On dispersed two phase flows past obstacles

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On Dispersed Two Phase Flows Past Obstacles

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door

HAORAN MENG

geboren te Beijing, CHINA

druk: wihro dissertatiedrukkerij, helmond.
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prof. dr. ir. A.A. van Steenhoven
en de copromotor
dr. C.W.M. van der Geld

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SUMMARY

SAMENVATTING

CURRICULUM VITAE

ACKNOWLEDGEMENTS
## NOMENCLATURE

### ROMAN LETTERS

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<td>base pressure coefficient (Eq.2.10)</td>
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<td>diameter of object lens (Sec.3.3)</td>
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<td>D_o</td>
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<td>D_s</td>
<td>diameter of bubble's shadow on photo</td>
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<td>diameter of a particle (or droplet or bubble)</td>
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<td>Unit</td>
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<td>frequency of vortex shedding (Sec. 5.2)</td>
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<td>acceleration of gravity (Sec. 2.2)</td>
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<td>time step (App. 4)</td>
<td>[s]</td>
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<td>gamma beam intensity through liquid</td>
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<td>gamma beam intensity through mixture</td>
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<td>$K_{1,2,3,4}$</td>
<td>coefficients (App. 5)</td>
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<td>$L_{lo}$</td>
<td>distance between lens and object (Sec. 3.3)</td>
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<td>$L_w$</td>
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<td>[m]</td>
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<td>$L_{w1}$</td>
<td>distance between window and lens (App. 9)</td>
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<td>$L_{2p}$</td>
<td>distance between two images of bubble</td>
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<td>length scales of flow regions for a wake</td>
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<td>number of projections (Sec. 3.2)</td>
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<td>$P_o$</td>
<td>upstream pressure (Sec. 5.2)</td>
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<td>$Q$</td>
<td>strength of a source (App. 4)</td>
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<tr>
<td>$R$</td>
<td>radius of a cylinder (Sec. 2.2)</td>
<td>[m]</td>
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<td>$R_{ci}$</td>
<td>radius of a particle (Sec. 3.2) &amp; (App. 2)</td>
<td>[m]</td>
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<tr>
<td>$Re$</td>
<td>cylinder Reynolds number ($Re=U_o D/\nu$) (App. 1)</td>
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</tr>
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</table>
Nomenclature

\( \text{Re}_d \)  \hspace{1cm} \text{cylinder Reynolds number } (\text{Re}_d = \frac{U_d D}{\nu})

\( \text{Re}_p \)  \hspace{1cm} \text{particle Reynolds number } (\text{Re}_p = \frac{U_p d}{\nu}) \quad \text{(App.3)}

\( R(t) \)  \hspace{1cm} \text{Lagrangian time correlation } \left( = \overline{v(t) v(t-t)} / \overline{v^2} \right)

\( r \)  \hspace{1cm} \text{coordinates } \text{(App.2)}

\( S_t \)  \hspace{1cm} \text{Strouhal number } (S_t = f_D / U_0) \quad \text{(Sec.5.2)}

\( t \)  \hspace{1cm} \text{instantaneous time}

\( t_c \)  \hspace{1cm} \text{counting-time constant } \text{(Sec.2.3)}

\( U \)  \hspace{1cm} \text{velocity component in } x \text{-direction } \text{(App.10)}

\( \overline{U} \)  \hspace{1cm} \text{velocity in particle moving direction } \text{(App.2)}

\( U_b \)  \hspace{1cm} \text{object distance of "bright spot" } \text{(Sec.3.3)}

\( U_{bub} \)  \hspace{1cm} \text{velocity of bubble } \text{(Sec.3.3)}

\( U_f \)  \hspace{1cm} \text{local instantaneous velocity of fluid}

\( U_o \)  \hspace{1cm} \text{mean fluid velocity upstream an obstacle}

\( U_p \)  \hspace{1cm} \text{local instantaneous velocity of particle}

\( U_r \)  \hspace{1cm} \text{relative velocity between two phases}

\( U_t \)  \hspace{1cm} \text{terminal velocity } \text{(Sec.2.3)}

\( U_w \)  \hspace{1cm} \text{velocity in the wake behind a cylinder}

\( U_{u2} \)  \hspace{1cm} \text{velocity of an oscillatory motion } \text{(App.2)}

\( U_{x,2,3} \)  \hspace{1cm} \text{x-direction velocity at various plane}

\( u_b \)  \hspace{1cm} \text{object distance } \text{(App.9)}

\( V \)  \hspace{1cm} \text{velocity component in } y \text{-direction } \text{(App.2)}

\( V_{b} \)  \hspace{1cm} \text{image distance of "bright spot" } \text{(Sec.3.3)}

\( V_{mi} \)  \hspace{1cm} \text{mass-in-flow velocity in } y \text{-direction}

\( V_t \)  \hspace{1cm} \text{measuring-table traversing speed } \text{(Sec.3.2)}

\( V_{y,2,3} \)  \hspace{1cm} \text{y-direction velocity at various plane}

\( \text{VAR} \)  \hspace{1cm} \text{variance value of a Gaussian function}

\( W \)  \hspace{1cm} \text{complex velocity } \text{(App.4)}

\( W_{tra} \)  \hspace{1cm} \text{equivalent width per trajectory } \text{(App.15)}

\( X \)  \hspace{1cm} \text{coordinates}

\( X_0 \)  \hspace{1cm} \text{x-coordinate of the separation point}

\( x' \)  \hspace{1cm} \text{velocity in } x \text{-direction } \text{(Sec.2.3)}

\( Y \)  \hspace{1cm} \text{coordinates}

\( y' \)  \hspace{1cm} \text{velocity in } y \text{-direction } \text{(Sec.2.3)}

\( y_{wb} \)  \hspace{1cm} \text{coordinates of the wake border } \text{(Sec.5.3)}

\( y_{owb} \)  \hspace{1cm} \text{coordinates of the outside wake border}

\( Z \)  \hspace{1cm} \text{coordinate in the complex plane } \text{(App.4)}

\( z \)  \hspace{1cm} \text{coordinate } \text{(App.4)}
### Greek Letters

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>α</td>
<td>Void fraction (Sec. 3.2) or angle (Sec. 2.2)</td>
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<tr>
<td>αₐ</td>
<td>Mean void fraction (Sec. 3.2)</td>
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</tr>
<tr>
<td>αₒ</td>
<td>Mean upstream void fraction</td>
<td>[-]</td>
</tr>
<tr>
<td>βₛ</td>
<td>Separation angle of flow past a cylinder</td>
<td>[°]</td>
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<tr>
<td>δ</td>
<td>Depth of penetration of a wave (App. 2)</td>
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<td>cₜ</td>
<td>Fraction of a sample step (App. 8)</td>
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<td>Stream function (App. 2)</td>
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<td>κ</td>
<td>Rate of shear (Sec. 2.2)</td>
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<td>Dynamic viscosity of fluid</td>
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<td>Dynamic viscosity of particle (Sec. 2.3)</td>
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<td>νₑ</td>
<td>Kinematic viscosity of fluid</td>
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<td>ρₑ</td>
<td>Density of a fluid</td>
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<td>ρₚ</td>
<td>Density of a particle</td>
<td>[kg/m³]</td>
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<tr>
<td>ρₘ</td>
<td>Density of a two-phase mixture (Sec. 3.2)</td>
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<td>σ</td>
<td>Surface tension (Sec. 2.2)</td>
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<td>τ</td>
<td>Instantaneous time (App. 2)</td>
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<tr>
<td>ζ</td>
<td>Basic transform plane (App. 4)</td>
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SUBSCRIPTS

c_0 zero velocity points (App.11)
c_{1,2} wake border (Sec.5.2)
f fluid (gas or liquid)
g gas (Sec.3.2)
i, j index for x−y coordinate
i_{10} input and output data
m mixture
\( p \) particle (solid particle, liquid drop, or gas bubble)
\( r, \theta \) index for spherical polar coordinates (App.2)
s separation point on a cylinder (Sec.5.2)
s_{1,2} starting and ending points
tra trajectory
x, y in the x− or y− direction
\( \prime \) calculated value (Sec.3.3)
\( \delta \) fluctuation term (Sec.5.2)
CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND AND OBJECTIVES

Dispersed phase two-phase flows are of fundamental importance in many natural physical processes as well as in a host of industrial operations. Rainfall, fermentation, and flotation are only a few of the phenomena in which dispersed phases play a primary role. Boiling, distillation, spray drying, and erosion analysis are examples of the engineering processes which rely on the knowledge of dispersed phase behavior in the system. In all these phenomena and processes, there exists a relative motion between the dispersed phase and the carrier fluid and therefore a strong interaction between these two phases. The phases are rarely, if ever, distributed uniformly over the field. Phase distributions are of primary interest in understanding the nature of the flows.

Dispersed two-phase flows past obstacles, which are typical non-uniform dispersed two-phase flow situations and the focus in this thesis work, are among the most familiar situations in industrial applications, including petroleum, power and process industry. An abstract picture of one typical flow situation is shown schematically in fig.1.1.

![Diagram](https://via.placeholder.com/150)

Fig.1.1 An abstract picture of one typical dispersed two-phase flow past an obstacle.
An uniform dispersed two-phase flow, in which the dispersed phase (either gas bubbles or liquid droplets or solid particles) move with their carrier fluid (gas or liquid), approaches to a (cylindrical) obstacle. Due to strong changes of both magnitude and direction of local velocities of the fluid flow (i.e. local fluid velocity gradients) and density difference between the dispersed phase and the fluid, the local phase distribution pattern changes markedly around the obstacle. The local concentration and dispersion of the dispersed phase affect the thermal-hydraulic transport process between both the dispersed phase with the fluid and the two-phase flow with the obstacle, such as heat transfer, dynamic fluctuation, corrosion process and so on. Consequently it concerns the efficiency, long-term reliability, and safety of the related industrial equipment. Two examples of such equipment are shown in fig.1.2.

(a) Shell-and-tube heat exchanger (b) Pressurized Water Reactor (PWR)

Fig.1.2 Two industrial examples related to dispersed two-phase flows past obstacles.

(a) where both heat transfer efficiency and corrosion problem are directly related to local phase distributions around circular tubes; (Kreith[1986])

(b) where under high pressure conditions, local phase concentrations of bubbly flows past top row tubes (U-bend region) affect not only heat transfer efficiency but also stress fluctuations, concerning reliability and safety of the system. (Cho[1993])
In contrast to a wide range of problems in engineering applications, quite little has been understood on phase distributions of dispersed two-phase flows past obstacles either theoretically or experimentally, probably because of the heterogeneity and the complexity of the problems.

Theoretical studies, either analytical modeling or numerical simulating, are mainly limited by the difficulties in describing a single-phase flow (the carrier flow) past an obstacle and in modeling the interaction between the dispersed phase and its carrier phase. While having been among the most extensively and intensively studied subjects in fluid mechanics for more than a century, a single-phase flow past a circular cylinder remains a subject that is not fully understood yet and still represents a very challenging research area. The subject relates not only to various problems encountered in a wide range of engineering applications but also to many fundamental phenomena such as separation of flow, production of turbulence and so on. While the Reynolds number \( \text{Re}_d = U_d D / \nu_f \) is the only representative and important one for identifying regimes of a single-phase flow past an obstacle as illustrated in appendix 1, more system parameters (such as the bubble to cylinder size ratio \( d_p/D \), the upstream void fraction \( \alpha_0 \), and so on) have to be taken into account for predicting accurately phase distributions of the dispersed two-phase flow past an obstacle. A complete numerical simulation for a complicated single-phase flow situation is already very difficult due to the capacity of computers, not to mention more complicated two-phase flow systems. Meanwhile, interaction between a dispersed phase and its carrier fluid, which is one of the most important bases for modeling two-phase systems (especially for dispersed two-phase systems), is far from fully understood. Detailed analyses require not only solving conservation equations but also introducing various specific theories such as the boundary-layer theory, vortex shedding theory and so forth.

Experimental investigations are mostly limited by the available experimental conditions (or setups) and the utilized measuring techniques. In general, a complete theoretical modeling is not possible for two-phase flows. Measurements still play primary roles in visualizing and investigating unknown flow behavior in a complicated system like a dispersed two-phase flow past a obstacle. The complexity and the variety of the system often require not only that careful considerations be given to ensure high quality data but also that specific arrangements be made for each individual situation.
This thesis concerns phase distributions in low-quality dispersed two-phase flows past obstacles and comprises a theoretical part of a more general nature and an experimental part highlighting bubbly flows past obstacles in vertical tubes.

The theoretical study aims at improving modeling algorithms for predicting phase distributions around obstacles as well as at gaining a better understanding of the interaction between the dispersed phase and its carrier fluid as a dispersed two-phase flow passes an obstacle. Utilizing the Lagrangian approach, which is more appropriate to the non-uniform low-quality dispersed two-phase flow situations where separation of phases occurs, the fluid phase is treated as a continuum and the trajectory of a single dispersed object in the fluid flow is predicted as a result of various forces acting on the object. The single-phase (or continuous-phase) carrier fluid flow past a obstacle is the starting point for the modeling of the interaction process between the two phases and is formulated based on empirical data. A well established full-form force balance equation is presented and subsequently solved to compute trajectories of the dispersed phase. From computation results, not only the magnitude of various interaction forces in the force balance but also the significance of main system parameters of the concerned system is illustrated and analysed. The influence of turbulence in the fluid on the entrainment process of bubbles into the turbulent wake behind the obstacle is also approximately investigated.

The experimental investigation is carried out in the framework of a research project of the Multiphase Flow Laboratory of the Eindhoven University of Technology. One of the main topics of the research project is to develop new measuring techniques or to improve existing ones for visualizing phase distributions of gas-liquid or vapour-liquid flows under high-temperature and high-pressure conditions. Accordingly, two measuring systems — one a gamma-ray densitometer system for general applications to provide local void fraction distribution measurements and the other a photographic system for quantifying properties of bubbles in a high-pressure tube — are developed. They are applied to take measurements of phase distributions of bubbly flows past obstacles over a wide range of experimental conditions, under both the atmospheric pressure and high pressures. Parallel to the corresponding theoretical studies, effects of the main system parameters on phase distributions of the bubbly flows are investigated. Special attention is paid to gas-liquid systems of small bubbles and small bubble to obstacle size ratios.
The conditions of the theoretical study and the experimental investigation are mostly similar to enable comparison (and thus validation) of the results.

1.2 OUTLINE OF THE TEXT

The thesis consists of six chapters and fifteen appendices. The appendices, which are related to one or several chapters, are presented as a whole at the end of the thesis. A brief outline of each of the chapters is now given.

Chapter 1 describes background, objectives, and outline of this thesis, including examples of engineering applications and problems in either theoretical or experimental work.

Chapter 2 analyzes the interaction between a dispersed phase and its surrounding fluid in an overall sense. A general form momentum equation to describe forces on a dispersed phase in an arbitrary fluid flow is introduced. The equation includes the steady drag force, the lift force, the added-mass force, the pressure-gradient force, the Basset history force and the buoyancy force. In a steady fluid flow past an obstacle, which is taken to be completely given by an inviscid solution excluding the turbulent wake region, the force balance equation is converted into a set of ordinary differential equations and then solved by the Runge–Kutta integration algorithm for computing trajectories of the dispersed phase as the dispersed two-phase flow passes the obstacle. By tracing trajectories of the dispersed phase in such a flow field, we can identify and analyze the magnitude of each individual force and the significance of system parameters (cylinder Reynolds number $Re_d$, particle to cylinder size ratio $d_p/D$, and particle to fluid density ratio $\rho_p/\rho_f$) in the interaction process. Computations cover a wide range of dispersed two-phase flows past obstacles. Besides bubble-liquid flows, which are the main topic of this thesis in connection to experiments, particle-gas (or droplet-gas) and particle-liquid flows are also discussed and presented in two parallel appendices respectively.

Chapter 3 presents two measuring systems: (1) a gamma-ray densitometer system for obtaining void fraction distribution measurements around obstacles in a vertical tube at both atmospheric pressure and high pressure, and (2) a photographic system for quantifying size, velocity, and concentration of bubbles in a high-pressure tube. For each of the measuring systems, three development stages of design, data interpretation, and system calibration are described.
Chapter 4 shows measurements of phase distributions of vertical bubbly flows behind obstacles with application of the two measuring techniques presented in chapter 3. The gamma-ray densitometer system obtains void fraction distribution measurements of bubbly flows behind the obstacles. Properties of bubbles upstream of obstacles are provided by a camera at the atmospheric pressure or by the photographic system at high pressures up to 70 bar. Bubble size is varied either by installing different system pressures or by adding a certain surface-active substance in water. Our study investigates effects of cylinder Reynolds number, bubble to cylinder size ratio, and upstream mean void fraction on void distribution patterns around cylinders in vertical bubbly flows, in particular gas–liquid two-phase flows with small bubble sizes and low void fractions.

Chapter 5 presents an attempt to predict the phase distribution of a bubbly flow past a cylinder under realistic conditions. Based on empirical data and analytical results in literature, a simple model is proposed to describe the wake region of the single-phase flow past the cylinder. The liquid turbulence is introduced in the bubble trajectory computations with a simplified scheme. In a similar fashion as in chapter 2, we follow bubble trajectories from the potential flow region to the wake region to investigate the magnitude of interaction forces and the influence of main system parameters in the interaction process between bubbles and the highly turbulent wake. Predictions are compared with corresponding measurements obtained in chapter 4.

Chapter 6 reviews the main conclusions and gives recommendations for further research.
CHAPTER TWO

COMPUTATION OF DISPERSED PHASE TRAJECTORIES IN FLUID FLOWS PAST OBSTACLES

2.1 INTRODUCTION

Computation of dispersed phase trajectories is of primary importance for studying a dispersed two-phase flow system, especially when local properties of the flow change strongly, as in the case of a dispersed two-phase flow past an obstacle. In the first place, dispersed phase trajectories are direct illustrations of distributions of the dispersed phase, resulted from interaction with its carrier fluid. In the second place, monitoring the separate steps of the (numerical) trajectory computations is an effective means for studying interaction processes between dispersed phase and fluid.

The interaction between the two phases is fundamental to and the most complicated part of modeling two phase flow systems. Detailed analyses require not only solving three conservation equations but also introducing various specific theories such as the boundary-layer theory, vortex shedding theory and so forth (Clift[1978]). For dispersed two-phase flows, the interaction between two phases is commonly expressed by a force balance on the dispersed phase, including various interaction forces. Each force represents an aspect of the total momentum transfer process and the forces together control relative motion between the dispersed phase and the fluid. Which of the forces are dominant in a certain process depends on the particular situation at hand.

For a dispersed two-phase flow past an obstacle, which is one of the typical non-uniform flow situations, many models have been employed or developed for computing particle trajectories and studying related interaction processes (e.g. Moris[1972], Pawlowiki[1984], Aihara[1986], and Kuo[1988]). Usually, a simplified force balance equation is applied and a limited number of forces (mostly only the steady drag force, where the drag coefficient is based on empirical data) is included. Other forces, especially the Basset history force which has a more complex form than other forces, are often assumed negligible. The assumptions simplify the force balance equation greatly and were quite true for certain situations. But extended to more general applications, such assumptions have to be justified.
In this chapter, the interaction process between a dispersed phase and its surrounding fluid is analyzed in an overall sense. Our study investigates the relative importance of various forces when the dispersed phase enters a typical non-uniform steady fluid flow—the dispersed two-phase flow around a circular cylinder. First, a general form force balance equation is introduced, that describes forces on a dispersed phase in an arbitrary fluid flow. The equation includes six individual forces: a steady drag force, a lift force, an added-mass force, a pressure-gradient force, a Basset history force and a buoyancy force. For simplicity, effects of interaction between dispersed phases and of deformation in a fluid dispersed phase are not considered here. Subsequently the force balance equation is converted to a set of ordinary differential equations and solved by the Runge-Kutta integration algorithm for computing trajectories of the dispersed phase in the fluid. The single-phase fluid flow is described by a potential flow model. Finally, the magnitude of each individual force and the significance of each important system parameter in the interaction process is identified and analyzed by tracing trajectories of the dispersed phase in such a flow field. Because of the general nature of the theoretical analysis, our studies cover not only bubble-liquid systems (which are of our primary topics relating to the parallel experimental investigation) but also particle-gas (or droplet-gas) systems and particle-liquid systems (which are discussed in appendices respectively).

2.2 INTERACTION FORCES ON THE DISPERSED PHASE

Henceforth we will use "particle" to represent the dispersed phase for convenience. In an arbitrary fluid flow, the momentum equation of the particle may be written in the following general form:

$$\rho_p V \frac{d\mathbf{U}_p}{dt} = \sum F_i$$

(2.1)

where \( V \) is volume of the particle and \( F_i \) is the \( i \)th kind of interaction force on the particle.

Forces acting on a particle can usually be classified into two categories: volume forces and surface forces. Volume forces act equally on all the matter within the particle and are proportional to the size of the particle. Surface forces arise from the action of matter outside the particle and act only in a thin layer adjacent to the boundary of the particle.
The total surface force is obtained by integrating stresses (both normal stresses and parallel stresses) over the surface of the particle. In steady fluid flows, the component of the total surface force parallel to the direction of motion of the particle is always negative (i.e., against the direction of the relative particle movement); accordingly it is usually called "drag". The component of the total surface force perpendicular to the relative direction of motion of the particle is called "lift".

2.2.1 Drag Force

A general expression of instantaneous drag was given by Clift[1978] as follows (originally proposed by Tchen[1947]; the deduction procedure is summarized in appendix 2).

\[ -F_{D_i} = \frac{\pi R^2 \rho_f C_d |U_{ri}|}{2} + \frac{\rho_f}{2} V C_a \frac{dU_{ri}}{dt} + 6 \pi \rho f C_b \sqrt{\frac{\rho_f U_f}{\mu_f}} \left( \frac{dU_{ri}}{dt} \right) \left( -\frac{d\tau}{\sqrt{t-\tau}} \right) \]

\[ -\rho_f V \left( \frac{dU_{ri}}{dt} - \nu_f \nu_f U_{ri} \right) \]

(2.2)

where \( F_{D_i} \) is orthogonal components of instantaneous drag \( F_{D_i} \); and \( U_{ri} (= U_{ri} - U_{fi}) \) and \( U_{ri} \) are those of relative velocity \( U_r \) and fluid velocity \( U_f \), respectively. For the definition of \( dU_{ri}/dt \), see note 2 after eq.(2.5).

On the right side of the equation, the first term is the "steady drag" (or Stokes' drag), which is derived from the "creeping flow approximation" applying at low particle Reynolds number (\( Re_p = 2RU/f \)). When \( Re_p \) is very small, the troublesome nonlinear convective acceleration term in the Navier-Stokes equation may be neglected; thus the solution of the momentum equation becomes much easier. The drag coefficient \( C_d \) is usually taken as an empirical function of the particle Reynolds number. For solid spheres in steady motion, drag can be easily taken from a "standard drag curve" or calculated by corresponding "standard drag equations" according to the value of \( Re_p \) (a well-defined table of recommended correlations was given in appendix 3, taken from Clift[1978]).

The second term is the "added mass force", which arises because acceleration of the particle requires acceleration of its surrounding fluid. The coefficient of the added mass term \( C_a \) is also an empirical coefficient, which accounts for differences from creeping flow and usually appears to depend only on relative accelerations. Limited empirical data are available only for very specific situations (e.g. a particle executes a rectilinear acceleration, Clift[1978]); Usually, \( C_a \) is taken as 0.5 for large \( Re_p \) situations.
The third term is usually called "Basset history integral" or "Basset history force", which indicates effects of the history of accelerations in which $t-\tau$ is the time elapsed since the past acceleration; the instantaneous acceleration is weighted by the square root of elapsed time $(t-\tau)$. The Basset history force accounts for the effects of the deviation in the flow pattern from a steady state and constitutes an instantaneous flow resistance of the particle. This force results from diffusion of vorticity from the particle. The significance of the Basset history force is not yet ascertained. Analytical results (e.g. Zimmels[1986]) show that its importance is likely to be small for dispersed two-phase systems only in cases of high particle-to-fluid density ratios or for systems involving a relatively low rate of change of accelerations and short relaxation times. It is only sure that when interest is not in transient response but rather in a statistically stationary response (such as desired when computing turbulent dispersion), the Basset term does becomes negligible (Ahmadi[1971]). For situations involving transient responses between particles and fluid flows (i.e. particles under high relative accelerations) and low particle-to-fluid density ratios (typically 1/1000 in a bubbly flow), the Basset term is certainly not negligible. For very specific situations (e.g. a particle executes a rectilinear acceleration, Clift[1978]), focus has been on providing empirical correlations for the Basset history integral coefficient ($C_h$) to account for differences from creeping flow, and it usually appears to depend only on relative accelerations. If there is no other alternative available, $C_h$ is usually taken as unity.

In general, flows are inhomogeneous (with presence of velocity gradients) and time dependent. So at least two new effects should be considered, under the restriction that the particle size is less than the shortest wave length in the flow (Corrsin[1956]). These two effects are:

1) a torque resulting from the spatial velocity gradient in the ambient fluid;
2) a force resulting from the static pressure gradient in the ambient fluid.

The first effect causes a side force (i.e. a lift), which will be discussed in next section. The second effect introduces a new term — the so-called "pressure-gradient force" as the last term in eq.(2.2). This "lumped" force is actually derived from the full Navier-Stokes equation of the fluid without presence of a particle; it represents the force required to accelerate the fluid which would occupy the space $V$ if the particle were absent.
2.2.2 Lift Force

The component of the total surface force perpendicular to the direction of particle motion is generally called "lift". There are two commonly used lifts found in literature. We shall call them "shear" lift and "rotation" lift.

The "shear" lift (also commonly called "Saffman lift") is due to velocity gradients in ambient fluid and is shown schematically in fig.2.1. A widely adapted expression was obtained by Saffman[1965] as follows,

\[ \mathbf{F_{L_s}} = C_{ls} \rho_l R^2 \sqrt{k \omega_l} (\mathbf{\ddot{e}_k} \times \dot{\mathbf{U}_r}) \] (2.3)

where \( \kappa \) is the rate of shear expressed in form of velocity gradient (e.g. in the potential flow region outside wake behind a cylinder, \( \kappa = |\partial U_f/\partial y| = |\partial U_f/\partial x| \), Morsi[1972]), \( \mathbf{\ddot{e}_k} \) denotes the unit vector of \( \kappa \) given by the "right-hand" rule from the angular velocity (\( \Omega \)), \( C_{ls} \) is the shear lift coefficient, and \( \mathbf{F_{L_s}} \) is in the direction normal to the direction of \( \dot{\mathbf{U}_r} \), as given by the "product" of \( \mathbf{\ddot{e}_k} \) and \( \dot{\mathbf{U}_r} \).

Fig.2.1 Schematic diagram showing "shear" lift.

Fig.2.2 Schematic diagram showing "rotation" lift.

The shear lift is significant only at \( Re_p < 1 \) or in viscous liquids. Fluid velocity gradients in simple shear flow will cause rotation of a spherical particle as well as differential pressures on two sides perpendicular to the moving direction of the particle. The shear lift coefficient \( C_{ls} \) varies with flow conditions; it is normally also derived experimentally for specified situations (for instance, Lawler[1971] has corrected the shear lift coefficient by fitting calculated trajectories of particles to the experimental results in a pipe flow). Saffman[1965] provided a value of the shear lift coefficient as \( 81.2/\pi \).
The "rotation" lift exists in a steady non-uniform rotational velocity field, which can be expressed in eq.(2.4) and is shown schematically in fig.2.2 (Auton[1988]).

\[ F_{Lr} = C_{lr} \rho_f V (\vec{\omega} \times \vec{U}_r) \]  

(2.4)

where \( C_{lr} \) is the rotation lift coefficient; and \( \vec{\omega} \) is the "rotation" in fluids and is generally expressed as \( \vec{\omega} = \nabla \times \vec{U}_r \). In potential flows, the rotation is zero.

The rotation lift is significant only in rotational flow situations which are weakly sheared and have a large Reynolds number, such as in the wake behind an obstacle. Strictly speaking, the rotation lift is also associated with "shear" or velocity gradients in the fluid; such "shear" appears as "rotation" of the fluid (or rotation of the solid particle relative to the fluid) in a system where inertial effects are dominant to viscous effects. Extensive discussions on this effect are based on analysis of unsteady non-uniform rotational flow situations (e.g. Auton[1983] &[1988]). The rotation lift coefficient (\( C_{lr} \)) was evaluated theoretically as 0.53 for a spherical particle.

Both lift forces discussed above are significant only in rather idealized or extreme flow situations, where the Reynolds number is either very big or very small. The gap between ranges of validity is wide and uncharted; there is no general expression of lift force found in literature, only correlations for very specific applications. For particles in a flow field around an obstacle, we shall simply utilize the "shear" lift in the region outside the wake behind the obstacle (which can be described by potential flow models at large Reynolds number) and utilize the "rotation" lift in the wake region which is characterized by large vorticity produced from separation of the boundary layer on the obstacle.

### 2.2.3 Force Balance Equation

Adding the forces discussed in the previous two sections, we can rewrite the momentum equation (2.1) in the following form:

\[ \rho_p V \frac{d\vec{u}}{dt} = \vec{F}_{sd} + \vec{F}_{sm} + \vec{F}_{pg} + \vec{F}_{base} + \vec{F}_{buoy} + \vec{F}_L \]  

(2.5)

where terms on the right side of the eq.(2.5) are defined as follows:
Steady drag force: 
\[ F_{sd} = -\frac{\pi R^2 \rho f C_d(U_r)}{2} |U_r| \]  
(2.5.1)

Added-mass force: 
\[ F_{am} = -\rho f V C_a \frac{dU_r}{dt} \]  
(2.5.2)

Pressure-gradient force: 
\[ F_{pg} = \rho f V \left( \frac{dU_r}{dt} - \nu f p U_r \right) \]  
(2.5.3)

Basset history force: 
\[ F_{basset} = -6\pi R^2 C_b \sqrt{\frac{\rho f \rho f}{\mu_f}} \int_0^t \frac{dU_r}{\sqrt{t-\tau}} \frac{d\tau}{\sqrt{t-\tau}} \]  
(2.5.4)

Buoyancy force: 
\[ F_{buoy} = (\rho_p - \rho f) V^2 g \]  
(2.5.5)

Lift force: 
\[ F_L = F_{Lx} = C_{1L} \rho f V (\omega \times U_r) \text{ or} \]  
\[ F_L = F_{La} = C_{1a}(\rho f R^3) \sqrt{\omega \cdot (\omega \times U_r)} \]  
(2.5.6)

\{ Notes: 
1) \( U_r = U_p - U_f \); 
2) \[ \frac{dU_r}{dt} = \frac{dU_p}{dt} - (\frac{\partial U_f}{\partial t} + U_p \frac{\partial U_f}{\partial t} + U_p \frac{\partial U_f}{\partial j}) \text{ (Clift[1978])} \} 

In principle, there is no limitation for applications of eq.(2.5), which presents a force balance on a particle in an arbitrary fluid motion. Accurate solutions for a specific situation (e.g. rectilinear acceleration of a rigid particle at high Reynolds number, Clift[1978]) may be obtained by deriving empirical correlations of coefficients in several individual forces (e.g. \( C_a \) in \( F_{am} \) and \( C_b \) in \( F_{basset} \)). In this thesis, \( C_a \) and \( C_b \) have been taken as 0.5 and unity respectively as indicated in the previous section, for generality.

Eq.(2.5) has a rather complete form in the sense that it encompasses most well known force terms. Each force represents a clear physical aspect of the total momentum exchange between the particle and its surrounding fluid. Some are dominant in a certain situations while some are in other cases. Most terms in the equation have a very complex form and it is difficult to analyze the significance of each individual force in the total interaction process. Without neglecting any term, eq.(2.5) can only be solved by numerical methods to derive instantaneous particle velocities (\( dU_p/dt \)). Additional equations, such as boundary conditions and initial conditions, are usually necessary. In the following sections, we shall put eq.(2.5) in a form suitable for numerical evaluation to compute particle trajectories. Then by tracking particles along their trajectories, we investigate the relative magnitude of individual forces in a typical non—uniform steady dispersed two—phase flow system —— a particle in a fluid flow past a cylinder.
2.3 COMPUTATION OF PARTICLE TRAJECTORY

At first, some of the terms in eq.(2.5) are discussed or simplified for the convenience of solving the equation. For a two-dimensional steady fluid flow past an obstacle at large Reynolds number, the local change of the fluid velocity \( \frac{\partial U_{fi}}{\partial t} \) equals zero; thus the fluid acceleration \( \frac{DU_{fi}}{Dt} \) only contains the convective part \( \frac{DU_{fi}}{Dt} = U_{fi} \frac{\partial U_{fi}}{\partial t} + U_{fi} \frac{\partial U_{fi}}{\partial j} \). In the region outside the wake behind the obstacle, which region is of our primary concern in this chapter, all terms concerning velocities of the fluid flow \( U_{fi}, \frac{\partial U_{fi}}{\partial t}, \frac{\partial U_{fi}}{\partial j} \), where \( i,j = x,y \) can be completely described by a potential flow solution (Parkinson[1970], see appendix 4); and the term \( V^2 U_f \) in eq.(2.5.3) equals zero. Furthermore, as discussed in section 2.2.2, only the shear lift is valid now and the rate of shear \( \kappa \) is expressed in form of velocity gradient as \( |\frac{\partial U_{ti}}{\partial j}| \) or \( |\frac{\partial U_{ti}}{\partial i}| \) (Morsi[1972]).

