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Deliverable D2.14

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Abstract

This report forms, in two parts, deliverable D2.14 of the TurboNoiseCFD project. It describes the recommended innovative “triple plane pressure” in-duct matching strategy (TPP) at inlet and bypass duct side, and it describes in detail the newly developed and favourably tested CAA code for the propagation and radiation problem at the exhaust side. Due to the presence of the jet, this solution is also innovative.

Both parts are written independently by the partners ISVR and TUE, although as a conclusion, in part II, the joint case study of a full calculational chain, from CFD data via the bypass duct into the far field radiation, is presented. In the by-pass duct TUE’s and ISVR’s propagation codes are compared. It is shown that the TUE semi-analytical multi-modal multiple scales solution for sound propagation in ducts compares extremely well with ISVR’s CAA solution. This is very important for the reliability of the matching method, which is based on this multi-modal multiple scales solution.
Deliverable 2.14 Part I.
Mode-matching Strategies
in Slowly Varying Engine Ducts

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Abstract

This report forms part 1 of the deliverable report D2.14 of the TurboNoiseCFD project. It summarises in a compact and concise way the matching strategy that finally resulted.

A matching method is proposed to connect the CFD source region to the CAA propagation region of rotor-stator interaction sound produced in a turbofan engine. The method is based on a modal decomposition across three neighbouring axial interfaces adjacent to the matching interface. The modal amplitudes are determined by a least-squares fit. By taking slowly varying modes the interface may be positioned in a duct section of varying cross section. Furthermore, the spurious reflections back into the CFD domain, which result from imperfect reflection-free CFD boundary conditions, can be filtered out by including both left- and right-running modes in the matching. Although the method should be applicable to a wider range of acoustic models, it is implemented and favourably tested for the slowly varying modes in homentropic potential flow in lined ducts. Homentropic potential flow is a very relevant model for the inlet side, and a good model for the bypass side if swirl or other types of vorticity are not dominant in the mean flow. By matching with density or pressure perturbations any contamination of residual non-acoustical vorticity is avoided.

I. 1 Introduction

Computationally it is inefficient, and practically as yet impossible, to describe the sound field, which is produced within a turbofan aero-engine duct by rotor-stator interaction of fan and OGVs, throughout the entire domain of interest by the same model.

In the aerodynamic regions of the source, i.e. in the vicinity of fan, gap, and OGV (figure 1), the unsteady parts of all flow variables such as pressure, velocity, density and possibly the thermodynamic variables are of the same order of magnitude as the steady (time-averaged)
The zones in a typical high-bypass ratio turbofan engine

parts. The governing model is therefore nonlinear, time-dependent, and rotational. It is described by some form of the compressible Navier-Stokes equations, possibly supplemented by a turbulence model. Especially in the acoustically most critical case of heavy blade loading, this model is mathematically so complicated that it can only be solved numerically by CFD methods. On the other hand, in the acoustic regions of inlet and bypass duct and free field, the unsteady components are small compared to the steady (mean) flow. The model here may be split up into a steady description of the mean flow and a separate linear model for the perturbations. In addition, other simplifications that do not seriously affect the sound propagation may be justified, such as the absence of viscosity, vorticity and entropy variations for instance. The acoustic field may therefore be resolved by more efficient CAA or (semi-)analytical techniques.

At the interface between both models, the source data from one is to be handed over to the other. Ideally, we would want to impose continuity of all variables and their derivatives at this interface. This is, however, not possible and the differences between the models cause a matching problem across the interface where the following issues have to be faced:

1. **Inconsistent field variables.** Pressure, density and velocity satisfy different equations in different models. So, for example, a pressure-velocity field consistent in one model may be just impossible to satisfy in the other. We may have to choose which variable is preferred, and cannot have both to be true.

2. **Residual numerical errors and spurious reflections.** The numerically obtained source field may have residual errors. In particular the assumed reflection-free boundary condition is usually imperfect, and the field available from the CFD solution at the interface will in reality only partly transmit due to some reflections. This reflected part should be recognized, otherwise the resolved field will be an overestimation of reality.

3. **False reflections.** When the difference in models is too great, this will in itself create false reflections that should not be included. This is an additional effect to the numeri-
ical reflections noted in item 2 above. From our experience at present, the imperfect reflection-free boundary condition of 2 appears to be most significant.

4. **False near field.** A perfect matching of the acoustic field to the given field at the interface is, at least in one variable, always possible, but this solution will almost certainly contain an acoustic near field that is completely an artefact of the method. In terms of duct modes, this is the field associated with the exponentially decaying cut-off modes. If the duct near the matching interface is smooth and the field is nearly linear, this part of the modal spectrum (the amplitudes of the cut-off modes) should be small in reality, because the acoustic field’s origin will be at the fictitious source distribution consisting of the rotor/stator blades and the nonlinear flow upstream, not locally. It is, however, impossible to quantify how small these modes should actually be, given the uncertainties of the source field. Therefore, we must try to limit these cut-off modes in a sensible way, but at the same time bear in mind their contribution to the flow-field.

In order to translate the aerodynamic source data into their acoustic equivalents, the following two methods have been considered by Wilson[1],[2] and Nijboer & Schulten[3] respectively:

(i) **Wave splitting.** If the interface is at a section of the duct that is parallel to the axis, while the mean flow is axially constant, the field can be solved by a sum of modes. By applying conditions of continuity of at least two variables (pressure and axial velocity in irrotational flow, pressure and pressure gradient in vortical flow), a linear system can be created that defines the modal amplitudes. The advantage of the method is that it is relatively flexible to assumptions made about the source, so long as modes can be constructed. The disadvantages are that the duct has to be locally straight, and that care must be taken that the two field variables are not inconsistent in the acoustic model, either because of the model differences or by the numerical discretisation error.

(ii) **Ffowcs Williams-Hawkings integrals over the blades.** It is argued that in the absence of shock waves associated to the rotor or stator blades, and in the absence of an important displacement effect due to the thickness of the blades, the dominating part of the source is the pressure distribution over the blades. These constitute a rotating distribution of dipoles, of which the radiated sound can be described by a Ffowcs Williams-Hawkings integral over the blades. The advantage of this method is that the pressure distribution can probably be relatively accurately determined. The disadvantages are that the Green’s function in the duct is required for the solution, which is only explicitly available for simple ducts and mean flows, while the scattering of the rotor field by the stator, and vice versa, is not included.

**I. 2 Triple-plane pressure matching method**

The above considerations brought us to the following matching method.

In order to be able to capture any type of source, we followed the idea of mode splitting
at the interface, which allows us at the same time to remove any spurious reflections in the CFD data. We assume a circular symmetric annular duct section. This is near a rotor not a restriction. By using the slowly varying modes of Rienstra[4],[5], we are able to avoid the restriction of a straight duct section and, therefore, it is possible to match half-way along the spinner (in the inlet duct). In the present implementation, these modes are solutions of a homentropic potential flow model. The generalisation to modes in flow with swirl[6],[7],[8], [9],[10] may be a next step, but is not done here yet.

The matching is done for a single variable in order to avoid any inconsistency. A convenient choice is the pressure because this is always available, while it is (practically) not affected by vortical contamination [1], which cannot be absorbed by a potential flow model. This is because convected relative density ($\rho$) perturbations only depend on the solenoidal part of the velocity ($v$),

$$\rho^{-1}\left(\frac{\partial}{\partial t} + v \cdot \nabla\right)\rho = -\nabla \cdot v,$$

while for isentropic perturbations, pressure and density fields are merely algebraically related.

