Stability of steel arches

Start document

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BCO 01.02
February 2001
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1 Introduction

In steel construction the objective is often to construct large spans with a relatively low weight. In doing so stability phenomena such as plate-, local-, overall- and lateral torsional buckling govern the design. Therefore an understanding of all stability phenomena is required. From this understanding practical design rules should be deducted to check the structure for stability.

The use of steel arches is becoming increasingly popular due to the following reasons:
- The appreciated esthetics of curved elements in contemporary architecture
- The mechanics of an arch is often favorable, especially when the main reaction force is compressive.
- Efficient production through rolling of standard sections
- The use of advanced computer based design tools.

Currently very few and limited design rules for steel arches are available in international steel construction codes. In literature, several publications have been dedicated to the stability phenomena of steel arches over the past 50 years. However, neither a general theory nor design rules have been adopted.

At the TUE research has been carried out on the stability of steel arches, such as in-plane stability [1] and out-of-plane stability [2]. As of yet, the studied load cases for the out-of-plane stability do not resemble practical load cases. Therefore, additional research is needed to come to design rules for arches used in practice.

1.1 Key words
Steel, arch, stability, design rules, structural behavior, finite element method, experimental research, numerical research.

1.2 Objectives
The objectives of the research project are formulated as follows:
- Studying the structural behavior of steel arches.
- Deducing design rules from the structural behavior.
- Validating the design rules through experimental and numerical research.

1.3 Restrictions
Not all possible configurations of loads, shape of the arch and support bearings can be studied. A selection of parameters will be made based upon a study of literature and upon practical demands. A number of parameters are set down, these may alter as the project evolves. These restrictions are:

- Behavior: overall stability and lateral torsional buckling. Local- and distortional buckling will not be studied. These two phenomena will make the analysis more complex whereas in practice one would choose the dimensions of the cross section such that they do not occur.

- Shape: circular. Circular arches are easy to produce by rolling and it is understood that these are used most often. As the project evolves other shapes might be considered as well.

- Structural system: free standing arch. In modern architecture more often a single, laterally unsupported arch is chosen. This research is aimed at these kind of structures.
• Load cases: several concentrated loads and distributed load.
  An arch is loaded most favorably in pure compression, however, that is not a common
  load case. Therefore the combination of bending and compression due to concentrated
  loads will be studied. As the number of concentrated loads increase one might consider
  it as a distributed load.

• Cross sections: Wide flange beams and tubes.
  In this research only arches produced from a single continuous element will be
  considered. Arches build-up from several elements will not be studied.

The above mentioned terminology is illustrated in the appendix.

1.4 Assumptions
As a starting point the work of previous projects carried out at the TUE is taken. Furthermore,
the dimensions, load configurations and support bearings of the studied arches will be selected
such that they resemble practical cases.

2 Earlier research at the TUE
Research into the stability of steel arches commenced at the TUE in 1996. A thorough study of
literature has been performed by Verstappen [1]. This study has been completed with FEM-
simulations of in-plane stability of single, circle-shaped arches. This study resulted in a simple
and efficient verification method.
It has been found that asymmetric buckling is normative for stability. To determine the stability
of the arch an equivalent straight beam is taken. This beam has a buckling length of halve the
length of the arch and a load which is equal to the normal force at a quarter length of the arch,
see figure 1.
Compared to the verification method for in-plane stability of arches given in the German code
DIN 18800, this method allows larger loads.

![Figure 1: Verification method by [1] and DIN](image1.png)

In 1998 a second study into the spatial stability of single, unbraced and laterally unsupported
arches was performed by Delrue [2]. Two load cases were studied: uniform compression and
uniform bending, see figure 2.