With the above provided information, we can now divide both side of the force balance equation (2.5) by \( \rho_f V \) and re-organize it into the following form,

\[
C_1 \frac{dU_{pi}}{dt} = C_{2i} U_{pi}^2 + C_{3i} U_{pi} + C_{4i} + C_{5i} U_{pj}
\]

where

\[
C_1 = \frac{\rho_p}{\rho_f} + C_a \tag{2.6.1}
\]

\[
C_{2i} = [\ast_i] \frac{3}{8R} C_{di} \tag{2.6.2}
\]

\[
C_{3i} = -[\ast_i] \frac{3}{4R} C_{di} U_{fi} + C_a \frac{\partial U_{fi}}{\partial i} \tag{2.6.3}
\]

\[
C_{4i} = [\ast_i] \frac{3}{8R} C_{di} U_{fi}^2 + \frac{DU_{fi}}{Dt} - \frac{9}{2R} \nu_f \frac{\pi}{\nu_f} \bar{t} \left( \frac{dU_{ei}}{dR} \right)_t \frac{dR}{d\bar{t}} + \left( \frac{\rho_p}{\rho_f} - 1 \right) g
\]

\[
- [\ast_2] C_{1s} U_{fj} \frac{3}{4\pi R} \sqrt{\nu_f} \frac{\partial U_{fi}}{\partial i} \tag{2.6.4}
\]

\[
C_{5i} = C_a \frac{\partial U_{fi}}{\partial j} + [\ast_2] C_{1s} \frac{3}{4\pi R} \sqrt{\nu_f} \frac{\partial U_{fi}}{\partial i} \tag{2.6.5}
\]
\{ Notes: \}

1) \([\pm] = \{ ' + ': \text{when } U_{pi} > U_{ri}, \)
\( ' - ': \text{when } U_{pi} < U_{ri} \}

2) \(g = 0\) for \(j\), where the direction of \(\vec{g}\) is assumed to be \(-\hat{t}\);

3) \([\pm_2]\) is determined by the "right–hand rule" from two vectors \(\vec{e}_k\)
and \(\vec{U}_r\), reading in formulae as follows:

\( [\pm_2] = '+', \text{ if } \frac{\partial U_{ri}}{\partial t} > 0 \text{ and } U_{ri} > 0 \text{ or if } \frac{\partial U_{ri}}{\partial t} < 0 \text{ and } U_{ri} < 0; \)
\( [\pm_2] = '-', \text{ otherwise. } \}

Then if we apply an trial–and–error iteration procedure to the troublesome implicit
particle acceleration \((dU_{pi}/dt)\) in the Basset history force term \((dU_{ri}/dt, \text{ as defined in}
note 2 after eq. (2.5))\), where the implicit \(dU_{pi}/dt\) at time \(t_k\) is extrapolated from values
at time \(t_{k-1}\) and \(t_{k-2}\), we can convert eq. (2.6) into a set of ordinary differential equations.
In an abstract form, we may write the following equations,

\[
\frac{dx'}{dt} = F_x(t_k, x_k, y_k, x_k', y_k') \quad \text{and} \quad \frac{dy'}{dt} = F_y(t_k, x_k, y_k, x_k', y_k') \tag{2.7}
\]

where for convenience \(x, y, x',\) and \(y'\) denote \(X_{pi}, X_{pj}, U_{pi}\), and \(U_{pj}\) respectively. This
set of equations can be solved with a numerical integration algorithm. In this manner,
the velocities of the particle \((x_k' \text{ and } y_k')\) are obtained.

Further on, from the definition of the fluid velocity

\[
dx_k/\ dt = x_k' \quad \text{and} \quad dy_k/\ dt = y_k' \tag{2.8}
\]

the coordinates \([x_k, y_k]\) of particles at time \(t_k\) may be calculated.

Eq. (2.7) and eq. (2.8) form a group of second–order two–dimensional ordinary differential
equations which describe the movement of a particle in a fluid flow. For a solution of
such equations, there are many kinds of numerical integration algorithms available.
Among them, the 4th–order Runge–Kutta algorithm is the most often applied and arguably even most useful algorithm (Press[1985]). An outline of such a algorithm
applied to our specific case is given in appendix 5.
Now we can compute trajectories of particles in a fluid flow from given initial conditions by implementing the above numerical integration algorithm. The required initial conditions are the initial position of a particle in the fluid field \([x_0, y_0]\) and the initial velocity of the particle \([x'_0, y'_0]\). The initial particle position can be taken easily from to the geometry of a flow field. The initial particle velocity is chosen as one of the following sets:

\( \begin{align*}
\text{a)} \quad x'_0 &= 0 \\
& \{y'_0 = 0 : \text{A particle is introduced into the flow with zero velocity. This may represent a situation where a new particle is introduced into the fluid.} \\
\text{b)} \quad x'_0 &= U_t \\
& \{y'_0 = 0 : \text{A particle moves in the direction of \(x\)-axis against the direction of gravity. The subscript "t" denotes a kind of terminal state of a steady particle motion which can be calculated by a simple model. One widely adopted model is the terminal velocity of a particle in creeping flow. With the total drag equal to the net buoyancy force on the particle, the following equation is provided (Batchelor[1967]),}
\end{align*} \)

\[ U_t = \frac{2gR^2(r_0 - p_f)}{3\mu_f} \left( \frac{1 + \mu_p/\mu_f}{2 + 3\mu_p/\mu_f} \right) \]  

(2.9)

where the particle viscosity is only meaningful when particles are liquid droplets. This may represent a situation where the particle density is close to the fluid density or the initial velocity of the fluid is small.

\( \begin{align*}
\text{c)} \quad x'_0 &= U_{fx}(x_0, y_0) \\
& \{y'_0 = U_{fy}(x_0, y_0) : \text{A particle is initially moving at the same speed as its carrier fluid. This may represent a situation where two phases are strongly coupled such as gas–water flows with a high velocity.} \\
\end{align*} \)

The numerical algorithm is realized in a computer program (written in TURBO PASCAL) to compute trajectories of particles in a fluid flow past an obstacle. Results include not only trajectories of particles (which are the results of interaction between the particles and the fluid field) but also a history of the interaction process itself (which is indicated by instantaneous values of particle velocities, particle accelerations and individual forces on particles). In this way we are able to provide valuable information about the interaction process between the particles and the (non–uniform steady) flow.
field. Consequently we gain a deeper understanding of many kinds of momentum transport mechanisms during the interaction process, such as energy dissipation, added mass, velocity gradient, diffusion of vorticity and so on. This subsequently may contribute to knowledge on corresponding heat and mass transport mechanisms.

2.4 RELATIVE IMPORTANCE OF DIFFERENT FORCES

In the force balance on a particle in an arbitrary fluid flow, we have included six interaction forces: the steady drag force, the added-mass force, the pressure-gradient force, the Basset history force, the buoyancy force and a lift force. In many computations of particle trajectories in literature, the equation is simplified and only the steady drag force and the buoyancy force are kept. The other forces, especially the Basset history force (and the lift force), are often assumed negligible because of their complexity and uncertain significance (White[1986]). This may be acceptable when particles are in an (approximately) rectilinear fluid motion with streamlines close to straight lines, but large errors may be introduced for particles in a non-uniform flow field where the convective changes in local fluid velocity are considerable.

It is not easy to see directly the magnitude of each interaction force as part of the total force balance on a particle from its expression in eq.(2.5). In a flow situation where all the forces are present, direct separate evaluation of the individual effects is often difficult, if not impossible. The evaluation of these forces relies on experimental correlations or theories which are developed only from special situations where a single effect (or an individual force) is dominant, such as single bubbles rising steadily in stagnant liquid (Clift[1978]). Due to the lack of analytical solutions, we can only gain a better understanding from computational results for a small number of selected typical cases.

A typical steady non-uniform dispersed two-phase system — a particle in the steady flow around a circular cylinder — is investigated here. In such a field, fluid velocities change strongly in direction as well as in magnitude as the fluid approaches the cylinder. The particles, which are assumed to have one of the three proposed initial conditions some distance upstream of the cylinder, tend to follow the path of the surrounding fluid, but the time delay of the particle response results in relative motion between particle and fluid. Any relative motion between the two phases in turn results in interaction forces.
To show the relative magnitude of individual interaction forces more clearly, force terms in eq.(2.5) are now first rewritten in the form shown in table 2.1.

Table 2.1 Exploded representation of relative magnitudes of different forces.

<table>
<thead>
<tr>
<th>Force</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{sd} )</td>
<td>( -\frac{1}{2}C_d \rho_f \pi R^2 )</td>
</tr>
<tr>
<td>( F_{am} )</td>
<td>( -C_a \rho_f \frac{4}{3}\pi R^3 \frac{dU_t^r}{dt} )</td>
</tr>
<tr>
<td>( F_{pg} )</td>
<td>( 1 \rho_f \frac{4}{3}\pi R^3 \left( \frac{dU_t^r}{dt} - \nu_t \pi^2 U_t \right) )</td>
</tr>
<tr>
<td>( F_{basset} )</td>
<td>( \frac{6}{\sqrt{\pi}} C_b \rho_f \sqrt{\nu_t} \pi R^2 ) ( \int_0^{t-\tau} \left( \frac{dU_t^r}{dt} \right)_\tau \frac{d\tau}{\sqrt{t-\tau}} )</td>
</tr>
<tr>
<td>( F_{buoy} )</td>
<td>( g \rho_p - \rho_f \frac{4}{3}\pi R^3 )</td>
</tr>
<tr>
<td>( F_{L_r} )</td>
<td>( C_{lr} \rho_f \frac{4}{3}\pi R^3 (\dot{\omega} \times U_t) )</td>
</tr>
<tr>
<td>( F_{L_a} )</td>
<td>( \frac{1}{\pi} C_{ls} \rho_f \sqrt{\nu_t} \pi R^2 \sqrt{\omega(\hat{e}_k \times U_t)} )</td>
</tr>
</tbody>
</table>

where

1) \( U_{rl} = U_{p1} - U_{f1} \)

2) \( \frac{dU_{fi}}{dt} = \frac{\partial U_{fi}}{\partial t} + U_{f1} \frac{\partial U_{fi}}{\partial t} + U_{fj} \frac{\partial U_{fi}}{\partial j} \)

3) \( \frac{dU_{p1}}{dt} = -\frac{\partial U_{p1}}{\partial t} + U_{p1} \frac{\partial U_{p1}}{\partial t} + U_{p2} \frac{\partial U_{f1}}{\partial l} \)
From table 2.1, the following observations may be obtained:

(1) Among above listed forces, only the buoyancy force is due to the universal gravity field. All other forces, however, are associated primarily with changes of the velocity field in forms of relative velocities ($U_r$), relative accelerations ($dU_r/dt$), or fluid and particle accelerations ($DU_r/Dt$ and $dU_p/dt$). Relative magnitudes of individual forces cannot be illustrated explicitly because of the implicit form of the terms containing velocities (either particle velocities or liquid velocities) and accelerations (either particle accelerations or liquid accelerations). For the flow field around the cylinder, the magnitude of changes of the velocity field ($U_r$, $dU_r/dt$, $dU_p/dt$ and $DU_r/Dt$) mainly depends on the upstream fluid velocity ($U_{fo}$) and the cylinder diameter ($2R$), i.e., the cylinder Reynolds number ($Re_d=U_{fo}D/\nu_f$).

(2) While $F_{am}$, $F_{pg}$, $F_{buoy}$, and $F_{Lr}$ are proportional to the volume of the particle ($\frac{4}{3}\pi R^3$), $F_{ad}$, $F_{basset}$, and $F_{Ls}$ are proportional to the cross-section (or surface) area of the particle ($\pi R^2$). If the size of a particle ($R$) increases, the first group ($F_{am}$, $F_{pg}$, $F_{buoy}$, and $F_{Lr}$) increase more rapidly than the second group ($F_{ad}$, $F_{basset}$, and $F_{Ls}$).

(3) As regards to effects of the properties of the fluid (density $\rho_f$ and viscosity $\nu_f$), terms in the fourth column are the same except for $F_{basset}$ and $F_{Ls}$. Considering that most common fluids (e.g. water and air at the atmospheric pressure) have not too different kinematic viscosities, the properties of a fluid may be assumed to have not too large an influence on the relative magnitude of interaction forces.

(4) Except the steady drag coefficient ($C_d$), which depends on the particle Reynolds number $Re_p=U_r d_p/\nu_f$, all other coefficients in the third column may be simply taken as constants given either empirically or analytically as discussed in the previous sections. Accordingly, the influence of all these coefficients ($C_b$, $C_a$, $C_{1v}$, and $C_{1h}$) on the relative magnitude of interaction forces are small.
(5) The steady drag coefficient $C_d$ may vary widely, depending on particle Reynolds number and nature of the dispersed phase. Only for solid spherical particles, $C_d$ is well known as presented in the correlations in appendix 3. In our cases, $Re_p$ lies mostly in the intermediate range ($250 < Re_p < 10^4$) and here $C_d$ is approximately constant (roughly 0.5) for a spherical solid particle. Accordingly, the relative magnitude of the interaction forces will hardly be influenced by the small variation of $C_d$.

(6) For gas bubbles, $C_d$ ranges from roughly 0.2 at low $Re_p$ ($Re_p=O(500)$) in pure water to about 3 at $Re_p$ values beyond $10^3$ (Clift[1978]) as a consequence of "flattening". For liquid droplets, the behaviour of $C_d$ lies generally between that for solid particles and for gas bubbles, which means that the value of $C_d$ may vary considerably, also depending on Weber number ($=U_t^2d_p\rho_f/\sigma_f$) for both droplets and bubbles. The just mentioned variations of $C_d$ reflect variations in the flow pattern around the particle. Consequently they point to possible changes in (the coefficients of) the added mass force and the Basset history force. In literature, however, almost no data were found about these changes.

(7) It may argued that the larger $C_d$ of a flattened bubble (or droplet) goes with larger velocities at the edge of the object and with stronger vorticity shedding, and hence with a large added mass force (coefficient) and a larger Basset history force (coefficient). From this point of view, the relative magnitude of steady drag, added mass force, and Basset history force may not change very much, compared to their relative magnitudes for a spherical particle. But a larger $C_d$ also leads to smaller values of $U_t$ and $dU_t/dt$; this results in a relatively smaller added mass force and Basset history force, considering the form of the related terms in eq.(2.5). Roughly speaking, the last effect is proportional to the square root of $C_d$ and does not exceed a factor of about two (again compared to the solid spherical particle).

In our calculations of particle trajectories and individual interaction forces, only the standard steady drag correlations for spherical solid particles were used. Probably the conclusions reached on this basis also hold, at least qualitatively, for other dispersed phase objects. In the high Reynolds number flows past obstacles, that were investigated experimentally (see next two chapters), observation has shown that the bubbles remained nearly spherical in almost all cases. To which degree the interface was immobile (an additional requirement) is not known however.
Inspection of eq.(2.5) shows that the upstream fluid velocity (U_{fo}) or the cylinder Reynolds number (Re_d=U_{fo}D/\nu_f), the particle size (d_p) or the particle-to-obstacle size ratio (d_p/D), and the particle-to-fluid density ratio (\rho_p/\rho_f) are the most influential system parameters in our calculations.

In connection with the experiments, which will be reported in the next two chapters, the results and analyses of bubble-liquid systems are our primary interest. They are presented in the next section. The results of the calculations for particle-liquid and for particle-gas systems are presented in appendix 6 and appendix 7 respectively. It is in particular the ratio of particle-to-fluid density (\rho_p/\rho_f) that is very different in these types of two phase flows. From now on, the words particle, drop, and bubble have their ordinary meaning.

2.5 CASE STUDY OF BUBBLE TRAJECTORY COMPUTATIONS

A bubble-liquid system is characterized by a very small density ratio (e.g. \rho_p/\rho_f=1/1000 for air bubbles in water at the atmospheric pressure). The two phases are strongly coupled and there exists strong interaction between the two phases when one phase changes either in time or in space.

Two sets of cases of gas bubbles moving up in a steady water flow past a cylinder are investigated. In each set of cases, either bubble size (d_p) or upward liquid velocity (U_{fo}) is varied to compute trajectories of bubbles as well as to obtain individual interaction forces along trajectories. Velocities and velocity gradients of the steady water flow field outside the wake behind the cylinder are given from the potential wake model (for details, see appendix 4). Here we do not discuss trajectories of bubbles after collisions with the cylinder or inside the wake and leave it for a detailed analysis in chapter 5. The buoyancy force is included in the momentum equation and the direction of the gravity is against the direction of the upward liquid velocity. Main parameters for computations are listed in table 2.2. Results of computations are shown in fig.2.3, fig.2.4 and fig.2.5.
Table 2.2 Main parameters for computing bubble trajectories.

(Computational results are shown in fig.2.3, fig.2.4, and fig.2.5)

<table>
<thead>
<tr>
<th>Case Nr.</th>
<th>$U_{fo}$ (m/s)</th>
<th>$d_p/D$</th>
<th>$\rho_p/\rho_f$</th>
<th>$Re_d (U_{fo}D/\nu_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3A</td>
<td>2.0</td>
<td>1/10</td>
<td>0.0012</td>
<td>80000</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1/40</td>
<td>0.0012</td>
<td>80000</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1/80</td>
<td>0.0012</td>
<td>80000</td>
</tr>
</tbody>
</table>

— results in fig.2.3A and fig.2.4;

| 2.3B     | 0.02           | 1/10    | 0.0012          | 800                 |
|          | 0.2            | 1/10    | 0.0012          | 8000                |
|          | 2.0            | 1/10    | 0.0012          | 80000               |

— results in fig.2.3B and fig.2.5;

(D=40mm, $\rho_f=1000/m^3$, $\nu_f=10^{-6}m^2/s$)

(Bubbles are introduced at the same speed as the water
at four positions of 0.2D, 0.4D, 0.6D, 0.8D
from x-axis and at a distance of 3D upstream from y-axis.)
Fig. 2.3A Trajectories of bubbles with various sizes.

\[ D = 40 \text{mm}; \ \rho_f = 10^3 \text{kg/m}^3; \ \rho_p/\rho_f = 0.0012; \]
index: (\( \ast \)) \( d_p/D = 1/10 \), \( (\ast) \ d_p/D = 1/40 \), \( (\ast) \ d_p/D = 1/80 \);

Fig. 2.3B Trajectories of bubbles with various upstream liquid velocities.

\[ D = 40 \text{mm}; \ \rho_f = 10^3 \text{kg/m}^3; \ \rho_p/\rho_f = 0.0012; \]
index: (\( \ast \)) \( U_{f_0} = 0.02 \text{m/s} \), \( (\ast) \ U_{f_0} = 0.2 \text{m/s} \), \( (\ast) \ U_{f_0} = 2 \text{m/s} \);
Fig. 2.4 Interaction Forces along Trajectory 3°, 3°, & 3° in fig. 2.3A;

index: (1) $F_{sd}$, (2) $F_{Lb}$, (3) $F_{pg}$, (4) $F_{am}$; (5) $F_{basct}$.
Fig. 2.5 Interaction Forces along Trajectory 3*, 3x & 3a in fig. 2.3B;

index: (1) Fsd, (2) F_Lp, (3) F_PG, (4) F_am, (5) F_basset.
2.5.1 Influence of Bubble Size

Fig. 2.3A shows three sets of trajectories of bubbles of different diameters ($d_p/D=1/10$, 1/40, and 1/80). Bubbles enter the non-uniform liquid field with their carrier liquid at rather high speed (2 m/s). When bubbles pass the cylinder, small bubbles tend to follow the streamlines closer and move further away than bigger ones. This means among others that big bubbles in the high-speed liquid flow around a cylinder have more chances to be drawn in the wake than small bubbles.

Interaction forces along the trajectory 3 of each set are shown in fig. 2.4. The steady drag force as usual plays a leading role among all interaction forces. In x-direction, it first resists the acceleration of bubbles driven by the buoyancy force far upstream, then accelerates bubbles when they pass the cylinder, thereafter decelerates bubbles again downstream. In y-direction, it pushes bubbles away from the cylinder upstream but changes its direction right after passing the cylinder. The shear lift force is very small compared with other forces in all three cases and thus may be negligible. The pressure-gradient force only contributes a part in y-direction when the bubble size is big. The added-mass force decreases gradually as the bubble size decreases. The Basset history force, in contrast to the added-mass force, takes a more and more significant part as the bubble size decreases. It changes almost in the reverse phase of the steady drag force. The added-mass force, the Basset history force, and the steady drag force are equally important in all three cases.

2.5.2 Influence of Upstream Liquid Velocity

Fig. 2.3B shows three sets of trajectories of bubbles entering the field with different liquid upstream velocity ($U_{fo}=0.02, 0.2, 2$ m/s). Bubbles in the slow liquid flow tend to move straight forward and are closer to the cylinder than those in the fast liquid flow. The large magnitude of relative velocities in the slow liquid flows reflects large magnitude of interaction forces on the bubbles, so the bubbles will be more likely to be "drawn" by the liquid to follow streamlines when the bubbly flow passes the cylinder. Interaction forces along the trajectory 3 of each set are shown are shown in fig. 2.5.
In x-direction, the steady drag force dominates all other forces, it balances the buoyancy force all the way. Both the shear lift force and the pressure-gradient force take negligible parts in the force balance. The added-mass force and the Basset history force decrease as the upstream liquid velocity decreases. The added-mass force is only significant at the beginning when the steady drag force changes rapidly. The Basset history force still plays a considerable role even when the upstream liquid velocity is very small.

In y-direction, all five forces need to be considered in liquid flows of low velocities. The steady drag force first pushes bubbles away from the cylinder upstream but changes its direction right after passing the cylinder. The added-mass force keeps a constant value relative to the steady drag force and changes in the reverse phase of the steady drag force upstream of the cylinder. The Basset history force takes more and more significant part as the upstream liquid velocity decreases and changes almost in the reverse phase of the steady drag force all the way. The pressure-gradient force only contributes a part when bubbles pass the cylinder in the slow liquid flow. The shear lift force increases very much as the upstream liquid velocity decreases and appears even to be the biggest of all interaction forces when bubbles pass over the cylinder in very slow liquid flows.

2.5.3 Concluding Summary

(1) Both bubble size and upstream liquid velocity have big influence on bubble trajectories and thus on the interaction forces between the two phases. The added-mass force and the Basset history force are of the same magnitude as the steady drag force and thus must be included in the force balance equation.

(2) The shear lift force is negligible except when the liquid velocity is very small.

(3) The pressure-gradient force only contributes a part when the bubble size is big or the upstream liquid velocity is small.

(4) The added-mass force decreases gradually as bubble size decreases and tends to keep a constant ratio to the steady drag force as the upstream liquid velocity decreases.

(5) The Basset history force takes a more and more significant part both when the bubble size decreases and when the liquid velocity decreases; it changes almost in the reverse phase of the steady drag force.
2.6 CONCLUDING REMARKS

In this chapter, we have carried out a numerical investigation on the interaction between a dispersed gas phase and a liquid when they pass a circular cylinder. A rather complete form force balance equation was first constructed, which encompasses of a steady drag force, an added–mass force, a pressure–gradient force, a Basset history force, a buoyancy force, and a shear lift force. The equation was converted into sets of second–order two–dimensional ordinary differential equations and was solved with use of the 4th–order Runge–Kutta integration numerical algorithm. As a first rough approximation, the standard drag curve for spherical rigid particles in a steady fluid flow was assumed to apply to the dispersed gas phase. By computing and tracking trajectories of such a dispersed gas phase in a liquid approaching to the cylinder, we could illustrate and analyze the magnitude of the different interaction forces in the total force balance and the influence of main system parameters on the interaction process between the two phases.

The results, which have been summarized in each related section and appendices, are not repeated here. Rather unexpectedly, several often assumed negligible "additional forces" (especially the troublesome Basset history force) were found to play important roles. Despite serious efforts, no method was found for quantifications of the results, e.g., in the form of critical values of dimensionless numbers.

Since a flow past a cylinder is a typical non–uniform case, results may qualitatively be extended to applications for many more general situations. The magnitude of different interaction forces and the influence of main system parameters provides a guidance for better understanding and modeling of dispersed two–phase systems.

Computational results concerning particle–liquid and particle/drop–gas flows are presented and discussed in appendix 6 and appendix 7 respectively, in the same fashion as in this chapter.
CHAPTER THREE

MEASURING SYSTEMS

3.1 INTRODUCTION

In general, for two-phase flows no complete theoretical modeling exists. Measurements still play a primary role in visualizing and investigating unknown flow behavior in a system. The complexity and variety of the system mostly require not only that careful considerations be given to ensure high quality data but also that specific arrangements be made for each particular situation.

Our investigation concerns the dispersed gas-liquid flow (or bubbly flow) around an obstacle in a vertical tube at atmospheric or high pressure. In the framework of this investigation, we have developed two measuring systems: a gamma-ray densitometer system for obtaining void fraction distribution measurements around the obstacles, and a photographic system for quantifying size, velocity, and location of bubbles in a high-pressure tube.

Development of each of the measuring systems may be roughly divided into four stages:

1. design and construction of the system;
2. derivation of the data processing or interpreting principle;
3. calibration of the new system;
4. application of the system to practical situations.

The first three stages are described in this chapter. Application of the system and measurement results are presented in the next chapter.
3.2 GAMMA–RAY DENSITOMETER SYSTEM

A variety of measuring techniques has been applied for measuring phase distributions of a dispersed gas–liquid flow in a vertical tube (see reviews by Jones[1976], Hewitt[1978], Jones[1983], and Cheremisinoff[1986]). Three common types are the photon–attenuation technique, the intrusive probe technique, and the optical probe technique. In contrast to intrusive (and most optical) probes, a photon–attenuation densitometer has no direct contact with either fluids or test–tubes, and hence produces neither disturbance to flows nor corrosion to the measuring hardware. In comparison with an optical probe, a photon attenuation densitometer has no refraction or reflection effect due to macroscopic density discontinuities (i.e. bubble surface in water). Furthermore, the sensor of a photon–attenuation densitometer is situated outside the test section and thus requires no complex structure for taking a large number of measurements at different positions within a short period of time.

The gamma–ray densitometer technique is one of the most widely used photon–attenuation densitometer techniques (e.g. Petrick[1958], Staub[1967], DeVuono[1980], and Lahey[1989]). This technique makes use of the observation that a homogeneous material absorbs monochromatic gamma–rays and the attenuated beam radiation intensity decreases exponentially with increasing absorption length at constant material density (Lassahn[1977]). In its simplest form, a gamma densitometer consists of a radiation source and a detector which together perform a linear scan over an object region containing an unknown two–dimensional density distribution. During this process, a collimated beam of radiation is attenuated by the object and sensed by the detector. A computer digitizes and stores the radiation transmission data at each of N locations along the linear scan. In this way, a projection or a view of line–average density of the object is provided.

Based on a single–beam gamma densitometer, which has been utilized for studying gas–liquid two–phase flows in the Laboratory of Multiphase Flows at Eindhoven University of Technology for years (e.g. Houf[1987] and Meng[1988,1990,1992,1993]), the new measuring system is equipped with a personal computer unit to achieve fully automatic control of measuring–table traversing movements for improving efficiency and safety. Much attention has also been given to data sampling and processing algorithms for deriving reliable and accurate void fractions from "raw" measurements. Through careful calibration tests on a simple mockup, the performance of the new measuring system was carefully evaluated before application.
3.2.1 System Configuration

A diagram of the gamma-ray densitometer system is shown in fig.3.1.

Fig.3.1 Diagram of the Gamma-ray Densitometer System.

A Cesium-137 isotope source (1 Curie) and its shielder, two collimators, and a detector are installed on a measuring-table. The detector is composed of a NaI(Tl) crystal integral assembly with a photomultiplier tube and a preamplifier. A gamma beam, emitted from the radiation source and collimated by the two collimators, strikes the crystal generating a "flash of light" or photons. After being amplified by an amplifier
and analyzed by a pulse–height analyzer, pulse signals from the detector are counted over a time interval through a multi–function interface card (the LabMaster). The installation of the time interval and the counting procedure are controlled by a program (GammaMca, written in Turbo Pascal) in the personal computer. A measurement of the line–average density over the region crossed by the gamma beam is thus obtained. A stabilizer is employed to stabilize amplified pulse spectrum against high voltage instability, amplifier gain drift, and pulse amplitude variation due to temperature sensitivity of the photomultiplier.

The measuring–table is driven by an electrical DC motor whose speed and direction of motion are controlled by GammaMca through a motor–driver unit and the LabMaster. The traversing speed of the measuring table is adjustable in the range of 0 to 0.25mm/s. During a traversing movement over the object region, GammaMca takes in measurements of line–average densities. The width of one measurement equals the product of the traversing speed and the so called "counting–time constant". After the measuring–table scans the entire object region, an array (or a projection, as it is usually called) of line–average densities is obtained. A two–dimensional image of line–average densities of the full object volume can further be provided by taking projections at various vertical positions through moving the measuring–table up and down.

The data digitizing and storing procedures of the new system is also fully computer–controlled in GammaMca with use of the multi–function interface card (the LabMaster). After installing counting–time constant (t_c), measuring–table traversing speed (V_t), number of samples (N_s), number of projections (N_p) and starting a measuring procedure, no further action is required until the procedure is finished. This enhances the number of measurements which may be taken in limited time, improves the safety of operation and permits the application of a strong gamma source.

To a large extent, four installation parameters (t_c, V_t, N_s, and N_p) determine the accuracy of a "raw" line–average density distribution measurement by the gamma–ray densitometer system. Among them, the counting–time constant (t_c) is the most influential parameter. On the one hand, the fundamental inaccuracy due to photon statistical fluctuations depends on the the beam intensity of the gamma source (I_s) and the counting–time constant (t_c), as shown in eq.(3.1).

$$\Delta (\ln I_s) = \frac{\Delta I_s t_c}{I_s t_c} = 1/\sqrt{I_s t_c} \quad (3.1)$$
where the relative deviation of a measurement $\Delta(\ln I_0)$ is proportional to the inverse of the root of $I_{tc}$. Accordingly, a large counting-time constant effectively reduces the statistical error in measurements. On the other hand, the resolution of line-average density distribution data, which depends mainly on the sample width (i.e. the width of the "strip region" over which data are averaged), is better served with a small $t_c$. The smaller the sampling width is, the higher the resolution of the distribution data will be. Therefore with a fixed measuring-table traversing speed (usually installed to be large for finishing a measuring procedure in a short time period), the counting time constant has to be small.

It is obvious that the requirements for these two criteria are contrary to each other. To find an optimal combination of above parameters for our measuring system and applications, we performed a series of preliminary tests. A set of empirical optimum settings are:

- Counting time constant ($t_c$) : 2.5s
- Measuring table traversing speed ($V_t$) : 0.2mm/s
- Sample width ($= t_c V_t$) : 0.5mm

Besides the above two parameters ($t_c$ and $V_t$), the number of samples per projection ($N_s$) and the number of projections ($N_p$) are chosen according to the size of an object volume and the limitation of total measuring time. For example, to cover entirely the flow inside a test tube with internal diameter of 40mm, we have to make $N_s$ at least as big as 80 so that the product $N_s V_t t_c$ is not smaller than 40mm. Within the permitted time interval for maintaining a stable flow condition in the test tube, $N_p$ is chosen to be as large as possible. According to $N_p$, the gamma beam scans the object region to-and-fro ($N_p / 2$ times) to take $N_p$ projections under one experimental condition. This in fact increases the time period for computing the time-average distribution data. In this way, the statistical fluctuation error is further reduced.

### 3.2.2 Data Processing Algorithm

Due to the random nature of a gamma radiation source and the erratic behavior of two phase flows, it is essential to formulate a good and effective data processing algorithm in order to derive accurate and reliable void fraction distributions from "raw" measurements.
The data processing procedure includes four major steps:

**Step 1: Extracting samples from raw data**

One example of raw measurements and of extracted samples is shown in fig. 3.2. Due to the flexibility of the long test tube (8m) and the rather rough transmission structure of the measuring-table, it is not certain that several projections would start from the same position where the internal edge of the tube wall lies. Any small shift between different projections would induce large deviations in time-average distribution data. Accordingly instead of installing a sample number as 80 just covering the flow region in the tube, we have chosen the sample number to be 120. From such an extended projection (or raw data), samples over the flow region can be extracted with reference to the measurement at the internal edge of the tube wall, where local maximum beam path of the tube wall always shows a clear local minimum counting rate measurement (or a "valley") in a set of distribution data.

---

**Fig. 3.2 Extracting samples from raw data.**
Step 2: Calculating averages of samples
Averages of samples from four projections (a number chosen considering the available measuring time and the experimental conditions concerned) are calculated as shown in fig.3.3. For an obviously symmetrical flow (time-average) in a symmetrical construction, such as a gas-liquid flow in a vertical tube, we actually only need measurements over one half of the object region. Accordingly we average two halves of the set of samples to obtain a symmetrical set of data. The results are equivalent to the averages of eight sets of data.

Step 8: Filtering noise in a set of distribution data
Considering that a (time-average) line-average density distribution of a flow changes gradually over a test-tube, we employ a low-pass filtering procedure for attenuating high frequency noise elements in the distribution data. A set of samples and the set of filtered data are shown in fig.3.4 (for the deduction procedure, see appendix 8). The filtering procedure, which acts to "smooth" an input, is a kind of "tap weight" procedure where the output is merely the average of neighbour values (Roberts[1987]).
Fig. 3.4 A set of distribution measurements before and after noise filtering.

**Step 4: Computing line–average void fractions**

The data shown in fig.3.2, fig.3.3, and fig.3.4 are all attenuated gamma intensity data (counts per unit time). The relationship between the line–average density, $\rho_m$, and the attenuated gamma intensity measurement, $I$, is given by the following equation (Lassahn[1977]),

$$\rho_m = - B \ln(I/I_0)$$  \hspace{1cm} (3.2)

where $I_0$ is a constant and $B=1/(K_aX_1)$. $X_1$ is the beam path length through the fluid and $K_a$ is the mass attenuation coefficient of the fluid. In principle, values for $B$ and $I_0$ in eq.(3.2) can be calculated precisely. However, such computations are mostly very complicated and time consuming. Usually, $B$ and $I_0$ are directly determined by empirical calibrations of two attenuated gamma intensity measurements with known densities. For a gas–liquid flow, density of the mixture is derived from measurements of $I_g$ and $I_f$ with known densities of pure gas ($\rho_g$) and of pure liquid ($\rho_l$). With the density definition for a two–phase mixture [$\rho_m=\alpha\rho_g+(1-\alpha)\rho_l$], eq.(3.3) is obtained:

$$\alpha = \frac{\ln I_m - \ln I_f}{\ln I_g - \ln I_f}$$  \hspace{1cm} (3.3)

where $\alpha$ is the line–average void fraction (for flow in a circular tube, it is actually
a chordal—average void fraction. Eq.(3.2) is the most common form of the attenuation equation (also called the "calibration equation"), which allows direct computation of the line—average void fraction data from three sets of measurements. Diagrams of three measurements (I_g, I_a, and I_f, which are measurements by the gamma—ray densitometer with the test—tube filled with pure gas, two—phase mixture, and pure liquid respectively) and the computed void fractions are shown in fig.3.5 and fig.3.6, respectively.