As matching of a single variable at a single axial plane is not enough to distinguish left- from right-running waves, the number of matching planes is extended to three. Two would be just enough of course, but three offers some excess of information that allows, via a least squares fit, a smoothing of errors and a better use of available information. Of course, one could take more matching planes than three; the idea remains exactly the same. Another advantage of a geometrically spread matching zone, is the possibility of limiting the amplitudes of the cut-off modes in a simple and systematic way without any resorting to ad-hoc assumptions about the actual source locations (see below).

The mean flow of the potential flow model is necessarily nearly uniform along a cross section. The axial velocity, mean density and mean sound speed are therefore chosen such that their cross sectional average is equal to the corresponding CFD mean flow component.

Finally, it may be noted that the present slowly varying modes are also valid for lined ducts, via the implementation of Myers’ soft-wall boundary condition[11]. Hence, the matching interface may be chosen equally in either a hard-walled or lined duct section.

I. 3 Equations for ideal fluid motion

As the interface refers to a jump in model, it is important to make clear what assumptions and approximations are made in our acoustic model, which forms the basis for the matching method. This will be explained in detail below.

For pressure $p$, density $\rho$, velocity vector $v$, deviatoric stress tensor $\tau$, temperature $T$, entropy $s$ and heat flux vector $q$ the equations for conservation of mass, momentum and
The energy of an ideal gas read[12]
\[
\left( \frac{\partial}{\partial t} + v \cdot \nabla \right) \rho = -\rho \nabla \cdot v,
\]
\[
\rho \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v = -\nabla p + \nabla \cdot \tau,
\]
\[
\rho T \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) s = -\nabla \cdot q + \tau : \nabla v.
\]

The variables are thermodynamically related with enthalpy \( h \), internal energy \( \epsilon \) and sound speed \( c \) by
\[
T \, ds = d\epsilon + pd\rho^{-1} = dh - \rho^{-1} dp, \quad h = \epsilon + p\rho^{-1},
\]
\[
p = \rho \mathcal{R} T, \quad d\epsilon = C_V dT, \quad dh = C_p dT,
\]
\[
ds = C_V \frac{dp}{p} - C_p \frac{d\rho}{\rho}, \quad c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \gamma \mathcal{R} T,
\]
\[
\mathcal{R} = C_p - C_V, \quad \gamma = C_p / C_V,
\]
where \( \mathcal{R} \) is a gas constant. For a perfect gas, the specific heats \( C_V \) (at constant volume) and \( C_p \) (at constant pressure) are constants, such that we can integrate the expression above to obtain
\[
s = C_V \log p - C_p \log \rho, \quad c^2 = \gamma p / \rho.
\]

In acoustics, the variations are too quick for heat conduction (Péclet number is large) while the viscous shearing forces are small (Reynolds number is large), leading to
\[
\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho v) = 0,
\]
\[
\rho \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v = -\nabla p,
\]
\[
\left( \frac{\partial}{\partial t} + v \cdot \nabla \right) s = 0.
\]

The last equation denotes a constant entropy along streamlines. If this constant is the same for all streamlines, the flow is homentropic. If the flow is irrotational (like in the inlet), we can introduce a velocity potential \( \phi \) with \( v = \nabla \phi \) and integrate the momentum equation to obtain Bernoulli’s equation
\[
\frac{\partial \phi}{\partial t} + \frac{1}{2} |v|^2 + \frac{c^2}{\gamma - 1} = G(t),
\]
where \( G \) is an unimportant function of time. The flow can now be split up into a mean flow plus small perturbations
\[
v := V + v, \quad p := P + p,
\]
\[
\phi := \Phi + \phi, \quad \rho := D + \rho,
\]
(while a sound speed perturbation serves no purpose, so we write \( c := C \)) and, after
linearisation, we obtain for the mean flow field (dimensionless, of course)

\[ \nabla \cdot (DV) = 0, \quad \gamma P = D^\gamma, \quad C^2 = \gamma P / D = D^{\gamma - 1}, \]

\[ \frac{1}{2} |V|^2 + \frac{C^2}{\gamma - 1} = E, \quad \text{a constant}, \]

and for the acoustic field

\[ \frac{\partial}{\partial t} \rho + \nabla \cdot (D \nabla \phi + \rho V) = 0, \]
\[ \frac{\partial}{\partial t} \phi + V \cdot \nabla \phi + p / D = 0, \quad p = C^2 \rho, \]

which simplifies to solving

\[ (\frac{\partial}{\partial t} + V \cdot \nabla) [C^{-2} (\frac{\partial}{\partial t} + V \cdot \nabla) \phi] = D^{-1} \nabla \cdot (D \nabla \phi). \]

**I. 4 Slowly varying modes**

When a slowly varying duct, given by

\[ R_1(X) \leq r \leq R_2(X), \quad X = \varepsilon x, \]

where \( \varepsilon \) is a small parameter, is the only cause of variation of the mean flow (this is an assumption), the mean flow variables depend essentially on the slow axial variable \( X \), rather than on \( x \), and application of the method of slow variation[13] leads to the asymptotic expansion

\[ V(X, r; \varepsilon) = U(X)e_x + \varepsilon V(X, r)e_r + O(\varepsilon^2), \]
\[ D(X, r; \varepsilon) = D(X) + O(\varepsilon^2), \]
\[ C(X, r; \varepsilon) = C(X) + O(\varepsilon^2), \]
\[ P(X, r; \varepsilon) = P(X) + O(\varepsilon^2), \]

where \( e_x \) and \( e_r \) are unit vectors in the axial and radial directions respectively. The leading-order mean flow field satisfies the equations

\[ U(X) = \frac{F}{D(X)(R_2(X)^2 - R_1(X)^2)}, \]
\[ \frac{1}{2} U(X)^2 + \frac{1}{\gamma - 1} D(X)^{\gamma - 1} = E. \]

The two parameters \( F \) and \( E \) determine the entire mean flow field and must be chosen to fit the CFD mean flow data as ideally as possible. The mean flow from the CFD must therefore be stripped of any swirling and vortical component to make the velocity nearly uniform axially. Given that \( \pi F \) is the mass flux through an axial plane, \( F \) can be found
by finding the average mass flux of the mean flow at the three axial planes given. The Bernoulli constant $E$ can be found by a similar averaging process of the mean density or, indeed, the mean pressure or mean sound speed if they are regarded as more suitable.

Once a mean flow consistent with the acoustic model and close to the steady part of the CFD data has been derived, the acoustic field is Fourier-decomposed in frequency and circumferential order. If $p(x,t)$ is periodic in time with period $2\pi/\Omega$, it can be written as

$$p(x,r,\theta,t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} p_m(x,r;n\Omega) e^{in\Omega t-im \theta}$$

and each Fourier component is given by

$$p_m(x,r;n\Omega) = \frac{\Omega}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi/\Omega} p(x,r,\theta,t) e^{im \theta - in \Omega t} \, d\theta \, dt.$$

As we will deal in the rest of this paper with a single Fourier component $p_m(x,r;\omega)$ only, no explicit reference will be made to the circumferential order $m$ or frequency $\omega$ dependence.