![Figure 2: Previously studied basic load cases](image2.png)

In literature several analytical models for calculating the eigenvalue are available for these two
load cases. Table 1 gives an overview of the available theories and the applicability of the
models. The eigenvalues from these models were compared to FEM-determined eigenvalues.
The model suggested by Rajasekaran appeared to be the best.
<table>
<thead>
<tr>
<th>researchers</th>
<th>year</th>
<th>theory</th>
<th>calculation method</th>
<th>effect of warping</th>
<th>cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timoshenko and Gere</td>
<td>1961</td>
<td>straight beam</td>
<td>direct</td>
<td>not included</td>
<td>closed, double symmetric</td>
</tr>
<tr>
<td>Vlasov</td>
<td>1961</td>
<td>straight beam</td>
<td>direct</td>
<td>included</td>
<td>double symmetric 1-section</td>
</tr>
<tr>
<td>Yoo</td>
<td>1982</td>
<td>straight beam</td>
<td>energy method</td>
<td>included</td>
<td>non-symmetric</td>
</tr>
<tr>
<td>Trahair and Papangelis</td>
<td>1987</td>
<td>curved beam</td>
<td>energy method</td>
<td>included</td>
<td>non-symmetric</td>
</tr>
<tr>
<td>Rajasekaran and Padmanabhan</td>
<td>1989</td>
<td>curved beam</td>
<td>virtual displacement</td>
<td>included</td>
<td>non-symmetric</td>
</tr>
<tr>
<td>Yang and Kuo</td>
<td>1989</td>
<td>curved beam</td>
<td>virtual displacement</td>
<td>included</td>
<td>non-symmetric</td>
</tr>
</tbody>
</table>

Further geometric and material non-linear FEM-simulations of several arch with different dimensions showed that it was possible to use the existing column buckling curves to check stability. The relative slenderness is determined with the slightly modified model of Rajasekaran for the eigenvalue. The verification rules are then as follows:

For uniform compression:

$$\frac{N_{c,u,d}}{\omega_{arch.out} \cdot N_{c,a,d}} \leq 1$$

For uniform bending:

$$\frac{M_{u,max,a,d}}{\omega_{arch.out} \cdot M_{y,u,d}} \leq 1$$

In which ‘$$\omega_{arch.out}$$’ is the buckling factor based on the existing column buckling curves.

These two studies will serve as a staring point for the investigation into the stability of steel arches.

### 3 Scope of research

In many of the previously performed research projects the objective was to determine the eigenvalue analytically. To be able to do this simplifications needed to be made and most researches examined the two load cases of figure 2. However, the boundary conditions of these two load cases are such that one could argue that these are curved members, rather than arches. Further simplifications are the absence of imperfections and the assumption of linear elastic material behavior, thus ignoring the yield range of steel.

If one would construct an arch one would choose a hinge on both sides, rather than a roller bearing on one side, or one would consider fixed supports. For these load cases it becomes increasingly complex to determine the eigenvalue analytically.

With the aid of FEM-analyses it is possible to determine the eigenvalue numerically. Therefore the FEM will be used to determine the eigenvalues.

The eigenvalue is an important value to determine the load at which a structure fails through the loss of stability, but it does not give the actual failure load. To determine the actual failure load two methods are available: through experiments or through numerical simulations.

A series of experiments will be set-up with the objective to calibrate a FEM-model. Once a satisfactory FEM-model is obtained, it will be used to carry out geometric and material non-linear simulations.
This way two data sets are obtained: a set of eigenvalues and a set of simulated failure loads. It will be studied how, from the relation between these two values, a verification rule for the stability can be derived.

4 Approach

A number of stages can be distinguished in this project:

Study of literature: Much is based upon the findings of the investigations performed at the TUE, see chapter 1. Additional studies will be performed in areas that are not covered (experimental research) and in recently published work.

Experiments: An investigation will be carried out to come to a test program and a test setup for a limited number of experiments. The experiments will serve to capture the spatial stability phenomena of steel arches. FEM-simulations will be used to make a selection of the dimensions of the specimen, the load configuration and the support bearings. After the tests have been completed all data on the material, imperfections and eccentricities will be included in the FEM-model to calibrate this model on the experimentally obtained behavior.

Theory: Stability theory and the mechanics of arches will be studied. It will be reviewed if analytical models can be found or deducted to describe the stability of steel arches. Furthermore it must be reviewed if workable solutions of these models can be obtained. A second part of the theoretical work is to draft a verification rule and test it on numerically gathered data.

FEM-simulations: A FEM-model will be developed and calibrated on the experiments. Once a reliable model has been obtained it will be used in a parameter study to identify which parameters and to what degree they influence the stability of steel arches.

Design rules: Stability theory and mechanics of arches will be used to derive design rules. Some aspects are not covered by the theoretical approach, but are included in the FEM-simulations (non-linear material behavior, etc.). In the proposed design rules, these aspects will be added as empirical data gathered by the parameter study.