![Fig.3.5 Three measurements (I_g, I_a, and I_f).](image1)

![Fig.3.6 Void fractions computed by the calibration equation.](image2)
3.2.3 System Calibration

To validate the gamma-ray densitometer system and the accuracy of chordal-average void fraction distribution measurements, a series of calibration tests are performed using a simple mockup. The main part of the mockup is a plastic cell matrix. By filling certain cells with water and leaving the rest with air, we are able to simulate a number of two-phase flow patterns with known void fraction distributions. Measurements by the gamma-ray densitometer system are then compared with the actual density distributions. Two of the results are shown in fig.3.7.

![Fig.3.7 Two results of calibration tests using a simple mockup.](image)

(a) Case I; (b) Case II;

In general, measurements and actual void fraction distributions agree with each other fairly well. Big deviations only appear at places where rapid changes of void fraction values occur. This is mainly due to the big sampling width compared with the small interval where such changes happen in the mockup. Measurements will certainly be improved if the sample width is decreased. But such sharp changes are not likely to occur in the practical situation where time-average measurements are taken. Accordingly, the gamma-ray densitometer system may be expected to provide reliable measurements of void fraction distributions for our applications.
3.3 PHOTOGRAPHIC SYSTEM

Without any disturbance to the flow, photographs provide us with the most objective and direct visualizations of local parameters (such as bubble size, concentration and velocity) of the dispersed phase in two-phase flows.

Among various photographic methods, which take advantage of changes of the refractive index due to density changes, the shadow-photograph method is the simplest optical manifestation of density changes (Hewitt[1978]). Light of a point source or a parallel beam of light passes through a two-phase flow in a test section. Because of the gradient variation of the refractive index normal to the light beam, deflections of adjacent rays will differ. They will converge or diverge, giving increased illumination or decreased illumination on the film placed behind a camera lens.

In the Laboratory of Multiphase Flow of Eindhoven University of Technology, instruments have been developed to provide both average and local vapor-water flow properties under high-pressure conditions. Among them, the photographic system is one of the most important. The present photographic system is the result of a long development. The earlier system described by van der Geld(1984, 1985) and van Manen(1988) suffered from insufficient illumination because of the small opening on the test tube and difficulty of focusing because of refraction effects of the thick window.

Instead of introducing some modifications to the old-photographic system, we have developed a new photographic system following the suggestion of Wijchers (Prins Maurits Laboratorium TNO, 1988). Based on a different image formation principle, we have redesigned the opening window on the test tube, the entire camera unit, and the light unit. A direct result from observation during the preliminary tests was a new principle for determining the longitudinal position of a spherical bubble from the size of a shadow and a bright spot on the film. This new principle makes it possible to obtain three-dimensional data on the position of bubbles in a water flow from a two-dimensional photograph. The performance of the new-photographic system and the new interpretation principle have been validated through a series of calibration tests.
3.3.1 System Configuration

A diagram and a photo of the photographic system are shown in fig. 3.8.

Fig. 3.8 Diagram and photo of the photographic system.

The photographic system is rather conventional and encompasses a camera unit and an illumination unit, mounted on an optical table. The illumination unit produces a beam of parallel light by placing a point light source at the focus point of a positive lens (f 30mm). The light source may be either a usual continuous light source or a flash light source which produces light pulses at a set frequency (a "nanolight" or a stroboscope). Through windows on two sides of the test section, the parallel light rays cross the two-phase flow region and then are received by the camera unit. Through a camera lens (or an object lens [f 35mm]), shadow images of bubbles are formed on the film in a camera body (PRACTICA MTL-180). The image distance, which is the distance from the object lens to the film, may be adjusted through a bellow extension mounted on an optical rail.

The optical table consists of two supporting boxes and a supporting table. The two supporting boxes, which can be finely adjusted in three directions, lift the camera unit and the illumination unit to a demanded position. Both boxes are made of thick steel plates (8mm thick) and have very high rigidity. The supporting table is firmly fixed on the test section.
The test-section is a vertical stainless-steel tube with length of 8.23m, inner diameter of 39mm, and thickness of 5.5mm. Pressure up to 250bar can be maintained in the tube and there are about 80 universal mounting points along the tube. The photographic system is mounted on the lower half of the tube for having better chances for clear photographs of vapour bubbles in their developing stage when the void fraction is high in the tube. To resist the highly corrosive conditions at high pressures, the windows are made out of sapphire and coated with a thin Si₃N₄ film in a careful deposition procedure (see van der Geld[1985]). A diagram of the window construction on one side of the tube is shown in fig.3.9.

![Diagram of the window construction](image)

**Fig.3.9** Diagram of the window construction.

### 3.3.2 Photographic Result Interpretation

In a bubble–water flow, bubbles are mostly spherical or elliptical. When a beam of parallel light rays passes a bubble, the bubble may be divided into two parts according to effects on the parallel light rays. The edge part of the bubble will deflect or diverge light rays strongly, resulting in a dark contour on the film. The central part of the bubble, on the other hand, will deflect or converge light rays slightly, resulting in a bright spot on the film. The first effect is the basis for the shadow–photograph principle, that has been widely used for years to visualize size and velocity of bubbles in bubble–water systems. The second effect, on the other hand, has been almost completely ignored and no discussions were found in literature.
During our preliminary experiments, we observed a useful relationship between the size of such a bright spot on photographs and the longitudinal position (or the optical axial position) of the bubble in the test-setup. From the size of the bright spot, the longitudinal distance from the bubble to the object lens may be calculated, following the principles of geometrical optics. Consequently, three dimensional information about a bubble in a liquid system may be derived.

![Image](image.png)

Fig. 3.10 Image formation of a parallel beam passing a bubble in water.

In geometrical optics, the most accurate way to calculate image formation is to trace the path of light rays through the entire system. This is simple in principle, but in practice it can become a formidable task if high accuracy is required (Klein[1984]). An elementary derivation is now given for a spherical bubble floating in water, with reference to fig.3.10.

When a beam of parallel light passes a spherical bubble in water, the edge part of the bubble diverges parallel light rays due to its curvature, resulting in a shadow-contour on the film. From the diameter of this shadow-contour \(D_s\), the diameter of the bubble \(D_0\) may be computed with the following equation:

\[
D_0 = D_s \frac{F_1}{L_{lf}-F_1} \quad (3.4)
\]
The central part, on the other hand, converges parallel light rays, resulting in a bright spot in the middle of the shadow-contour on the film. In terms of geometrical optics, this part of the bubble actually can be considered as an "virtual optical lens". From the film, light rays emerge as if they came from a point light source (we name it the "virtual light point") placed on the focal point of this virtual lens. The focal length ($F_o$) of this virtual lens can be approximately calculated (Morton[1984]):

$$F_o = \frac{D_o N_d}{4(N_d-1)}$$

(3.5)

where $N_d$ is the ratio of refraction indices of this virtual lens to the ambient media. For a gas bubble in water, $N_d=1/1.33$ and hence $F_o=-0.76D_o$, where the minus sign indicates that the gas bubble functions as a negative lens in water.

If we take the distance between bubble and object lens ($L_{10}$) as the longitudinal position of the bubble on the optical axis (i.e. the axis of the object lens), a virtual light point will appear at an object distance $U_b=L_{10}-F_o$ (because of the thick windows, we must correct $U_b$ for refraction effects to nonparallel rays; for details, see appendix 9). A light cone seems to be emitted from the virtual light point and is bordered by the lens rim. Its image appears as a bright spot on the film. The diameter of the bright spot ($D_b$) may be computed from the following relation:

$$D_b = D_1 - \frac{|L_{10}-V_b|}{V_b}$$

(3.6)

where $D_1$ denotes the physical diameter of the object lens and $V_b$ is the image distance of the virtual light point, determined from $F_1$ and $U_b$ by the following conjugation law:

$$\frac{1}{U_b} + \frac{1}{V_b} = \frac{1}{F_1}$$

(3.7)

Combining eq.(3.4) through eq.(3.7) yields the longitudinal position of the bubble ($L_{10}$):

$$L_{10} = \frac{F_1 L_{10} D_1}{D_1 - V_b} - \frac{N_d D_b F_1}{4(N_d-1)(L_{10}-F_1)}$$

(3.8)

where the sign "±" (induced from the term $|L_{10}-V_b|$ in eq.(3.6)) is determined according to the following conditions:

"+", if $L_{10}>V_b$; \quad "−", if $L_{10}<V_b$; \quad ($L_{10}=U_b-F_o$, if $L_{10}=V_b$)
Based on the Inverse Square Law of Illumination (Morton[1984]), the sign of the term \(|L_{1f}-V_b|\) can also be judged directly from bubble images on the film according to the lightness of bright spots relative to that of the bright background. The bright background indicates the maximum dimension of the lighted region in the tube; it is proportional to the size of the minimum aperture stop \((D_b)\). As a rule, when the bright spot is just a bright point, \(L_{1f}=V_b\); when the bright spot is brighter than the bright background, \(L_{1f}<V_b\); Otherwise, \(L_{1f}>V_b\).

With eq.(3.8), information on the longitudinal position (i.e. the third dimension) of the bubble is provided in addition to the bubble size (in two dimensions) calculated from its shadow size on the photo by eq.(3.3). In this way we can interpret a two dimensional photo into a three dimensional picture of a bubble in a liquid flow.

Up to now our discussion has been limited to a bubble lying on the optical axis of the system. When a bubble is not on the axis of the system, the focal point introduced by the bubble becomes an off-axis virtual light source. There will be a gradual loss of light as the virtual light point moves off the axis. A cone of light rays from an off-axis light point to the object lens will not be transmitted entirely but be partially cut off by the rim of the lens (which is also the aperture stop). This is called vignetting in term of the geometrical optics, which is illustrated in fig.3.11. In this case, we should take the required measurements \((D_u\) and \(D_b)\) on the "complete part" of the bubble image on the film. Subsequent computation is done in the same way as for an on-axis bubble.

![Fig.3.11 Off-axis light source and vignetting.](image-url)
In addition to the measurements indicated above, the instantaneous velocity of the bubble may be found by using a flashing light source (a "nanolight" unit or a stroboscope). Both flashing light sources can provide a train of light pulses at a set frequency \( f_n \). By coordinating the light flashing frequency and the camera diaphragm opening time, a number of image pulses is registered on one single photograph. From the distance the bubble has traveled between two light pulses, the bubble velocity (\( U_{\text{bub}} \)) may be calculated by the following equation:

\[
U_{\text{bub}} = f_n L_{2p}
\]  

(3.9)

where \( L_{2p} \) is the distance between two images of the same bubble measured directly from the photo.

3.3.3 System Calibration

For the validation of the photographic system and the novel 3D—interpretation principle, three series of calibration tests were performed with glass spheres and air bubbles. The calibration setup, which simulates situations in the high—pressure tube, is shown in fig.3.12. The test section is a rectangular glass container wrapped entirely with black tapes, except for two windows on two sides. A continuous light source (i.e. an incandescent—filament lamp) is employed during the first and the second series of experiments and a nanolight unit or a stroboscope during the third.

![Diagram of the calibration test setup.](image)
In the first series of experiments, glass spheres were used as transparent objects. The glass spheres are 2 or 3 mm in diameter and have a refraction index of 1.51. Glass spheres therefore act as positive lenses whereas air bubbles act as negative ones. But both obey the same rules of image formation given in the previous section.

To check eq. (3.4), with which the object size \( D_o \) is computed from the measurement of its shadow size \( D_s \) on the photo (where \( D_s \) is independent of the longitudinal position of the object), a photo of three glass spheres (one 2mm and two 3mm) was used as shown in fig.3.13. Comparison between computations \( (D_o') \), calculated from measurement \( D_s \) and actual \( D_o \), as listed in Table 3.1, yields very satisfactory agreement.

![Fig.3.13 A photo of three glass spheres (D_o=2mm or D_o=3mm) in a calibration test.](image)

| Index | \( D_s \)(mm) | \( D_o \)(mm) | \( D_o' \)(mm) | \( |\Delta D_o| / D_o \) |
|-------|-------------|-------------|--------------|-----------------|
| (1)   | 6.6±0.1     | 2.00        | 1.99         | 0.5%            |
| (2)   | 10.0±0.1    | 3.00        | 3.02         | 0.7%            |

* \( D_o \) and \( D_o' \): actual and computed object sizes respectively;

* \( L_1=151\text{mm}, F_1=35\text{mm}, \Delta D_o=D_o-D_o'. \)
Eq.(3.8) is validated through a series of photos, taken with fixed distance (L_{lf}) between the object lens and the film. The camera unit was shifted step by step along the leading rail. Longitudinal positions of the object (L_{10}') were calculated with eq.(3.8) from measurements of shadow-contour sizes (D_s) and bright spot sizes (D_b). The series of photos and the comparison between actual and computed longitudinal positions (L_{10} and L_{10}') are shown in fig.3.14 and Table 3.2, respectively.

Fig.3.14 Photos for validating the computation of bubble longitudinal positions.
(object: 3mm glass spheres).
Table 3.2 Comparison of computed with actual object sizes (glass spheres).

| Index | $L_{10}$ (mm) | $L_{10}'$ (mm) | $|\Delta L_{10}|/L_{10}$ |
|-------|---------------|----------------|----------------------|
| (1)   | 57.60         | 59.11          | 2.6%                 |
| (2)   | 54.60         | 55.83          | 2.3%                 |
| (3)   | 51.60         | 53.07          | 2.8%                 |
| (4)   | 48.60         | 48.64          | 0.1%                 |
| (5)   | 47.10         | 47.15          | 0.1%                 |
| (6)   | 45.60         | 45.36          | 0.5%                 |
| (7)   | 44.10         | 44.10          | 0.0%                 |
| (8)   | 42.60         | 42.83          | 0.5%                 |
| (9)   | 39.60         | 41.10          | $D_b > D_s$          |
| (10)  | 36.60         | 41.10          | $D_b > D_s$          |

* $L_{10}$ and $L_{10}'$: actual and computed longitudinal positions respectively;

* $D_o=3$ mm, $L_{1f}=151$ mm, $F_1=35$ mm, $\Delta L_{10}=L_{10}'-L_{10}$.

In the second series of experiments, a bubble injector was employed. Since a free rising bubble does not move in a straight line, the longitudinal position of a bubble could not be kept all the way to be the same as when it left the injector. Accordingly photos had to be taken in the neighbourhood of the mouth piece of the injector where bubbles were not spherical. Measurements of sizes of shadow contours ($D_b$) and bright spots ($D_b$) were taken on one of the normal cross-sections of the bubble image on the photo with the assumption that such a cross-section has a circular shape. Longitudinal positions of bubbles ($L_{10}'$) were again calculated by eq.(3.8) and then were compared with actual ones ($L_{10}$), taken to be identical to the position of the mouth piece of the bubble-injector. Examples of photos and a list of comparison are shown in fig.3.15 and Table 3.3. It is remarkable that deviations from spherical shape do not seriously affect the accuracy of the method.
Fig. 3.15 Photos for validating computations of bubble longitudinal positions.
(object: air bubbles at mouth of injector).

Table 3.3 Comparison of computations with actual object positions (air bubbles).

| Index | $L_{10}$ (mm) | $L_{10}'$ (mm) | $|\Delta L_{10}|/L_{10}$ |
|-------|---------------|----------------|--------------------------|
| (1)   | 50.60         | 50.52          | 0.2%                     |
| (2)   | 49.60         | 50.79          | 2.4%                     |
| (3)   | 47.60         | 46.64          | 2.2%                     |
| (4)   | 45.60         | 44.95          | 1.4%                     |
| (5)   | 44.60         | 42.51          | 4.7%                     |
| (6)   | 43.60         | 42.01          | 3.6%                     |
| (7)   | 41.60         | 40.26          | 3.2%                     |
| (8)   | 39.60         | 39.88          | 0.1%                     |
| (9)   | 37.60         | 36.61          | 2.6%                     |
| (10)  | 35.60         | 34.64          | 2.7%                     |

$L_{10}$ and $L_{10}'$: actual and computed longitudinal positions respectively;

$L_{1f}=151\text{mm}, \, F_1=35\text{mm}, \, \Delta L_{10}=L_{10}'-L_{10}$.
In the third series of experiments, two or three flashes from the nanolight were registered on one photo during one opening of the camera diaphragm with a certain combination of the camera shutter time and the nanolight frequency. Accordingly two or three images of some passing bubbles were recorded on one photograph. From eq.(3.9), velocities of bubbles thus were computed. Two examples are shown in fig.3.16.

Fig.3.16 Two photos of calibration tests for determining bubble velocities.  

(1): $f_n=375\text{Hz}$, $\Delta L_2p=1.8\text{mm}$, $\Rightarrow U_b=0.675\text{m/s}$)  
(2): $f_n=375\text{Hz}$, $\Delta L_2p=1.2\text{mm}$, $\Rightarrow U_b=0.45\text{m/s}$)

All three series of experiments have shown very satisfactory results. Relative deviations of computations from actual bubble sizes are less than 1.0%, as shown in Table 5.1. Relative deviations of computations to actual bubble longitudinal positions are all within 5.0%, as shown in Table 3.2 and Table 3.3. Bubble velocities obtained are close to reality, too.

The photographic system is meant for applications in a high-pressure tube. At high pressures, bubbles are much smaller than under atmospheric pressure. Therefore the shape of the bubbles will be closer to spherical. This is in favour of the measuring strategy.
3.4 CONCLUDING REMARKS

A gamma-ray densitometer system for obtaining measurements of void fraction distributions over a vertical tube and a photographic system for quantifying bubbles in a high pressure tube have been developed. Both measuring systems may serve the investigation of phase distributions and related flow pattern transitions of gas-liquid two-phase flows in a vertical high pressure tube.

Based on a conventional single-beam gamma densitometer, much effort has been dedicated to automation of the control system and processing algorithms of "raw" measurements. The new system has a number of unique features for our special applications to derive reliable and accurate void fraction distributions in vertical tubes from "raw" gamma-beam intensity measurements. It compares well with similar systems in literature. Through a series of carefully designed calibration tests on a simple mockup, the new measuring system and the developed data processing procedures were evaluated; measurements of void fraction distributions over a vertical tube provided by the system proved to be reliable, efficient, and of satisfying accuracy.

The new photographic system was introduced to obtain shadow-photographs of bubbles in a special situation — a high-pressure vertical tube (Maximum pressure: 250 bar; Tube internal diameter: 39mm; Window diameter: 6mm). With this system and related interpretation principles, not only size and velocity of a bubble may be computed from its shadow images on one photo but also the longitudinal position of the bubble on the optical axis may be estimated from the size of the bright spot in its shadow image. The relation for estimating bubble longitudinal positions is to the author's knowledge introduced for the first time. It provides extra information from a usual shadow photograph in a very simple manner. Performance of the new system and validity of the interpretation principles have been examined through three series of calibration tests, using calibration test-setups simulating situations in the high pressure tube. Very satisfactory results have been obtained, relative deviations between computations and actual sizes or longitudinal positions being generally smaller than 1% to 5%. Velocities obtained by using a flashing nanolight in the third calibration series are also agree well with realistic values under the same conditions.
CHAPTER FOUR

MEASUREMENTS OF PHASE DISTRIBUTIONS AROUND OBSTACLES

4.1 INTRODUCTION

In contrast to the large amount of literature on phase distributions of gas-liquid flows in either horizontal or vertical tubes, only a few reports have been found on gas-liquid flows past obstacles, probably because of their heterogeneity and complexity. For such situations, a complete theoretical modeling or numerical simulation is not yet possible (Song[1990]). Measurements still play primary roles in visualizing and investigating the unknown behavior in such complicated situations.

Some experimental investigations of phase distributions around obstacles have been published (Hulin[1982], Salcudean[1983], Inoue[1986], and Robinson[1988]). Hulin et al. studied experimentally the vortex emission behind obstacles with trapezoidal cross-section in vertical air-water flows. A two-dimensional pseudo-image of the void fraction distribution in the vortex street was obtained by using an optical fiber probe moved stepwise in the tube. Salcudean et al. investigated the effects of an obstruction on the void fraction distributions in a horizontal flow. An image of local void fractions was constructed from measurements by using an optical probe and rotating the test section. Robinson et al. reported void distributions in bubbly flow through an array of rods inclined relative to the average flow direction. Qualitative results and analysis were provided mainly based on photographic observations. The most extensive study is the work by Inoue et al.. They provided a rather overall picture of void distribution patterns around circular cylinders in vertical bubbly flows, where effects of mean liquid velocity, cylinder size, and upstream mean void fraction on void distribution patterns were illustrated. Local void fractions were measured, using an electrical impedance needle equipped with a traversing device.
In all above cases, experiments were performed under the room temperature and the atmospheric pressure. The bubble size is between 3mm and 5mm, under which dimension the shape of bubbles differs generally much from the ideal spherical shape. For measurements of time-average local void fractions, either an optical probe or an electrical impedance needle was employed. Generally these intrusive probe measuring techniques have yielded rather good results. But such an intrusive probe and its support affect local phase distribution patterns, especially when bubble size is small or an obstacle is of the same size as the bubbles.

In the present study, void fraction distribution patterns around circular cylinders in vertical bubbly flows are measured using the non-intrusive gamma-ray densitometer technique described in section 3.2. Compared with the work mentioned above, our experiments are performed not only at the atmospheric pressure but also at high pressures (up to 70bar). At the high pressures, a number of special features are obtained (such as small bubble sizes, closer density-viscosity values between vapour and water, and so on), which will be discussed in section 4.3. These special features not only provide valuable experimental findings which are important for many high-pressure industrial applications (and which to author's knowledge have not been found in literature) but also extend the range of some primary system parameters (such as the bubble-to-cylinder size ratio (dp/D) and the upstream mean void fraction (α₀) — the subjects of our investigation) which are difficult or often impossible to be achieved by experiments at the atmospheric pressure.

In this chapter, effects of the cylinder Reynolds number (Reₜ=UₒD/νₜ), the bubble size (dp) or the bubble-to-cylinder size ratio (dp/D), and the upstream mean void fraction (α₀) on void distribution patterns around cylinders in vertical bubbly flows are investigated on two test loops —— an atmospheric-pressure loop and a high-pressure loop. In particular, our experimental investigation highlights gas-liquid two-phase flows with various and small bubble-to-cylinder size ratios, either by setting different system pressures (at the high pressures) or by adding a surface-active substance in the water (at the atmospheric pressure). In addition to the gamma-ray densitometer, an ordinary camera or video and the specific photographic system described in section 5.3 are employed to provide basic information on individual bubbles some distance upstream of the obstacle.
4.2 MEASUREMENTS AT ATMOSPHERIC PRESSURE

4.2.1 Atmospheric-pressure Loop

Experiments at room pressure were performed on an atmospheric-pressure loop, which is shown schematically in fig. 4.1.

![Diagram of the atmospheric-pressure loop.](image-url)
The test-section is a 8 meter long transparent perspex circular tube with an internal diameter of 40mm. Air is injected through a sintered tube located at the bottom of the vertical tube. A mixer is installed right above the air injector to obtain a uniform distribution of bubbles. Forced water flows are controlled with a bypass line around the pump; the volume flow rate is measured with a differential-pressure meter. The maximum mean flow velocity is around 2.5m/s. The air flow rate is controlled via the air pressure and measured with a ROTA-meter. One of three circular cylinders (D=11mm, 6mm, and 2mm, made of perspex, brass, and stainless-steel, respectively) is installed at the position of 5 meter above the mixer.

With the gamma-ray densitometer system, measurements of void fraction distributions in the wake region behind a cylinder were taken over a range of 3D from the central line of the cylinder (y=0 to 3D). An ordinary camera and a video camera were utilized for taking photos or videos through the transparent test-section in the atmospheric-pressure loop. The photos or the video films are used both to estimate basic parameters of bubbles (such as bubble size and bubble velocity) and to visualize void distribution patterns qualitatively. In addition, void fraction distributions over the vertical tube without presence of a cylinder in the test tube (which is equivalent to the void fraction distributions at some distance upstream the cylinder at the same flow rate) were measured with the gamma-ray densitometer system. The chordal-average void fraction distributions over the circular test tube are approximately uniform over the central part of the circular tube at high flow rates, satisfying our assumption of an uniform distribution of void fractions upstream the cylinder (for a detailed discussion, see appendix 10).

Among the three system parameters (Re_d, dp/D, and α_o), which are varied to illustrate their effects on void fraction distributions around cylinders, the mean upstream void fraction (α_o) may only be varied within a very limited range in order to maintain a uniformly distributed bubbly flow in the test-tube. Accordingly experiments at the atmospheric pressure are performed mainly while varying the other two parameter (Re_d and dp/D). Experimental conditions with different Reynolds numbers (Re_d) are easily obtained by varying the size of cylinder (D) or the volume flow rate (ρfU_fo), covering a Re_d range of from 5000 to 25000. Although experimental conditions of different ratios of bubble size to cylinder size (dp/D) may be provided by varying the size of cylinder (D), the range of dp/D is very narrow, because of the small diameter of the test tube. To show effects of dp/D on void fraction distributions, we need air bubbles of various sizes in the atmospheric loop.
Several methods were tried out to decrease the size of bubbles by adding surface-active substance in the water (under the atmospheric pressure). It was found that the mean bubble size can be effectively decreased from 4mm to 0.5mm by adding 0.4% ethyl alcohol in water. Surface tension and viscosity of a mixture of ethyl alcohol and water are slightly decreased compared to that of pure water. Such an effect would certainly decrease the departure size of bubbles from the mouth of the air-injector. But the main reason is the Marangoni effect which greatly decreases the chance of coalescence between bubbles after they leave the air-injector. It has been shown by Chester(1991) that a coalescence process of two bubbles can be roughly divided into three stages: collision, film drainage, and rupture. In the presence of a small quantity of surface active substance like ethyl alcohol in an air-water system, local mass concentration varies along the surface of bubbles when two bubbles collide with each other. The local surface tension along the bubble surface follows changes of the local mass concentration. The tiny difference in surface tension along the bubble surface will dramatically increase the drainage film thickness between two bubbles so a rupture film thickness can hardly be reached. In this way, a coalescence process of two bubbles can not be completed and the size of bubbles remains close to the size at departure from the air-injector.

Two typical groups of experimental results at various cylinder Reynolds numbers (Re_d), various ratios of bubble diameter to cylinder diameter (d_p/D), and the same upstream mean void fraction (α_o) are shown in fig.4.2 and fig.4.3 respectively. Each of which includes a contour diagram and a 3D mesh illustration of ratios of local void fractions to the upstream mean void fraction, and a shadow photo as a rough visualization of the concerned void concentration pattern. In each group, one parameter (either Re_d or d_p/D) was varied while keeping the others constant.

4.2.2 Effects of the Cylinder Reynolds Number

In fig.4.2, cylinder diameter (D) is 11mm. In all three cases, upstream mean void fractions (α_o) are 4%, bubble sizes (d_p, which is taken by averaging 20 samples from video for each corresponding case) are about 4mm and the upstream mean water velocity (U_fo) varies from 0.46m/s to 2.27m/s.
Chapter 4

4.2.1 $Re_d=5100$ ($U_f=0.46\,m/s$)

4.2.2 $Re_d=14000$ ($U_f=1.27\,m/s$)

4.2.3 $Re_d=25000$ ($U_f=2.27\,m/s$)

Fig. 4.2 Influence of the cylinder Reynolds number ($Re_d$) on void distributions behind cylinder at the atmospheric pressure.

(In all three cases: $d/D=1/3$, $D=11\,mm$, $\alpha_0=4\%$)

(Along the test-tube, measurements start from $x=0.5D$; one projection per $0.5D$ further)

(a) Contour lines of ratio of local void fractions to the upstream mean void fraction ($\alpha/\alpha_0$);

(b) Shadow photo as a rough visualisation of void concentrations;

(c) 3D mesh diagram of ($\alpha/\alpha_0$).
Measurements of Phase Distributions around Obstacles

4.3.1 \( d_p/D = 1/2 \)

\( (D=6\text{mm}, U_0=2.33\text{m/s}) \)

4.3.2 \( d_p/D = 1/3 \)

\( (D=11\text{mm}, U_0=1.27\text{m/s}) \)

4.3.3 \( d_p/D = 1/12 \)

\( (D=6\text{mm}, U_0=2.33\text{m/s}) \)

Fig. 4.3 Influence of the bubble to cylinder size ratio \( (d_p/D) \) on void distributions behind cylinder at the atmospheric pressure.

(In all three cases: \( Re_d=14000, \alpha_0=4\% \))

(Along the test-tube, measurements start from \( x=0.5D \); one projection per 0.5D further)

(a) Contour lines of ratio of local void fractions to the upstream mean void fraction \( \alpha/\alpha_0 \);

(b) Shadow photo as a rough visualization of void concentrations;

(c) 3D mesh diagram of \( \alpha/\alpha_0 \).
The three cases show rather similar void distribution patterns. A local void fraction region, in which local void fractions are about three times the value in the upstream main flow, appears in the wake region at a distance of 1 to 1.5 diameter downstream of the cylinder. Transient flow patterns are clearly visible on video films. Periodic vortex streets were observed most distinctly in the second case ($Re_d=14000$). Bubble size downstream of the cylinder is a bit smaller than that upstream.

The void fraction distribution patterns are shown to be closely related to the vortex shedding patterns of single-phase flows past a cylinder at the same $Re_d$ range. In a single-phase flow past a cylinder, cylinder Reynolds number ($Re_d=U_0D/\nu$) is the characteristic dimensionless number for classification of the flow pattern around the cylinder. A wide range of $Re_d$ (from 300 to 300000) is defined as the subcritical range, in which a vortex street changes gradually from a linear von Karman vortex street to a nonlinear highly turbulent vortex street as $Re_d$ increases. The separation angle lies at 80 degree to 85 degree from the front stagnation point on the cylinder, and vortices produced in the separation region on the surface of a cylinder are shed away downstream almost at a constant frequency. To a certain extent, bubble concentration patterns visualize time-average vortex shedding patterns as shown clearly on the shadow photos in fig.4.2.2(b). The similarity of the void fraction distribution patterns indicates obviously that void fraction distributions downstream of a cylinder are mainly determined by vortex shedding patterns.

But with existence of bubbles, the vortex shedding process is suppressed through the intensive momentum transport between bubbles and vortices. As the upstream mean water velocity becomes smaller, this suppression effect becomes more dominant. Bubbles entrapped in vortices dissipate a large percentage of the energy of the vortex street and thus the two rows of a vortex street do not appear as in single-phase flows at such a low $Re_d$. Similarly void distribution patterns, which have single high concentration core regions behind the cylinder, are shown in all three cases, covering a rather wide $Re_d$ range.

A slight change of bubble size upstream and downstream the cylinder can be explained due to large velocity gradients in vortex core where bubbles are more likely to be broken up into smaller ones than to coalesce with each other (Hulin[1986]). A more uniformly distributed bubbly flow thus obtained downstream of the cylinder.
4.2.3 Effects of the Bubble to Cylinder Size Ratio

In fig. 4.3, all three cases have the same upstream mean void fraction ($\alpha_0$) of 4% and the same cylinder Reynolds number ($Re_d$) of 14000. The cylinder diameter ($D$), the mean upstream liquid velocity ($U_{fo}$), and the bubble-to-cylinder diameter ratio ($d_p/D$) are respectively 6mm & 2.33m/s 1/2, 11mm & 1.27m/s 1/3, and 6mm & 2.33m/s 1/12.

The bubble to cylinder size ratio is seen to have large influence on void fraction distribution patterns around a cylinder. In the wake region behind the cylinder, the peak value of local void fraction is much lower for a flow with small size bubbles ($\alpha/\alpha_0=1.7$ at $d_p/D=1/12$, in fig. 4.3.3) than that for a flow with big size bubbles ($\alpha/\alpha_0=3.0$ at $d_p/D=1/3$, in fig. 4.3.1). As $d_p/D$ decreases, the high void fraction region in the wake becomes smaller but the pair of vortex streets show up more separate and distinct as high void region; see fig.4.3.

In the theoretical studies presented in chapter 2, bubble trajectory computations have shown an increasing difference between liquid streamlines and bubble trajectories as bubble size increases. As a result, large bubbles tend to move closer to the wake border (i.e. a free streamline started from the separation point) in the potential flow region than small ones and thus are more likely to be drawn by large eddies into the wake region. On the other hand large bubbles, once in the wake, may be less influenced and dispersed by the turbulence or vortices in the wake. Both effects presumably contribute to the high peak void fraction in the wake found for large bubbles.

The effects of the bubble-to-cylinder size ratio on void fraction distribution patterns are even more striking in the two cases shown in fig.4.4 (Photos in X and Z direction for the first case are shown in fig.4.5). In both cases, upstream mean void fraction, cylinder diameter, and mean upstream velocity are the same ($\alpha_0=4\%$, $D=2$mm, and $U_{fo}=1.5m/s$; so $Re_d=3000$) but bubble diameters are respectively 4mm and 0.5mm. Thus $d_p/D$ is respectively 2/1 and 1/4.
Fig. 4.4 Void distributions behind small cylinder (in flows with large/small bubbles).
(In both cases: $D=2\text{mm}$, $\alpha_0=4\%$, $Re_d=3000$; Vertical direction, one projection per $0.5D$)
(a) Contour lines of ratio of local void fraction to the upstream mean void fraction ($\alpha/\alpha_0$);
(b) 3D mesh diagram of ($\alpha/\alpha_0$).