Each component is now a wave propagating in a slowly varying medium (both geometry and mean flow are to leading order functions of $X$ only). This can be approximated by an application of the WKB method (a variant of the method of multiple scales) where each radial mode is assumed to adapt itself to the varying surroundings, without any intermodal coupling[4]. The resulting field can then be written as a summation of left- and right-running modes in the following way

$$p(x,r) = \sum_{\mu=-\infty}^{\infty} A_{\mu} \psi_{\mu}(X,r) e^{-i \int_{\sigma}^{\epsilon} k_{\mu}(\epsilon \sigma) d\sigma},$$

where $\mu = 0$ is excluded. The functions $\psi_{\mu}$ represent the basis functions for the right-running ($\mu > 0$) and left-running ($\mu < 0$) slowly-varying pressure modes which take the form[4]

$$\psi_{\mu}(X,r) = -i D(X)(\omega - k_{\mu}(X) U(X)) \left[ N_{\mu}(X) J_m(\alpha_{\mu}(X)r) + M_{\mu}(X) Y_m(\alpha_{\mu}(X)r) \right],$$

where $J_m$ and $Y_m$ are Bessel functions of order $m$. The slowly-varying radial eigenvalues $\alpha_{\mu}$ are found from the hard or soft wall boundary condition[11] and the axial eigenvalues $k_{\mu}$ can be subsequently derived for each $\alpha_{\mu}$ via a slowly varying dispersion relation (with the right sign convention of the square root for right, i.e. $\mu > 0$, and left, i.e. $\mu < 0$, running modes). The functions $N_{\mu}$ and $M_{\mu}$, which are determined by a solvability condition, dictate how the modal amplitude changes axially through the duct and can be suitably normalized.
I.5 The matching method in detail

Suppose that at three axial planes $x = x_0, x_1$ and $x_2$ (where $x_0 < x_1 < x_2$), representing the matching zone (figure 2), the pressure $p(x, r)$ can be written by a finite sum of each plane’s slowly varying basis functions as follows:

$$p(x, r) = \sum_{\mu = -M}^{M} A_\mu \psi_\mu(X, r) e^{-i \int_{x_0}^{x} k_\mu(r) \, dr}.$$

Hence, given the Fourier-decomposed $(\omega, m)$-component of the pressure data $P_0(r)$ at $x = x_0$, $P_1(r)$ at $x = x_1$ and $P_2(r)$ at $x = x_2$, we can write

$$\sum_{\mu = -M}^{M} A_\mu \psi_\mu(X_0, r) = P_0(r), \quad (4a)$$

$$\sum_{\mu = -M}^{M} A_\mu \psi_\mu(X_1, r) e^{-i \int_{x_0}^{x_1} k_\mu(r) \, dr} = P_1(r), \quad (4b)$$

$$\sum_{\mu = -M}^{M} A_\mu \psi_\mu(X_2, r) e^{-i \int_{x_0}^{x_2} k_\mu(r) \, dr} = P_2(r). \quad (4c)$$

Evidently, it will not be possible to find amplitudes $A_\mu$ that satisfy this over-determined set of equations exactly. However, a best fit may be determined by a least squares approach, although some care is needed to prevent exponentially large terms at the zone ends from unbalancing the least squares minimisation. The diagram in figure 3 shows two modes approaching the three-plane interface from either side. If these modes are cut-off, $\text{Im}(k_\mu) \neq 0$, then their respective amplitudes will decay exponentially as they pass through the interfaces. Hence, if they both have amplitudes of $O(1)$ at the plane $x_0$, then at $x = x_2$ the amplitude of the right-running cut-off mode will be exponentially small and the amplitude of the left-running mode will be exponentially large. This leads to an imbalance in the minimisation process, where the cut-off modal amplitudes are in general too large, and the errors do not become spread evenly across all the axial planes. A typical consequence

![Fig. 2. Sketch of typical Fan/OGV geometry with interfaces up and downstream.](image)
Fig. 3. Sketch of the amplitude decay of cut-off modes across three axial planes.

of this can be the appearance of oscillations or wiggles in the CAA pressure and velocity fields.

One satisfactory solution to the problem is to rescale the left- and right-running amplitudes and, reciprocally, rescale the corresponding basis functions, at whichever axial plane they take their maximum value in the least squares procedure. In other words, they are rescaled to the axial plane nearest their apparent source. Hence, in our diagram, the amplitude of the left-running mode is rescaled to be $\Theta(1)$ at the end interface $x_2$ and to be exponentially small at $x = x_0$. Note that this does not involve any manual adaptation or post-processing of the amplitudes. We just temporarily redefine amplitudes and basis functions such that the penalty in the least squares method becomes effectively much greater for cut-off modes at the outer axial planes. The bonus of this rescaling is that while it limits the cut-off modes within the matching zone, it suppresses in a simple and systematic way the false near field created at the zone, but never suppresses more than what is actually present in reality.

The rescaling for the left-running amplitudes is conveniently given by introducing new amplitudes and basis functions as follows.

\[
B_\mu = \begin{cases} 
A_\mu & \text{if } \mu > 0, \\
A_\mu e^{-i\int_{x_0}^{x_2} k_\mu(\sigma) d\sigma} & \text{if } \mu < 0,
\end{cases} \tag{5a}
\]

\[
\xi_\mu(r) = \begin{cases} 
\psi_\mu(X_0, r) & \text{if } \mu > 0, \\
\psi_\mu(X_0, r) e^{-i\int_{x_0}^{x_2} k_\mu(\sigma) d\sigma} & \text{if } \mu < 0,
\end{cases} \tag{5b}
\]
\[ \xi_{\mu}(r) = \begin{cases} \psi_{\mu}(X_1, r) e^{-i \int_{x_0}^{x_1} k_{\mu}(r) \, dr} & \text{if } \mu > 0, \\ \psi_{\mu}(X_1, r) e^{-i \int_{x_1}^{x_2} k_{\mu}(r) \, dr} & \text{if } \mu < 0, \end{cases} \] 
\[ \chi_{\mu}(r) = \begin{cases} \psi_{\mu}(X_2, r) e^{-i \int_{x_2}^{x_0} k_{\mu}(r) \, dr} & \text{if } \mu > 0, \\ \psi_{\mu}(X_2, r) & \text{if } \mu < 0. \end{cases} \]

Consequently, a more balanced system is obtained:

\[ \sum_{\mu=-M}^{M} B_{\mu} \xi_{\mu}(r) = P_0(r), \quad (6a) \]
\[ \sum_{\mu=-M}^{M} B_{\mu} \xi_{\mu}(r) = P_1(r), \quad (6b) \]
\[ \sum_{\mu=-M}^{M} B_{\mu} \chi_{\mu}(r) = P_2(r). \quad (6c) \]

The least squares procedure can now be applied as follows. Multiply left and right hand sides of equations (6) with the respective complex conjugated basis functions \( \xi^*_{\nu}, \zeta^*_{\nu}, \) and \( \chi^*_{\nu}, \) and integrate across the duct to obtain

\[ \sum_{\mu=-M}^{M} B_{\mu} \int_{R_1}^{R_2} \xi_{\mu}(r) \xi^*_{\nu}(r) r \, dr = \int_{R_1}^{R_2} P_0(r) \xi^*_{\nu}(r) r \, dr, \quad (7a) \]
\[ \sum_{\mu=-M}^{M} B_{\mu} \int_{R_1}^{R_2} \zeta_{\mu}(r) \zeta^*_{\nu}(r) r \, dr = \int_{R_1}^{R_2} P_1(r) \zeta^*_{\nu}(r) r \, dr, \quad (7b) \]
\[ \sum_{\mu=-M}^{M} B_{\mu} \int_{R_1}^{R_2} \chi_{\mu}(r) \chi^*_{\nu}(r) r \, dr = \int_{R_1}^{R_2} P_2(r) \chi^*_{\nu}(r) r \, dr, \quad (7c) \]

or in matrix form,

\[ \mathcal{M} a = p_0, \quad \mathcal{N} a = p_1, \quad \mathcal{Q} a = p_2, \quad (8) \]

where

\[ \{ \mathcal{M} \}_{\nu \mu} = \int_{R_1}^{R_2} \xi_{\mu}(r) \xi^*_{\nu}(r) r \, dr, \quad (9a) \]
\[ \{ \mathcal{N} \}_{\nu \mu} = \int_{R_1}^{R_2} \zeta_{\mu}(r) \zeta^*_{\nu}(r) r \, dr, \quad (9b) \]
\[ \{ \mathcal{Q} \}_{\nu \mu} = \int_{R_1}^{R_2} \chi_{\mu}(r) \chi^*_{\nu}(r) r \, dr, \quad (9c) \]
\[ \{ p_0 \}_{\nu} = \int_{R_1}^{R_2} P_0(r) \xi^*_{\nu}(r) r \, dr, \quad (9d) \]
\[ \{p_1\}_\nu = \int_{R_1}^{R_2} P_1(r) \zeta_\nu^*(r) r \, dr, \quad (9e) \]

\[ \{p_2\}_\nu = \int_{R_1}^{R_2} P_2(r) \chi_\nu^*(r) r \, dr, \quad (9f) \]

\[ \{a\}_\mu = B_\mu. \quad (9g) \]