The emphasis of the project should be to first carry out a thorough search of literature. With the obtained knowledge a FEM-model will be built and a test setup and test program conceived. At the conclusion of the tests, the FEM-model can be calibrated and used for a parameter study. The results of the parameter study, combined with the present theory on the stability of steel arches, should lead to one or more reliable design rules.
To put all of this in some perspective the following comparison is made. In *The Structure of Scientific Theories*, Suppe [3] describes the received view on theories. In [4] a graphical representation of the founding and justification of theories (theory X) is given, which is called the empirical cycle. This cycle is an idealized model and is illustrated in figure 3. Justification of a theory can be done in two ways:

- Inductive: from the observation (I) via empirical laws (II) to the theory (III)
- Hypothetical-deductive: from the theory (III) hypotheses (IV) are deducted which are checked on the observations (I).

![Figure 3: the empirical cycle by Suppe](image)

Without going too deep into this subject, we will use an analogy to the empirical cycle for this research project. The terms of this project are substituted into the empirical cycle and are given in italic print in figure 4.

Now, one can see that experiments are used for confirming (calibrating) the FEM-analysis. In turn, the FEM-analysis is a basis for the general structural theory of arches. This is the inductive part of the empirical cycle.

By using the mechanics of arches one can deduct design rules. With these, the experiments can be predicted. This is the hypothetic-deductive part of the empirical cycle.

![Figure 4: Analogy to the empirical cycle](image)

A large part of this research is going to be inductive in nature. Once a substantial amount of data has been gathered a design rule can be deducted and tested on the gathered data.
5 Verification method

A few ideas exist about in which direction a verification method needs to be sought. The first is to draft such a verification rule that existing column buckling curves can be used. This has been utilized in the two, in chapter 1, discussed research projects.

Buckling length: For the in-plane stability a simpler straight column is checked for buckling in which the mechanics of the arches are expressed in the buckling length.

Relative slenderness: For the out of plane stability the relative slenderness is determined for the arches. With the relative slenderness the existing column buckling curves can be used to determine the load at which stability governs.

Some further direction to investigate is the Merchant-Rankine postulate.

 Merchant-Rankine: The Merchant-Rankine postulate is used in sway frames and is defined as follows: \[
\frac{1}{F_{\text{critical}}} = \frac{1}{F_{\text{buckling}}} + \frac{1}{F_{\text{plastic}}}
\]

The thus predicted failure load \(F_{\text{critical}}\) is always lower than the upper bounds of failure (the buckling load or the rigid plastic failure load).

6 Planning

This project has commenced in the fall of 1999 and will last for four years. A general planning has been made and is shown in table 2. All major stages of this project are listed in the planning.

<table>
<thead>
<tr>
<th>Year</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dates</td>
<td>Sept 99 – Aug 00</td>
<td>Sept 00 – Aug 01</td>
<td>Sept 01 – Aug 02</td>
<td>Sept 02 – Aug 03</td>
</tr>
<tr>
<td>Month</td>
<td>1-6</td>
<td>7-12</td>
<td>13-18</td>
<td>19-24</td>
</tr>
<tr>
<td>1 Study of literature</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Preliminary investigation experiments</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>3. Experimental research</td>
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<tr>
<td>4. Development and calibration of FEM-model</td>
<td></td>
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<tr>
<td>5. Analytical research</td>
<td></td>
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<tr>
<td>6. FEM-simulations</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7. Writing of thesis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Planning of the project
References

[1] Verstappen, I.
Toetsingsregels voor stalen bogen
Technische Universiteit Eindhoven, BKO 96.10, 1996, (Consists of two parts).

[2] Delrue, S.
Out-of-plane stability of steel arches

[3] Suppe, F.
The Structure of Scientific Theories,

Inleiding wetenschapsfilosofie
Appendix
**Definitions**

**Curved member:** Member in the shape of part of either a circle, catenary, parabola or of the sine function, without horizontal (or radial) restraint at one support in the plane of loading.

**Arch:** Member in the shape of part of either a circle, catenary, parabola or of the sine function with horizontal restraints in the plane of loading.

