Multirate methods for the transient analysis of electrical circuits

Citation for published version (APA):

Document status and date:
Published: 01/01/2005

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.
Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.


Multirate methods for the transient analysis of electrical circuits

A. Verhoeven* 1, A. El Guennouni2, E.J.W. ter Maten1,3, and R.M.M. Mattheij1
1 Eindhoven University of Technology, Den Dolech 2, 5600 MB Eindhoven, The Netherlands
2 Yacht B.V., The Netherlands
3 Philips Electronics Nederland B.V., Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands

Multirate integration is an important tool to increase the speed of the transient analysis of circuits. This paper shows an approach for the “Compound-Fast” multirate algorithm how to control the errors at the coarse and the refined time-grid by means of the independent stepsizes of these grids.

1 Introduction

Analogue electrical circuits are usually modelled by differential-algebraic equations of the following type:

\[\frac{d}{dt} [q(t, x)] + j(t, x) = 0,\]

where \(x \in \mathbb{R}^n\) represents the state of the circuit. A common analysis is the transient analysis, which computes the solution \(x(t)\) of this non-linear DAE along the time interval \([0, T]\) for a given initial state.

In the classical circuit simulators, this Initial Value Problem is solved by means of implicit integration methods, like the BDF-methods. Each iteration, all equations are discretized by means of the same stepsize.

Often, parts of electrical circuits have latency or multirate behaviour. Latency means that parts of the circuit are constant during a certain time interval. Multirate behaviour means that some variables are slowly varying, compared to other variables. In both cases, it would be attractive to integrate these parts with a larger timestep [2].

2 Partitioned multirate BDF methods

For a multirate method it is necessary to partition the variables and equations into an active (A) and a latent (L) part. This can be done by the user or automatically. Let \(B_A \in \mathbb{R}^{n_A \times n}\) and \(B_L \in \mathbb{R}^{n_L \times n}\) with \(n_A + n_L = n\) be the partition mappings with the orthogonality properties: \(B_A B_A^T = I, B_L B_L^T = 0, B_A B_L^T = 0, B_L B_A^T = I\). Then the variables and functions can be split in active (A) and latent (L) parts:

\[
\begin{align*}
x &= B_A^T x_A + B_L^T x_L, \\
q(t, x) &= B_A^T q_A(t, B_Ax, B_Lx) + B_L^T q_L(t, B_Ax, B_Lx), \\
j(t, x) &= B_A^T j_A(t, B_Ax, B_Lx) + B_L^T j_L(t, B_Ax, B_Lx).
\end{align*}
\]

Now equation (1) is equivalent to the following partitioned system:

\[
\begin{align*}
\frac{d}{dt} [q_A(t, x_A, x_L)] + j_A(t, x_A, x_L) &= 0, \\
\frac{d}{dt} [q_L(t, x_A, x_L)] + j_L(t, x_A, x_L) &= 0.
\end{align*}
\]

In contradiction to classical integration methods, multirate methods integrate both parts with different stepsizes or even with different schemes. Besides the coarse time-grid \(\{T_n, 0 \leq n \leq N\}\) with stepsizes \(H_n = T_n - T_{n-1}\), also a refined time-grid \(\{t_{n-1,m}, 1 \leq n \leq N, 0 \leq m \leq q_n\}\) is used with stepsize \(h_{n,m} = t_{n,m} - t_{n,m-1}\) and multirate factors \(q_n\). If the two time-grids are synchronized, \(T_n = t_{n,0} = t_{n-1,q_n}\) holds for all \(n\). There are a lot of multirate approaches for partitioned systems [1, 4, 5] but we will consider the "Compound-Fast" version of the BDF methods. This method first integrates the complete circuit at the coarse grid. Afterwards only the active part is integrated at the refined grid, while the latent interface variables are interpolated. Multirate BDF methods need both coarse and refined Nordsieck vectors \(\tilde{Y}^n, \tilde{X}^n, \tilde{P}^n, \tilde{Q}^n\) and \(\bar{Y}^{n,m}, \bar{X}^{n,m}, \bar{P}^{n,m}, \bar{Q}^{n,m}\), which represent the coarse and refined Predictor and Corrector polynomials for \(x(t)\) and \(q(t, x(t))\) [3]. Note that for the refined vectors only the active part is stored. Algorithm (1) shows the structure of Partitioned "Compound-Fast" Multirate BDF methods.

* Corresponding author: e-mail: averhoev@win.tue.nl, Phone: +31 40 247 4847, Fax: +31 40 244 2489
Algorithm 1 Partitioned "Compound-Fast" Multirate BDF algorithm

**Compound phase**
Initialize integration method
While $T_{n-1} < T$ do:
  Compute coarse predictor Nordsieck vectors
  Initialize compound step
  Solve $x^n$ from nonlinear equation
  Update coarse corrector Nordsieck vectors
  Compute (weighted) error norm $\dot{\rho}_C^n$ and interpolation error norm $\dot{\gamma}_I^n$
  If $\dot{\rho}_C^n < \text{TOL}_C$ and $\dot{\gamma}_I^n < \text{TOL}_I$ do:
    $T_n = T_{n-1} + H_n$
  Perform refinement phase
  Update coarse corrector Nordsieck vectors for active part
  Compute next timestep $H_{n+1}$ and order $K_{n+1}$
  $n = n + 1$
  else do:
    Reduce timestep (or order)
  End
End