The least squares approach aims to find a set of amplitudes \(a\) that minimizes the cost function
\[
\|Ma - p_0\|^2 + \|Na - p_1\|^2 + \|QA - p_2\|^2. \quad (10)
\]

If we search for the vector \(a\) that minimizes the above cost function \((10)\) and use the hermitian property of \(M, N\) and \(Q\) we get \(^1\) the following equation for \(a\),
\[
(M^2 + N^2 + Q^2)a = Mp_0 + Np_1 + Qp_2 \quad (11)
\]

and this equation is easily solved by standard numerical techniques. The actual reflected amplitudes \(A_\mu\) for \(\mu < 0\) are then recovered from the resulting \(B_\mu\) after the error minimisation is completed.

To exclude the reflected modes in the above analysis is easy: all that is needed is to change the summation limits from
\[
\sum_{\mu=-M}^{M} \text{ to } \sum_{\mu=1}^{M}.
\]

and restrict the set of basis functions to the ones that are outgoing. The problem remains identical, except that the size of the vectors and matrices are \(M\) and \(M \times M\) respectively.

I. 6 Testing the TPP matching strategy

The method was tested in a number of cases kindly provided by the partners of the project. We present here some representative examples supplied by Rolls-Royce and DLR.

I. 6.1 Rolls-Royce Fan/OGV in bypass duct

The Rolls-Royce CFD test data is based on the FANPAC engine geometry (figure 4) used in the TurboNoiseCFD project for investigating noise generated by rotor/stator interac-

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\(^1\) Introduce the complex vectorial inner product \([x, y]\), which is equal to the ordinary inner product with \(y\) complex conjugated: \([x, y] = (x, y^*)\). Then the hermitian property of \(M\) implies \([Mx, y] = [x, M^*y] = [x, My]\). Each squared distance in the cost function \((10)\) becomes now like \(\|Ma - p\|^2 = [Ma - p, M^*a - M^*p] = [a, M^2a] - [a, Mp] - [p, a] + \|p\|^2\). If we vary around \(a\) by substituting \(a + \epsilon b\), we find for \(O(\epsilon)\) that the variation of \(\|Ma - p\|^2\) is \(2 \text{Re} \{[M^2a - M^2p, b]\}\). If we look for stationary values of cost function \((10)\) for any vector \(b\), the result \((11)\) for \(a\) is obtained.
Fig. 4. Fan/OGV testcase engine geometry. Rolls-Royce supplied eleven axial planes located at the OGV-to-bypass-duct interface. The duct has hard walls.

Clearly, the engine duct geometry varies significantly along its axis, but in the region of the duct behind the stator, the duct is more-or-less parallel, with radial variations of $\mathcal{O}(10^{-4} \text{ m})$.

Fig. 5. Bypass Fan/OGV testcase. Axial and azimuthal mean flow profiles at $x = 0.340 \text{ m}$.

Rolls-Royce supplied data for eleven axial planes equally spaced between $x = 0.320$ and $x = 0.360$ at the stator-to-bypass-duct interface. Perturbation data was provided for a single frequency, $ka = 40.89$ (2BPF), and a single circumferential wavenumber, $m = -13$. 
Observe in figure 5 that the mean flow has hardly any swirl but contains vorticity (as expected). Stripping this mean flow to create a uniform flow leads to a mean axial Mach number of $M = 0.44$ and the average hub-to-tip ratio across the interfaces is 0.649. Solving the eigenvalue problem for the acoustic model predicts five cut-on radial modes to be present. Using the supplied CFD perturbation data, traditional wavesplitting methods mentioned in the introduction of imposing continuity of pressure (ignoring reflections) and of imposing continuity of pressure and axial velocity, described as the $[P + V]$ method, were compared to the TPP matching strategy across all eleven axial locations. Amplitudes for each radial mode were obtained from each of these methods as the mode’s SPL (sound pressure level) at the outer casing. These results are presented in figures 6 to 9 and the main observations are described below.

On performing the matchings, it soon became clear that there is significant vorticity contamination of the velocity field and just as significant reflections in this case. The results in figure 6 for the TPP method, for instance, indicate that the reflected amplitudes of the cut-on modes are about 20% of the amplitude of their transmitted counterparts. Matching without reflections in this case therefore leads to a significant overestimation (as expected) of the transmitted acoustic field.

The strength of the vortical part of the flow field compared to the acoustical (irrotational) part can be clearly seen by reconstructing irrotational axial velocity profiles from the obtained modal amplitudes using the CAA model and comparing these to the original CFD data. In the homentropic potential flow model, the irrotational axial velocity $\frac{\partial \phi}{\partial x}$ can be
Fig. 7. Bypass Fan/OGV test case. Reconstructed axial velocity profiles (real parts) from traditional \([P + V]\) wavesplitting methods applied to data at each of the three neighbouring planes \(x = 0.340\) m, \(x = 0.344\) m and \(x = 0.348\) m (dotted lines) and from the TPP matching method applied across all three planes (solid line). Using the homentropic potential flow model, the reconstructed profiles are all compared at the plane \(x = 0.344\) m to the original CFD velocity data (triangles).

obtained from the axial velocity modes \(\{u_\mu\}\) that correspond to the pressure modes of the form (2). To the order of the approximation, the axial velocity modes are given by

\[ u_\mu(X, r) = \frac{k_\mu(X)\psi_\mu(X, r)}{D(X)(\omega - U(X)k_\mu(X))}. \tag{12} \]

These are translated to the required axial plane \(x\), before summing them to give the complete profile (at the current frequency and circumferential wavenumber of interest). The real parts of such reconstructed axial velocity profiles from traditional \([P + V]\) wavesplitting and the TPP matching method compared at a single axial plane are shown in figure 7. The most striking observation one can make from the profiles is that the TPP strategy, which uses no velocity information, can only predict, and hence only support, (irrotational) velocity perturbations one tenth of the magnitude of the ones observed in the data.

The observation that the vortical part of the axial velocity perturbations is greater than the solenoidal part naturally affects the accuracy of the traditional \([P + V]\) wavesplitting methods significantly. The axial velocity profiles from performing a \([P + V]\) wavesplitting at each neighbouring plane in figure 7, disagree with one another’s solution when compared at a chosen plane because they assume a solenoidal behaviour in the velocity. Yet further
Fig. 8. Bypass Fan/OGV testcase. Amplitudes (SPL) at the outer casing of the transmitted radial modes predicted by traditional $[P + V]$ wavesplitting methods at five different axial locations.

Fig. 9. Bypass Fan/OGV testcase. Amplitudes (SPL) at the outer casing of the transmitted radial modes predicted by the TPP matching method at five different axial locations.
evidence can be seen in the plot of transmitted modal SPL at the outer casing derived by applying the \([P + V]\) wavesplitting at five different axial locations in figure 8.