Fig. 4.5 Shadow photos in X and Z directions (of the first case, fig. 4.4.1).
Fig. 4.4 Void distributions behind small cylinder (in flows with large/small bubbles).
(In both cases: D=2mm, α₀=4%, Reₐ=3000; Vertical direction, one projection per 0.5D)
(a) Contour lines of ratio of local void fractions to the upstream mean void fraction (α/α₀);
(b) 3D mesh diagram of (α/α₀).

4.4.1 d₀/D=2/1 (d₀=4mm) 4.4.2 d₀/D=1/4 (d₀=0.5mm)

Fig. 4.5 Shadow photos in X and Z directions (of the first case, fig.4.4.1).
In the first case (fig.4.4.1, where \( d_p/D = 2/1 \)), an air cavity is clearly observed behind the cylinder (fig.4.5). Large (4mm) bubbles apparently intrude easily into the wake region behind the tiny (2mm) cylinder. Presumably the vortices, which are commonly assumed to have the same diameter as the cylinder, are not capable of dispersing the large bubble downstream. As more air (or air bubbles) are entrapped behind the cylinder, the local void fraction increases up to a certain critical value. Subsequently bubbles coalesce and form a cavity. Again presumably the interface between cavity and liquid coincide with the free streamline of the potential flow which detaches from the separation points (see fig.4.5).

In the second case with 0.5mm bubbles (fig.4.4.2, where \( d_p/D = 1/4 \)), no cavity is observed. The small bubbles almost follow the streamlines of the potential flow and move away somewhat from the wake; see e.g. fig.2.3A. Hence peak local void fractions in the wake are low. In the wake region bubbles apparently are taken downstream and no cavity is formed.

**4.3 MEASUREMENTS AT HIGH PRESSURES**

**4.3.1 Special Features at High Pressures**

Measurements in the high-pressure loop are the main part of our experimental investigation. At high pressures, vapour-water bubbly flows past a cylinder have a number of special features:

1. The water kinematic viscosity decreases sharply as the system pressure increases (e.g. from \( \nu_1 = 1 \times 10^{-4} \text{m}^2/\text{s} \) at \( P = 1\text{bar} \) to \( \nu_1 = 0.12 \times 10^{-4} \text{m}^2/\text{s} \) at \( P = 70\text{bar} \), Kaye[1973]). Accordingly, a large \( Re_d \) condition may be obtained by installing a cylinder of small size. The influence of the tube wall on the wake is thus reduced.

2. The vapour bubble size also decreases as the system pressure increases (e.g. from \( d = 1.61\text{mm} \) at \( P = 1\text{bar} \) to \( d = 0.19\text{mm} \) at \( P = 70\text{bar} \), Hsu[1976]). Small size bubbles are certainly more likely to be of a spherical shape than big ones. Such a feature is important for the validation of the theoretical modeling in which bubbles are generally assumed to be spherical.

3. For long vertical tubes, the relative pressure change due to the hydraulic head is greatly reduced at a large reference system pressure (e.g. in a 10m long vertical
tube, the ratio of the bottom pressure to the top pressure is 2/1 at \( P=1 \) bar while \( 71/70 \) at \( P=70 \) bar. As a result, two-phase flows are more stable in a high-pressure vertical tube than in an atmospheric-pressure vertical tube.

(4) The water in the high-pressure loop is carefully conditioned by deaerating and demineralizing. Salts and minerals are chemically extracted. For such a purified system, the interfacial mobility can significantly delay both the formation of an attached eddy and the wake shedding from moving bubbles (Clift[1978]). Not only the shape of the bubble is thus further stabilized but also the upstream condition is closer to the condition (i.e. bubbles in a liquid potential flow) as assumed in our modeling.

All above features are of strong interest in the vapour-water bubbly flows at high pressures that are frequently encountered in high performance industrial equipment. Most of them also favour comparison with theoretical predictions which will be discussed in next chapter.

### 4.3.2 High-pressure Loop

A diagram of the high-pressure loop is shown in fig.4.6. The loop encompasses a preheater, a test-section, a steam drum, a condenser, a subcooler, and a pump. The water, which is deaerated and demineralized before operations, is circulated in the loop by the pump. In the preheater, which includes a variable transducer unit with a maximum heating power of 500kW, a percentage of water is evaporated into vapour to produce a vapour-liquid two-phase mixture flowing upward to the test-section as mentioned earlier. The test-section is a stainless steel tube with the length of 8.23m, the internal diameter of 39mm, the wall thickness of 5.5mm, and a thermal insulation layer enclosing the tube. It may be used to act as an evaporator-tube by supplying current heating power directly to the test-section through a dual transformer/rectifier unit. During our experiments, only a small quantity of heating power (around 3kW) was supplied to the test-section to compensate exterior heat loss to the ambient air, so a saturated vapour-water flow condition is maintained in the test-section. In the steam drum, the vapour and the water are separated. The vapour is then condensed in the condenser and is fed back to the preheater together with the water from the steam drum. If necessary, the subcooler can also be put into action to supply subcooled water to the preheater. The subcooler was not active during test runs described in this chapter. The loop may be operated at a maximum pressure of 250 bar.
There are about 80 connection points on the test-section. Universal mounting units have been designed so that various kinds of sensors may be interchanged and take measurements at different locations along the test-section. Main system parameters, such as pressure, temperature, and water mass flow rate, are regulated and monitored with a number of pressure gauges, thermocouples, and orifice mass flow meters in the related components during operations.
The cylindrical obstacle is mounted in one of the universal mounting units in the test-section at a location of 5m above the preheater. It consists of a 2mm stainless steel cylindrical obstacle and a pair of supporting rods (see fig.4.7). These rods lift the obstacle over the mounting unit in order to avoid the influence of the massive mounting unit on void fraction distribution measurements by the gamma-ray densitometer system.

![Diagram of the cylinder installation on the test-section.](image)

By scanning the test-section at various locations downstream of the cylinder, the gamma-ray densitometer system provides a two-dimensional pseudo-image of void fraction distributions behind the cylinder. Through a specially designed window located about 2m below the cylinder, the new photographic system was utilized to take photos providing information on bubbles upstream of the cylinder. In addition, four local void probes (LVP, Van der Geld[1985]) were employed at about 15D upstream of the cylinder to measure local void fractions for each experiment. The average of the four values measured is taken as the upstream mean void fraction ($\bar{\alpha}_0$).

In the next section, first information on vapour bubbles upstream of the cylinder is shown as obtained with the photographic system at two system pressures (P=30bar and P=70bar). Measurements of void fraction distributions in the wake region behind the cylinder are subsequently then studied in a similar way as at atmospheric pressure. Two typical groups of experimental results are shown and discussed respectively with varying upstream water velocity ($U_{ro}$) and with different upstream mean void fraction ($\bar{\alpha}_0$).
4.3.3 Vapour Bubble Size at High Pressures

There is not much information (neither experimental nor theoretical) on vapour bubbles under high pressure conditions in a forced convection evaporator loop. Available data mostly concerns pool boiling (up to 10 bar) or the departure of bubbles from a heated plate (see van Straalen[1979] and Hsu[1976]).

In pool boiling, it has been established that the bubble departure is relatively independent of heat flux but is strongly dependent on the size of cavity and the system pressure. As the system pressure increases, the bubble departure size decreases. For a smooth heating plate, an empirical correlation given by Hsu[1976] is written as:

\[ d_p = 1.614 \left( \frac{1}{\sqrt{P}} \right) \]  

(4.1)

where \( d_p \) denotes the bubble departure size [mm] and \( P \) the system pressure [bar]. In graphic form, the relation between \( d_d \) and \( P \) is shown in fig.4.8.

![Fig.4.8 The bubble departure diameter versus the system pressure.](image)

For a saturated forced convection boiling flow, the shape of a bubble hardly changes after departure from the heating surface. This is now illustrated by two typical photographic results, taken respectively at 30bar and 70bar and shown in fig.4.9.
According to eq. (4.1), $d_p=0.29\text{mm}$ at $P=30\text{bar}$ and $d_p=0.19\text{mm}$ at $P=70\text{bar}$. Results from photographs in fig. 4.9 are rather close to the calculations. To a certain extent, this validates our assumptions that the size of a fully developed bubble differs not much from the bubble departure size in a forced convection boiling at high pressures.

### 4.3.4 Effects of the Cylinder Reynolds Number

Three typical experimental results, each including a contour diagram and a 3D mesh illustration of ratios of local void fractions to the upstream mean void fraction, are shown in fig. 4.10. Experiments were performed under a system pressure of 70bar (water kinematic viscosity ($\nu$) is $0.127\times10^{-6}\text{m}^2/\text{s}$). The cylinder diameter is 2mm and the bubble size was found from photos to be approximately 0.2mm. In all three cases, the upstream mean void fraction ($\alpha_o$) is about 10%; a rather high value was required for maintaining a stable vapour–water flow over a long period (typical 2 hours) in the test tube. The upstream mean water velocity ($U_{f0}$) was varied to be 0.85m/s, 1.5m/s, and 2.9m/s, so flow conditions at three various cylinder Reynolds numbers ($Re_d = 13400, 24400, \text{and} 45700$) were obtained.
Measurements of Phase Distributions around Obstacles

4.10.1 $\text{Re}_d=13400$ ($U_{fo}=0.85\text{m/s}$)

4.10.2 $\text{Re}_d=24400$ ($U_{fo}=1.5\text{m/s}$)

4.10.3 $\text{Re}_d=45700$ ($U_{fo}=2.9\text{m/s}$)

Fig. 4.10 Influence of the Reynolds number ($\text{Re}_d$) on void distributions behind the cylinder at high pressure (70bar).

(All cases: $P=70\text{bar}$, $D=2\text{mm}$, $d_0=0.3\text{mm}$, $\alpha_0=10\%$)

(Vertical direction, one projection per D)

(a) Contour lines of ratio of local void fractions to the upstream mean void fraction ($\alpha/\alpha_0$);
(b) 3D mesh diagram of ($\alpha/\alpha_0$).
Those three cases show rather similar void distribution patterns. A peak local void fraction region ($\alpha/\alpha_0 = 2.4$ to $1.8$) can clearly be observed downstream of the cylinder. As $Re_d$ increases, the center of the peak region moves downstream only slightly from $3D$ to $3.5D$ from the center of the cylinder but the length of the peak region itself elongates in the downstream direction from $0.2D$ to $1.5D$. Further downstream of the cylinder, the central peak local void fraction region develops gradually into two high void areas. The position of the two high void rows shifts to downstream from $y=4D$ to $y=5.8D$ as $Re_d$ increases from 13400 to 45700.

The above observations can be explained in the same way as in Section 4.2 by analyzing the interaction processes between bubbles and the single-phase flow field around a cylinder. Such interaction processes occur between bubbles and the velocity field in the potential flow region and between bubbles and vortices in the wake region. In a single-phase liquid flow, the separation region shifts slightly to the rear part of the cylinder as $Re_d$ increases. The wake behind the cylinder becomes narrower and more turbulent. Consequently bubbles, which are small at the high pressure and therefore are more likely to move along the wake border (a free streamline starting from the separation point) once they enter the separation region, move closer and faster to the flow axis downstream of the cylinder. In the wake, the suppression effect of bubbles on the turbulence (or the vortices) becomes weaker (compared with large bubbles). The effect of the vortices (and dispersion to the downstream) on bubbles becomes stronger; the measurements in fig.4.10 have a rather small bubble to cylinder size ratio ($d_p/D = 1/10$). Accordingly the peak local void fraction (or the "holdup" of bubbles) decreases and is elongated to further downstream of the cylinder as $Re_d$ increases. Two high void areas are thus distinctly reflecting the pattern of two rows of vortex streets in the corresponding $Re_d$ range of single-phase flows.

4.3.5 Effects of the Upstream Mean Void Fraction

A group of experiments were performed at system pressure of 30bar ($\nu_1=0.14\times10^{-6} m^2/s$) to illustrate the effect of upstream mean void fraction ($\alpha_0$) on void distributions around a cylinder ($D=2mm$). With a flow of $Re_d=25700$ ($U_{fo}=1.8m/s$), measurements of void fraction distributions were taken at two vertical positions ($Y=1.5D$ and $Y=2.5D$ from the center of the cylinder) with four values of $\alpha_0$ (4%, 8%, 12%, and 20%). The upstream mean void fraction ($\alpha_0$) is varied by changing the heat input to the preheater. Two sets of measurement results are shown respectively in fig.4.11.1 and fig.4.11.2.
Obviously void fraction distribution patterns do not vary much as $\alpha_0$ changes. Close to the cylinder ($Y=1.5D$), void fraction distributions at different $\alpha_0$ show almost the same pattern. At a distance further downstream the cylinder ($Y=2.5D$), which is actually already very close to the region where a peak void fraction appears as shown in fig.4.10, void fraction distribution patterns at various $\alpha_0$ differ slightly from each other. Although two sets of measurements only cover a limited range of the wake region, results still illustrate evidently the same trend as observed by Inoue(1986): the upstream mean void fraction hardly affects the void fraction distribution pattern behind a cylinder. More precisely, the level of the void fraction increases with the upstream value but the spatial differences remain the same.
In both experiments by Inoue (1986) and by us, a rather small ratio of bubble diameter to cylinder diameter \((d_p/D)\) was presented (1/10 and 1/7, respectively). When the size of bubbles is much smaller than the dimension of vortices produced in the separation region on the surface of the cylinder, void fraction distribution patterns in the wake should more or less follow the vortex shedding patterns of the water flow. The curves in fig 4.11 suggest that under these circumstance the bubbles first are concentrated in the potential flow region outside the wake border (the free streamline starting from the separation point, see also fig.2.3A and fig.2.3B) and subsequently diffuse into the wake (due to the entrainment process and vortices/turbulence, see next chapter).

### 4.4 CONCLUDING REMARKS

An experimental investigation on phase distributions behind obstacles in vertical bubbly flows has been carried out rather extensively on two test loops (the atmospheric-pressure loop and the high-pressure loop) and is reported in this chapter.

The photographic systems, both an ordinary camera for application on the atmospheric-pressure loop and our new photographic system for application on the high-pressure loop, were utilized to provide information on bubbles in the upstream main flow as well as a means to roughly visualize void distribution patterns. In particular, photos of bubbles under high pressure condition are believed to be obtained for the first time. The mean diameter of vapour bubbles in a vertical saturated water flow is very small at high pressure \((d_p=0.2\text{mm at } P=70\text{bar})\) and is close to the departure bubble diameter calculated by the empirical relation \((Hsu[1976])\).

Main results are two-dimensional pseudo-images of void fraction distributions in the wake behind cylinders, obtained by using the (non-intrusive) gamma-ray densitometer system. Directly from measurements, effects of three characteristic parameters — the cylinder Reynolds number \((Re_d)\), the ratio of bubble-to-cylinder size \((d_p/D)\), and the upstream mean void fraction \((\alpha_0)\) — on void fraction distribution patterns around a cylinder are illustrated and analyzed. Main observations are:

1. Void fraction distribution patterns to a large extent are similar to vortex street patterns of corresponding single-phase flows under the same \(Re_d\). A high void fraction region, in which local void fractions are as big as 2 to 3 times the upstream mean value \((\alpha_0)\), is observed to lie in the wake behind the cylinder.
(2) As $Re_d$ increases, the high void region tends to elongate itself to the downstream direction and the peak value of local void fractions decreases slightly, probably due to increased intensity of turbulence in the wake.

(3) Among the three characteristic parameters, $dp/D$ is found to be the most important parameter to determine void fraction distribution patterns of a bubbly flow past a cylinder. As $dp/D$ decreases, the distance from the center of the high void region to the center of the cylinder increases from 1D to 1.5D under a relatively large $dp/D$ ratio (=1/3) to 3D to 3.5D under a relatively small $dp/D$ ratio (=1/10). Two high void areas, which somewhat reflect the pattern of the vortex streets of a single-phase flow past a cylinder in the concerned $Re_d$ range, are distinctly observed under small $dp/D$ ratio (=1/10).

(4) The decisive effect of $dp/D$ on void fraction distribution patterns behind a cylinder is shown even more evidently by experiments in which the size of the bubbles is bigger than the diameter of the cylinder. Bubbles are trapped in the wake region and form a cavity behind the cylinder even at a very low velocity of water flow.

(5) Void fraction distribution pattern does not vary much as $\alpha_0$ changes. This results are in accordance with measurements by Inoue(1986).

In particular, compared with the relevant group of measurements under the atmospheric pressure condition in fig.4.2, high pressure measurements show differences on two points, mainly because of the small ratio of bubble size to cylinder size ($dp/D=1/10$):

(a) In fig.4.2, the peak void fraction regions lie at a distance of 1D to 1.5D from the center of the cylinder, which is much smaller than that of 3D to 3.5D in fig.4.10. In the wake, small bubbles are affected (and dispersed) by turbulence (or vortices) much stronger than big ones. As a result, the peak void fraction region for small size bubbles not only is farther away from the cylinder but also has a lower $\alpha/\alpha_0$ value than that for big size ones at the same $Re_d$.

(b) In fig.4.10, the elongation of the peak void region is observed as $Re_d$ increases. Such a process is not so readily discerned in fig.4.2 since it involves large bubbles.
With a much bigger $d_p/D$ ratio ($=1/3$) in fig.4.2, the suppression effect of bubbles on turbulence in the wake is much stronger than which with the small $d_p/D$ ratio ($=1/10$) in fig.4.10. Mixing effect (or cross flow-axis effect) is to a large extent suppressed by the presence of big bubbles. Accordingly, the resulting elongation effect becomes certainly less visible in fig.4.2 than in fig.4.10.

It is noted that measurements of void fractions in the range from $x=-D/2+1\text{mm}$ to $x=D/2+1\text{mm}$ are of low accuracy because of the interference of the metal obstacle with the gamma beam (with a diameter of 2mm), as shown with the calibration tests. This has a rather serious effect on measurements with small obstacle ($D=2\text{mm}$ in the high pressure experiments). Measurements at high pressure are therefore only valid beyond $x=D$ (where lies the central line of the gamma beam) downstream of the cylinder, while measurements at atmospheric pressure are valid for the region closer to the cylinder (e.g. $x=2/3D$ for 6mm cylinder). However, considering the region where the local peak void fraction lies at both conditions ($1D-1.5D$ for atmospheric pressure measurements and $3D-3.5D$ for high pressure measurements), the obtained conclusions on the pattern of void distributions behind the cylinder are not affected.

Further to the theoretical results of the interaction process between bubbles and velocity field in the potential flow region (discussed in chapter 2) and between bubbles and vortices/turbulence in the wake region (preliminary analysis, more details are presented in next chapter), an attempt has been made to interpret measurements of void distribution around cylinder under various conditions.
CHAPTER FIVE

PREDICTIONS OF PHASE DISTRIBUTIONS AROUND OBSTACLES

5.1 INTRODUCTION

Both our own experimental investigations presented in chapter 4 and measurements in literature (e.g. Hulin[1982], Inoue[1986]) have shown high concentrations of bubbles in the wake region of a cylinder. It is obvious that bubbles are drawn into the wake by the action of some momentum transport processes between bubbles and the liquid flow field around the cylinder.

As compared with the few experimental investigations mentioned above, even less studies on the prediction of phase distributions of a bubbly flow past a cylinder were found. This is mainly due to a lack of knowledge of the highly turbulent single-phase liquid flow past a cylinder and of the complicated interaction process between a bubble and the liquid field and other bubbles. Most theoretical studies have an explanatory nature based on empirical data (e.g. Hulin[1982], Inoue[1986]). Some are limited to highly idealized flow situations such as bubble entrapment by a line vortex (Thomas[1983]). Even for numerical approaches, a description of a single-phase flow past a cylinder is still a quite challenging subject at high $Re_d$, where a very dense computational grid is needed to resolve the details of small-scale turbulence and thin boundary layers (Song[1990]). Predictions of two-phase flows past a cylinder are even more difficult and require higher computer capacities.

For a better understanding of the phase distribution mechanism of a dispersed two-phase flow past a cylinder, we shall extend our theoretical analysis of chapter 2 to the wake region. This will facilitate interpretation of the new experimental data of chapter 4. Bubble entrainment processes are studied and phase distributions in a bubbly flow past a cylinder are predicted under the experimental conditions of chapter 4.
In section 5.2, we first complete the description of the (time-average) velocity field of a liquid flow past a cylinder. As discussed in chapter 1, the flow field around a cylinder may conveniently be divided into a potential flow region and a wake region if the Reynolds number lies in the subcritical range \( (3 \times 10^2 < \text{Re}_D < 3 \times 10^5) \), which covers our experiment investigations. In addition to the description of the potential flow region in appendix 4, the velocity field in the wake region is formulated in a semi-empirical way, basing the turbulent characteristics on available empirical data. It is important to note here that the approximate semi-empirical approach is sufficient since void distributions do not depend on details of the flow field.

In section 5.3, we apply a very simplified scheme of turbulence to introduce an additional turbulent action on bubbles in the (close) wake. We follow bubble trajectories from the potential flow region to the wake region to investigate the magnitude of interaction forces and the influence of main system parameters on phase distributions in the wake. This approach provides a semi-quantitative explanation and a better understanding of bubble entrainment mechanisms in the wake.

In section 5.4, the predictions from section 5.3 are compared with the experimental results presented in chapter 4.

5.2 DESCRIPTION OF THE SINGLE-PHASE WAKE OF AN OBSTACLE

5.2.1 Identification of Flow Regions and Regimes

For a single-phase flow past a cylinder in the subcritical Reynolds number range \( (\text{Re}_D = 3 \times 10^2 - 3 \times 10^5) \), flow separation has a laminar nature but the wake is turbulent. The most striking flow phenomenon is the periodic shedding of vortices and the subsequent formation of two vortex streets downstream of the cylinder. The Strouhal number, \( \text{St} \), that represents the frequency of vortex shedding \( (\text{St}=f_sD/\text{U}_0, \) where \( f_s \) is the frequency of vortex shedding), remains nearly constant at a value of 0.2 within this subcritical Reynolds number range. So remains the coefficient of drag on the cylinder \( (C_{\text{drag}} \) approximately equals 1.2).

Conventionally, three wake regions — close-, near- and far-wake — are distinguished based upon downstream distance from the cylinder. The close-wake extends up to 10D of the cylinder; the near-wake extends further downstream to a distance of some 50D, and the far-wake further downstream.
In the far—wake region, the pressure is not much different from that in the free stream. Without "direct" effects of the cylinder on the flow, the far wake satisfies the self—preserving condition (i.e. the similarity of structure of turbulence and kinematics in the free turbulent region). Together with another special feature of the wake flow, namely it being relatively narrow in directions traverse to the main—flow direction, this self—preserving condition forms a basis for the application of several classical and phenomenological theories, such as boundary—layer approximation theory, Prandtl's mixing—length theory, and so on. The equations of motion (i.e. the Navier—stokes equations) can be reduced to a set of ordinary differential equations and solved to obtain velocities laterally distributed according to a Gaussian profile (Hinze[1975]).

As compared with the far—wake region, the near—wake and close—wake region is much more difficult to describe because of very complicated phenomena such as formation and breakdown of the vortex streets and transition from laminar to turbulent flow. Our focus is on the close—wake region which is the region where bubbles tend to concentrate and which our experimental results (given in chapter 4) have covered. A simple model is presented here, based mainly on experimental observations and theoretical analyses in literature (Gerrard[1966], Papailiou[1974], Perry[1982], and Kourta[1987]). Extended from the well—known Gaussian profile for describing mean local velocities in the far—wake region, the model yields velocity profiles in the close—wake region. These velocity profiles are the starting points for studying momentum transport processes in a dispersed two—phase flow past a cylinder in later sections.

Figure 5.1 Identification of flow regimes in the close—wake region after Kouta(1987).

L₁, appearance of instability in the shear layer; L₂, length of the dead fluid zone;
L₃, starting of the wake vortex; L₄, final length of the wake formation region.
We first give a brief description of single-phase flow characteristics in such a region. A clear identification of flow regimes was given by Kourta(1987) from experimental results covering a rather wide Reynolds number range (2x10^3 < Re_d < 6x10^4). His results are shown schematically in fig.5.1 and are discussed for two (lower or higher) Reynolds number sub-ranges.

In the lower Reynolds number range (2x10^3 < Re_d ≤ 1.6x10^5), vortices are not formed directly at the rear part of the cylinder. Vortices produced from the separating boundary layer on the cylinder surface are shed downstream along a shear layer (or a mixing layer) and become unstable at L_1, thus starting formation of a vortex street. Small-scale vortices control the mixing-layer growth and are carried by the larger regular vortices of the main vortex shedding, giving rise to the complex interaction mechanism of transition. A dead-fluid zone, in which local velocities are nearly zero, is observed behind the cylinder. Both the locations of the end of the dead-fluid zone (L_2) and the appearance of instability (L_1) move upstream as the Reynolds number increases. The length of the formation region (L_4) is taken to be determined by the position of the maximum intensity of the fluctuation on the central line of the wake (and of U_w(x,0) equals zero). This length is found to decrease monotonically as Re_d increases (see appendix 13).

In the higher Reynolds number range (1.6x10^5 < Re_d < 6x10^4), flows are intermittent and turbulent. The dead-fluid zone no longer exists and vortices are formed directly at the back of the cylinder. The characteristic lengths are not clearly distinct any more.

Two characteristic lengths (the dead fluid zone length L_2 and the wake formation length L_4) are utilized for formulating our model to describe the velocity distribution in the close-wake region.

5.2.2 Formulation of Velocity Distributions in the Close-wake

Although very little quantitative information is found on mean velocity distribution in the close-wake region, it is well known that U-profiles (U: x-component of local velocity) have negative components in the wake formation region, corresponding to reverse circulations, and then develop gradually into a Gaussian velocity profile in the near- and far-wake region. Even in the region very close to the cylinder (x=4D), the
U-profile still resembles the Gaussian profile (Papailiou[1974]). Accordingly we assume a Gaussian profile to approximately describe x-components of mean local velocities in the close-wake by adding a constant (negative) velocity component to the Gaussian-form velocity profile of a turbulent wake. The profile is defined by eq.(5.1) and shown schematically in fig.5.2.

\[
U_w(x,y) = U_o \left\{ 1 - C_1[x] \sqrt{D/x} \exp\left( -\frac{U_o Y^2}{4 \varepsilon_w x} \right) \right\} + C_2 [x] 
\]

where \(U_w(x,y)\) has a negative value when it represents the reversed flow part; \(x\) and \(y\) are coordinates with regard to the origin at the center of the cylinder; \(U_o\) is the upstream velocity (or undisturbed velocity); \(\varepsilon_w\) is the eddy viscosity for a turbulent wake flow given by experimental data. Part <1> is the velocity profile (Gaussian form function) for describing a turbulent far-wake (Hinze[1975]) and part <2> is the added component with a negative value. \(C_1[x]\) and \(C_2[x]\) are both functions of \(x\). To derive these two unknown coefficients at each \(x\), we need two additional relations.

One relation is the continuity condition of local velocity in \(x\)-direction at the wake border which is obtained from the potential flow solution. The value of \(U_p(x,y_{cl})\) computed by the eq.(5.1) should be equal to that obtained by the potential flow solution \(U_p(x,y_{cl})\) at the same point on the wake border. Accordingly, we have the following relation,

\[
U_o \left\{ 1 - C_1(x) \sqrt{D/x} \exp\left( -\frac{U_o Y^2 e_{cl}}{4 \varepsilon_w x} \right) \right\} + C_2[x] = U_p(x,y_{cl}) 
\]

Another relation is obtained from the balance of momentum in an imaginary control plane as shown in fig.5.3 and eq.(5.3).
where the potential flow region and the wake region are separated by the line $S_1C_1E_1$ which is the wake border defined by the potential flow solution; Coordinates $x_a, y_a, y_b$ are chosen to be large ($x_a=-6D, y_a=y_b=4D$ in our computations) so the velocities at $A_1$ and $B_1$ are close to the undisturbed velocity ($U_0$); Turbulent fluctuations are accounted for in eq.(5.8) below.

The meaning of the various terms in the equation is as follows.

The first term and third term on the left-hand side are respectively the momentum flux across the plane $A_1A_2$ and two planes $B_1C_1B_2C_2$. The first term on the right-hand side is the momentum flux across two planes $A_1B_1$ and $A_2B_2$. The velocity components in the above three terms [$U_1(x,y), U_2(x,y), V_2(x,y)$ and $U_3(x,y)$] are given by the potential flow solution. Since the velocity components $U(x,y)$s are all positive in the potential flow region, the sign "$| |$" for a absolute value can be abandoned in the three terms (i.e., $|U(x,y)|=U(x,y)$) in the equation.
The second term on the right-hand side is the drag force on the cylinder. It is defined by the following equation

$$F_{\text{drag}} = 0.5 C_{\text{drag}} \rho U_o^2$$  \hspace{1cm} (5.4)$$

where the drag coefficient $C_{\text{drag}}$ is obtained directly from empirical data and is approximately a constant ($C_{\text{drag}} = 1.2$) in the entire subcritical Reynolds number range.

The great advantage of the momentum balance (eq. (5.3)) is the fact that the (steady) drag coefficient $C_{\text{drag}}$ is accurately known from the empirical data. This justifies to some extent the entire approach.

The last term on the right-hand side in eq. (5.3) is due to the pressure difference at two sides of the control plane in x-direction ($A_1A_2$ and $B_1B_2$). In the potential flow region, local pressure $P(x,y)$ can be easily derived from the Bernoulli’s equation,

$$P(x,y) + 0.5 \rho U_o^2 = P_0 + 0.5 \rho U_o^2$$  \hspace{1cm} (5.5)$$

where $P_0$ and $U_0$ are the pressure and velocity in the free stream (or the undisturbed flow); $U(x,y)$ and $V(x,y)$ are given from the potential solution, which is adopted since the Reynolds number is high (typically $10^4$) in our cases. In the wake region, however, local pressures are much more difficult to define. For simplicity, pressure distributions are assumed based on experimental facts as much as possible. It is noted that although the pressure field is connected to the velocity field via the Navier-Stokes equation, the uncertainty in the equation (e.g. the turbulent viscosity) is so high that this route would not lead to a pressure field with a higher accuracy. It is well known that local pressure has a constant value over the rear half surface of the cylinder (the base pressure $P_b$, which is a somewhat lower than the free-stream pressure) and recovers gradually from the base value to the free-stream value some distance downstream of the cylinder. Beyond the wake formation region (i.e. $x \geq L_4$), the pressure may be expected to be uniform over the wake. Based on such facts and the definition of the characteristic lengths ($L_2$ and $L_4$), a profile of local pressure along the x-axis is proposed as follows:

$$P(x,0) = P_b + (x-R)[P_p(L_4,y_{cl})-P_b]/(L_4-R), \quad \text{when } R < x \leq L_4;$$

$$P(x,0) = P_p(x,y_{cl}), \quad \text{when } x > L_4; \hspace{1cm} (5.6)$$

where $P_p(x,y_{cl})$ is the pressure on the wake border (the free streamline starting from the separation point in the potential flow region) and is provided from eq.(5.5). Further at each x position, the local pressure is proposed to increases from $P(x,0)$ to $P_p(x,y_{cl})$ linearly, following the relation $P(x,y) = P(x,0) + [P_p(x,y_{cl})-P(x,0)]y/y_{cl}$.\n
The second term on the left-hand side of eq.(5.3) is the momentum flux across the plane \( C_1 C_2 \) in the wake region. With the existence of high-level turbulence in the flow in the close-wake region, effects of velocity fluctuation components in local velocity \( (U_w = U_w + U_w') \) must be considered in this term. More precisely, it should be written in the following form:

\[
\int_{y_c1}^{y_c2} \rho t U_w(x_c,y) |U_w(x_c,y)| dy = \int_{y_c2}^{y_c1} \rho t [U_w(x_c,y)+U_w'(x_c,y)] [U_w(x_c,y)+U_w'(x_c,y)] dy
\]

(5.7)

With little quantitative information on the turbulent term \( (U_w') \), which is expected to be quite big in the close-wake region, it is very difficult to quantify the \( |U_w(x_c,y)+U_w'(x_c,y)| \) term and thus to derive a solution from eq.(5.3). For an easier mathematical treatment, we introduce a factor \( \varepsilon(x) \) and rewrite eq.(5.7) as follows:

\[
\int_{y_c1}^{y_c2} \rho t [U_w(x_c,y)+U_w'(x_c,y)] [U_w(x_c,y)+U_w'(x_c,y)] dy = \int_{y_c2}^{y_c1} \rho t U_w(x_c,y) |U_w(x_c,y)| dy + \varepsilon(x) \int_{y_c2}^{y_c1} \rho t U_0^2 dy
\]

(5.8)

where the factor \( \varepsilon(x) \) may be considered as some kind of relative intensity of turbulence, which is conventionally defined by \( (U_w')^2/U_0^2 \). The difference between the conventional definition and the factor \( \varepsilon(x) \) is that the sign of \( U + U' \) is sometimes different from the sign of \( U \) (e.g. for \( y > y_{co} \) in the close-wake region). It is noted that other definitions might be adopted to accommodate for the turbulent fluctuations without affecting results.