**Shapes of arches:** Arches can have the shape of one of the functions of figure 5, or of any third or higher order polynomial. In order to have just compression in the arch, each load case has a corresponding shape:

<table>
<thead>
<tr>
<th>load case</th>
<th>shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>- radial load</td>
<td>(part of a) circle</td>
</tr>
<tr>
<td>- evenly distributed load</td>
<td>parabola</td>
</tr>
<tr>
<td>- constant load per unit length of the arch</td>
<td>catenary (tension)</td>
</tr>
</tbody>
</table>

![Figure 5: Shapes of arches](image)

**Snap-through buckling:** A shallow arch loaded in compression in which the axial shortening is of such magnitude that the arch is pressed into a straight member after which it flips into the inverse shape and is loaded in tension.
On the next pages some illustrations of the use of arches are given. At first some examples of bridges are shown, then of arches used in buildings and finally an example of an arch used as building.

Below a classification is given for arches which is used to order the illustrations

Type of member comprising the arch:

**Continuous element:** Arch manufactured from one continuous element, such as a tube or boxed beam or wide flange beam.

**Arch truss:** Three or more continuous elements with bracing in between forming an arch.

Configuration of arches:

**Single arch:** For an arch made of one continuous element the lateral stability is obtained from the supported structure through the hangers or ties. For an arch truss the lateral stability is largely obtained through the lateral bending stiffness of the arch itself.

**Multiple arches:** Two, or more, arches parallel to each other offer the possibility to derive lateral stability from bracing in between the arches.
Arches in bridges

1
Single arch from continuous element
Lateral stability is obtained from the ties.

Oriente Station, Lisbon, Portugal
Architect: Santiago Calatrava
Year: 1998

2
Multiple arches from continuous elements
Lateral stability is obtained from bracing in between the two arches.

G-Mex center bridge, Manchester, Great Britain

3
Single arch from continuous element
Lateral stability is obtained from the ties

River Yarra bridge, Melbourne, Australia

4
Single arch truss
Lateral stability is obtained by the lateral bending stiffness of the truss.

Navajo bridge, Arizona, USA
Arches in buildings

5

Single arch truss

Blaak tram, subway and railway station,
Rotterdam, the Netherlands

Architect: Harry Reijnders
Year: 1987 - 1990

6

Single arch from continuous element

Kölner arena, Cologne, Germany

Architect: Peter Böhm
Year: 1998

7

Braced multiple arches from continuous elements

Ludwig Erhard Haus,
Berlin, Germany

Architecture:
Nicholas Grimshaw & Partners Ltd.
Year: 1998
Arch as building

Single arch from continuous element

This arch is built from concrete. This example is used to show an arch which only has to support itself. Therefore it is built in the shape of the catenary.

Thomas Jefferson expansion memorial,
Saint Louis, Missouri, USA.

Architect: Eero Saarinen

Year: 1959-1964

Source of the photographs in the appendix

1 Dagowin la Poutré


5 Figure de liaison, Station de tramway, métro, Rotterdam, Technique et Architecture, June-July 1995, no. 420, p. 88

6 Ulrich Brinkmann, Auf Dock gelegt, Die Deutzer Kölnarena, Bauwelt, Vol. 89, no. 44, pp. 2482-2487
   Photo: Lukas Roth, Cologne

7 Mathias Remmele, Das Bogen-Haus, Ludwig Erhard Haus in Berlin, Bauwelt, Vol. 89, no. 44, pp. 2470-2477
   Photo: Werner Huthmacher, Berlin

8 L’arco-balcone di St. Louis rivisitato
   Architettura; cronache e storia, June 1979, vol. 25, no. 6(284), pp. 352-353
Bibliography

A
Antman, S.S.; Marlow, R.S.
*New phenomena in the buckling of arches described by refined theories*, International journal of solids and structures, vol. 30, no. 16, 1993, pp. 2213-2241

Austin, W.J.; Ross, T.J.
*Elastic buckling of arches under symmetrical loading*, Journal of the structural division, vol. 102, no. ST5, 1976, pp. 1086-1095

Austin, W.J.
*In-plane bending and buckling of arches*, Journal of the structural division, vol. 97, no. ST5, 1971, pp. 1575-1592

Austin, W.J.; Ross, T.J.; Tawfik, A.S. et al

B
Bochenek, B.; Gajewski, A.
*Multimodal optimalization of arches under stability constraints with tow independent design function*, International journal of solids and structures, vol. 25, no. 1, 1989, pp. 64-74

C
Chucheepsakul, S.; Buncharoen, S.; Huang, T.