Given that the amplitudes of the cut-on modes should be constant in the homentropic potential flow acoustic model, the amplitudes obtained from the traditional wavesplitting method show an enormous variability, even to the order of \(10 \sim 15\) dB for the fourth and fifth radial modes. Indeed the resolution of the higher order cut-on modes appears to be strongly affected by the vorticity contamination.

The comparison of the \([P+V]\) wavesplitting modal amplitudes to the almost constant transmitted amplitudes, with differences much less than 1 dB, obtained by the TPP matching method (figure 9) seems to be strong proof of the superior ability of the TPP matching method to capture and extract the acoustically behaving part of such a complicated CFD solution extremely well. The TPP matching method, in effect, has not only matched the data but synchronised the two models, leading to a greater confidence that the model jump is not too severe in this case.

As a final comparison of matching in the bypass duct case (figure 10), another set of CFD data produced for the same bypass geometry was matched using the TPP matching method and compared to a Rolls-Royce in-house wavesplitting method[1],[2] and the acoustic analogy approach of Ffowcs Williams-Hawkings[3]; both of these methods were briefly described in the introduction. Very good agreement is obtained for the first three radial modes in all cases, although the agreement is nowhere near as good for the fourth and fifth radial modes. Indeed, the in-house wavesplitting method is unable to resolve these higher-order radial modes, possibly due to the strong vorticity effects described above. The TPP matching method does not suffer from such difficulties and, due to the indirectness of the acoustic analogy method and the consistency of our results, it is reasonable to postulate that the TPP result is probably the most accurate in obtaining the fourth and fifth modal amplitudes.

![Fig. 10. Bypass Fan/OGV test case. Comparison of the TPP matching method with an in-house wavesplitting method and the acoustic analogy approach of Ffowcs Williams-Hawkings.](image_url)
I. 6.2  **DLR Inlet matching on a spinner**

For exactly the same engine geometry as the Rolls-Royce case (figure 11) DLR were able to supply sets of CFD data at three axial planes for inlet-duct matching (upstream of the fan in the figure). The case presented here is at a frequency of $ka = 21.5$ (1BPF) and circumferential wavenumber $m = -26$, where the average axial Mach number is taken to be $M = 0.35$. This testcase enabled us not only to tackle the difficult task of matching on the spinner, where the axial variation of the duct is highly significant, but also to examine the ability of the TPP matching method to deal with cut-off modes correctly (there are no cut-on radial modes in this case). Data was supplied at axial planes $x = -0.06, x = -0.05$ and $x = -0.04$ and the transmitted and reflected modal amplitudes (SPL) at the outer casing of the plane furthest upstream are shown in figure 12. Modal reflections appear to be fairly negligible in this case (20 dB lower than the transmitted amplitudes for lower radial orders) and the modal amplitudes drop smoothly with increasing radial order, in line with physical expectations. The final figure (figure 13) shows a comparison of the CFD pressure data (imaginary part) with that derived from our homentropic potential flow model, using the obtained modal amplitudes. The agreement appears to be excellent, with both CFD and analytical models capturing the exponential decay of the sound field away from the source.

![Figure 11. Fan/OGV testcase engine geometry. DLR supplied three axial planes located at the fan-to-bypass-duct interface. The duct has hard walls.](image)

I. 7  **Conclusions**

In order to make progress in the prediction of turbomachinery noise by direct calculation with CFD methods of the noise sources, like rotor-stator interaction between fan and OGV or between compressor and turbine stages, it is necessary to decouple the noise generation
Fig. 12. Inlet testcase. Transmitted and reflected modal amplitudes (SPL) at the outer casing from TPP matching.

from the propagation effects. Otherwise, the available computational resources will be not sufficient for many years to come.

By our Triple-Plane Pressure (TPP) matching method, we have shown that it is possible to take the CFD noise field from planes in the vicinity of the CFD domain boundaries and extract in a robust manner the information required to input to CAA or analytical models, which are designed to describe the propagation and radiation processes more efficiently.

The method is both robust and flexible, as it is based on a least squares fit that in principle allows the matching of any flow variable at any number of planes. A subtle scaling of the amplitudes proves to be sufficient to suppress the exponential coupling of cut-off modes between the matching zone ends, and at the same time limits in a simple and systematic manner the occurrence of the false near field that is bound to occur when a pressure distribution at a single interface acts as an acoustic boundary condition.

By using slowly varying modes, there is no geometrical restriction to the location of the interface. It is indeed possible to apply the matching at any diverging or converging part of the duct. Perhaps the only locations that should be avoided for matching are close to areas where either cut-on cut-off transition (in hard-walled ducts) or near cut-on cut-off transition (in lined ducts) of a mode may occur[14,15].
Fig. 13. Inlet testcase. Comparison of CFD imaginary pressure data (dotted lines) with the modal solutions derived from the TPP matching method (solid lines) at all three axial planes - first 50 radial modes included.

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References


Deliverable D2.14 Part II.
In-Duct Matching, Propagation and Radiation

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Abstract

This report forms part 2 of the deliverable report D2.14 of the TurboNoiseCFD project. It describes the process by which acoustic waves propagating inside the bypass duct of a generic aircraft engine are admitted into a region closer to the exit plane of the duct and radiating to the acoustic far field. The emphasis is placed on the best practices to ensure accurate prediction of far field directivity. The far field directivity is taken as an important criterion of the quality of the prediction. The proposed numerical method has three ingredients: a matching process to admit acoustic waves into the in-duct propagation region; near field propagation inside the duct and diffraction at the lip of the exhaust duct; an integral surface for far field directivity. The quality of the proposed method is illustrated through a case study. A representative exhaust duct geometry is selected to conduct a joint exercise with Technical University of Eindhoven (TUE), in which TUE calculates the in-duct propagation using their multi-scale code. At a matching plane inside the duct, the acoustic waves are admitted into the Southampton computational aeroacoustics (CAA) model as part of the inputs on the boundary of the computation domain. There is an overlapping region inside the duct where solutions exist for both TUE and Southampton models, which allows cross validation of both methods. Results so far suggest that the current method has promise.

II. 1 Introduction

The aim of part of the workpackage 2 is to develop a numerical method for sound radiation from ducts. These include both intake and exhaust ducts. The exhaust duct radiation problem is, by nature, more difficult. In the case of radiation from either a bypass duct or a core nozzle, there are added issues associated with the presence of a shear layer between the exhaust flow and the external stream. The presence of the shear flow would lead to a different noise radiation pattern from that of a simple mean flow. A correct simulation is not trivial as there are additional features which so far have not been accounted for. Thus far little information is placed in open domain on this problem.
The present method is based on the CAA approach. CAA is concerned predominantly with obtaining time-accurate numerical solutions, through the use of long-time accurate time-integration strategies and high-order spatial discretization schemes. In pursuing this approach, we have in mind the problem of modal radiation from bypass and exhaust ducts of aircraft engines. In the near field in and around the duct exit, we model the noise propagation through solutions of the linearised Euler equations (LEE). LEE has certain advantages in that they admit entropic, vortical and acoustic disturbances, and allow for non-uniform mean flows. For the CAA methods, a finite computational domain is required, leading to the requirements of non-reflective acoustic boundary conditions and a radiation model, generally in the form of an integral representation. In this project we assume an axisymmetric mean flow. Under the assumption of axisymmetric mean flow, the disturbances can be represented by a series of Fourier components in the azimuthal direction. For a particular azimuthal mode, a system of two-dimensional equations can be used. Hence we are solving the so-called 2.5D form of linearised Euler equations. For far field radiation, we employ the Ffowcs Williams–Hawkins integral.