The qualitative identification of flow regimes for the close-wake region early in this section has indicated clearly that the turbulent intensity and consequently \( \varepsilon(x) \) have a maximum at \( x = L_4 \), where the wake formation process is completed according to the measurements by Kouta(1987). Point F (the end of the wake formation zone) is actually a transient point where \( U_w(x,0) \) equals zero. Accordingly, we have eq.(5.9)

\[
U_0 \left\{ 1 - C_1(L_4) \sqrt{D/L_4} \exp\left( - \frac{U_0 y_{co} (L_4)^2}{4 \epsilon_{w,L_4}} \right) \right\} + C_2(L_4) = 0
\]

(5.9)
There are three unknown coefficients \( C_1(L_4), C_2(L_4), \) and \( \varepsilon(L_4) \) in a set of three equations (eq.(5.2), eq.(5.3) and eq.(5.9)). In order to satisfy all these requirements and not to introduce arbitrariness, this set of equations is solved numerically. This provides the value of \( \varepsilon(L_4) \). In addition, Hinze(1975) provides the value of \( \varepsilon(50D) \) in the far–wake region: approximately 0.05. If now the relative turbulence intensity is assumed to vary smoothly between these limits, a profile of the kind

\[
\varepsilon(x) = \begin{cases} 
\varepsilon(L_4)^2/(L_4-x_s_1)(x-x_s_1) & \text{when } x_s < x < L_4; \\
\varepsilon(L_4) + (\varepsilon(50D) - \varepsilon(L_4))(x-L_4)/(50D-L_4) & \text{when } x > L_4.
\end{cases}
\]

(5.10)

is deduced. Here the (relative turbulence intensity) factor \( \varepsilon(x) \) has been assumed to increase from 0 at \( x=x_s_1 \) to \( \varepsilon(L_4) \) at \( x=L_4 \) following a parabolic function and to decrease from \( \varepsilon(L_4) \) at \( x=L_4 \) to \( \varepsilon(50D) \) at \( x=50D \) following a linear function. This assumption is based on the qualitative descriptions of turbulence in wake flows in literature (e.g. Townsend(1949), Hinze(1975), Landahl(1986)).

In the close–wake region, where the flow is non–preserving, the turbulence intensity has been described only qualitatively (Chang[1970]). The level of the turbulent intensity (or more precisely the turbulent energy) is generally high in this region. Starting from the separation point, turbulence increases as the instability of the shear layer increases and the vortex street grows. Local turbulence reaches the highest level when the vortex street is completely formed. This highest level is then gradually and slowly reduced downstream, mainly due to the dissipation and the diffusion in the wake flow. In the sequel, the functional relationship of \( \varepsilon(x) \), given in eq.(5.10), was varied to examine its influence on particle trajectories. Since the extreme values are fixed, however, it is hardly surprising that the influence appears to be very small indeed.

With a known profile of \( \varepsilon(x) \), eq.(5.3) contains only two unknown variables \([C_1(x) \text{ and } C_2(x)]\) at each \( x \) plane (\( x \geq R \) — the cylinder is required to be completely enclosed in the control plane for application of the momentum flux balance). Together with eq.(5.2), eq.(5.3) is solved to obtain two coefficients \([C_1(x) \text{ and } C_2(x)]\) of the velocity profile at each \( x \) plane in the wake region. Subsequently, \( x \)–components of local velocities \( U_w(x,y) \) in the wake region (\( x \geq R \)) are calculated from eq.(5.1).
The $y$–component of a local velocity $V_w(x,y)$ may be derived from $U_w(x,y)$ according to the continuity equation,

$$\frac{\partial U_w(x,y)}{\partial x} + \frac{\partial V_w(x,y)}{\partial y} = 0$$

(5.11)

where $V_w(x,y)$ is calculated step by step from the wake axis ($V_w(x,y)=0$ at $y=0$) to the required $y$ position at a fixed $x$ position.

In general, $V_w(x,y_{ci})$ calculated by eq.(5.11) is negative but the $V_p(x,y_{ci})$ calculated from the potential flow solution is positive on the wake border. A discontinuity of $y$–components of a local velocity appears. Such a discontinuity may be eliminated but this requires a laborious remodeling of the potential flow region and further a good description of the entrainment process across the wake border. In a first calculation (the velocity field, see fig.5.4.1), it was found that the remodeling of the potential flow region did not have much influence on the bubble trajectories; therefore it was abandoned.

Based on experimental observations, Papailiou(1974) has described the entrainment process as follows:

*The origin of the surface convolutions is the vortices of the turbulent vortex street. A vortex draws fluid from the region outside the street into the wake as it rotates, by the action of turbulent shearing forces. Part of the fluid entrained remains within the vortices, leading to their growth, while the remainder diffuses into the turbulent core by the action of the turbulence. The entrainment mechanism can be thought of as a "pumping" process which brings fluid into the wake, shown as the widening of the turbulent vortices and the core. The entrainment is very strong in the wake formation region. As the vortices move downstream, they are diffused by the action of the turbulence, and thus lose their strength, so that the rate of entrainment is slowed.*

Obviously the process so complicated that only a very schematic description is feasible at present. The entrainment or "pumping" process actually takes place in some region around the border $y_{ci}(x)$. The extent of this region is indicated in fig.5.5 (where $y_{wb}(x)$ is equal to $y_{ci}(x)$) and is defined in eq.(5.13) (more detailed discussions on fig.5.5 and eq.(5.13) are given in section 5.3.1).

The precise location of the outer boundary is in fact relatively unimportant since the entrainment process dampens out rather quickly for $|y(x)|>y_{ci}(x)$, where the $y$–component (mean) velocity profile is continuous everywhere, such that the apparent
discontinuity is smoothed out. The deviation $V_{w}(x,y)-V_{p}(x,y)$ on the wake border where $y=\gamma_{c1}$ henceforth named as "mass-in-flow" profile $V_{ml}(x,\gamma_{c1})$ gradually decreases as $y$ increases for $|y(x)|>\gamma_{c1}(x)$. This is due to the mean value of the $y$--component velocity fluctuation profile that is superposed on local liquid velocities in the wake for $\gamma_{c1}(x)<|y(x)|<\gamma_{owb}(x)$. This fluctuation profile is given by

$$
\begin{align*}
V_{ml}(x,y) &= \frac{y-\gamma_{c1}}{(\gamma_{owb}-\gamma_{c1})} V_{ml}(x,\gamma_{c1}) & \text{when } y_{c1} < |y| < \gamma_{owb} \\
V_{ml}(x,y) &= 0 & \text{when } |y| > \gamma_{owb}
\end{align*}
$$

(5.12)

The superposition of $V_{ml}(x,y)$ on local velocities for $y_{c1}<|y|<\gamma_{owb}$ affects slightly the conditions of mass continuity (eq.(5.11)) and momentum balance (eq.(5.3)). The first is accounted for by the mean value of the $x$--component velocity fluctuation profile. After adding these new components, the whole procedure described above is repeated in order to satisfy the momentum balance. Fortunately the "mass-in-flow" profile is weak enough (Values of $V_{ml}(x,y)$ are mostly much less than 10% of local $y$--component liquid velocity almost everywhere, as shown in fig.5.4.4) to guarantee very quick convergence.

5.2.3 Discussion on Implementation Points and Computation Results

Until now, the velocity field in the close--wake of a single--phase flow past a cylinder has been described in a semi--empirical way. Possibly the description does not give a very accurate quantitative description of the single--phase wake flow, but it surely provides a convenient tool to investigate the main parameters of importance to particle trajectory studies. It is noted that the only alternative with the present state of knowledge would have been a resort to numerical packages such as FLUENT (or PHOENICS) and interpolating the discrete values.

There are three implementation points that need to be further discussed:

**Point 1: Velocity field description of close--wake from $x=x_{s1}$ to $x=R$**

Application of above computation algorithm for deriving local velocities in the close--wake region has been limited to the range of $x \leq R$. For the range of the close--wake region from $x=x_{s}$ to $x=R$, the local pressure distribution changes rapidly and the friction along the surface of the cylinder must be also excluded from the drag term ($F_{drag}$) in the balance of momentum flux. To avoid this complex and unknown situation, we treat this part of the wake region independently in a simple way, presented in detail in appendix 11.
Point 2: Modification of velocity profiles close to the flow axis and the cylinder
The above velocity distribution (either only the $x$-components of local velocities when $x>R$ or the absolute values of local velocities when $x_e<x<R$) in the close-wake region has been assumed to be in the form of a Gaussian function. Such an assumption is generally in good agreement with experimental data except in the region very close to the cylinder, where a reverse flow is involved. For instance, it is well known that the mean velocity on the flow axis decreases from zero at the rear stagnation point to a maximum negative value some distance downstream, increases gradually to be positive further downstream, and finally recovers to a value close to the undisturbed velocity in the far-wake region. The Gaussian-form profile in our model correctly predicts the last two trends but fails to show the first trend. In the present model, the maximum negative value of velocity on the the axis is located at the rear stagnation point where the average pressure has the lowest value over the corresponding $x$-plane. To correct this unrealistic picture, we introduce a simple modification procedure on the reverse flow part of derived velocity profiles in the region of $x<L_2$. The entire procedure, which is described in detail in appendix 12, amounts to selecting a fit profile for the $U_w$-velocity component different from the Gaussian profile.

Point 3: Empirical coefficients introduced in the above deduction procedure
In addition to the two coefficients ($\beta_s$ and $C_p$) that we have discussed in section 2.2, three more empirical coefficients ($\epsilon_w$, $L_2$, and $L_4$) are required for the solution of the defined equations. The eddy viscosity $\epsilon_w$, which to some extent plays a part in determining the shape of the velocity profile defined by eq.(5.1), is relatively unimportant in our model because of the existence of the coefficient $C_1(x)$. We simply derive the required value corresponding to the Reynolds number by interpolating or extrapolating from the pair of available experimental data (e.g. $\epsilon_w=0.6$ when $Re_d=1080$; and $\epsilon_w=2.0$ when $Re_d=5215$; Papailiou[1974]). The length of the dead fluid zone $L_2$ decreases as the Reynolds number increases, as has been discussed earlier in this section. When $Re_d>16000$, no dead fluid zone is observed; then $L_2=R$. When $2000<Re_d<16000$, $L_2$ is again derived with respect to the Reynolds number simply by interpolating or extrapolating from a pair of experimental data ($L_2=D$ when $Re_d=4800$; and $L_2=1.3D$ when $Re_d=2400$; Kouta[1987]). The final length of the wake formation region $L_4$ has been extensively investigated by Gerrard(1966). It can be directly obtained from the corresponding empirical correlation curves with respect to the Reynolds number (Gerrard[1966], the empirical correlation is given in appendix 13).
With above additional implementation, the velocity field of a single-phase flow past a cylinder can be fully obtained by solving the given equations numerically (The self-developed program is called PDAC for predicting Phase Distribution Around Cylinder, written in TURBO PASCAL). A typical set of computational result of the velocity field of a single-phase flow past a cylinder, which is directly related to one of our experimental conditions we have discussed in chapter 4, is shown in fig.5.4.

In fig.5.4.1, local velocity vectors are shown at 10 x-locations (and 8 y-positions) from the flow axis to the surface of the cylinder to the line of y=2D over the entire flow field. A more detailed structure of local velocity profiles in the close-wake region is given in fig.5.4.2. The computed profile of the relative turbulence intensity factor $\epsilon(x)$ and the corresponding "mass-in-flow" profile $V_{\text{m1}}(x) = [V_\infty(x,y_c) - V_p(x,y_c)]$ are illustrated in fig.5.4.3 and fig.5.4.4 respectively.

Qualitatively, the computational results shown in fig.5.4 are quite close to what is usually observed in experiments and numerical simulations (e.g. Durgin[1971], Song[1990]). Some deviations of local velocities might appear in the region close to the surface of the cylinder.

In fig.5.4.3, the maximum value of the relative turbulence intensity factor $\epsilon(L_4)$ at the end of the wake formation region is about 0.57, which indicates a rather high turbulence level in the wake formation region. Such a high value may be justified by the arguments given above. In order to investigate the influence of this value, it was artificially decreased to different values (e.g. 0.57/2, 0.57/4) in a few runs. Resulting liquid velocity profiles are quite similar to each other and much as in fig.5.4.1.

In fig.5.4.4, the "mass-in-flow" velocities $V_{\text{m1}}(x)$ are all negative, as expected. The "mass-in-flow" velocity ($V_{\text{m1}}(x)$) has the largest value in the wake formation region and decrease sharply further downstream. The computational results of $\epsilon(x)$ and $V_{\text{m1}}(x)$ are in accordance with previous qualitative descriptions and discussions.

The developed model of a single-phase flow field around a cylinder serves as the basis for our further analysis of momentum transport processes between two phases when a second phase (i.e. dispersed phase — gas bubble, liquid droplet, or solid particle) is introduced into the flow. It is considered to be adequate, notwithstanding the possible difference between model and reality as regards the local velocities very close to the rear half of the cylinder. It is likely that the dispersed phase can hardly move into or out of this region, and experimentally only low bubble concentrations are found here.
Fig. 5.4.1 Calculated local velocities in the flow field around a cylinder.
(with the potential flow region corrected for "mass-in-flow" at the wake boundary)

Fig. 5.4.2 Detailed structure of calculated local velocities in the close-wake region;

Fig. 5.4.3 Profile of relative turbulence intensity factors $\epsilon(x)$;

Fig. 5.4.4 Profile of mass-in-flow components $V_{\text{m}}(x,y_1)$;
$[V_{\text{m}}(x,y_1)=V_w(x,y_1)-V_P(x,y_1)]$

Fig. 5.4 Calculated local velocities of a flow past a circular cylinder.

$(Re=13400; \beta_x=80,045^\circ; C_{\mu b}=-0,96; \epsilon_w=1,25m^2/s; L_2=0,6D; L_4=1,7D)$
5.3 COMPUTATION OF BUBBLE TRAJECTORIES IN THE WAKE

Based on the results of the previous section and in a similar way as in chapter 2, we now compute bubble trajectories in the wake region of a liquid flow past a cylinder.

In the general studies in chapter 2 on dispersed two-phase flows past obstacles, the magnitude of various forces on bubbles and the importance of main system parameters on bubble trajectories were quantified and analyzed. Due to strong gradients in the liquid velocities around an obstacle (both in magnitude and in direction) and very small gas to liquid density ratio, all interaction forces except the shear lift force, are important in the total force balance. With several usually neglected forces (e.g. the Basset history force) in the force balance are included, bubble trajectories run closer to the cylinder and to the wake border than without these "additional" forces. However without taking into account the effects of turbulence, bubbles are only found to move into the wake at quite a distance downstream of the cylinder where the mean streamlines enter the wake; and additionally in the case of very low liquid upstream velocities and large relative bubble sizes (i.e. bubbles rising in an almost stagnant liquid). These predictions are far from the empirical results which show high concentrations of bubbles behind the cylinder in the wake. Apparently the effect of turbulence in the liquid flow must be considered in order to obtain more realistic predictions. The turbulence causes the bubbles to diffuse into the wake.

Before investigating the effects of the liquid turbulence on void distributions behind a cylinder, we first have to face again the problem of the complexity of the single-phase liquid flow past a cylinder and the lack of empirical data. To concentrate on our focus in this thesis, the phase distributions of a dispersed two-phase flow past a cylinder, we handle the problem of the single-phase flow description in a similar way as in section 5.2. This is to say that we base our semi-quantitative description mainly on empirical observations and characteristic analytical results found in literature.

5.3.1 Specification of Turbulence in Single-phase Flows

The liquid velocities \((U_i[x,y])\) change both in time and in space in the wake of a cylinder. In general, turbulent fluctuations of the liquid velocity are of a random nature and therefore change in time according to a Gaussian distribution function at each of
locations. Such a distribution function is defined by two parameters: a mean value (MEAN) and a variance value (VARI). Making use of two features of turbulence in the wake of a liquid flow past a cylinder discussed in section 5.2 — the "mass-in-flow" profile $V_{\text{in}}[x]$ along the wake border and the relative turbulence intensity factors $\xi[x]$ in the wake — we may quantify the two required local parameters ($\text{MEAN}[x,y]$ and $\text{VARI}[x,y]$) in the following steps:

**Step 1: Specify two characteristic regions of the wake behind the cylinder.**

Characteristics of the close-wake of a single-phase flow past a cylinder have been repeatedly described and discussed (e.g. Bloor[1964], Papailiou[1974], Hinze[1975], Kouta[1987]), based mostly on experimental facts.

In the close-wake region, often called the developing region of the wake flow, two shear layers originate from the boundary layer on the sides of the cylinder. Characteristics of the wake flow may be represented at best as mutual effects between the shear layers and the "discrete" vortices (i.e., locally, the periodic liquid velocity fluctuations) which in the two (Von Karman) vortex streets have a stable, alternating, position. An instantaneous picture of the wake flow behind a cylinder has been given by Hinze(1975) and shows two characteristic regions: a central region of continuous turbulence (I) and a transient region of periodic (or intermittent) turbulence (II), as shown in fig.5.5.

![Fig.5.5 Two regions for classifying fluctuation of liquid velocity.](image-url)
Predictions of Phase Distributions Around Obstacles

The inner wake border (WB), which we define as the free streamlines bounding the external potential flow in the inviscid wake flow model (see appendix 4) is equivalent to the positions where the nature of velocity fluctuations changes from intermittent turbulence to continuous turbulence as the hot-wire moves to the flow axis (Bloor[1964]). Without finding any empirical or analytical data but with knowing that the vortex street grows gradually from the separation point to downstream and is further featured by the large-eddy motion beyond the wake formation region, we assume a profile of the outer wake border (OWB) as follows:

\[ y_{owb}(x) = y_{wb}(x) + y_{wb}(L_4) \left( \frac{x-x_s}{L_4-x_s} \right) \quad \text{when } x_s < x < L_4; \]
\[ y_{owb}(x) = 2y_{wb}(x) \quad \text{when } x > L_4 \quad (5.13) \]

Here the outer wake border \( y_{owb}(x) \) is considered to enclose the region in which the influence of the vortex street on the liquid velocities is significant.

**Step 2: Quantify turbulence parameters (MEAN\([x,y]\) & VAR\([x,y]\))**

With little quantitative information on velocity fluctuations (or turbulence) in the wake behind a cylinder available, we now try to make use of some results obtained in section 5.2 to quantify the turbulence characteristic parameters (MEAN\([x,y]\) and VAR\([x,y]\)), referring to some qualitative descriptions in literature (e.g. Durgin[1971], Hinze[1975]). In the wake behind the cylinder, measurements of velocity fluctuations have shown these to be continuous in the central region (Region I) and then to decrease sharply toward the outer border in the transient region (Region II). Considering the nature of the relative turbulence intensity factors \( \xi(x) \) defined in section 5.2, we accordingly assume a profile of the variance (VAR\([x,y]\)) of a Gaussian distribution for describing fluctuations of the local liquid velocity as follows:

\[ \text{VAR}(x,y) = U_0\xi(x), \quad \text{when } y < y_{wb}(x) \quad (I) \]
\[ \text{VAR}(x,y) = U_0\xi(x) \left( \frac{y-y_{owb}(x)}{y_{wb}(x)-y_{owb}(x)} \right) \quad \text{when } y_{wb}(x) \leq y < y_{owb}(x) \quad (II) \]

\[ (5.14) \]

where the variance (VAR\([x,y]\)) equals \( U_0\xi(x) \), as given by eq.(5.8). It is noted that the functional relationship given in eq.(5.14) will be varied in order to investigate its influence on bubble trajectories.
The mean (\( \text{MEAN}[x,y] \)) of a Gaussian distribution for describing fluctuations of the local liquid velocity has in principle different values for \( x \)- and \( y \)-component of the velocity. With little net flux in \( x \) direction due to effects of the local velocity fluctuation, the mean (\( \text{MEAN}[x,y] \)) of the \( x \)-component of the velocity may be taken as zero in both region I and region II (in fig.5.5). In the \( y \) direction, however, a net flow across the wake border was obtained from mass conservation due to the action of local velocity fluctuations in the liquid. Such a net flow has been explained by the entrainment mechanism and given in the form of a "mass-in-flow" profile \( V_m(x,y) \) as in eq.(5.12) border. Accordingly \( \text{MEAN}(x,y) \) is taken here the same as \( V_m(x,y) \) in eq.(5.12).

**Step 3: Include effect of turbulence into bubble trajectory computations**

Based on the above quantification of fluctuations of local liquid velocities in the wake behind a cylinder, we now include the effect of velocity fluctuations into the bubble trajectory computations by adding a random value of \( U_1'(x,y) \) to the liquid velocity \( U_1[x,y] \) at each location \([x,y]\), as shown by eq.(5.15).

\[
U_p[x,y]-U_1[x,y] = U_p[x,y] - (U_1[x,y]-U_1'[x,y])
\]

\[
= (U_p[x,y]-U_1[x,y]) + U_1'[x,y]
\]

\[
<1> + <2> \quad (5.15)
\]

where \( U_1[x,y] \) is the time-averaged local velocity of liquid; \( U_1'[x,y] \) is the fluctuation component of the liquid velocity. Term \(<1>\) in the equation is the usual relative velocity term in the steady drag while term \(<2>\) induces additional contributions to the force balance on the bubble due to the velocity fluctuations.

The random value (\( U_1'(x,y) \)), which is provided by a random-number generator in the computer program (PDAC), is defined by the \( \text{MEAN}[x,y] \) and \( \text{VAR}[x,y] \) of a Gaussian distribution function obtained from the step 2, for \( x \)- and \( y \)-component of fluctuation of the liquid velocity respectively. (A justification of the random generation procedure is shown in appendix 14)

In this way, effects of liquid velocity fluctuations may be included into bubble trajectory computations by using the expression of the local relative velocity given in eq.(5.15) into the force balance equation (eq.(2.5)), in which all forces but the gravity force on a bubble are related to the relative velocity between bubble and liquid.
With a number of typical runs in the rest of this chapter, effects of liquid velocity fluctuations and other parameters on bubble trajectories in a flow past a cylinder are investigated.

5.3.2 Numerical Implementation

As shown above, the effects of liquid velocity fluctuations are introduced into particle trajectory computations in an ad-hoc manner. No correlations for liquid velocity fluctuations where applied, the computations depend on e.g. the random number generator in the program. Some of the hence more important numerical features in the program are briefly discussed below.

Grid system: For application of the Lagrangian approach to compute particle trajectories, no fixed grid system is needed. In convenience of storing and handling of computation results, a special procedure is used to extract data at x-coordinates with equal intervals (Δx has been chosen as 0.1D in all the computations).

Timestep: In computing particle trajectories by integrating a set of ordinary differential equations, a flexible timestep is used that adjusts itself automatically in the program to satisfy the limitation of the length step: 0.01D<\sqrt{\Delta x^2+\Delta y^2}<0.05D and \sqrt{\Delta x^2+\Delta y^2}<d_p. Usually the first limitation is dominant; it automatically yields a turbulent "frequency" of roughly U_0/0.1D as the dominant one. This may be too high a frequency for large bubbles (say, larger than 0.1D); possibly these are (too) weakly influenced by the artificial turbulence.

Random number generator: A routine (RAN2, namely a portable random number generator) in Numerical Recipes software package (Press[1985]) is utilized in the program PDAC. Computations were carried out on a HP-286 personal computer.

5.3.3 Effects of Some Assumptions

In the deduction procedure of section 5.2, some model assumptions for constructing a time-average velocity field of a single-phase wake flow are proposed because of a lack of available quantitative information. With one parameter varied and all others kept constant, the effect of these model assumptions on the constructed model are now briefly illustrated.
**Chapter 5**

**x-component velocity profile:** The Gaussian-form velocity profile (part $<1>$ in eq.(5.1)) is valid in the far-wake region (typically 50D from the cylinder). With little information available on velocity profiles in the close-wake region, the Gaussian-form profile is extended to be the basis of the liquid velocity profile in the close-wake region. Part $<2>$ is added in eq.(5.1) to account for the deviations in far-wake and close-wake regions. To limit unknown variables in the assumed profile ($C_1[x]$ and $C_2[x]$ in eq.(5.1)), we have proposed part $<2>$ to be a constant at each $x$ coordinate. An alternative, namely $C_2(x)[0.5y_{wb}[x] - y[x]]$, was also tried out and was found to yield results far beyond reality, with too large reversed flow components close to the flow axis. It is possible that the additional term (part $<2>$) should have a more complex form to account for larger deviations between velocity profiles in the far-wake region and in the close-wake region. But this would introduce more than two unknown variables, which requires additional relations to obtain solutions. Taking the single-phase wake flow description merely as a starting point for further qualitative studies on dispersed two-phase systems, we have limited our efforts on single-phase wake flow descriptions and considered the current form of eq.(5.1) to be sufficient for this purpose.

**Pressure profiles:** Pressure distributions in the close-wake region have been assumed as eq.(5.6), based mainly on experimental data. Besides the forms of local pressure profiles (linear) shown in section 5.2, several different kinds of forms (e.g. a parabolic profile) were also examined. Computation results of velocity profiles are found to be affected very much by the pressure profiles. In principle, the complicated pressure field in the close-wake can only be accurately determined with the aid of the Navier-Stokes equations. However, the averaging procedure itself is of an approximate nature and there is no point in taking much effort for a fine analysis of the pressure field. Accordingly, the pressure field in the close-wake region is given the form of section 5.2, which is consistent with experimental observations and is considered to be sufficiently accurate for our purpose.

**e(x) profile:** As shown in section 5.2, the maximum value of $e(x)$ is first derived at the end of the wake formation region ($x=L_4$). Then to both sides of $x=L_4$, $e(x)$ is assumed to decrease following either a parabolic profile or a linear profile. The factor $e(x)$ may be considered as some kind of mean relative intensity of turbulence. The turbulence in the close-wake region has a very complicated
structure. Both the value of $\epsilon(L_4)$ and the form of decreasing profiles might differ from reality. With specific selections of $\epsilon(L_4)$ (e.g., half of what is calculated) and forms of decreasing profiles, several runs were performed to examine the effect of $\epsilon(x)$ on computed velocity profiles. Rather surprisingly, neither $\epsilon(L_4)$ nor the form of decreasing profiles are found to have big impact on computed single-phase velocity profiles.

Correlations between various parameters: In our model, liquid velocity fluctuations are introduced locally, following a Gaussian function for describing a random variable. For simplicity, no correlations between system parameters have been taken into account. In general, correlations (such as which between $U[t_{k-1}]$ and $U[t_k]$ or $\sqrt{\Delta x^2+\Delta y^2}$ and $\Delta t$, and so on) have to be obtained by solving additional conservation equations and mostly have very complex form (as refer to the extensive discussion by Hinze[1975]). Among various common correlations, the Lagrangian time correlation (defined as $R_1(t)=\overline{v(\tau)v(\tau-t)/v^2}$) is important for our case of bubble trajectory computations. According to the character of the $R_1(t)$ and our model, the magnitude of liquid velocity fluctuations should be decreased with consideration of $R_1(t)$.

5.3.4 Effects of Simulated Liquid Velocity Fluctuations

Fig.5.6 shows two sets of trajectories of bubbles computed without and with taking into account the effect of liquid velocity fluctuations. Main parameters for the computations are the following: $Re_d=13400$; $d/D=1/10$. In addition, instantaneous bubble velocities and interaction forces along one particular trajectory in the two cases are shown respectively in fig.5.7 and fig.5.8.

Without including the effect of liquid velocity fluctuations in the force balance on bubbles, trajectories of bubbles in fig.5.6(a) appear to be "parallel" to the wake border and lie only in the potential flow region, as to be expected. No bubbles enter the wake region behind the cylinder. Bubble velocities in fig.5.7(a) change gradually when bubbles pass the cylinder except during the "sliding" phenomenon over the cylinder as discussed in the next section.
Fig. 5.6 Computations of bubble trajectories to show effects of liquid velocity fluctuation.
(a) without and (b) with effects of liquid velocity fluctuation;

Fig. 5.7 Computations of bubble velocities along trajectories in Fig. 5.6.
(a) without and (b) with effects of liquid velocity fluctuations;
Fig. 5.8 Interaction forces on bubbles along trajectory 2 in fig.5.6.
index: (1) $F_{\text{drag}}$; (2) $F_{\text{am}}$; (3) $F_{\text{basset}}$; ($F_{\text{pg}}$ and $F_{\text{L}}$ are negligible in the force balance)

(a) without and (b) with effects of liquid velocity fluctuations;
In the force balance of a bubble along trajectory 2 in fig.5.8(a), the steady drag force ($F_{sd}$) and the added-mass force ($F_{am}$) are dominant when the bubble is between $x=-D$ and $x=D$ around the cylinder. The Basset history force ($F_{basset}$) takes an important part in the force balance further downstream ($x>D$). The pressure-gradient force ($F_{pg}$), the lift force ($F_{L\nu}$, vorticity lift in the wake) and the buoyancy force ($F_{buoy}$) are negligible (generally in a magnitude of 1/100 of the steady drag force) in the force balance.

Including the effect of liquid velocity fluctuation in the force balance on bubbles leads to dramatic changes in the pattern of bubble trajectories downstream of the cylinder. See fig.5.6(b) compared to fig.5.6(a). Bubbles entering the extended wake region, are affected by liquid velocity fluctuation and tend to move into the wake in random paths. As a result of this, fig.5.7(b) exhibits strong fluctuations in bubble velocity. In the region where the liquid velocity field goes through rapid changes in the close-wake behind the cylinder, liquid velocity fluctuations induce dramatic changes in bubble velocities and bubble trajectories. Because of the relative small magnitude of the bubble velocity component in $y$-direction compared to that in $x$-direction, effects of liquid velocity fluctuations have more significant impact in $y$-direction than in $x$-direction. Accordingly, both bubble trajectories and bubble velocities change more randomly in $y$-direction than in $x$-direction.

Among the interaction forces, $F_{pg}$, $F_{L\nu}$ and $F_{buoy}$ are again negligible in the force balance. The other three forces (i.e. $F_{sd}$, $F_{am}$, and $F_{basset}$) are of the same order of magnitude in the force balance. Whereas magnitudes of the steady drag force and the added-mass force increase only slightly compared with those in fig.5.8(a), the magnitude of the Basset history force increases dramatically (The values in fig.5.8(b) are almost 100 times as big as those in fig.5.8(a)). Reasons for this will be discussed in this section later.

In fig.5.8(b), the steady drag force and the added-mass force change more or less with the same magnitude but anti-phased. As a random liquid velocity fluctuation introduces a large relative velocity (reflected by large $F_{sd}$) between a bubble and the liquid flow, the bubble would endure immediately a large relative acceleration (reflected by large $F_{am}$) to follow the change of its ambient liquid flow. To a certain extent, the relation between $F_{sd}$ and $F_{am}$ illustrates a cause–and–effect interaction process through which bubbles are forced to follow changes in the turbulent liquid wake.
With $F_{ad}$ and $F_{am}$ having a compensating opposing effect on each other, the Basset history force draws bubbles into the wake as shown clearly by the majority of negative values downstream of the cylinder in $y$-components of either $V_y$ in fig.5.7(b) or $F_{basset}$ in fig.5.8(b).

This is illustrated even more clearly with a set of computation results without including $F_{basset}$ in the force balance, as shown in fig.5.9 corresponding to these in fig.5.6(b). Bubbles penetrate the wake to a lesser extent. Above results show a significant effect of the Basset history force on a bubble in a turbulent liquid flow with liquid velocity fluctuations dependent on locations. This can also be explained by analysis in section 2.2.1, the Basset term becomes important when the situation is one of transient response.

![Fig.5.9 Computations of bubble trajectories without including $F_{basset}$ as compared with fig.5.6(b).](image)

The integral result of all past relative accelerations of a bubble in the wake (i.e. the Basset history force, see appendix 2) draws bubbles into the wake core. Obviously this is close to what is observed in experiments as will be further discussed in section 5.4. Whether the present form of the history force still holds in turbulence is not proven here. It has merely been applied to investigate the consequence. The consequences are considerable.

There are also cases in which bubbles tend to move away from the wake as shown in fig.5.6(b). This is obviously due to some incidental large positive random liquid velocity fluctuation in $y$-component (which might be unrealistic) along the wake border close to the cylinder, indicated as large $V_y$ values in fig.5.7(b). Once a bubble is "pushed" into the potential region, local "weak" liquid velocity gradients can not force the bubble to adjust to the streamlines very quickly (but bubbles indeed are bent towards the streamlines, though on a timescale beyond the scope of the figures). The number of bubbles moving away from the wake, however, should be much smaller than those who
move into the wake. Such a probability of bubbles move into or away from the wake depends to a large extent on the modeling of the "sliding" phenomenon, which is discussed in next section.

5.3.5 Influence of the Sliding Phenomenon

A major uncertainty in bubble trajectory computations lies in the "sliding" phenomenon. Bubbles, which collide with the cylinder from upstream (mostly large ones), have been observed to slide over the surface of the cylinder and to be released into the liquid flow again in the neighbourhood of the single-phase separation points. The exact point of release has a large influence on bubble trajectories further downstream.

To illustrate the influence of the release condition, we carried out a numerical test to compute eight bubble trajectories with various release positions (instead of what was used in section 5.2 — the separation point $X_i = X_s$ & $Y_i = Y_s$). Results are shown in fig.5.10.

![Fig.5.10 Bubble trajectories to show influence of the release position.](image)

The results in fig.5.10 do show a rather significant influence of the release position on bubble trajectories downstream of the cylinder. Most remarkably, this is only the case when the Basset history force is taken into account. When the Basset history force is not included in the calculations, the bubbles move smoothly more or less along the wake border and enter the wake at quite some distance downstream of the cylinder. This validates again the proposition that the Basset history force is needed to get bubbles into the close-wake, as discussed in section 5.3.4.
No literature was found in which the sliding phenomenon and the release condition are studied thoroughly. Here we have only illustrated the considerable influence of those effects on bubble trajectory computations and thus on the void fraction computations in the wake. Considering the intermittent nature and complexity of the phenomena involved, much work is certainly required for improving the modeling to provide more realistic and accurate predictions.

5.3.6 Preliminary Remarks on the Influence of $d_p/D$ and $Re_d$

Although there are a number of uncertainties in our model as discussed above, it still can illustrate qualitatively the effects of primary system parameters (the bubble to cylinder size ratio, $d_p/D$; and the cylinder Reynolds number, $Re_d$) on bubble trajectories. These effects are preliminarily discussed below.

