 - U_{in}(10^7 t) &= 0, \\
g(V_1, V_2) - g(V_2, V_3) - C \frac{dV_2}{dt} - \frac{1}{R} V_2 &= 0, \\
g(V_{N-1}, V_N) - g(V_N, V_{N+1}) - C \frac{dV_N}{dt} - \frac{1}{R} V_N &= 0, \\
g(V_N, V_{N+1}) - C \frac{dV_{N+1}}{dt} - \frac{1}{R} V_{N+1} &= 0,
\end{align*}
$$

$$
g(V_o, V_b) = \begin{cases} 
(I_s e^{ \frac{V_o - V_T}{V_r} } - 1) & \text{if } V_o - V_b > 0.5 \\
0 & \text{otherwise}
\end{cases}
$$

$$
U_{in}(t) = \begin{cases} 
20 & \text{if } t \leq 10 \\
170 - 15t & \text{if } 10 < t \leq 11 \\
5 & \text{if } t > 11
\end{cases}
$$

The state of the diode chain model consists of 302 elements but there is a lot of redundancy. The numerical solution (nodal voltage in each node) on the time interval $[0, 70 \text{ ns}]$ is computed by the Euler Backward method with fixed stepsizes of $0.1 \text{ ns}$. TPWL is able to reduce the model to small sizes with an acceptable error (see Fig. 2 (left)). Most of the time the relative error of TPWL is lower than the chosen error bound $\varepsilon = 0.025$. Furthermore, for higher order reduced models a smaller number

Copyright line will be provided by the publisher

* Corresponding author  E-mail: Jan.ter.Maten@nxp.com
of linearization points (LP) is used than for the reduced models with lower order, as the local systems with higher orders are more accurate. E.g., for a reduced model of order 100 we have used 42 LPs and for smaller reduced models 60 LPs.

POD (without MPE) is also able to reduce this nonlinear model to size 10. The POD models are, as expected, (much) more accurate than the TPWL ones (see Fig. 2 (right)), but are considerably slower to simulate than the TPWL models (see the corresponding extraction and simulation times in Table 1). POD was significantly speeded up by combining the POD with MPE and by keeping the Jacobian matrices constant as much as possible.

Table 1 Comparison of performances of TPWL and of POD.

<table>
<thead>
<tr>
<th>Model</th>
<th>( r )</th>
<th>Extr. time</th>
<th>Sim. time</th>
<th>Model</th>
<th>( r )</th>
<th>Extr. time</th>
<th>Sim. time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>302</td>
<td>0</td>
<td>80</td>
<td>POD</td>
<td>10</td>
<td>302</td>
<td>80</td>
</tr>
<tr>
<td>TPWL</td>
<td>10</td>
<td>290</td>
<td>1.1</td>
<td>POD</td>
<td>20</td>
<td>302</td>
<td>80</td>
</tr>
<tr>
<td>TPWL</td>
<td>25</td>
<td>285</td>
<td>1.5</td>
<td>POD + MPE</td>
<td>10</td>
<td>32</td>
<td>84</td>
</tr>
<tr>
<td>TPWL</td>
<td>50</td>
<td>206</td>
<td>2.3</td>
<td>POD + MPE</td>
<td>20</td>
<td>55</td>
<td>89</td>
</tr>
</tbody>
</table>

Fig. 2 Numerical results diode chain, showing the errors for TPWL (at the left) and for POD (at the right).

References