Over the period of the project, the numerical method development was accompanied by a series of benchmark test case studies. These include two-dimensional wave propagation in a duct to assess non-reflecting boundary conditions, high-order spinning mode propagation in an infinite duct to establish points-per-wavelength requirement, high-order spinning mode radiation from an unflanged duct to establish far field directivity requirement, and sound radiation from an exhaust duct to account for the effect of shear layer. Analytical solutions exist for all of the above cases, which can be used to compare with the numerical computation. Hence the development work is validated at every steps. Finally lessons learnt from these benchmark case studies are used in a generic bypass duct radiation case study.

II. 2 Description of Matching, Propagation and Radiation

The basic problem is illustrated in Fig 14. In the near field in and around the duct exit, we model the noise propagation through LEE solutions. Under the assumption of axisymmetric mean flow, the disturbances can be represented by a series of Fourier components in the azimuthal direction. For a particular spinning-mode, a system of two-dimensional equations can be used[1]. To admit the acoustic waves propagating inside the duct into the CAA propagation region, a matching processing is required, in the form of an absorbing zone. The absorbing zone also fulfills the task of damping outgoing spurious waves[2]. The far field directivity is estimated through an integral surface solution; the surface is placed inside the near field CAA propagation region[3,4] (see also Fig. 15). These main ingredients of the numerical methods will now be described separated below.
II. 2.1 Propagation

Assuming small perturbations about a steady mean flow, acoustic wave propagation can be described by the linearised Euler equations. Solutions of the full 3D equations are expensive. If we were to assume that the acoustic disturbances are restricted to the blade passing
frequency and harmonics and propagate on an axisymmetric mean flow field without swirl, it would be possible to write the disturbances in terms of a Fourier series, e.g. for the axial velocity disturbance $u'$

$$u' = u'_m(x, y) e^{ikt-im\theta}.$$  \hspace{1cm} (13)

It is now possible to express the overall disturbances in terms of the sum of individual components. Each individual component (mode) can be described by a set of 2D equations. Generally the equations are solved in complex space. However, if we utilise Eq. (13), then it is possible to arrive at a new set of equations:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho' u_0 + \rho_0 u')}{\partial x} + \frac{\partial (\rho_0 v' + \rho' v_0)}{\partial y} - \frac{m \rho_0 w'_i}{k y} - \frac{\rho_0 v' + \rho' v_0}{y} = 0, \hspace{1cm} (14)$$

$$\frac{\partial u'}{\partial t} + \frac{\partial (u_0 u')}{\partial x} + v_0 \frac{\partial u'}{\partial y} + v' \frac{\partial u_0}{\partial y} + \rho_0 \frac{\partial p'}{\partial x} = 0, \hspace{1cm} (15)$$

$$\frac{\partial v'}{\partial t} + u_0 \frac{\partial v'}{\partial x} + u' \frac{\partial v_0}{\partial x} + \rho_0 \frac{\partial (v_0 v')}{\partial y} + \rho_0 \frac{\partial p'}{\partial y} = 0, \hspace{1cm} (16)$$

$$\frac{\partial p'}{\partial t} + u_0 \frac{\partial p'}{\partial x} + v_0 \frac{\partial p'}{\partial y} + u' \frac{\partial p_0}{\partial x} + v' \frac{\partial p_0}{\partial y} + \frac{\gamma}{\gamma} \left( \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} - \frac{m}{k y} w'_i \right) + \frac{\gamma}{\gamma} \left( p_0 v' + p' v_0 \right) = 0, \hspace{1cm} (17)$$

where superscript (’) and subscript (0) are perturbation and mean properties respectively. $\rho$, $p$, $u$, $v$, $w$ are density, pressure, axial, radial and azimuthal velocities respectively. $x$ and $y$ are axial and radial coordinates, and $\theta$ is the azimuthal angle. For convenience, the spinning mode number ($m$) is dropped from the subscript of the variables in Eqs. (14) to (18). The fluid is modelled as a perfect gas. Eq. (17) is cast in terms of $w'_i(= \partial w/\partial t)$. Solutions of the new 2.5D equations only require two extra arrays for $w^i_{(i)}$ and $w^i_{(i+1)}$, where $i$ stands for time step. The boundary treatment for $w'_i$ is the same as that for $w'$. The $w'$ velocity is recovered after each time step.

**II. 2.2 Radiation**

An integral solution of the FW-H equation is implemented numerically to allow the near and far field noise levels to be determined efficiently. The FW-H formulation is attractive in comparison to other integral methods as it permits the passage of hydrodynamic disturbances through the integration surface without affecting the acoustic field, and therefore affords a greater degree of flexibility in positioning the surface than the Kirchhoff method[5]. The particular integral solution implemented is known as formulation 1A following Farassat [4].
The in-duct matching with TUE is illustrated in Fig. 16, which shows a region near the exit of a typical bypass duct.

The matching between the TUE and Southampton solutions is realised through an absorbing zone arrangement\cite{2}. To match the TUE solutions, the incoming acoustic waves are specified as a target in the absorbing zone region (red zone in Fig. 16). This takes the form of an explicit formulation:

\[
\tilde{Q}^{n+1} = \overline{Q}^{n+1} - \sigma(x)(\overline{Q}^{n+1} - \tilde{Q}_{\text{target}})
\]

The solution vector is explicitly damped after each time step. \(\overline{Q}^{n+1}\) is the solution vector computed after each time step. The damping coefficient, \(\sigma\), varies smoothly according to the function,

\[
\sigma(x_b) = \sigma_{\text{max}} \left| 1 - \frac{x_b - L}{L} \right|^\beta
\]

where \(L\) is the width of the buffer zone, \(x_b\) is the distance from the inner boundary of the buffer zone and \(\sigma_{\text{max}}\) and \(\beta\) are set coefficients which determine the shape of the damping function.

The target, i.e. TUE input, has the general form of:

\[
p' = A \left[ J_m(k_r y) + c_1 Y_m(k_r y) \right] \Re \left\{ e^{i\omega t - ik_a(x-x_1) - im\theta + i\phi_1} \right\}, \quad y_{\text{inner}} < y < y_{\text{outer}}
\]

\[
u' = \frac{k_a p'}{\rho_j(k - k_u u_j)},
\]

\[
u' = \frac{A}{\rho_j(k - k_x u_j)} \frac{d}{dy} \left[ J_m(k_r y) + c_1 Y_m(k_r y) \right] \Re \left\{ e^{i\omega t - ik_a(x-x_1) - im\theta + i\phi_1 + i\pi/2} \right\},
\]
\[ w'_j = -\frac{mkA}{\rho_j y(k - k_x u_j)} \left[ J_m(k_r y) + c_1 Y_m(k_r y) \right] \text{Re} \left\{ e^{i\omega t - i\omega_k(x - x_1)} - i\omega \theta + i\varphi_1 + i\pi/2 \right\}, \]

where \( j \) stands for the duct flow condition, \( J_m \) is the Bessel function of the first kind and \( Y_m \) is the Bessel function of the second kind. The modal amplitude \( A \) and phase \( \varphi_1 \) are calculated from the data supplied by TUE. The axial mode number \( k_a \) and cut-on ratio \( \xi \) is calculated from

\[ k_a = \frac{k}{\beta^2} \left( -M_j + \sqrt{1 - \xi^{-2}} \right) \quad \text{and} \quad \xi = \frac{k}{k_r \beta}, \]

where \( \beta = \sqrt{1 - M_j^2} \). \( k_r \) is solved through iteration by,

\[ \frac{J^*_m(y_{\text{inner}} k_r)}{Y^*_m(y_{\text{inner}} k_r)} = \frac{J^*_m(y_{\text{outer}} k_r)}{Y^*_m(y_{\text{outer}} k_r)}, \tag{19} \]

where the superscript \( * \) stands for derivative (\( d/dy \)), \( y_{\text{inner}} \) and \( y_{\text{outer}} \) are the duct inner and outer wall diameters respectively.