(1) Influence of the bubble to cylinder size ratio

Fig.5.11 shows three sets of bubble trajectories with different bubble-to-cylinder diameter ratios ($d_p/D=1/3$, 1/10, and 1/30), while all other parameters are same. As $d_p/D$ decreases, the effect of liquid velocity fluctuation becomes more and more apparent. Although the smaller bubbles tend to move further away from the wake than bigger ones (in the potential flow region, see section 2.4.1), the smaller bubbles are much more affected by the action of velocity fluctuations.

The bubble to cylinder size ratio ($d_p/D$) also indirectly indicates the relative ratio of bubble dimension to the (local) fluctuation amplitude of the flow field, as resulting from periodic velocity fluctuations (i.e. vortex shedding). The fluctuation amplitude may be estimated to be of the order of the cylinder diameter (but somewhat smaller). When $d_p/D$ is very small, the bubbles are obviously more easily drawn into the wake through the action of liquid velocity fluctuations or the entrainment process. Especially in the early period after "shedding" bubbles have re-enter the main flow, small bubbles are drawn into the wake close to the cylinder. Bubbles in this region may incidentally be pushed toward the cylinder as a result of the combined action of all forces. The insensitivity of large bubbles to imposed velocity fluctuations may result from the relatively high frequency of the latter (see the discussion on timestep in section 5.3.2). In reality, however, only small bubbles survive in such a sharply-varying velocity gradients in the close-wake region (Hulin[1986]). Large bubbles are deformed and tend to break up. These realistic features are not included in the present computations, that merely try to model the bubble entrainment process.
Fig. 5.11 Calculated bubble trajectories at different bubble to cylinder size ratios.
(a) $d_p/D=1/3$; (b) $d_p/D=1/10$; (c) $d_p/D=1/30$. ($Re_d=13400$)
Fig. 5.12 Calculated bubble trajectories at different cylinder Reynolds numbers. 

(a) $Re_d=13400$; (b) $Re_d=24400$; (c) $Re_d=45700$. $(d_p/D=1/10)$
(2) Influence of the cylinder Reynolds number

Fig. 5.12 shows three sets of bubble trajectories with different Reynolds number ($Re_d = 13400, 24400, \text{ and } 45700$), and all other parameters constant. Patterns of bubble trajectories under different Reynolds numbers hardly vary with each other.

As has been discussed in section 5.2, flow patterns of single-phase liquid flows vary little over the entire subcritical Reynolds number range ($10^4 < Re_d < 10^6$). The flow separation condition (i.e. the separation angle) on a cylinder surface and the base pressure at the rear half of the cylinder, which determine formation and development of vortex shedding processes, are almost constant. Bubble trajectories and void distribution patterns are fully dependent on the vortex shedding processes under the same ratio of bubble size to cylinder size ($d_p/D$).

The independence of bubble trajectories on $Re_d$ adds to the conclusions in the previous section: Bubble trajectories and redistributions under the present modeling assumptions appear to be mainly dependent on the bubble to cylinder size ratio.

5.3.7 Concluding Remarks

(1) Realistic bubble trajectories, with a number of bubble trajectories entering the wake of a cylinder, are only obtained if the effect of liquid velocity fluctuations (or turbulence in the liquid) is simulated and some kind of sliding phenomenon for colliding bubbles is taken into account.

(2) The steady drag force and the added-mass force are usually counteracting each other. The Basset history force plays a dominant role in drawing bubbles into the wake. Compared with these three forces, the lift force, the pressure gradient force and the buoyancy force are negligible in the force balance.

(3) A preliminary qualitative check on the influence of the bubble to cylinder size ratio ($d_p/D$) and the cylinder Reynolds number ($Re_d$) are carried out, in a similar way as the theoretical analysis in chapter 2 and the experimental investigations in chapter 4. It shows that effects of these two main system parameters ($d_p/D$ and $Re_d$) on bubble trajectory computations with considerations of liquid velocity fluctuations are as follows:

a) With varying $d_p/D$, which approximately represents the relative magnitude of bubble dimension to (local) fluctuation amplitude, patterns of bubble trajectories vary significantly. Only larger bubbles tend to enter the wake further downstream.
For this conclusion, however, the role of the frequency of the fluctuations requires further investigations.

b) In the subcritical Reynolds number range, the cylinder Reynolds number \( (Re_d) \) has little influence on the bubble trajectories.

5.4 COMPARISON OF PREDICTIONS AND MEASUREMENTS

For the investigation of a common but complicated flow situation like a bubbly flow past a cylinder, predictions and measurements are of equal importance. The measurements (in chapter 4) are taken under such conditions that the main parameters (e.g. \( Re_d \) or \( d_p/D \)) vary substantially. Whereas the range of the measurements is often limited by the ability of the utilized measuring technique and the operation range of the used experimental setup, the applicability of the model is much less restricted. However, the accuracy of the descriptions is dependent on the available knowledge (e.g. single-phase flow and driving forces). A close connection between predictions and measurements is difficult to obtain.

Comparison of predictions and measurements of phase distributions of a bubbly flow past a cylinder is here mainly based on results of the present investigation because of a lack of other information in literature (either theoretical or experimental).

Computation of void fraction distributions from trajectories is easy in principle but laborious (see appendix 15). On the other hand, bubble trajectory pictures like those in section 5.3 already clearly indicate where high void fraction regions are to be expected. For the present analysis, the trajectories shown are therefore sufficient. The significance of the main system parameters (such as the cylinder Reynolds number \( Re_d \) or the bubble to cylinder size ratio \( d_p/D \)) for phase distributions is easily investigated by varying the studied parameter with the others unchanged. In section 5.3, several sets of bubble trajectories are computed under the same conditions in the corresponding experiments in chapter 4. Comparison of the computation results (section 5.3) and the experiment results (chapter 4) yields:

(1) While measurements clearly reveal high void concentrations in the wake core as in fig.4.2, 4.3 and 4.10, predictions show that bubbles can enter the wake only when the effect of liquid velocity fluctuations, some kind of sliding phenomenon and the Basset history force are included in the computations.
(2) Both measurements and computations show little change in the phase distribution patterns of a bubbly flow past a cylinder if the cylinder Reynolds number is changed in the subcritical range.

(3) Measurements show the bubble to cylinder size ratio \((d_p/D)\) to be an important parameter for bubbly flows past a cylinder. They show a tendency for the peak void concentration region in the wake core to shift further downstream as \(d_p/D\) decreases (from \(1D-1.5D\) at \(d_p/D=1/3\) to \(3D-3.5D\) at \(d_p/D=1/10\) in fig.4.3 and fig.4.10). Preliminary computations show a similar effect of the bubble to cylinder size ratio on bubble trajectories. Bubbles are more strongly carried into the wake for small \(d_p/D\) than for large \(d_p/D\) (as shown in fig.5.11). It is noted that turbulence may explain the diffusion of bubbles into the wake, but can not explain the existence of peak void concentration regions.

In order to obtain a more realistic picture of void distribution around a cylinder, the following should be considered:

(1) Description of a single-phase flow past a cylinder, which is the starting point of our modeling approach (i.e. the Lagrangian approach to predict phase distributions by bubble trajectory computations), is still insufficient and incomplete. As discussed earlier in this thesis, a time-average single-phase flow past a cylinder is still one of the most difficult problems in fluid mechanics, involving many fundamental but complicated phenomena such as boundary-layer separation, vortex-shedding, turbulence production, and so on. Empirical data will be indispensable for a long time to come.

(2) To be close to realistic conditions under which measurements are taken, bubble trajectory computations must include such effects as the deformation of bubbles and the interaction between bubbles. Furthermore, the influence of bubbles on the single-phase flow should also be considered, especially if bubble size is relatively large. Also the force balance used for bubble trajectory computation might be improved, e.g., by including effects of surface tension and surface active agents.

(3) Simulation of velocity fluctuations or turbulence either in the single-phase flow field or between the two phases should be improved by including sufficient correlations for common descriptions of turbulence.
CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

The study has focused on phase distributions in low quality dispersed two phase flows around obstacles. It consists of a theoretical part of a more general nature and an experimental part highlighting bubbly flows around a cylinder in vertical tubes. Concluding remarks are put forward at the end of each section and are summarized below.

6.1 MAIN RESULTS

Theoretical Analyses
Theoretical analyses are based on the Lagrangian approach, which is most convenient for the general—nature study on interaction forces between a dispersed phase and a single phase flow. By computing trajectories of a dispersed phase in a single phase flow approaching a cylinder, we illustrate the general computation method and analyze the magnitude of interaction forces in the force balance of the dispersed phase and the influence of system parameters on the interaction process.

Magnitude of interaction forces in the total force balance of the dispersed phase
Particle—liquid systems, particle— or drop—gas systems, and bubble—liquid systems are studied. New results are obtained concerning several "additional" forces that are often assumed negligible. In particular, the troublesome Basset history force appears to play a very significant role in the force balance. (sec.2.4 and sec.5.3)

Influence of main system parameters
Two parameters, namely the cylinder Reynolds number ($Re_d$) and the bubble to cylinder size ratio ($d_p/D$) are investigated in a practical range of values. The bubble to cylinder size ratio is found to be the most important system parameter for determining bubble trajectories and thus void fraction distributions.
Experimental Investigation

A gamma-ray densitometer system for measuring void fraction distributions in a vertical tube is developed. Also a photographic system for quantifying bubbles in a high pressure tube is implemented. Both measuring systems are applied to the investigation of phase distribution patterns of gas (vapour)—liquid two—phase flows around a cylinder in a vertical tube at atmospheric and high pressures.

Gamma-ray densitometer for local void fraction measurements

The system is based on a conventional single—beam gamma densitometer. Much effort is given to processing algorithms and automation of the control system. The system has a number of unique features for our special applications and permits the derivation of reliable void fraction distributions from "raw" gamma—beam intensity measurements. Through carefully designed calibration tests, measurements with the system are proved to be efficient and accurate. The system was successfully utilized to obtain two—dimensional distributions of void fractions in the wake of cylinders in vertical tubes.

Photographic system in a high pressure evaporator tube

The new photographic system was implemented to obtain shadow—photographs of bubbles in a high—pressure vertical evaporator tube. With this system and the related interpretation principles, not only size and velocity of a bubble may be computed from its shadow images on one photo but also the longitudinal position of the bubble on the optical axis can be estimated from the size of the bright spot in its shadow image. The relation for estimating bubble longitudinal positions is to the author's knowledge introduced for the first time. It provides extra information from a usual shadow photograph in a very simple manner. Performance of the new system and validity of the interpretation principles were examined through calibration tests and proved to be very satisfactory. The system was utilized to provide information of bubbles in upstream main flows as well as a means of rough visualization of void distribution patterns under high pressures. In particular, photos of small bubbles under high pressure condition are believed to be obtained for the first time.
Phase distribution measurements behind obstacles in vertical bubbly flows

From void fraction distribution measurements provided by the gamma-ray densitometer, the effects of three characteristic parameters — the cylinder Reynolds number \( (Re_d) \), the bubble to cylinder size ratio \( (d_p/D) \), and the upstream mean void fraction \( (\alpha_0) \) — on void fraction distribution patterns around a cylinder are studied. Void fraction distribution patterns appear to be somewhat similar to vortex street patterns of corresponding single-phase flows under the same \( Re_d \): the void fraction is high in regions where the vortex streets or turbulence occur in the wake behind the cylinder. Among the dominating parameters, the \( d_p/D \) is found to be the most important parameter determining void fraction distribution patterns. The decisive role of \( d_p/D \) is most evident in experiments in which the bubble size is bigger than the cylinder size. Bubbles are trapped in the wake region and form a cavity downstream of the cylinder even at a very low upstream velocity. The void fraction distribution pattern does not vary much as \( \alpha_0 \) changes.

Comparison of Predictions and Measurements

Comparison of predictions and measurements of phase distributions of a bubbly flow past a cylinder is here mainly based the results of the present investigation, because of the scarcity of information in literature.

Prediction of phase distributions around obstacles

Due to a lack of knowledge concerning the highly turbulent single-phase flow past (and in particular behind) a cylinder and concerning the complicated interaction process between a bubble and the flow field and other bubbles, predictions of phase distributions of a bubbly flow past a cylinder are more of a qualitative and explanatory nature than of a quantitative kind. This also applies to the comparison. As a starting point for studying bubble trajectories in the wake, the time-average single-phase velocity field around an cylinder was described in a semi-empirical manner. The description, though approximate, yields a first indication of the phase distributions. It is found that qualitative agreement between predictions and experiments is only obtained if the effect of liquid velocity fluctuations (or turbulence in the liquid) is somehow simulated. Also some kind of sliding phenomenon has to be taken into account. The classical expression for the Basset history force is found to play a dominant role in drawing bubbles into the wake core.
Qualitative comparison of predictions and measurements
Both measurements and computations show little change in the phase distribution patterns of a bubbly flow past a cylinder if the cylinder Reynolds number (Re_d) is changed in the subcritical range. The interaction process between bubbles and local liquid velocity fluctuations is apparently of fundamental importance to bubble segregation. A preliminary qualitative check on the influence of the bubble to cylinder size ratio (d_p/D) and the cylinder Reynolds number (Re_d) are carried out in section 5.3.6, in a similar way as the theoretical analysis in chapter 2. Similar conclusions are drawn as from the experimental results. The check yields again a significant effect of the bubble to cylinder size ratio (d_p/D) on bubble trajectory computations. But for predictions of higher accuracy, the time-dependent nature of a wake flow need to be considered.

6.2 DISCUSSIONS AND RECOMMENDATIONS

On Experimental Investigations
Considering the nature and complexity of a dispersed two-phase flow past an obstacle, experimental investigations will still play an important (to some extent also determining) role in the future to study or visualize complicated and novel phenomena in such a system. For a quantitative comparison with theoretical predictions, specific purpose experiments are necessary, requiring both careful design of new test setups and utilization of more adequate measuring techniques. The most useful will probably be the visualization of particle or bubble trajectories in such a way that more precise information on the Basset history force is obtained.

On Theoretical Predictions
The theoretical results in this thesis have a rather general nature and are of fundamental interests. Considering the heterogeneity and complexity of a dispersed two-phase flow past an obstacle, the predictions of void fraction distributions by the present model are preliminary and more of a qualitative nature than of a quantitative one. Much more effort is required to obtain predictions that can quantitatively be compared with experimental results.
Description of single-phase flow around a cylinder

The description of a single-phase flow past a cylinder, which is the starting point of our Lagrangian approach to predict phase distributions by bubble trajectory computations, is still incomplete. First the description of a single-phase flow past a cylinder is still one of the most difficult problems in fluid mechanics, involving many fundamental but complicated phenomena. Our semi-empirical approach is based on experimental data as much as possible; for the time-averaged flow field they are proved to be rather insufficient. New additional experimental (and may be numerical) data are required.

Bubble trajectory computations in the wake

As discussed previously, liquid velocity fluctuations and sliding phenomenon play an important role in providing more realistic predictions of void concentrations in the wake core. However, they both could only be modeled (or assumed) quite coarsely. More attention should be paid in further work to analyze and model the processes.

Basset history force in turbulent flows

The significant role of the Basset history force in the force balance on a particle in the potential flow region can well be explained. But it is rather surprising to find that the Basset history force plays an important role in phase distributions in the wake region. Whether the Basset history force still holds in strong turbulence for gas bubbles (when the deformation of bubbles need to be taken into account for large bubbles; e.g., \( d_p = 4 \text{mm} \) at atmospheric pressure) might be questioned and should be investigated.

Necessary considerations in bubble-liquid systems

To be close to realistic conditions under which measurements are taken, bubble trajectory computations must include effects such as the deformation of bubbles and the interaction between bubbles. Furthermore, the effect of bubbles on the single-phase flow should also be considered, especially if bubble size is relatively large. The force balance used for bubble trajectory computation might need amendment, e.g. for the lift force and the influence of a wall.
REFERENCES


APPENDIX 1 MAJOR IDENTIFIABLE REGIMES OF SINGLE-PHASE FLOWS PAST A CIRCULAR CYLINDER

A flow around a circular cylinder mainly depends on the Reynolds number ($Re_d = U_0D/\nu$), which indicates the ratio of the inertia effect to the viscous effect. Figure A1.1 shows major identifiable regimes as presented in many textbooks on fluid mechanics (e.g. Chang[1970]; Massey[1979], Chen[1987], Munson[1990]).

At very low Reynolds numbers ($Re_d<5$), the flow does not separate. The inertia forces are negligibly small and the flow adheres to the surface of the cylinder. Streamlines may be predicted from the potential-flow theory. As $Re_d$ is increased, the inertia forces become appreciable and the flow separates to form a pair of recirculating eddies in the rear of the cylinder. As $Re_d$ is further increased, the shedding eddies become elongated in the flow direction; their length increases linearly with Reynolds number until the flow becomes unstable at $Re_d$ of the order of 40. The vortices then break away to form a periodic staggered vortex street which stretches a considerable distance downstream the cylinder. This vortex street is laminar and is generally referred to as von Karman vortex–street. At a $Re_d$ up to $3\times10^4$, the boundary layer is laminar over the front part of the cylinder. Over the rear part, the layer separates and breaks up into a turbulent wake. Throughout a wide Reynolds number range between $3\times10^2$ and $3\times10^8$, which is usually called the subcritical range, the vortices are shed downstream at a constant frequency (i.e. a constant Strouhal number $= 0.2$) and the separation points lie in an angular position between 80 and 85 degree measured from the direction of the oncoming flow. At a Reynolds number of about $3\times10^5$ (the exact value depends on free stream turbulence and surface roughness), the kinetic energy of the fluid in the laminar boundary layer over the forward part of the cylinder is sufficient to overcome the unfavorable pressure gradient without separating. The flow in the boundary layer becomes turbulent while it is still attached. The separation points move backward and the closing of the streamlines reduces the size of the wake. At a higher Reynolds number, the vortex street is re–established.
Fig.A1.1 Regimes of single-phase flows past a circular cylinder. (from Chen[1987]; original source Lienhard[1966])
APPENDIX 2 DERIVATION OF DRAG ON A PARTICLE IN AN ARBITRARY FLOW

For a fluid, which is incompressible and with constant viscosity, the Navier–Stokes equation is written as follows (Batchelor[1967]):

$$\frac{\partial \mathbf{U}_f}{\partial t} + (\mathbf{U}_f \cdot \text{Grad}) \mathbf{U}_f = -\frac{1}{\rho_f} \text{Grad}(p) + \nu_f \Delta \mathbf{U}_f \tag{A2.1}$$

Analytical solutions are available only in creeping flow (or Stokes’ flow) situations where the non–linear inertial term \((\mathbf{U}_f \cdot \text{Grad})\mathbf{U}_f\) is small and thus can be ignored compared with other terms in the equation. Eq.(A2.1) then is simplified to:

$$\frac{\partial \mathbf{U}_f}{\partial t} = \frac{1}{\rho_f} \text{Grad}(p) + \nu_f \Delta \mathbf{U}_f \tag{A2.2}$$

Especially for axis–symmetrical creeping flow situations, if we derive the curl of both sides of the equation (where \(\text{Curl(Grad(p))} = 0\) and \(\text{Curl(} \Delta \mathbf{U}_f) = 0\)), eq.(A2.2) may further be simplified to:

$$\frac{\partial (E^2 \psi)}{\partial t} = \nu_f E^4 \psi \tag{A2.3}$$

where \(\psi\) is the stream function and \(E^2\) in spherical polar coordinates is defined as follows:

$$E^2 = \frac{\partial}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial}{\partial \theta}$$

Then with boundary conditions of

1. uniform flow remote from the particle,
2. no flow across the surface of the particle,
3. continuity of tangential velocity across the surface of the particle, and
4. continuity of tangential stress across the surface of the particle,
Eq. (A2.2) can be solved for a spherical particle which executes a rectilinear oscillatory motion relative to the remote fluid. The main deduction steps are as follows (Landau[1987] and Clift[1978]):

**Step 1:** A spherical particle is considered executing a purely rectilinear oscillatory motion relative to remote fluid and its velocity is given as

\[ U(t) = U_0 e^{-i\omega t} \]  
(A2.4)

where \( \omega \) is the frequency of oscillation.

**Step 2:** By analogy the solution for steady creeping flow,

\[ \psi = -U \frac{\sin^2 \theta}{2} \left( \frac{R^3}{2r} \frac{2(2+3\mu_p/\mu_t)}{2(1+\mu_p/\mu_t)} - \frac{(\mu_p/\mu_t)R^3}{2r^3(1+\mu_p/\mu_t)} \right) \]

which is adapted from the Hadamard–Rybizynski solution (Clift[1978]), that may be assumed to take the form, relative to the particle, of

\[ \psi = f(r) e^{-i\omega t} \sin^2 \theta \]  
(A2.5)

**Step 3:** Combining eq.(A2.5) and eq.(A2.3), we can derive \( f(r) \) by solving the equation with known boundary conditions. So the stream function is obtained in the following form:

\[ \psi = -U \frac{\sin^2 \theta}{2} \left( r^2 - \frac{R^3}{2} \left( \frac{3R\delta(1+i)R+i\delta}{r} - (1+i\delta) e^{(i-1)(r-R)/\delta} \right) \right) \]  
(A2.6)

where \( \delta = \sqrt{2\nu t/\omega} \), which is the depth of penetration of a transverse wave perpendicular to the direction of oscillation of the sphere and may be regarded as a characteristic length scale for diffusion of vorticity generated on the particle surface into the surrounding fluid (Landau[1987]). In other words, the motion of the fluid caused by the oscillation of the sphere is rotational in a certain layer round the sphere; the motion changes rapidly (exponential to \( \delta \)) to a potential flow at larger distance. The depth of penetration of the rotational flow is of the order of \( \delta \).
Step 4: The velocities $U_r(\equiv \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \theta})$ and $U_\theta(\equiv \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r})$ are calculated immediately from $\psi$. Consequently the shear stresses $\sigma_{rr}'(=2\mu r \frac{\partial U_r}{\partial r})$ and $\sigma_{r\theta}'(=\mu \left(\frac{\partial U_r}{r \partial \theta} + \frac{\partial U_\theta}{\partial r} - \frac{U_\theta}{r}\right))$ may be calculated. The drag on the particle is then obtained by integrating the normal and shear stresses over the surface of the particle, as shown in the following equation

$$F_{d\omega} = \oint_S (-p \cos \theta + \sigma_{rr}' \cos \theta - \sigma_{r\theta}' \sin \theta) \, dS$$  \hspace{1cm} (A2.7)

Furthermore, for a spherical particle, we get

$$-F_{d\omega} = 6\pi \mu RU + \frac{V_2}{2} \frac{dU}{dt} + 3\pi R^2 \left(2\mu \rho \frac{dU}{dt} + \delta \rho \frac{dU}{dt}\right)$$  \hspace{1cm} (A2.8)

Step 5: This result is then generalized to an arbitrary rectilinear motion. With the total velocity as the sum of all frequency components ($U_\omega$) written in the form of a Fourier integral

$$U(t) = \int_\infty^{-\infty} U_\omega e^{-i\omega t} \, d\omega$$  \hspace{1cm} (A2.9)

the total drag may be written as an integral of the drag component $F_{d\omega}$ in the same way as for velocity components because of their linear relation (shown in eq. (A2.9)) as follows:

$$F_d = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_{d\omega} \, d\omega$$  \hspace{1cm} (A2.10)

It is extended to be in the following form

$$-F_d = 6\pi \mu RU + \frac{V_2}{2} \frac{dU}{dt} + 6R^2 \sqrt{\pi \rho} \mu t \int_\infty^{-\infty} \frac{dU}{d\tau} \frac{d\tau}{\sqrt{\tau - \tau_0}}$$  \hspace{1cm} (A2.11)

where $R$ is the particle radius, $U$ is the particle instantaneous velocity relative to the fluid and $F_d$ is the instantaneous total drag on the particle.
On the right side of eq. (A2.11), the first term is the Stokes' drag, which is derived from the "creeping flow approximation" applying at low particle Reynolds number (Re_p = 2RU/\nu_f). When Re_p is very small, the troublesome nonlinear convective acceleration term in the Navier–Stokes equation may be neglected; thus the solution of the momentum equation becomes much easier. The second term is the "added mass" contribution, which arises because acceleration of the particle requires acceleration of its surrounding fluid. The third term is usually called "Basset history integral" or "Basset history force", which indicates effects of the history of accelerations in which t−τ is the time elapsed since the past acceleration; the instantaneous acceleration is weighted by a square root of elapsed time (t−τ). The form of the Basset history integral results from diffusion of vorticity from the particle.

Eq. (A2.11) is only applicable to situations of creeping flows (or Stokes' flow as it's usually called). For a description of instantaneous drag on a spherical particle in an arbitrary flow (high Re_p), there have been two different approaches (Clift [1978]). The first is to ignore the Basset force term (and usually also the added mass term). The instantaneous drag is derived by correlating the Stokes' drag term in eq. (A2.11) using an acceleration dependent coefficient of drag C_d. This approach has some justification for large density ratio (\rho_p/\rho_f) particle–fluid flows, which are characterized by slow variation of velocity with acceleration. The second is to extend the results of creeping flows to more general flows empirically. The form of eq. (A2.11) is taken as the basic form. The basic equation then is modified by introducing a number of coefficients to the related terms in eq. (A2.11). Efforts are made on deriving empirical correlations for such coefficients for a specific situation. Accordingly, a general expression of instantaneous drag on a particle in an arbitrary fluid flow was extended from eq. (A2.11) and is presented as follows (Clift [1978], originally proposed by Tchen [1947]):

\[
-F_{d1} = \frac{\pi R^2 \rho f C_d U_{ri} |U_r|}{2} + \frac{\rho_f V}{2} C_a \frac{dU_{zi}}{dt} + 6R^2 C_b \sqrt{\pi \rho_f \mu_f} \int_0^t \left( \frac{dU_f}{dt} \right)_t^{t-\tau} \frac{d\tau}{\sqrt{t-\tau}}
- \rho_f V \left( \frac{dU_{fi}}{dt} - \nu_f \frac{d^2U_{fi}}{dt^2} \right)
\]

(A2.12)

where F_{d1}, U_{ri}, and U_{fi} are orthogonal components of instantaneous drag \( \tilde{F}_d \), relative velocity \( \tilde{U}_{ri} \), and fluid velocity \( \tilde{U}_{fi} \), respectively.
**APPENDIX 3 TABLE OF RECOMMENDED DRAG CORRELATIONS**

(For a solid spherical particle in a steady fluid flow)

\[(\text{Re}_p = 2R|U_p-U_f|/\nu_f, w = \log_{10}\text{Re}_p; \text{Source of data: Clift}[1978])\]

<table>
<thead>
<tr>
<th>Rep Range</th>
<th>Cd Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rep &lt; 0.01</td>
<td>(C_d = \frac{3}{16} + 24/\text{Re}_p)</td>
</tr>
<tr>
<td>0.01 (\leq) Rep (\leq) 20</td>
<td>(C_d = \frac{24}{\text{Re}_p}(1 + 0.1315\text{Re}_p^{0.82-0.05w}))</td>
</tr>
<tr>
<td>20 (\leq) Rep (\leq) 260</td>
<td>(C_d = \frac{24}{\text{Re}_p}(1 + 0.1935\text{Re}_p^{0.6105}))</td>
</tr>
<tr>
<td>260 (\leq) Rep (\leq) 1500</td>
<td>(\log_{10}C_d = 1.6435 - 1.1242w + 0.1558w^2)</td>
</tr>
<tr>
<td>1500 (\leq) Rep (\leq) 1.2(\times)10(^4)</td>
<td>(\log_{10}C_d = -2.4571 + 2.5558w - 0.9295w^2 + 0.1049w)</td>
</tr>
<tr>
<td>1.2(\times)10(^4) (\leq) Rep (\leq) 4.4(\times)10(^4)</td>
<td>(\log_{10}C_d = -1.9181 + 0.6370w - 0.0636w^2)</td>
</tr>
<tr>
<td>4.4(\times)10(^4) (\leq) Rep (\leq) 3.38(\times)10(^5)</td>
<td>(\log_{10}C_d = -4.3390 + 1.5809w - 0.1546w^2)</td>
</tr>
<tr>
<td>3.38(\times)10(^5) (\leq) Rep (\leq) 4(\times)10(^5)</td>
<td>(C_d = 29.78 - 5.3w)</td>
</tr>
<tr>
<td>4(\times)10(^5) (\leq) Rep (\leq) 10(^8)</td>
<td>(C_d = 0.1w - 0.49)</td>
</tr>
<tr>
<td>10(^8) (\leq) Rep</td>
<td>(C_d = 0.19 - 8\times10^4/\text{Re}_p)</td>
</tr>
</tbody>
</table>
APPENDIX 4 POTENTIAL–FLOW SOLUTION OF A FLUID FLOW
PAST A CYLINDER

The Reynolds number is a measure of the ratio of a representative magnitude of inertia forces to that of viscous forces. When the Reynolds number is much larger than one (either velocity very high or viscosity very small), viscous forces play only negligible parts in the equation of motion over nearly all the flow field except a thin boundary layer. In cases in which separation of the boundary layer from a rigid boundary does not occur, the flow field tends to have a form close to an inviscid fluid. Even in cases of flows of a real fluid in which separation of the boundary layer does occur, there is still a large part of the flow which locally is not affected significantly by the viscosity of the fluid and to which the inviscid–fluid theory may be applied with defined separation boundaries. Analysis of flows of an inviscid fluid is much simpler than that of a viscous fluid.

With the approximation of an inviscid fluid, the mean characteristics of a separated potential flow over a two-dimensional bluff body may be treated analytically by means of free-streamline theories. Several models have been presented (e.g. Roshko[1954], Woods[1955], Parkinson[1970], Kyta[1977]). The basic idea is to employ holograph methods to transform a separation shear layer to a free streamline on another plane bounding the external potential flow. Certain properties of the free streamline (such as the tangential velocity along the line) and separation conditions (such as the separation angle) are given from experimental data. Thus the external flow problem is completely specified and can be solved using conformal transformation of complex velocities \( w(z)=U_0(z)+iV_0(z) \) and complex potential planes \( z=x+iy \), with the complex potential \( \omega(z)=\phi(z)+i\psi(z) \) as a fundamental independent variable. In the present work, the model from Parkinson(1970) is applied. The key steps are described briefly.

**Step 1: Transformation between a physical plane and a basic plane**

Fig.A4.1 shows schematically the transformation of a two-dimensional potential flow from a physical plane \( z \) to a basic plane \( Z \). The flow, which approaches to a circular cylinder with a upstream velocity of \( u_0 \), separates symmetrically at points \( s_1 \) and \( s_2 \) on the surface of the cylinder \( c \) in the physical plane.
Potential-flow Solution of A Fluid Flow past a Cylinder

The transformation is defined by the following analytic function,

\[ z = F(Z) = Z - \cot(\alpha) - \frac{1}{Z - \cot(\alpha)} \]  

(A4.1)

With this function, the upstream part \( s_1s_2 \) of the cylinder contour \( c \) is mapped conformally from the corresponding part of the circle \( C \) in the \( Z \)-plane; the angles of intersection of curves are doubled at \( s_1 \) and \( s_2 \) in the \( z \)-plane and the complete circle \( C \) is mapped onto the slit \( s_1s_2 \). Because of the doubling of angles at the critical points, stagnation streamlines leaving \( S_1 \) and \( S_2 \) in the \( Z \)-plane become tangential separation streamlines at \( s_1 \) and \( s_2 \) in the \( z \)-plane. The part of the actual cylinder contour \( c \) downstream of separation at \( s_2 \) is in the wake and is ignored.

Angle \( \alpha \) in the \( Z \)-plane in fig.A4.1 and in the transformation function in eq.(A4.1) is directly related to the separation angle \( \beta_s \) in the \( z \)-plane. If the \( Y \)-coordinate of the critical point \( S_1 \) in the \( Z \)-plane is taken as unity, we can immediately have the value of the radius of the circle \( C : R = \csc(\alpha) \). Then from eq.(A4.1), coordinates of points \( s_1 \) and \( o \) are easily derived to be \( 2i \) and \( -0.5\cot(2\alpha) \). The center of the cylinder (point \( o \)) is on the \( x \)-axis while the separation point (point \( s_1 \)) is on the \( y \)-axis. From the coordinates of these two characteristic points, a relation between \( \alpha \) and \( \beta_s \) is given as follows:

\[ \alpha = 0.5(\pi - \beta_s) \]  

(A4.2)

where the separation angle \( \beta_s \) is obtained directly from empirical data.
Eq. (A4.1) is a conformal transformation, by which the complex potential $\omega(z)$ should have same values at corresponding points in the $z-$ and $Z-$planes. Accordingly, the complex velocity $w(z)$ in the $z-$plane may be given in terms of the complex velocity $W(Z)$ in the $Z-$plane by eq. (A4.3),

$$w(z) = W(Z)/F'(Z) \quad \text{(A4.3)}$$

where $F'(Z) = 1 + \frac{1}{|Z - \cot(\alpha)|^2}$, easily derived from eq. (A4.1).

**Step 2: Determination of complex velocities in the $Z-$plane**

In the $Z-$plane, the basic flow past the circle $C$ is a familiar combination of the uniform flow in the direction of the real axis plus the flow from a suitable doublet at the origin of the circle $C$. To this the induced flow from the surface double sources of strength $2Q$ symmetrically located at angle $+\delta$ on the contour $C$ and from their image sinks $(-2Q)$ at the origin are added. The complex potential of the resulting flow is

$$\omega(Z) = U_0 Z + \frac{U_0 R^2}{Z} + \frac{Q}{\pi} \left[ \ln(Z - Re^{i\delta}) + \ln(Z - Re^{-i\delta}) - \ln Z \right] \quad \text{(A4.4)}$$

and the corresponding complex velocity is

$$W(Z) = \frac{d\omega}{dz} = U_0(1 - \frac{R^2}{Z^2}) + \frac{Q}{\pi} \left[ \frac{1}{Z - Re^{i\delta}} + \frac{1}{Z - Re^{-i\delta}} - \frac{1}{Z} \right] \quad \text{(A4.5)}$$

In eq. (A4.5), there are two unknown variables ($Q$ and $\delta$), which requires two additional relations for a solution.