**Refinement phase**
Initialize refinement phase
While $t_{n-1,m-1} < T_n$ do:
  Compute refined predictor Nordsieck vectors
  Initialize refinement step
  Interpolate $\tilde{x}_L^{n-1,m}$ from $\tilde{X}^n$
  Solve $X_L^{n-1,m}$ from nonlinear equation
  Update refined corrector Nordsieck vectors for active part
  Compute error norm $\dot{\gamma}_A^{n-1,m}$
  If $\dot{\gamma}_A^{n-1,m} < \text{TOL}_A$ do:
    $t_{n-1,m} = t_{n-1,m-1} + h_{n-1,m}$
    Compute next timestep $h_{n-1,m+1}$ and order $k_{n-1,m+1}$
    $m = m + 1$
  else do:
    Reduce timestep (or order)
  End
End

3 Multirate error control

Although the orders $K_n$ and $k_{n,m}$ can be variable, from now on they are assumed to be fixed at $K_n = K, k_{n,m} = k$. The local discretization error $\delta^n$ of the compound phase still has the same behaviour $\delta^n = O(H_n^{k+1})$. This error can be estimated by $\delta^n$ using the Nordsieck vectors for $q^T$: $\delta^n = c \frac{H_n^{k+1}}{T_n - T_{n-1}} \left[ \tilde{Q}_n^n - \tilde{B}_n^n \right]$. Now $\tilde{r}_C^n = ||B_A \dot{\gamma}_C^n|| + \tau ||B_A \delta^n||$ is the used weighted error norm, which must satisfy $\dot{r}_C^n < \text{TOL}_C$. At the refined time-grid the DAE has been perturbed by the interpolated latent variables. The local discretization error $\delta^n$ is defined as the residue after inserting the exact solution in the BDF scheme of the refinement phase. However during the refinement instead of $\delta^n$ the perturbed local error $\delta^n$ is estimated. During the refinement each step $X_L^{n-1,m}$ is computed from the following scheme:

$$\alpha_{n-1,m} q_A(t_{n-1,m}, x_A^{n-1,m}, \tilde{x}_L^{n-1,m}) + h_{n-1,m} a_{n-1,m} (t_{n-1,m}, x_A^{n-1,m}, \tilde{x}_L^{n-1,m}) + \beta_{n-1,m} = 0.$$ (4)

Here $\beta_{n-1,m}$ is a constant which depends on the previous values of $x_A$ and $\tilde{x}_L$. Let $\beta_{n-1,m}$ be the constant for exact values of $x_L$, then we assume that $\beta_{n-1,m} = \beta_{n-1,m}^L = \beta_{n-1,m}^L + \frac{\partial}{\partial x_L^n} (x_L^{n-1,m} - x_L(t_{n-1,m}))$. Using the notation $t = t_{n-1,m}, h = h_{n-1,m}, \alpha = \alpha_{n-1,m}, \beta = \beta = \beta_{n-1,m}$, the error $\dot{\gamma}^{n,m}$ satisfies

$$B_A \dot{\gamma}^{n,m} = \alpha q_A (t, x_A(t), x_L(t)) + h j_A (t, x_A(t), x_L(t)) + \beta + h K_{n-1,m} (x_L(t) - x_L^{n-1,m})$$ (5)

Here $\rho^{n,m}$ is the interpolation error at the refined grid and $\tilde{K}_{n-1,m}$ is the coupling matrix which satisfies $\tilde{K}_{n-1,m} = \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t)) + \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t)) + \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t)) + \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t)) + \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t)) + \frac{\partial}{\partial x_T^n} (x_A(t), x_L(t))$.

The coupling matrix $\tilde{K}_{n-1,m}$ can be estimated by

$$\tilde{K}_n = \tilde{K}_n \left[ \frac{\partial}{\partial x_T^n} (T_n, x^n) - \frac{\partial}{\partial x_T^n} (T_n, x_T^{n-1}) \right] B_T^T + A_B \frac{\partial}{\partial x_T^n} (T_n, x^n) B_L^T.$$ (6)

The perturbed local discretization error $B_A \tilde{\gamma}^{n,m}$ behaves as $O(H_n^{k+1})$ and can be estimated in a similar way as $\delta^n$. Let $L$ be the interpolation order, e.g. $L = K$, then it can be shown that $||B_A \tilde{\gamma}^{n,m}||$ is smaller than $\dot{\gamma}_I^n = \frac{H_n^{k+1}}{T_n - T_{n-1}}$.

$$||B_A \tilde{\gamma}^{n,m}|| \leq \dot{\gamma}_A^{n-1,m} + \dot{\gamma}_I^n.$$ (7)

If $\dot{\gamma}_I^n < \text{TOL}_I$ and $\dot{\gamma}_I^{n-1,m} \leq \text{TOL}_A$ then $\dot{\gamma}_I^n < \text{TOL}_A + h \text{TOL}_I = \text{TOL}_A$. The weighting factor $0 < \sigma < \frac{1}{h_{n-1,m}}$ is chosen such that $(\dot{\gamma}_I^n / \text{TOL}_C) \leq (\dot{\gamma}_I^n / \text{TOL}_L)$. Adaptive stepsize control can be used to keep $\dot{\gamma}_I^n = O(H_n^{k+1})$ and $\dot{\gamma}_I^{n-1,m} = O(h_{n-1,m}^{k+1})$ close to $\text{TOL}_I$ and $\theta \text{TOL}_A$ respectively, where $0 < \theta < 1$ is a safety factor.

References


Copyright line will be provided by the publisher