II. 3 Numerical Methods

The LEE solver uses an optimised prefactored compact scheme[6] for spatial derivatives and a 4th-order 4/6 stage explicit Runge-Kutta scheme[7] for time integration. The compact scheme is a development of the formally 6th-order compact scheme by Hixon[8]. A slip-wall boundary condition is applied to the duct wall and buffer zones are placed around the boundary of the physical computational domain to provide non-reflecting boundary conditions. Inside the duct, a buffer zone (with a width of at least one wavelength, \( \lambda_a \)) is used to absorb the spurious waves as well as acts as a wave admission region, where the incident wave is used as reference. Elsewhere in the surrounding buffer zones, the waves are damped to zero at the buffer zone outer boundary. The basic methodology has been validated against a range of test cases, including a Gaussian pulse passing though a 2D cylinder and linear wave propagation problems. The 2D method developed for single spinning-mode propagation is also been validated against sound radiation from an unflanged duct[1]. In the unflanged duct study, we established point-per-wavelength requirements for duct radiation computation using the current method. These requirements are employed in the project and reported below.

A 3D integral surface needs to be constructed to evaluate the far field noise level. For the computation of the \( m \)th mode, we use a total of \( m \times N \) patches in the azimuthal direction, where \( N \) is time steps per modal period.
II. 4 Towards Better Far Field Prediction

II. 4.1 Absorbing boundary conditions

To ensure best in-duct matching with TUE calculation, it is necessary to minimise the spurious reflections on the inflow boundary, which would contaminate the solution. The explicit condition described earlier was found to be the most effective among a number of boundary conditions proposing to be non-reflective. These include an implicit formulation[2], an implicit damping with artificial convection[9], Thompson’s characteristic boundary conditions[10] and perfect matched layer (PML) condition[11].

Figs. 17(b), (c) and (d) give the wave patterns of a generic test case. This test case is the same as that employed in the workpackage 1, i.e. two-dimensional cut-on and downstream/upstream propagating acoustic waves. The basic setup of the problem is given in Fig. 17(a). In the inflow zone we prescribe a sinusoidal pressure wave moving from left to right:

\[ p' = 89 e^{i\omega t - ik_x y} \]  

(20)

By varying the cut-on ratio, we can vary the wave frequency and the wave angle of the input wave. The wave angle, \( \theta \), is defined as the angle made between the wavefront normal and the normal of the boundary edge. Details of the study can be found in Richards, et al[2].

Fig. 17(b) reveals an interesting feature of the explicit formulation. Inside the outflow buffer zone, the effective wave angle is gradually reduced. This compares with the results for the implicit formulation (Fig. 17(c)) and the implicit+local convection formulation (Fig. 17(d)), where the angle of the wavefront is kept. This is more so in Fig. 17(d) than in Fig. 17(c). Non-reflecting or absorbing conditions work well with waves of small angle. Hence for large wave angle (or small cut-on ratio), the explicit formulation has an advantage over the other formulations. Indeed the results presented in Fig. 18 support this assertion. It shows that the explicit formulation performs better at all the angles (or cut-on ratios) tested. Finally we should mention that the so-called PML condition was found to introduce unstable solutions at the PML boundary and computation domain proper. An effective reduction of time step is therefore required to allow for a stable solution. Furthermore we observe no advantages of PML conditions over the other formulations in terms of in-duct matching performance.
II. 4.2 Computation Grid Requirement

Once a satisfactory matching is achieved in the in-duct matching zone, i.e. absorbing zone, The waves should be allowed to propagate towards the exit plane of the duct without significant reduction in strength or being distorted.
Fig. 18. Reflection with varying wave angle, $\theta$. Buffer grid points are fixed at 20 points.

(a) SPL level on the duct wall; $m = 30$, $n = 1$, $k = 40$, $\xi = 1.23$.

(b) SPL decay per wavelength.

Fig. 19. Axial resolution requirement for duct modal propagation. $2\pi/(k \Delta y) = 15.2$.

To test the resolution requirement, the propagation of a single mode inside a circular duct with hardwall boundary condition is tested[1]. A quantitative description of the resolution requirement is given in Fig. 19(a), where the pressure decay on the wall is plotted over an axial distance of about 9 wavelengths ($\lambda_a$). In obtaining the results in Fig. 19, the cell distribution in the radial direction is fixed at an average $2\pi/(k_r \Delta y) = 15.2$ with a stretched grid (compression ratio of 1.05 and concentration of grid points near the wall). It can be seen that with lower $2\pi/(k_a \Delta x)$, a larger decay is experienced along the axial direction. The solution improves as $2\pi/(k_a \Delta x)$ is increased.
Defining a criterion for resolution is an arbitrary process. If we specify a requirement of sound pressure level (SPL) decay per wavelength, \( |d(\Delta \text{SPL}_{w}, \text{dB})/d(x/\lambda_a)| < 0.1 \text{ dB} \), an axial resolution of between 7 and 8 appears to be adequate (Fig. 19(b)).

In order to test the grid resolution requirement in the radial direction, we have used an axial resolution of \( 2\pi/(k_a \Delta x) = 10.7 \) and changed the cell distribution in the radial direction. To meet the decay criterion, a minimum resolution near the wall must be satisfied. Tests suggest that a minimum value of \( 2\pi/(k_r \Delta y) = 30 \) is required near the wall (not shown).

II. 4.3 Far Field Directivity

A critical criterion to judge the quality of a duct radiation prediction ought to be the far field directivity. It is to be expected that some major shortcomings in a radiation prediction method will manifest themselves in terms of deterioration in the far field directivity prediction. Therefore it is recommended that any duct radiation method should first pass a quality test in terms of far field directivity prediction. There exist generic cases where analytical solutions are available, e.g. sound radiation from an unflanged duct (Homicz and Lordi[12]) and sound radiation from an exhaust duct[13]). Fig. 20 illustrates a reasonable computation of far field directivity of an unflanged duct radiation. The two features to look for are the range of observation angle and the dynamic range of pressure. By applying the best practices reported in this and previous deliverable reports, the dynamic range of the prediction is now more than 100 dB.

![Fig. 20. Sensitive indicator of prediction quality: far field directivity of radiation from an unflanged duct. \( m = 13, n = 1, k = 23, M_{\infty} = -0.5 \).](image)

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A common feature of the far field directivity prediction is the slight deterioration in the pressure level at higher observation angles. This deterioration is only relative to the main radiation peak prediction, which is always good if the best practices are employed. The radiation at the higher angle is inevitably linked to the edge of the duct. Hence in addressing the quality of the prediction, we need to look at two issues: (a) the treatment of the duct exit and (b) the grid resolution near the exit. To help clarify the problem, we conduct two exercises: one with different duct edge geometry and another with different grid resolution near the duct edge and placement of the integral surface.

For the duct edge geometry study, the modal radiation test case is that of $m = 13$, $n = 1$, $k = 23$, $M_\infty = 0$. Two types of duct edge are used: a rectangular edge of finite thickness and a chamfered wedge. Results in terms of the far field directivity are given in Fig. 21. Compared with the datum case of zero wall thickness, the 15 deg wedge does not introduce any significant changes in the radiation pattern (Fig. 21(a)). Changes in the angle of the wedge do not alter the main radiation peak and the interference dip angle. The ‘leakage’ in the low observation angle as discussed earlier is not affected (not shown). At high observation angle, there is a perceptible change in the far field directivity with a small bump appearing at around 105 deg. The rectangular exit has a bigger effect on the second radiation peak. Increasing the thickness of the wall from 4% to 8% of the duct diameter shifts the predicted pressure level up by 1.5 dB. The angle, thought, remains the same. Again, the main influence is in the high observation angle range rather than the low observation angle range (not shown).