One relation is from the zero velocity condition at the critical point $S_1$. The upstream flow from the surface sources creates symmetrical surface stagnation points and these are located at $S_1$ and $S_2$ by setting $W(Z) = 0$ there ($Z = Re^{i\alpha}$), from which a relation between $Q$ and $\delta$ is obtained,

$$Q = 2\pi U_0 \csc(\alpha) [\cos(\delta) - \cos(\alpha)] \quad \text{(A4.6)}$$
Another relation is the Bernoulli's equation,

\[ P + \frac{1}{2} \rho U|W(Z)|^2 = P_0 + \frac{1}{2} \rho U_0^2 \]  \hspace{1cm} (A4.7)

which relates the empirical base pressure \( P_b \) (or the base pressure coefficient \( C_{pb} \)) at the critical point \( S \) to the free-stream pressure \( P_0 \) and the free-stream velocity \( U_0 \) in the following form:

\[ k_b = \frac{|W(Z)|}{U_0} = \sqrt{\frac{1 - \frac{P_b}{P_0}}{\frac{1}{2} \rho U_0^2}} = \left( 1 - C_{pb} \right)^{1/2} \]  \hspace{1cm} (A4.8)

After substitution of eq.(A4.5) into eq.(A4.8) and some trigonometric manipulation, we can derive a relation between \( \delta \) and \( k_b \) as follows:

\[ \cos \delta = \cos \alpha + \frac{\sin \gamma \alpha}{k_b} \]  \hspace{1cm} (A4.9)

From eq.(A4.6) and eq.(A4.9), the wake strength \( Q \) and the angle \( \delta \) may be derived. As a result, the complex velocity \( W(Z) \) in the \( Z \)-plane can now be calculated from eq.(A4.5).

**Step 3: Computation of velocities in the \( z \)-plane**

To calculate the velocity at a point in the \( z \)-plane, we first have to calculate the corresponding coordinate in the \( Z \)-plane, i.e., a reverse transformation from \( z \) to \( Z \). From eq.(A4.1), we can derive the following relation,

\[ Z = \cot \alpha + \frac{1}{2} \left[ z \pm (z^2 + 4)^{1/2} \right] \]  \hspace{1cm} (A4.10)

This function is not an one-to-one transformation as in eq.(A4.1) but an one-to-two transformation. The condition for justifying the '+/' sign in eq.(A4.10) is not clear. Based on the fact that values at points in the \( z \)- and \( Z \)-planes almost have the same magnitude, we would simply take the one (of two possible values in \( Z \)-plane) which is closer to the corresponding point in \( z \)-plane during computations.
For each \( Z \), velocity \( W(Z) \) in the \( Z \)-plane is calculated from eq.(A4.5) and then is transformed to \( w(z) \) in the \( z \)-plane by eq.(A4.3).

As shown in the above deduction procedure, two empirical parameters (the separation angle \( \beta_s \) and the base pressure coefficient \( C_{pb} \)) are required for solving the equations. In the subcritical range of the Reynolds number, the separation angle increases only slightly from 80° to 85° and the base pressure coefficient is almost a constant. During computations, we simply assume that the separation angle increases linearly from 80° at \( Re_d=300 \) to 85° at \( Re_d=3\times10^5 \); The base pressure coefficient is taken as \(-0.96\), according to Parkinson(1970).

In summary, we can now provide a complete description of local (mean) velocities in the potential flow region from a set of installation parameters (the uniform upstream velocity \( U_0 \) and the cylinder radius \( r \)). Such a description of a single-phase flow past a cylinder is not our primary topic but necessary for computation of particle trajectories and analysis of interaction forces between the two phases.
APPENDIX 5 4TH-ORDER RUNGE–KUTTA ALGORITHM

(for solving 2nd-order two-dimensional ordinary differential equations)

The basic idea of the Runge–Kutta algorithm is to propagate a solution over a time interval (or a timestep) by combining the information from several Euler-style steps (each involving one equation to advance a solution over a timestep) and then using the information obtained to match a Taylor series expansion up to some higher order. To advance a solution from \( t_k \) to \( t_{k+1} = t_k + h \), we may have an Euler’s step as follows:

\[
x_{k+1} = x_k + hF'(t_k, x_k)
\]

where \( h \) is the timestep; \( F'(t_k, x_k) \) defines a relation for the first-order differential of \( x_k \) at \( t_k \) and \( x_k \) and \( t_k \) (\( dx_k/dt = F'(t_k, x_k) \)).

Eq. (A5.1) actually includes a first-order Taylor series expansion. If we utilize a fourth-order Taylor series expansion, we may write the following equation,

\[
x_{k+1} = x_k + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) + O(h^5)
\]

where

\[
\begin{align*}
k_1 &= hF'(t_k, x_k) \\
k_2 &= hF'(t_k + \frac{h}{2}, x_k + \frac{k_1}{2}) \\
k_3 &= hF'(t_k + \frac{h}{2}, x_k + \frac{k_2}{2}) \\
k_4 &= hF'(t_k + h, x_k + k_3)
\end{align*}
\]
The error term in eq. (A5.2) is the fifth power of the timestep. In the same manner for a second-order two-dimensional case, we may obtain solutions as follows:

\[
\begin{align*}
    x_{k+1} &= x_k + hx'_k + \frac{h}{8}(K_1 + K_2 + K_3) \\
    y_{k+1} &= y_k + hy'_k + \frac{h}{8}(M_1 + M_2 + M_3) \\
    x'_{k+1} &= x_k + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \\
    y'_{k+1} &= y_k + \frac{1}{6}(M_1 + 2M_2 + 2M_3 + M_4) \\
\end{align*}
\]

(A5.3)

where

\[
\begin{align*}
    K_1 &= hF_x(t_k, x_k, y_k, x'_k, y'_k) \\
    M_1 &= hF_y(t_k, x_k, y_k, x'_k, y'_k) \\
    K_2 &= hF_x(t_k + \frac{h}{2}, x_k + \frac{h}{2}x'_k, y_k + \frac{h}{2}y'_k, x'_k + \frac{K_1}{2}, y'_k + \frac{M_1}{2}) \\
    M_2 &= hF_y(t_k + \frac{h}{2}, x_k + \frac{h}{2}x'_k, y_k + \frac{h}{2}y'_k, x'_k + \frac{K_1}{2}, y'_k + \frac{M_1}{2}) \\
    K_3 &= hF_x(t_k + h, x_k + hx'_k + \frac{h}{4}K_1, y_k + \frac{h}{2}y'_k + \frac{h}{4}M_1, x'_k + \frac{K_3}{2}, y'_k + \frac{M_3}{2}) \\
    M_3 &= hF_y(t_k + h, x_k + hx'_k + \frac{h}{4}K_1, y_k + \frac{h}{2}y'_k + \frac{h}{4}M_1, x'_k + \frac{K_3}{2}, y'_k + \frac{M_3}{2}) \\
    K_4 &= hF_x(t_k + h, x_k + hx'_k + \frac{h}{2}K_1, y_k + hy'_k + \frac{h}{2}M_1, x'_k + K_3, y'_k + M_3) \\
    M_4 &= hF_y(t_k + h, x_k + hx'_k + \frac{h}{2}K_1, y_k + hy'_k + \frac{h}{2}M_1, x'_k + K_3, y'_k + M_3) \\
\end{align*}
\]
APPENDIX 6 MAGNITUDE OF INTERACTION FORCES
IN NON–UNIFORM PARTICLE–LIQUID SYSTEMS

Particle trajectory computations are of direct interest in many practical situations. Many models have been employed for computing particle trajectories and studying related interaction process in non–uniform system, e.g., Petrov(1988), Kallo(1989), and Zhung(1989). Simple momentum equations are usually applied, which mostly include only a steady drag force where the drag coefficient is often based on empirical data. Other forces, especially the Basset history force, are often assumed negligible.

Using the momentum equation in general form, we now investigate the interaction process when a particle–liquid flow approaches a circular cylinder. Our aim is to shed more light on the conditions under which the various forces may be neglected, in particular in flows around obstacles. Buoyancy forces are excluded from the momentum equation for the sake of generality. The flow liquid field is assumed steady and is defined by the inviscid potential flow solution of the equation of motion, without considering the wake flow behind the cylinder since our primary goal is for comparison with results of other investigations mentioned above. Trajectories of particles are computed from some distance upstream the cylinder. By tracking the trajectories of particles, the magnitude of each individual force and the significance of main system parameters in the two–phase interaction process may be found and analyzed qualitatively.

In a particle–liquid system, the density of the solid particle mostly is rather close to that of the liquid. Accordingly the two phases are coupled quite tightly and have small relative velocities when they execute a rectilinear motion. Separation of the two phases is expected to be small when they approach the cylinder. An example of the instantaneous particle–, liquid–, and relative–velocities and accelerations along a particular trajectory in fig.A6.2A (for which the parameters will be given in the next paragraph) is shown in fig.A6.1A and fig.A6.1B. It is clearly shown how a particle responds to changes of the liquid motion when it approaches the cylinder.
Fig.A6.1A Velocities along Trajectory Line 3(b) in fig.A6.2A.

\( V_x \text{ (m/s)} \)

\begin{align*}
1 & \quad 2 \\
-2D & \quad -D \\
0 & \quad 0 \\
1D & \quad 2D \\
\end{align*}

\( V_y \text{ (m/s)} \)

\begin{align*}
1 & \quad 2 \\
-2D & \quad -D \\
0 & \quad 0 \\
1D & \quad 2D \\
\end{align*}

a) \( X \)-Components  
b) \( Y \)-Components

\( U_{fo}=4 \text{m/s}; \ D=25 \text{mm}; \ dp/D=0,01; \ \rho_f=10^3 \text{kg/m}^3; \ \rho_p/\rho_f=2 \)

Index: (1) Liquid velocity; (2) Particle velocity; (3) Relative velocity;

Fig.A6.1B Accelerations along Trajectory Line 3(b) in fig.A6.2A.

\( A_x \text{ (m/s}^2) \times 10^3 \)

\begin{align*}
1 & \quad 2 \\
-2D & \quad -D \\
0 & \quad 0 \\
1D & \quad 2D \\
\end{align*}

\( A_y \text{ (m/s}^2) \times 10^3 \)

\begin{align*}
1 & \quad 2 \\
-2D & \quad -D \\
0 & \quad 0 \\
1D & \quad 2D \\
\end{align*}

a) \( X \)-Components  
b) \( Y \)-Components

\( U_{fo}=4 \text{m/s}; \ D=25 \text{mm}; \ dp/D=0,01; \ \rho_f=10^3 \text{kg/m}^3; \ \rho_p/\rho_f=2 \)

Index: (1) Liquid acceleration; (2) Particle acceleration; (3) Relative acceleration;
Among many parameters which determine trajectories of particles in such a non-uniform system, the ratio of particle density to liquid density ($\rho_p/\rho_l$), the particle diameter ($d_p$) and the upstream liquid velocity ($U_{fo}$) play the most important roles. $U_{fo}$ decides the magnitude of changes of the liquid velocity field while $\rho_p/\rho_l$ and $d_p$ determine the magnitude of the response of particles to changes of the liquid flow. Below we will illustrate and discuss influence of each of the three system parameters on trajectories of particles and magnitude of interaction forces.

Three sets of trajectories of particles have been computed for different values of the three system parameters. Results are shown in fig.A6.2A, A6.2B, and A6.2C. Main parameters are listed in table A6.1.

Table A6.1 Main parameters for computing particle trajectories in liquid flows past a cylinder.

<table>
<thead>
<tr>
<th>Case Nr.</th>
<th>$U_{fo}$ (m/s)</th>
<th>$d_p/D$</th>
<th>$\rho_p/\rho_l$</th>
<th>$Re_d (U_{fo}D/\nu_l)$</th>
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<td>100000</td>
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<td>4.0</td>
<td>1/100</td>
<td>2</td>
<td>100000</td>
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<td>1/100</td>
<td>1.2</td>
<td>100000</td>
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<td></td>
</tr>
<tr>
<td>A6.2B</td>
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<td>100000</td>
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<td>100000</td>
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<tr>
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<td>100000</td>
</tr>
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<td></td>
</tr>
<tr>
<td>A6.2C</td>
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<td>2</td>
<td>100000</td>
</tr>
<tr>
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<td>1/10</td>
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</tr>
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<td></td>
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</tr>
</tbody>
</table>

(D=25mm, $\rho_l=1000kg/m^3$, $\nu_l=10^{-4}m^2/s$)

(Particles are introduced at the same speed as water at four positions of 0.1D, 0.2D, 0.4D, 0.6D at a distance of 3D upstream from the y-axis.)
Fig. A6.2A Trajectories of Particles of Various Densities;
D = 25mm; $\rho_l = 10^3$kg/m$^3$; $U_f_0 = 4$m/s; $d_p/D = 0.01$;
index: (●) $\rho_p/\rho_l = 10$, (⋆) $\rho_p/\rho_l = 2$, (▲) $\rho_p/\rho_l = 1.2$;

Fig. A6.2B Trajectories of Particles of Various Sizes;
D = 25mm; $\rho_l = 10^3$kg/m$^3$; $U_f_0 = 4$m/s; $\rho_p/\rho_l = 2$;
index: (●) $d_p/D = 0.01$, (⋆) $d_p/D = 0.05$, (▲) $d_p/D = 0.1$;

Fig. A6.2C Trajectories of Particles for Different Liquid Velocities;
D = 25mm; $\rho_l = 10^3$kg/m$^3$; $\rho_p/\rho_l = 2$; $d_p/D = 0.01$;
index: (●) $U_f_0 = 4$, (⋆) $U_f_0 = 2$, (▲) $U_f_0 = 0.4$; [m/s]
Fig. A6.3A Interaction Forces along Trajectory $3^*$, $3^*$ & $3^*$ in fig. A6.2A; index: (1) $F_{sd}$, (2) $F_{lg}$, (3) $F_{pg}$, (4) $F_{am}$; (5) $F_{basset}$ (see eq. (2.5))
Fig. A6.3B Interaction Forces along Trajectory $3^*$, $3^*$ & $3^\wedge$ in fig. A6.2B;
index: (1) $F_{sd}$ (2) $F_{Ls}$ (3) $F_{pg}$ (4) $F_{ami}$ (5) $F_{basset}$ (see eq. (2.5))

Fig. A6.3C Interaction Forces along Trajectory \(1^*, 2^*, 3^*\) & \(3\) in fig. A6.2C; index: (1) \(F_{sd}\), (2) \(F_{Lg}\), (3) \(F_{pg}\), (4) \(F_{am}\), (5) \(F_{basset}\) (see eq. (2.5))
A6.1 Influence of Particle Density

Fig.A6.2A shows three groups of particle trajectories computed with different particle densities ($\tau=\rho_p/\rho_l=10, 2, 1.2$). Particles of small density will follow to some extent the liquid streamlines and hence will not collide with the cylinder except in a very small region near the stagnation point. For particles which do not collide with the cylinder, trajectories of high density particles tend to spread away from the cylinder while those of low density converge towards the $x$ axis. It is easy to see the reason when looking at eq.(2.6), where the particle densities appear only on the left side of the equation. The bigger $\rho_p$ is, the smaller the particle accelerations will be. High density particles have slower responses than low density ones and consequently their trajectories deviate more from streamlines of the liquid.

Fig.A6.3A shows forces along three of the trajectories (shown in fig.A6.2A) on particles of different densities, computed from the same initial position. The steady drag force, which is mainly proportional to the square of the relative velocity, is dominant for high density particles and plays a decreasing role as the particle density decreases. Low density particles response to the liquid much faster and therefore will have smaller relative velocities than the high density ones. The shear lift force, which is dependent on both the liquid velocity gradient and the relative velocity, appears always very small compared to other forces and thus can be ignored safely without affecting the force balance. This also indicates that the effect of liquid velocity gradients is much smaller than the effect of liquid accelerations in such non-uniform particle-liquid flows. The pressure-gradient force, which is actually the 'driving force' on the particle by the non-uniform ambient liquid and depends fully on liquid accelerations, increases as the particle density decreases. While the particle density decreases, the liquid field does not change. The effect of the 'stable' pressure-gradient force thus becomes more significant compared to the 'decreasing' drag force. The added-mass force, which depends on the relative acceleration concerning effects of both the liquid non-uniformness and the response of the particle, has a rather constant influence and always plays a considerable role in the force balance. The Basset history force, which accounts for the history of particle accelerations and is very difficult to analyze directly due to its implicit form, has a considerable contribution in all three cases, especially for medium density particles. It is also observed that the Basset history force increases its influence if the difference between the the drag force and pressure-gradient force decreases.
A6.2 Influence of Particle Size

Fig.A6.2B shows three sets of particle trajectories computed with different particle diameters (d/D=0.01, 0.05, and 0.1). It can be seen that the size of particles has very big influence on the force balance. Small particles tend to move over the cylinder and bigger particles obviously have much higher chance to collide with the cylinder than the small ones. The smaller the particles are, the less the deviation from the liquid streamlines, as is to be expected.

Fig.A6.3B shows the forces along three of the trajectories (shown in fig.A6.2B) of particles of different diameters, computed from the same initial position. Among the five forces, the pressure-gradient force increases very much as the particle size increases and becomes dominant for very big particles. The added-mass force shows a similar trend. This is simply because the above two forces are proportional to the volume of the particle (d^3) whereas the other three forces are proportional to the surface area of the particle (d^2). Accordingly the steady drag force plays a diminishing role as the particle size increases. The shear lift force is very small again and can be ignored safely. The Basset history force, which still plays a very considerable role in the force balance, not only changes congruously with the drag force again but also is of the same order as the drag force, especially so in front of the cylinder. The Basset history force is shown again to have an increasing effect as the difference between the pressure-gradient force and the drag force decreases, the same observation we made previously.

A6.3 Influence of Upstream Liquid Velocity

Fig.A6.2C shows three sets of particle trajectories computed with different upstream liquid velocities (U_in=4, 2, and 0.4m/s). It is seen that the upstream liquid velocity has only a small effect on the force balance. With high upstream liquid velocities, particle trajectories are a bit closer to the cylinder than those with low ones.

Fig.A6.3C shows the forces along three of the trajectories (shown in fig.A6.2C) of particles in liquid flows with different upstream velocities, computed from the same initial position. Slow liquid flows induce both small relative velocities and relative accelerations. As a result, the effect of either the steady drag force or the added-mass force decreases as the upstream liquid velocity decreases. The liquid acceleration, on the other hand, will maintain its relative significance as the upstream liquid velocity decreases. Accordingly the pressure-gradient force plays a dominant role in all three cases. The shear lift force is negligible again. It is no surprise to see that the Basset
history force increases greatly as the upstream liquid velocity decreases. It is almost of the same magnitude as the pressure-gradient force when the upstream liquid velocity becomes very small. This is obviously because it takes longer for the particles to pass the cylinder as the liquid moves slower, so the Basset history force (which is the only one related to time in the force balance on the particle) increases as the upper limit of the Basset history integral increases.

**A6.4 Concluding Summary**

(1) Particle density and diameter are the dominant factors for determining particle trajectories. High density (or large) particles tend to collide with the cylinder more easily than low density (or small) ones;

(2) The influence of the upstream liquid velocity on the force balance is rather small.

(3) The drag force is only superior over other forces in the force balance if the ratio of the particle density to the liquid density is very large.

(4) The shear lift force is very small and can be safely ignored;

(5) The added-mass force only takes an important part in the force balance when the particle size is large;

(6) The pressure-gradient force is generally a dominant force. Its magnitude relative to other forces increases if the particle densities is decreased or if the particle size and the upstream liquid velocity is increased;

(7) The Basset history force appears to change approximately congruously with the steady drag force and plays a considerable role in the force balance. The effect of the Basset history force in the force balance increases if the difference between the pressure gradient force and the steady drag force decreases. It appears that no other forces is important than the Basset history force if a particle is continuously accelerated.

Since the particle-liquid flow approaching to a circular cylinder represents a quite common non-uniform particle-liquid system, the above conclusions can probably be generalized to other non-uniform particle-liquid systems.
APPENDIX 7 MAGNITUDE OF INTERACTION FORCES IN NON-UNIFORM PARTICLE/DROP-GAS SYSTEMS

Particle-gas or drop-gas two-phase flows pass a cylinder are very usual in the process and chemical industry. Predictions of particle trajectories in such situations is of a considerable importance in many studies, such as: 1) heat transfer between a gas-liquid mist flow and a cylindrical object; 2) efficiency of filters in capturing particles from a carrier fluid; 3) the erosion damage by a gas-solid or gas-droplet flow in steam or gas turbines; and so on.

Many models have been employed to compute particle trajectories in a gas flow around a cylinder, e.g., Morsi (1972), Pawlowiki (1984), and Aihara (1986). Simple momentum equations are mostly constructed, which usually include only the steady drag force and sometimes the buoyancy force. The force terms caused by the pressure-gradient effect, the added-mass, and the Basset history force are assumed to be negligible, except when the density of fluid is equal to or greater than the density of the particle (White [1986]). This assumption simplifies the momentum equation greatly and is quite true for uniform flow situations. But for non-uniform flow situations, such as particles injected into a flow around a cylinder, deviations may result.

For a particle (or a drop) moving in a gas flow, the surrounding gas exerts forces to keep the particle following the gas flow in response to the relative velocity and the relative acceleration between the particle and the gas. All forces depend on the relative velocity or the relative acceleration or the gas acceleration. The velocities and the accelerations show very different patterns, depending on the nature of the flow fields (uniform or non-uniform). So are the magnitude of forces in the force balance on the particle.

When a gas flow is steady and uniform, where the gas acceleration and the gas velocity gradient equal zero, the shear lift force and the pressure-gradient force both equal zero and the relative acceleration equals the particle acceleration. Consequently the relative acceleration is much smaller than relative velocity as the ratio of particle density to gas density \( \gamma = \rho_p / \rho_g \) is very large (usually of the order of the 1000). As a result the added-mass force and the Basset history force are negligible compared to the steady drag force. Therefore it is a rather valid assumption to include only the steady drag force in the force balance equation, as many cases found in literature.
But when a gas flow is non-uniform, the gas acceleration may be quite large due to gas velocity gradients. So the relative acceleration will differ from the particle acceleration. The forces ignored in the previous situation ($F_{am}$, $F_{pg}$, and $F_{basset}$), which depend on either $dU/dt$ or $dU_f/dt$, may take significant parts in the force balance. So does the shear lift force because $k$ is not zero any more.

To illustrate the magnitude of forces in the interaction process between a particle and a non-uniform gas flow, we now employ the force balance equation to compute trajectories of particles and to track the interaction process along the trajectories in a similar manner as we did in the previous section. The buoyancy force is excluded again in the force balance equation for simplicity and generality. The gas field is steady and calculated by the general inviscid solution without considering a wake behind the cylinder. The magnitude of the interaction forces and the significance of the two main system parameters (the particle diameter $d_p$ and the upstream gas velocity $U_{fo}$) are illustrated and analyzed.

Two groups of computations of particle trajectories, including either only a steady drag force or all the interaction forces in the force balance, are shown in fig.A7.1 and fig.A7.2.

Table A7.1 Main parameters for computing particle trajectories in gas flows past a cylinder.

<table>
<thead>
<tr>
<th>Case Nr.</th>
<th>$U_{fo}$ (m/s)</th>
<th>$d_p/D$</th>
<th>$\rho_p/\rho_f$</th>
<th>$Re_d (U_{fo}D/\nu_f)$</th>
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<td></td>
<td>0.04</td>
<td>1/100</td>
<td>833</td>
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</table>

(D = 25mm, $\rho_f = 1.2$kg/m$^3$, $\nu_f = 10^{-5}$m$^2$/s)

(Drops are introduced at the same speed as gas at four positions of 0.1D, 0.2D, 0.4D, 0.6D at a distance of 3D upstream from the y-axis.)
Fig. A7.1 Trajectories of particles in a gas flow approaching to a cylinder; 

D=25mm; U_\text{ref}=4m/s; (a) d_\text{p}/D=0.001; (b) d_\text{p}/D=0.01; (c) d_\text{p}/D=0.1;

index: 
(a) F_{sd}, F_{L_d}, F_{am}, F_{pg}, and F_{basset} are all included;
(b) Only F_{sd} is included.
Fig. A7.2 Trajectories of particles in a gas flow approaching to a cylinder;

D = 25mm; d_p/D = 0.01; (a) $U_{f0} = 4m/s$; (b) $U_{f0} = 0.4m/s$; (c) $U_{f0} = 0.04m/s$;

Index: 

- (a) $F_{sd}$, $F_{Ls}$, $F_{am}$, $F_{pg}$, and $F_{basset}$ are all included;
- (c) Only $F_{sd}$ is included.
Fig.A7.3A X—components of Forces along Trajectories in fig.A7.1.
index: (1) $F_{sd}$; (2) $F_{Lg}$; (3) $F_{am}$; (4) $F_{pg}$; (5) $F_{basset}$.

Fig.A7.3B Y—components of Forces along Trajectories in fig.A7.1.
index: (1) $F_{sd}$; (2) $F_{Lg}$; (3) $F_{am}$; (4) $F_{pg}$; (5) $F_{basset}$.
Fig. A7.4A  X—components of Forces along Trajectories in fig.A7.2.
index: (1) $F_{sd}$; (2) $F_{Lg}$; (3) $F_{ami}$; (4) $F_{pgi}$; (5) $F_{basset}$.

Fig. A7.4B  Y—components of Forces along Trajectories in fig.A7.2.
index: (1) $F_{sd}$; (2) $F_{Lg}$; (3) $F_{ami}$; (4) $F_{pgi}$; (5) $F_{basset}$.
A7.1 Influence of Particle Size

Fig. A7.1 shows three groups of particle trajectories with three different particle sizes \( \frac{d_p}{D}=0.001, 0.01, 0.1 \). Each group consists of two sets of particle trajectories computed from four initial positions, including either only the steady drag or all interaction forces in the force balance equation. It clearly shows a deviation between the trajectories computed with or without the "additional forces" \( F_{am}, F_{pg}, \) and \( F_{basset} \). The deviation becomes large when the particle size is big.

Fig. A7.3A and A7.3B show the forces on particles along one particular trajectory in each of those three groups (the trajectory 3s in fig. A7.1). Among all the interaction forces, the effect of the steady drag force is dominant in all cases just as expected. The shear lift force, whose ratio to the steady drag force is proportional to \( d_p \) as shown in table 2.1, increases gradually as \( d_p \) increases. The magnitude of the \( F_{Ls} \) and \( F_{Ls} \) mainly depend on the ratio of the relative velocities \( U_{rj}/U_{ri} \) and \( U_{ri}/U_{rj} \). For particles injected into a \( x \)-direction gas flow, \( U_{rj} \) is quite small but \( U_{ri} \) is big, especially for large particles. As a result, fig. A7.3 shows that \( F_{Ls} \) are relatively small compared with other three forces but \( F_{Ls} \) become more and more dominant as \( d_p \) increases. The sign of \( F_{Ls} \) is decided by the multiplication of vectors \( \vec{e}_k \) and \( \vec{U}_{rj} \), where \( \vec{e}_k \) fully depends on the gas flow field. In \( x \)-direction, it enforces the change of \( F_{sd} \) except for a short period when the particle move close to the cylinder. In \( y \)-direction, it "lifts particles up" both in front of and behind the cylinder and "pulls particles down" as the particles pass the cylinder. At the early stage of particle acceleration, it is even bigger than the steady drag force and is the main cause for deviations of two sets of trajectories in fig. A7.1. The added—mass force and the pressure—gradient force, whose ratios to the steady drag are both proportional to the particle diameter, increase greatly as \( d_p \) increases. And since \( dU_{rj}/dt = -dU_{rj}/dt \) for particle—gas non—uniform systems as we have discussed, the above two forces both mainly depend on the gas flow field and have the same sign. Their added contribution in the total force is approximately equivalent to one and half times pressure—gradient force.

The Basset history force, whose ratio to the steady drag is proportional to \( d_p \), increases gradually as \( d \) increases. In \( x \)-direction, it decelerates particles reverse to \( F_{sd} \) on the two sides of the cylinder. For small particles, which have very fast response to the gas flow and can achieves same velocity as gas in front of the cylinder, the negative peak appears at the early stage as shown in fig. A7.1B. But for big particles, which have a much slower response to the gas flow and the distance from initial position to the cylinder is too short
for particles to reach the same speed as the gas, no negative peaks appear. In y-direction, $F_{\text{Basset}}$ enforces the steady drag force all the way and changes almost at the same pace as $F_{\text{ad}}$. This makes $F_{\text{Basset}}$ contribute very much to the deviations of trajectories in fig.A7.1, especially at the early stage of particle acceleration. In general, the Basset history force plays a more significant role in the force balance than the other three additional forces for small particles as shown in fig.A7.3.

**A7.2 Influence of Upstream Gas Velocity**

Fig.A7.2 shows three groups of particle trajectories with three different upstream gas velocities ($U_{fo} = 4, 0.4, 0.04 \text{ m/s}$). Each group consists of two sets of particle trajectories computed from four initial positions, including either only the steady drag or all the interaction forces in the force balance equation. Although the small upstream gas velocity will not any more satisfy the requirement of large Reynolds number fluid flow for applying the inviscid solution, we still include such cases for providing a qualitative illustration.

Fig.A7.4A and A7.4B show forces on particles along one particular trajectory in each of those three groups (the trajectory 3s in fig.A7.2). As the gas upstream velocity decreases, the magnitude of both the gas velocities and the gas velocity gradients around the cylinder decrease. Accordingly, the magnitude of the relative velocities and relative accelerations decrease. So do the ratios of the four additional forces to the steady drag force. As a result, differences of trajectories decreases. Among all the interaction forces, the effect of the steady drag force is dominant again in all cases. Among the other four additional forces, the added-mass force and the pressure-gradient force are small and negligible. The shear lift force doesn't play important role for this mediate-size particle situation as we have just concluded. It is the Basset history force which has significant influence on the force balance and accounts the main part for the deviations of trajectories computed with or without including the additional forces in fig.A7.2. Especially at the early stage, the Basset history force reduces the acceleration effect by the steady drag in x-direction and enforces the "lifting up" effect of the steady drag in y-direction. Consequently trajectories of particles are pushed away further from the cylinder in addition to the steady drag force. The influence of the Basset history force decreases as the upstream gas velocity decrease.
A7.3 Concluding Summary

(1) The influences of the shear lift force, the added-mass force, the pressure-gradient force, and the Basset history force on the particle trajectories can be ignored when the particle size or the upstream gas velocity is small.

(2) When the particle size increases, the relative magnitude of the shear lift force in the total force balance increases. For situations involving large size particles, effects of the shear lift force is significant and thus must be considered.

(3) The Basset history force is the most significant additional force (in addition to the steady drag force) in the total force on particles except when the particle size is very large. Effects of the Basset history force can only be ignored for situations involving very small-size particles (d_p/D=1/1000) or very slow gas flows (U_{fo}=0.04m/s).

(4) The added-mass force and the pressure-gradient force have same effect in the force balance. Their combined effect is approximately equivalent to three times the added-mass force for spherical particles. Even their combined effect is still small and thus can be ignored safely.

(5) All additional forces tend to enforce the steady drag force, so trajectories of particles computed by the general form force balance move close to the gas flow streamlines than those by the momentum equation with only the steady drag force included.

Since particle-gas flow past a cylinder is a typical non-uniform case, we can expect that the conclusions are also applicable to many other cases.
APPENDIX 8 FILTERING NOISES IN DISTRIBUTION MEASUREMENTS

In the majority of measurement situations, the physical quantity being measured has a value which is either constant or changing only slowly with time. In these circumstances, the most common types of signal distortion are high-frequency noise components, and the type of signal processing element required is a low-pass filter. In this appendix, we present a simple low-pass filtering function, which was first found incidentally during our program development and then formulated according to the digital signal analysis theory.

As shown in section 3.2.2, the first step in the data processing is to extract samples from a set of "raw" discrete measurements. With reference to the internal edge of the tube wall, which coincides with the local minimum (as a "valley") in a line-average density distribution measurement, two border lines are moved one (discrete) measurement by one (discrete) measurement on the screen (controlled from the keyboard in the program) to extract required samples just covering the concerned region from the set of raw measurements. However, such a procedure mostly does not extract a set of samples exactly covering the flow region in the tube (i.e. the first and the last samples do not both lie on edges of internal tube walls) because of the nature of the discrete data and the rather thick gamma beam (2mm in diameter). A secondary procedure is thus needed to move the border lines in smaller (or finer) steps. This is done by applying an extrapolation procedure as shown below:

\[
\begin{align*}
\text{Move thread lines } \delta \text{ to the right:} & \quad I_2[k] = I_1[k] + \delta(\bar{I}_1[k+1] - I_1[k]); \\
\text{Move thread lines } \delta \text{ to the left:} & \quad I_2[k] = I_1[k] + \delta(\bar{I}_1[k-1] - I_1[k])
\end{align*}
\]  

(A8.1)

where \(I[k]\) denotes the \(k\)-th sample in the set of discrete measurements and \(\delta\) the fraction of a sample width (for simplicity, the sample width is taken as unity here).

It was a bit astonishing to discover that such a fine extraction procedure would not only enable us to extract a set of samples exactly covering the required region but also help smooth the distribution data. While causing little distortion on the shape of a set of measurements, this procedure attenuates high frequency noise fluctuations very effectively.
If we consider such a pair of movements (to the right once and then back to the left once) as one filtering step (or the set of data passing a filter once), we can derive the following equation by combining two relations in eq. (A8.1):

\[ I_2[k] = \delta(1-\delta)I_1[k-1] + ((1-\delta)^2 + \delta^2)I_1[k] + \delta(1-\delta)I_1[k+1] \]  

(A8.2)

If we propose an arbitrary set of discrete data as shown as \( I_1[k] \)s in fig. A8.1, we will get \( I_2[k] \)s after one movement to the left and then \( I_3[k] \)s after one movement to the right. The "smoothing" effect of such procedures is illustrated obviously in fig. A8.1.