D
Dawe, D.J.

Delrue, S.F.; Snijder, H.H.; Bijlaard, F.S.K.
*Out-of-Plane Inelastic Buckling and Strength of Steel Arches*, Discussion by (see authors), Journal of Structural Engineering, 1998, pp. 352-353

Delrue, S.F., Snijder, H.H.; Bijlaard, F.S.K.
*Toetsingsregels voor stalen bogen (2), Stabiliteit*, Bouwen met Staal, vol. 148, 1999, pp. 54-57

G
Gjelsvik, A.; Bodner, S.R.

Goddon, W.G.

H
Harrison, H.B.
Karami, G.; Farshad, M.; Banan, M.R.  

Kee, C.F.  
*The Design of the Unbraced Stabbogen Arch*, The Structural Engineer, 1959, pp. 265-270

Kee, C.F.  
*Lateral Inelastic Buckling of Tied Arches*, Journal of the Structural Division, vol. ST1, 1961, pp. 23-39

Kuo, S.R.; Yang, Y.B.  

Koo, S.L.; Haque, M.J.  

Palkowski, S.  

Papangelis, J.P.; Trahair, N.S.  

Papangelis, J.P.; Trahair, N.S.  

Papangelis, J.P.; Trahair, N.S.  

Papangelis, J.P.; Trahair, N.S.  

Papangelis, J.P.; Trahair, N.S.  

Plaut, R.H.  
*Buckling of shallow arches with supports that stiffen when compressed*, Journal of engineering mechanics, vol. 116, no. 4, 1990, pp. 973-976

Qaqish, S.; Haddadin, A.  
Rajasekaran, S; Ramm, E.

Rajasekaran, S.; Padmanabhan, S.

Sakimoto, T.; Komatsu, S.

Sakimoto, T.; Sakata, T.

Scholz, H.

Schreyer, H.L.; Masur, E.F.

Stüssi, F.;
Lateral Buckling and Vibration of Arches, IABSE Publications, vol. 7, 1944, pp. 327-343

Tokarz, F.J.
Experimental Study of Lateral Buckling of Arches, Journal of the Structural Division, vol. ST2, 1971, pp. 545-559

Trahair, N.S.

Trahair, N.S.

Trahair, N.S.; Papangelis, J.P.

Vacharajittiphan, P; Trahair, N.S.

Van der Bij-Verstappen, I.; Snijder, H.H.; et al
Verstappen, I.; Snijder, H.H.; Bijlaard, F.S.K.

Vlahinos, A.S.; Ermopoulos, J. Ch.; Wang, Y.C.

Wicks, P.J.

Wolde-Tinsaie, A.M.; Foadian, H.

Yabuki, T; Vinnakota, S.

Yabuki, T.; Vinnakota, S; Kuranishi, S

Yabuki, T.; Vinnakota, S; Arizumi, Y

Yang, Y.B.; Kuo, S.R.

Yang, Y.B.; Kuo, R.S.

Yang, Y.B.; Kuo, S.R.; Yau, J.D.

Yarimci, J.A.;Yura, J.A.;Lu, L.W.

Yong-Lin Pi; Trahair, N.S.

Yong-Lin Pi; Trahair, N.S.
Yoo, C.H.

Yoo, C.H.; Pfeiffer, P.A.

Yoo, C.H.; Pfeiffer, P.A.
*Buckling of curved beams with in-plane deformation*, Journal of structural engineering, vol. 100, no. 2, 1984, pp. 291-300
Stability of Steel Arches

Problem statement:
The use of steel arches is becoming more popular for aesthetic reasons and due to easier design- and calculation software. The most commonly used steel arches are slender for which stability governs. Three types of stability can be distinguished:
- In plane buckling
- "snap-through" buckling
- Out-of-plane stability

In Dutch and European codes no verification methods are given to check stability.

Objective:
- To study the structural behaviour
- To develop verification methods
- To validate the verification methods by means of experiments and numerical simulations.

Approach:
- Literature search
- Developing test set-up and performing experiments
- Simulating the structural behaviour of tested arches with the Finite Element Method (FEM)
- Building analytical models for verification methods
- Parameter study to develop design equations

FEM-model of arch
Two failure modes
Arch susceptible to out-of-plane stability

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