To test the effect of the integral surface placement, we use the $m = 13$, $k = 30$ and $M_\infty = -0.5$ case. A series of spherical integral surfaces are constructed, centered on the centre of the exit of the duct. Results in terms of the far field directivity are given in Fig. 22(a).
There is very little difference in the main radiation peaks and the interference dip angles, suggesting a degree of insensitivity of the prediction to the surface placement. There are some slight changes in the low observation angle ‘leakage’ and additional ‘wiggles’ at the high observation angle as the integral surface is moved away from the duct. This feature suggests that the dispersion errors of the numerical scheme contribute to the low angle ‘leakage’. The overall effect, though, is rather small.

For the resolution study, we use the $m = 13$, $k = 30$ and $M_{\infty} = 0$ case. The unstretched calculation has a grid with $2\pi/(kr\Delta y) = 13.5$ and $2\pi/(ka\Delta x) = 11.3$. The stretched calculation concentrates cells near the wall and the duct exit, with $2\pi/(ka\Delta x_{\text{min}}) = 25$ and $2\pi/(kr\Delta y_{\text{min}}) = 30$, while maintaining the minimum resolution requirement away from the duct exit. Results are given in Fig. 22(b). It can be seen that the main radiation peak is predicted well, as well as the low angle values. However, a discrepancy is introduced at the higher angle and this discrepancy becomes progressively larger as the observation angle is increased. This result reproduces the deterioration in the far field directivity prediction observed earlier in the project. This suggests that the insufficient resolution near the duct exit is the main cause of the discrepancy.

The above exercises are not trivial. Only one mode is calculated here. In practice, there exist a multitude of modes. To avoid cross contamination and achieve good prediction over a wide range of angles, correct resolution must be achieved at the exit of the duct.
II. 5 A Joint Case Study with TUE

II. 5.1 Mean Background Flow

The background mean flow of the generic bypass duct test case is illustrated in Fig. 23 in terms of Mach number. The matching plane with TUE is located at $x = 0.55\, \text{m}$. Local Mach number is 0.338. The exhaust stream is issued into a stationary environment. We note the relatively large variations in duct geometry and hence local flow near the inner wall of the bypass duct approaching the the exit plane of the bypass duct.

Fig. 23. Mean Mach number distribution of the generic test case.

II. 5.2 Incoming Waves

The incoming waves come from the TUE multi-scale computation. A total of five radial modes exist. These form the input on the inflow boundary of the Southampton CAA computation and the target in the absorbing zone.

<table>
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<th>$n$</th>
<th>$f$</th>
<th>$k$</th>
<th>$k_r$</th>
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<td>23.7191</td>
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Table 1
Summary of the Incoming Waves, $m = -13, k = 28.3179$
II. 5.3 In-Duct Comparison with TUE

The generic case study allows for a region near the exit plane of the bypass duct where predictions exist for both TUE and Southampton computations. This arrangement enables validation of the current method. The in-duct pressure profiles at five locations are presented Fig. 24 together with the TUE results. It can be seen that the agreement is very good at $x = 0.55 \text{ m}$, $0.7 \text{ m}$ and $1.0 \text{ m}$. These positions are located inside the duct in a section where the duct geometry is slow-varying. Near the exit plane of the duct, discrepancies start to appear. This is as expected as the relatively large variation of the duct geometry invalidates the underlining assumption of the TUE multi-scale code. The Southampton CAA computation also admits the diffracted waves from the lip of the duct into the duct region, which makes the comparison much more difficult. It should be noted that there appears a slight deviation from the TUE incoming wave at $x = 0.55 \text{ m}$ where the matching between the TUE and Southampton takes place. The deviation exists near the inner wall of the duct. The cause of the deviation is attributed to the reflected waves inside the duct propagating upstream and out of the inflow matching zone.

![Fig. 24. Comparison with TUE in-duct computation at five locations.](image)

In Figs. 25-27, contributions from three radial modes are plotted together with the TUE predictions. Fig. 25 shows the $n = 1$ radial mode. The major activity is concentrated near the outer wall of the duct. It can be seen that the wavefronts are kept well until near the lip of the duct. For the $n = 3$ radial mode, the modal shape near the lip of the duct shows a large deviation from the TUE prediction, particular near the inner wall. For the $n = 5$ radial mode, the mode is just cut-on and the CAA prediction shows a large difference between the TUE and Southampton results on the whole of the exit plane.

Despite the differences between the individual radial mode predictions near the exit plane, the combined wave pattern (Fig. 28) shows a remarkable agreement between the TUE and Southampton predictions, up until $x = 2.2 \text{ m}$. This gives confidence to the current method.
II. 5.4 Near Field Propagation and Far Field Directivity

Selected near field wave patterns are given in Fig. 29. For the $n = 1$ radial mode, a radiation peak at 50.3 degrees is observed. There exists a shadow interference dip angle at 55.8 degrees and a second radiation peak at 60.5 degrees. The second radiation peak is formed by the edge diffraction of the duct mode. However the main radiation peak is formed between the 60.5 degree peak and another peak at 38.7 degrees, which is due to reflections off the afterbody surface of the bypass duct. The reflected waves propagate through the shear layer and experience a deflection to a higher observation angle. For the $n = 3$ radial mode, the reflections off the afterbody surface are much prominent. The reflected radiation peaks would then propagate through the shear layer and be directed to higher observation angles. For the last radial mode, $n = 5$, there is little radiation to the far field as the mode is just cut-on.

The predicted far field directivity is shown in Fig. 30. It can be seen that the overall directivity pattern in far field is dominated by the $n = 1$ radial mode below $\theta = 50.3$ degrees. However, the overall directivity has a main radiation peak at 62.4 degrees which is not dominated by any of the radial modes. Rather it is a combination of all the radial modes. It is interesting to observed that the overall far field directivity does not reveal sharp interference dip angles as observed in the earlier exhaust duct computations where the shear layer is modelled as a vortex sheet[6]. There is a broadening of the directivity content over the
II.6 Summary Remarks

(a) It is recommended that the far field directivity is used as the critical parameter to judge the quality of a duct radiation prediction method.

(b) To facilitate in-duct matching with duct propagation code, a buffer zone boundary condition based on an explicit formulation is employed and found to give better performance than other methods.

(c) For the high-order CAA method, results suggest that a minimum of 7 to 8 points per wavelength is required in both the axial and the radial directions to capture the correct wave propagation inside the duct. However, an additional minimum points per wavelength requirement of 30 near the wall is also needed for the high-order spinning-mode propagation.

(d) The method developed in the workpackage 2, based on solutions of the linearised Euler equations (LEE) for propagation inside the duct and near field and acoustic analogy for radiation, is a promising method. A generic test case study with TUE proves the validity of the method.

Fig. 26. Comparison of Southampton (a) with TUE in-duct computation (b); n=3.
Fig. 27. Comparison of Southampton (a) with TUE in-duct computation (b); n=5.

References


Fig. 28. Comparison of Southampton (a) with TUE in-duct computation (b); n=1-5.


Fig. 29. Near field propagation; (a) $n = 1$, (b) $n = 3$, (c) $n = 5$, and (d) $n = 1 - 5$. 
Fig. 30. far field directivity; (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, and (d) $n = 4$. 