![Fig. A8.1 Fine extraction of samples from raw measurements.](image)

An explanation for such a smoothing effect lies in the theory of digital signal analysis. For a Input/Output description of a filter, we first rewrite eq. (A8.1) as follows:

\[ I_0[k] = \sum_{i=-1}^{1} \{ h[i] I_1[k-i] \} \]  

(A8.3)

where \( I_1 \) and \( I_0 \) are one sample before and after a filtering step, respectively; \( h[i] \)s are the transfer–functions in the following form:

\[ h[-1]=\delta(1-\delta) \quad h[0]=(1-\delta)^2+\delta^2 \quad \text{and} \quad h[1]=\delta(1-\delta) \]  

(A8.4)
To look at the frequency response of such a filtering step, we only need to transform the function $h[l]$ from time domain to the frequency domain by the discrete Fourier transformation (DFT, $H[e^{j\omega}] = \sum h[l]e^{-j\omega l}$) as follows:

$$H[e^{j\omega}] = \sum_{l=-\infty}^{\infty} \{h[l]e^{-j\omega l}\} = h[-1]e^{j\omega} + h[0] + h[1]e^{j\omega}$$  \hbox{(A8.5)}

Combining eq.(A8.4) and eq.(A8.5), we have the following equation:

$$H[e^{j\omega}] = (1-\delta)^2 + \delta^2 + 2\delta(1-\delta)\cos(\omega) \hbox{ (\omega=0..\pi)}$$  \hbox{(A8.6)}

If $\delta$ is taken as 0.1 (i.e. 10% of one sample width), we have eq.(A8.7) and the corresponding function curve in fig.A8.2.

![Fig.A8.2 Frequency response function (1 filtering step).](image)

$$H[e^{j\omega}] = 0.82 + 0.09 \cos(\omega)$$  \hbox{(A8.7)}

In fig.A8.2, it is shown that the transfer-function ($h[l]$) we have just derived acts as a "low-pass filter" to attenuate high frequencies. According to Roberts[1987], the transfer-function $h[l]$ is a kind of "tap weight". The resulting output is merely the average of its neighbour values (or the average of the present, past, and future values in a time domain). Thus the filter (or filtering function) acts to "smooth" the input. In other words, the frequency response of the filter attenuates high frequencies.
Moreover, instead of filtering a set of data once as it was done in eq.(A8.2), we can filter data with the same filtering function n times until satisfactory results are obtained. The equivalent transfer-function (eq.(A8.8)) of n times filtering can be easily derived in benefit of the linear (also shift-invariant) property of the system. Eq.(A8.8) is shown as follows:

\[ H_n[\omega] = \{H[\omega]\}^n = \{(1 - \delta)^2 + \delta^2 + 2\delta(1 - \delta)\cos(\omega)\}^n \quad (A8.8) \]

The corresponding frequency response function of \( n = 1, 2, 8, 20, 50, \) and 100 are illustrated in fig.A8.3 with \( \delta = 0.1 \) (which is chosen by a "try-out" procedure, depending on particular cases). As \( n \) is increased, the attenuation effect on high frequencies becomes stronger and stronger. At very high \( n \), part of high frequencies is almost cut off.

![Frequency response functions with varying number of filtering steps (n).](image)

In general, we need a tryout procedure with different \( n \) for each specific application. For our situation (the void fraction distribution measurements over a vertical tube), it has been found appropriate to have \( n = 20 \) for calibration measurements (\( I_g \) and \( I_l \)) and \( n = 8 \) for two-phase flow measurements.
APPENDIX 9 COMPENSATION OF WINDOW REFRACTION EFFECT

For the high pressure (± 150 bar) and highly corrosive conditions inside the test tube, windows on the test tube must have very high resistance both physically and chemically. Accordingly two thick (13mm) sapphire cylindrical windows are used for the application of the photographic system.

On the side of the illumination unit, parallel light rays pass through the window perpendicularly. They only lose strength to some extent but will not change their direction of propagation. On the side of the camera unit, not all light rays enter and leave the window perpendicularly. In fact, a "bright spot" on a photo results from deviation of parallel light rays passing a bubble in water. When light rays do not enter the window perpendicularly, a large refraction effect occurs due to the high refraction index (1.77) of the sapphire window. The effect induces a deviation in the object distance ($u_b$) and must be compensated for in the computations of image formation.

An optical diagram to illustrate the window refraction effect is shown in fig.A9.1.

Fig.A9.1 Optical diagram of the window refraction effect.
The deviation of the object distance ($u_b$), which is the distance from the object lens to the object, is given by the following relation:

$$\Delta u_b = u_b' - u_b = (L_w + L_{w1} + \frac{0.5D_1 - L_{w1}\tan \theta_1 - L_{w2}\tan \theta_2}{\tan \theta_3}) - u_b \quad (A9.1)$$

where $u_b'$ and $u_b$ are actual and computed object distance, respectively; $L_w$ denotes the thickness of the window and $L_{w1}$ the distance from the window to the object lens; $D_1$ is the diameter of the object lens; $\theta_1$, $\theta_2$, and $\theta_3$ are determined by eq. (A9.2) and eq. (A9.3) as follows:

$$\theta_1 = \arctan \left( \frac{D_1/2}{u} \right) \quad (A9.2)$$

$$n_{air}\sin \theta_1 = n_{sapphire}\sin \theta_2 = n_{water}\sin \theta_3 \quad (A9.3)$$

where $n_{air}$, $n_{sapphire}$, and $n_{water}$ are refraction indexes of air, sapphire, and water. Since $n_{sapphire}(=1.77) > n_{water}(=1.33) > n_{air}(=1.0)$, $\Delta u_b$ derived from eq. (A9.1) should be always positive. Eq. (A9.3) is in fact the refraction law in geometrical optics (Klein[1986]), assuming water on the left side of the window and air on the right side.

When analyzing photos obtained by the photographic system, we first calculate the object distance ($u_b = |L_{10} - F_{ol}|$) from the size of a bright spot ($D_b$) according to the equations defined in section 3.3.2. Then we can calculate a compensation $\Delta u_b$ from eq. (A9.1) and add $\Delta u_b$ to $u_b$ for an actual object distance ($u_b'$).
APPENDIX 10 VOID FRACTION DISTRIBUTIONS OVER A VERTICAL TUBE

Both the atmospheric-pressure loop and the high-pressure loop consist of a main vertical test-section of the circular shape. Over a vertical tube of circular cross-section, flows generally have axis-symmetrical nature. A set of void fraction distribution measurements by the gamma-ray densitometer system is actually a projection of chordal-average void fractions over the circular tube. Without existence of an obstacle, void fraction distribution measurements are at first taken under various mean velocities of water flows. These measurements may be considered approximately to represent upstream void fraction distributions in the presence of an obstacle under the same mean velocity of water flow. Three examples of void fraction distributions at various mean upstream liquid velocities are shown in fig.A10.1.

Fig.A10.1 Chordal-average void fraction distributions over a circular vertical test tube. (without presence of an obstacle; $U_o=0.5, 1.2, \text{ and } 2.4 \text{m/s}$)
Local void fractions are shown to be uniformly distributed over the central part of a circular tube under high mean water velocities. Near the tube wall, bubbles tend to migrate toward the tube wall to form a high void concentration region. In reference to the axis-symmetrical nature of a vertical flow in a circular tube, such local high void fraction values would lift up level of chordal-average void fraction measurements ($\Delta \alpha/\alpha$ is estimated to be about 10%) but would have little disturbance on the distribution patterns of chordal-average void fraction over the tube.

For experiments to measure void fraction distributions around a cylinder, the cylinder is installed on a central line of a cross-section of a test tube. With small ratios of cylinder diameter to tube diameter, especially for experiments performed in the high-pressure loop where the cylinder diameter is much smaller than the tube internal diameter, void fraction distribution measurements covers a relative narrow part in the center of the tube. Upstream the cylinder, a uniform void fraction distribution profile may be expected at high $U_0$. In the wake region behind the cylinder, high void concentration near the tube wall is observed not to happen in the direction of the gamma beam, mainly due to a dead-fluid zone appearing in the corner between the obstacle and the tube wall as shown on photos in fig.4.5 (X-direction). To a large degree, this "by-effect" will eliminate the influence of the high void region near the tube wall on void fraction distribution measurements.
APPENDIX 11 VELOCITY FIELD OF THE CLOSE–WAKE FORM X=Xs1 TO X=R

From x=x_{s1} to x=R, local velocities change strongly (but nevertheless should be continuously) both in magnitude and direction. Such a character can be probably at best represented by the continuity of zero–velocity points y_{co}(x), which may be considered as mean positions of the centers of vortices produced around the separation point. Accordingly, we assume a zero–velocity point line y_{co}(x) which changes gradually from the value y_{co}(R) at x=R (given by the solution of eq.(5.1) from U_w(x,y_{co}(R))=0) to y_{co}=y_{s1} at x=x_{s1}, i.e.,

\[ y_{co}(x) = y_{co}(R) + (x-x_{s1}) \frac{y_{s1} - y_{co}(R)}{R-x_{s1}} \]  

(A11.1)

in which y_{co}(x) is taken as \sqrt{R^2-x^2} when y_{co}<\sqrt{R^2-x^2}, representing the fact that the production of vortices takes place actually over a region on the surface of the cylinder for large Reynolds number flows (Song[1990]).

From the assumed profile of y_{co}(x), two coefficients C_1(x) and C_2(x) can be derived much easier from two relations of U_w(x,y_{co}(x))=0 and eq.(5.2) than from eq.(5.3) and eq.(5.2). In accordance with the character of the close–wake region from x=x_{s1} to x=R, namely rapid changes of local velocities both in magnitude and in direction, it would be more appropriate here to assume that absolute values of local velocities follow the relation in eq.(5.1) instead of only x–components of local velocities. The "mass–in–flow" components at the wake border are assumed to decrease from V_m1(R,y_{s1}) at x=R to zero at the separation point following a linear function and are added to the corresponding velocity components V_p(x,y_{co}) when absolute values of velocities at the wake border are substituted into eq.(5.2). Consequently computations from the corresponding profile defined by C_1(x) and C_2(x) are taken to be the absolute values of local velocities.

Direction of local velocities depending on y is assumed to change gradually from \theta_1 (in a tangential direction along surface of the cylinder) to \pi+\theta_{co} on the zero–velocity line when y_1<y\leq y_{co} and from \theta_{co} to \theta_2 on the wake border when y_{co}<y\leq y_2, as shown schematically in fig.A11.1 and defined by eq.(A11.2).
Then low velocities with components $U_w(x,y)$ and $V_w(x,y)$ are given by the following equation,

$$U_w(x,y) = |U_w(x,y)| \cos[\theta(x,y)]$$

$$V_w(x,y) = |U_w(x,y)| \sin[\theta(x,y)]$$

(A11.3)

where $|U_w(x,y)|$ denotes the absolute value of a local velocity.

With above defined equations, local velocities in the close–wake region are fully defined.
APPENDIX 12 MODIFICATION OF VELOCITY PROFILES CLOSE TO THE CYLINDER

Various experimental data have revealed clearly the fact that velocity on the flow axis $U_w(x,0)$ has the maximum negative value at the end of dead fluid zone ($x=L_2$) and increases linearly to zero from $x=L_2$ to $x=R$ for a single-phase flow past a cylinder (e.g. Nishioka[1974], Coutanceau[1977], Bouard[1980]). On the surface of the cylinder, velocity equals zero according to the no-slip condition. A new-form profile is then assumed for describing velocity distributions in the reverse flow part; it is defined by eq.(A12.1) and shown schematically in fig.A12.1.

$$U_w(x,y)_r = C_{r1}[x]y^2 + C_{r2}[x]y + C_{r3}[x]$$  \hspace{1cm} (A12.1)

where $C_{r1}[x]$, $C_{r2}[x]$, and $C_{r3}[x]$ are unknown coefficients which require three conditions.

Fig.A12.1 Old(Gaussian)— and new—form velocity profiles of the reverse flow part.
The first condition is the assumed velocities at the lower border (either the flow axis or the surface of the cylinder). It reads

\[
\begin{align*}
U_w(x,0)_r &= C_3[x] = U_w(L_3,0) \frac{x-R}{L-2R} \quad \text{when } x \geq R \\
U_w(x,\sqrt{R^2-x^2})_r &= C_2[x] = 0 \quad \text{when } x < R
\end{align*}
\]

(A12.2)

The second condition is zero velocity condition at the upper border \((y=y_\text{co}, \text{the zero velocity line derived from eq.(5.1)})\). It gives

\[
C_3[x]y_\text{co}[x]^2 + C_2[x]y_\text{co}[x] + C_3[x] = 0
\]

(A12.3)

The third condition aims that the new-form velocity profile still satisfy the momentum flux balance on the corresponding control plane. Concerning the related term in eq.(5.3), we have

\[
\int_{y_\text{co}}^{y_0} \rho U_w(x,y)_r|U_w(x,y)_r|dy = \int_{y_0}^{y_\text{co}} \rho U_w(x,y)|U_w(x,y)|dy \quad \text{(A12.4)}
\]

where \(y_\text{co}=0 \text{ when } x \geq R \text{ and } y_\text{co}=\sqrt{R^2-x^2}, \text{ when } x < R\).

From eq.(A12.2), eq.(A12.3), and eq.(A12.4), three coefficients \(C_3[x], C_2[x], \text{ and } C_3[x]\) are derived. While a more realistic form of velocity profiles in the reverse flow part is re-calculated from eq.(A12.1), the condition of the momentum flux balance over the concerned control plane is still satisfied. As has been mentioned above, the entire procedure merely amounts to selecting another fitting profile for the same conservation laws.
APPENDIX 13 EMPIRICAL CORRELATION CURVE OF $S_tL_4/D$–$Re_d$

Fig. A13.1 Empirical correlation curve of the length of $S_tL_4/D$ versus $Re_d$ (Gerrard[1966]).

St: the Strouhal number (approximately equal to 0.21 in subcritical $Re_d$ range);
$L_4$: the wake formation length;
D: the cylinder diameter;
$Re_d$: the cylinder Reynolds number ($Re_d=U_0D/\nu$).
APPENDIX 14 JUSTIFICATION OF RANDOM GENERATION PROCEDURE IN BUBBLE TRAJECTORY COMPUTATIONS

To include effects of random fluctuations of liquid velocities in bubble trajectory computations in a fluid past a cylinder, we make use of a numerical algorithm (or a subroutine) of "Numerical Recipes" (Press[1985]) in the program PDAC for generating random numbers, which are in the form of a Gaussian (normal) distribution. As an illustration and justification, a set of numerical tests were performed. Computation results are shown in fig.A14.1 below.

Fig.A14.1 Justification of Random Number Generation procedure in Bubble Trajectory Computations

(a) Trajectories of bubbles (Image trajectories to the flow axis are also shown);
(b) Bubble velocities along trajectories.
Eight bubble trajectories are computed from the same initial condition (3 cylinder diameter upstream the cylinder (X₀=−3D), 1/4 cylinder diameter from the flow axis (Y₀=D/4), and at the bubble terminal velocity (U₀=U_{t0})) in a vertical bubbly flow toward the cylinder. The cylinder Reynolds number (Re_cylinder) is 13400; and the ratio of bubble size to cylinder size (d/D) is 1/10. Trajectories and bubble velocities along the trajectories are shown respectively in fig. A14.1(a) and fig. A14.1(b).

Quite satisfactory results are provided. In the potential flow region, where no random fluctuations of local liquid velocities are included in bubble trajectory computations, eight trajectories are in coincidence with each other. Upward moving bubbles, which are close to the flow axis, would collide to the cylinder in computations. It is not very clear how bubbles would further move over the cylinder surface and then are released into the main flow. Based on high-speed video films, a procedure is assumed: Collided to the cylinder, bubbles first deform elastically, then wriggle (or rotate) forward over the cylinder surface, reentry the main flow as arriving at the separation point (x_s). Beyond the separation point, bubbles move into the extended wake region in which effects of random fluctuations of local liquid velocities are included in the force balance as we discussed in section 5.3. With existence of the random fluctuations, which are shown clearly in bubble velocities along trajectories in fig. A14.1(b), bubble trajectories differ from each other. Some of them can even move across the flow axis in accordance with experimental data.
APPENDIX 15 ALGORITHM OF DERIVING LOCAL VOID FRACTIONS FROM BUBBLE TRAJECTORIES

Trajectories of bubbles are usually called Lagrangian data for describing phase distributions of a dispersed two-phase flow past a cylinder. The Lagrangian description, which obtains typical traveling paths of bubbles in the liquid flow field, is convenient for a qualitative analysis of interaction between the bubbles and the liquid flow field but is difficult to compare with experimental measurements. Accordingly, there is usually a much stronger interest in Eulerian data than in Lagrangian data in practice. The Eulerian data, which are provided as local void fractions for a gas-liquid flow in the geometrical space, show the average behavior of a two-phase system more directly than the Lagrangian data and can be related directly to the experimental measurements. For a close comparison with measurement provided in chapter 4, we now present an algorithm to transform trajectories of bubbles to void fraction distribution data (i.e., transformation from Lagrangian data to Eulerian data).

The basic idea of the algorithm is to divide the flow field into small cells, compute a large number of trajectories of bubbles which are injected into the liquid flow some distance upstream the cylinder, and derive average local void fraction in a particular cell by counting the residence time of all bubble trajectories which pass through the cell. In detail, we now illustrate below.

Fig.A15.1 Coordinate definitions of the bubble-liquid system;
Step 1: Define coordinates of the bubble–liquid system (see fig. A15.1)

The two–dimensional flow field, which is enclosed by [X_s, Y_s], [X_s, Y_e], [X_e, Y_e], and [X_e, Y_s], is first divided into \((N_x \times N_y)\) cells. If a cell is represented by its left–down corner coordinate, we have

\[
\begin{align*}
X_c[i] &= X_s + i(X_e - X_s)/(N_x - 1) \\
Y_c[j] &= Y_s + j(Y_e - Y_s)/(N_y - 1)
\end{align*}
\]  

(A15.1)

where \(i = 1..N_x\) and \(j = 1..N_y\). The widths of two sides of a cell respectively are

\[
\begin{align*}
\Delta X &= (X_e - X_s)/(N_x - 1) \\
\Delta Y &= (Y_e - Y_s)/(N_y - 1)
\end{align*}
\]  

(A15.2)

Step 2: Compute a (large) number of trajectories of bubbles

At the upstream edge of the field \((X = X_s)\), which has been chosen at quite a distance upstream the cylinder, the bubbly flow has uniformly distributed local void fractions (i.e. the upstream average void fraction \(\alpha_0\)) and bubbles may assumed to move at their terminal velocities (as discussed in chapter 2). If the number of uniformly distributed trajectories at \(X = X_s\) is chosen as \(N_{tra}\) (as big as possible), the equivalent residence width \((W_{tra})\) per bubble trajectory is calculated by

\[
W_{tra} = \alpha_0 (Y_e - Y_s)/N_{tra}
\]  

(A15.3)

With given main system parameters (cylinder diameter \(D\), liquid flow upstream velocity \(U_{10}\), and bubble diameter \(d_p\)), \(N_{tra}\) trajectories are now computed from defined initial conditions by the Runge–Kutta integration procedure (given in chapter 2, the program PDAC) until the downstream edge of the field \(X = X_e\). Once a trajectory passes a \(i\)th border line \((X = X_c[i])\), local coordinates \((X_{tra[i,j]}, Y_{tra[i,j]})\), local instantaneous bubble velocity \((U_{tra[i,j]}, V_{tra[i,j]})\), and local instantaneous time \((t_{tra[i,j]})\) are stored in the corresponding locations in two–dimensional matrix.
Step 3: Calculate average void fractions over each cell

For an arbitrary cell ([i,j]), average void fraction is calculated by summing residence lengths of all bubbles trajectories which penetrate the cell, given as follows:

\[
\alpha_{ij} = \frac{W_{tr}}{\Delta X \Delta Y} \sum_{k=1}^{N_{tra}} (U_{tra[i,k]} \Delta t_{tra[i,k]})
\]

(A15.4)

where \(U_{tra[i,k]}\) and \(\Delta t_{tra[i,k]}\) are the average velocity and the residence time that \(k\)th trajectory passes the cell. Going through the \(i\)th row of data matrix \((X_{tra}, Y_{tra}, U_{tra}, V_{tra}, t_{tra} = [i,1,N_y])\), the program first justify whether a trajectory penetrate the cell. If a trajectory does penetrate the cell (possibly enters and leaves the cell on any of the four borders of the cell), parameters at the entrance \((X_{in}[k], Y_{in}[k], U_{in}[k], V_{in}[k], t_{in}[k])\) and at the exit \((X_{out}[k], Y_{out}[k], U_{out}[k], V_{out}[k], t_{out}[k])\) are interpolated from corresponding computation results on the \(i\)th and \((i+1)\)th border lines in the 2D data matrix. Seven different entry & exit patterns are considered and shown schematically in fig.A15.2.

Fig.A15.2 Entry & exit patterns of bubble trajectories through a cell.
Accordingly, $U_{\text{tra}[i,k]}$ and $\Delta t_{\text{tra}[i,k]}$ are derived by the following expressions

\[
U_{\text{tra}[i,k]} = 0.5 \left( \sqrt{U_{\text{out}[k]}^2 + V_{\text{out}[k]}^2} + \sqrt{U_{\text{in}[k]}^2 + V_{\text{in}[k]}^2} \right)
\]
\[
\Delta t_{\text{tra}[i,k]} = t_{\text{out}[k]} - t_{\text{in}[k]}
\]

(A13.5)

In this way, local void fractions ($\alpha[i,j]$) in the flow field can be derived from bubble trajectories. The accuracy of local void fraction date depends mainly on the number of trajectories used. Often processing time of the computer sets rather narrow limits here.
SUMMARY

The study concerns phase distributions in low quality dispersed two phase flows around obstacles. Such flows are of practical significance in many engineering disciplines, in particular the process and power industry. Obstacles affect flow patterns in such a manner that they induce a re-distribution or a separation of the two phases. This results in quite remarkable changes of phase distribution patterns and often influences greatly the efficiency of physical transport processes and the reliability of related industrial components.

The work consists of a theoretical part of a more general nature and an experimental part highlighting bubbly flows in vertical tubes.

Theoretical predictions of phase distributions around obstacles are based on the Lagrangian approach, which is more appropriate to the non-uniform flow fields in which separation of phases occurs and is more convenient for analyzing the interaction process between the two phases when they pass the obstacle.

The interaction between a dispersed phase and its surrounding fluid is first analyzed in a overall sense. A general form momentum equation to describe forces on a dispersed phase in an arbitrary fluid flow is introduced. The equation includes the steady drag force, the lift force, the added-mass force, the pressure-gradient force, the Basset history force and the buoyancy force. In a steady fluid flow past an obstacle, which is taken to be completely given by an inviscid, potential solution excluding the turbulent wake region, the force balance equation is converted into a set of ordinary differential equations and then solved by the Runge-Kutta integration algorithm for computing trajectories of the dispersed phase. By tracing trajectories of the dispersed phase, we can identify and analyze the magnitude of each individual force and the significance of system parameters (the obstacle Reynolds number, the particle-to-obstacle size ratio, and the particle-to-fluid density ratio) in the interaction process. Computations cover a wide range of dispersed two-phase flows past obstacles. Besides bubble-liquid flows, which are the main topic of this work in connection to experiments, particle-gas (or droplet-gas) and particle-liquid flows are also discussed.
Then an attempt is made to predict the phase distribution of a bubbly flow past a cylinder under realistic conditions. Based on empirical data and analytical results in literature, a simple model is proposed to describe the wake region of the single-phase flow past the cylinder. The liquid turbulence is introduced in bubble trajectory computations with a simplified scheme. In a similar fashion as in the potential flow region, we follow bubble trajectories from the potential flow region to the wake region to investigate the magnitude of the forces and the influence of main system parameters in the interaction process between bubbles and the highly turbulent wake.

Measurements of phase distributions around cylinders are taken in two set-ups, one at atmospheric pressure and one at elevated pressure.

The utilized measuring systems — a gamma-ray densitometer system for obtaining void fraction distribution measurements around obstacles at both atmospheric pressure and elevated pressure, and a photographic system for quantifying size, velocity, and concentration of bubbles in a high-pressure tube — are presented. For each of the measuring systems, three development stages of design, data interpretation, and system calibration are described.

With application of the gamma-ray densitometer system obtains void fraction distribution measurements of bubbly flows behind the cylinders. Properties of bubbles upstream of the cylinders are provided by a camera at the atmospheric pressure or by the photographic system at high pressures up to 70 bar. Bubble size is varied either by installing different system pressures or by adding a certain surface-active substance in water. Our experiments yield effects of cylinder Reynolds number, bubble to cylinder size ratio, and upstream mean void fraction on void distribution patterns around cylinders in vertical bubbly flows, in particular such with small bubble sizes and low void fractions.

Finally predictions and measurements are compared. Generally agreement is fair but a higher accuracy in measurements and predictions remain desirable for quantitative verifications. Also the eventual trajectories of bubbles colliding with the cylinder and sliding for some distance along its surface remain uncertain.

Since a bubbly flow past a cylinder is one of the typically non-uniform dispersed two-phase cross-flows, the results may with some caution be extended to more general situations.
SAMENVATTING

Het onderzoek heeft betrekking op de stroming rond een obstakel van disperse twee-fasenmengsels met een lage volumefractie van de disperse fase. Zulke stromingen zijn in vele gebieden van de techniek van belang, in het bijzonder in de procesindustrie en bij de vermogensopwekking. Obstkakels beïnvloeden de stroming vaak op een zodanige wijze dat zelfs een herverdeling of een scheiding van de twee fasen optreedt. Dit leidt tot aanmerkelijke veranderingen in de fasenverdeling en heeft vaak een grote invloed op de afloop van transport processen en op de betrouwbaarheid van industriële apparatuur.

Het werk bestaat uit een theoretisch gedeelte van meer algemene aard en een experimenteel gedeelte dat zich op bellenstromingen in verticale pijpen richt.

De theoretische voorspellingen van de fasenverdeling rond obstakels worden gebaseerd op een Lagrangian benaderingswijze. Deze benaderingswijze is toegesneden op niet uniforme stromingsvelden waarin scheidin van de fasen optreedt en leent zich gemakkelijk voor het ontdelen van de wisselwerking tussen de twee fasen wanneer deze het obstakel passeren.

De wisselwerking tussen gedispergeerde fase en omringende stoomstof wordt eerst zo volledig mogelijk geanalyseerd. Daartoe wordt een impulsvergelijking ingevoerd die het evenwicht van krachten op de gedispergeerde fase beschrijft in een willekeurige stroming. De vergelijking omvat de stationaire wrijvingskracht, de liftkracht, de toegevoegde-massa kracht, de kracht als gevolg van de drukgradient, de Basset geheugenkracht, en de hydrostatische kracht. Voor de stationaire stroming langs een obstakel, waarvoor een wrijvingsloze potentiaaloplossing wordt aangehouden behalve in het zoggebied, wordt de evenwichtvergelijking omgezet in een stelsel gewone differentiaalvergelijkingen, die daarna, met een integratie-algoritme volgens Runge-Kutta worden opgelost voor het berekenen van de baan van de gedispergeerde fase. Door de gedispergeerde fase langs zijn baan te volgen kan de grootte van elke individueel kracht gevonden en geanalyseerd worden, als ook de rol in de wisselwerking van de systeem parameters (het getal
van Reynolds betrokken op het obstakel, de verhouding van de grootte van de gedisperseerde objecten en het obstakel, en de verhouding van de dichtheden van objecten en stoomstof. De berekeningen strekken zich uit over een groot bereik van gedisperseerde twee-fasenstromingen rond obstakels. Naast bel-vloeistofstromingen, die het hoofdonderwerp zijn van de studie en de experimenten, worden ook deeltjes/druppels-gasstofstromingen en deeltjes-vloeistofstromingen besproken.

Vervolgens wordt een eerste poging gedaan om de fase-verdeling van een bellen-stroming langs een cylinder te voorspellen, onder realistische omstandigheden. Voor het zogebied van de een-fase stroming langs de cylinder wordt een eenvoudig model voorgesteld, gebaseerd op empirische gegevens en analytische resultaten uit de literatuur. De turbulentie van de vloeistof wordt met een vereenvoudigd schema in de baanberekeningen meegenomen. Op dezelfde wijze als in het gebied van de potentialstroming worden de bellen gevolgd in hun baan van het gebied van de potentialstroming naar het zogebied teneinde de grootte van de krachten en de invloed van de belangrijkste systeemparameters te onderzoeken bij de wisselwerking tussen de bellen en het zeer turbulente zog.

Metingen van de fasen-verdeling rond cylinders zijn uitgevoerd in twee opstellingen, een voor atmosferische druk en een voor hoge druk.

De twee toegepaste meetystemen worden beschreven — een gamma-straal dichtheids-meetysteem voor het bepalen van de bellen fractie rond een obstakel, zowel bij atmosferische als bij hogedruk; en een fotografisch systeem waarmee de afmeting, snelheid en concentratie van bellen in een pijp kunnen worden gevonden. Voor beide meetystemen worden het ontwerp, de gegevensverwerking en de ijking toegelicht.

Met de gamma-straal dichtheidsmeter zijn in een verticale bellenstroming bellenfractieverdelingen achter een cylinder gemeten. De eigenschappen van de bellen stroomopwaarts zijn bij atmosferische druk met een camera bepaald en bij hogere drukken, tot 70 bar, met het fotografische systeem. De afmeting van de bellen is gevarieerd door verschillende drukken in te stellen of door een oppervlakte-actieve stof aan het water toe te voegen. De experimentele resultaten
Samenvatting

laten de invloeden zien van het getal van Reynolds betrokken op de cilinder, van de verhouding tussen bellen- en cilinder-diameter, van de stroomopwaartse bellenfractie op de bellenverdeling, in het bijzonder zulke met kleine bellen en een lage bellenfractie.

Tenslotte worden de voorspellingen en de meetingen vergeleken. In grote lijnen is de overeenstemming goed, maar zowel bij de meetingen als bij de voorspellingen is nog een hogere nauwkeurigheid nodig om een kwantitieve vergelijking mogelijk te maken. Ook is de uiteindelijke baan onzeker vaan bellen die met de cilinder botsen en dan over enige afstand langs het oppervlak glijden.

Extrapolatie van de bevindingen naar bellenstromingen langs andere obstakels behoort tot de mogelijkheden.
The author of this thesis was born in Beijing, China, on January 17, 1964.

After ten-years education in primary and secondary school in Beijing, he was admitted to Xi'an Jiaotong University in September, 1981. In 1985 he got his Bachelor's degree on mechanical engineering and won a postgraduate scholarship for studying at one of the Dutch universities.

After one year intensive English training at Beijing Institute of Foreign Languages, he came to Eindhoven University of Technology (EUT) and registered as an undergraduate at the Faculty of Mechanical Engineering in September, 1986. Two years later, he received his "Ingenieur" title (Master of Science) on mechanical engineering. Thereafter, he followed a two—years design/research training program on fundamental mechanics in the Laboratory of Multiphase Flow and Heat Transfer at EUT. From December, 1990 to September, 1992, he was employed as a research assistant in the same laboratory, aiming at finishing a Ph.D thesis under supervision of prof. C.W.J. van Koppen and Dr. C.W.M. van der Geld.

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STELLINGEN

behoorende bij het proefschrift

On dispersed Two phase Flows past Obstacles

van

H. Keng

(Dec. 2, 1993)
1. When modeling any inhomogeneous dispersed two phase flow systems, the postset history force can not be neglected in the total drag on the dispersed phase.

(This thesis, chapter 2 and 5)

2. The significant effect of the particle to obstacle size ratio on phase distribution patterns of a dispersed two-phase flow past an obstacle indicates possible substantial errors when only the (arithmetic) average diameter of the dispersed phase is used in numerical simulations for polydispersed two phase flows, as it is quite often done.

(This thesis, chapter 2, 4, and 5)

3. The techniques of particle image analysis [Ref.1], which have been developed rapidly in the last decade, provide contact-free and whole-field measurements of particles and are perfectly applicable to the Lagrangian tracking of bubble motion in dispersed two phase flows, in particular for the validation of our theoretical predictions in chapter 2.


4. The knowledge on the "sliding phenomenon" is almost totally missing; Research on this topic is mandatory for further advance of the modeling of dispersed two-phase flows past obstacles.

(This thesis, chapter 5)

5. An advantage of the Lagrangian approach over the Eulerian approach is that the computer storage requirements do not increase significantly with the number of particle size groups [Ref.2]. This advantage will bring the Lagrangian approach to the fore in the modeling of dispersed two-phase flow systems.


6. While progresses in any special field of science is best assured by logical extensions of knowledge from known fundamentals, new questions (or research topics), on the contrary, are often and better introduced from uncertain bases by bold assumptions.
7. As we keep enjoying so much the convenience in our lives provided by science and technology, there seems to be no alternative to deeper scientific understanding and further technology development to help us solve the problems of environment and resource shortages.

8. One of the differences between western and eastern attitude to life is on personal successes and achievements: in western society, they are more accepted as to be based on natural talents; while in eastern society, they are more regarded as to be the result of consistently hard working.

9. Without a substantial well-educated and hard-working middle class, the western-style democracy does no better than to cause political chaos and economical turmoil for a country.

10. The Chinese philosopher Confucius said 2500 years ago: "Men are not born to be good, but taught to be good"; we may argue that a good, continually-improved compulsive education system is the key for any nation to solve current problems and to create a better future.

11. Considering the tremendous influence on society of propaganda with the aid of modern communication media, it is a pity that such a powerful "tool" has been more effectively used for bad purposes than for good aims in history.

12. One of the basic properties of the "living" nature must be the rhythm; you can feel this strongly when you stand on the beach at dusk or walk in the forest at dawn.