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General equilibrium and international trade
with natural exhaustible resources

Jan H. van Geldrop
Cees A.A.M. Withagen

Eindhoven University of Technology
Department of Mathematics and Computing Science
P.O. Box 513
5600 MB Eindhoven
The Netherlands

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The Netherlands
ABSTRACT

Mostly the pricing of raw materials from an exhaustible resource is analyzed within the context of a partial equilibrium. It is argued here that there are good reasons for a general equilibrium analysis. We present an international trade model with exhaustible resources and derive the characteristics of the general equilibrium.

1. INTRODUCTION

The pricing of raw material from natural exhaustible resources is an important real world issue, as has intermittently and painfully become apparent during the oil crises, the collapse of the tin cartel etcetera. Not surprisingly economic theory has put considerable effort in developing models to explain price movements on the markets for raw materials, taking into account a variety of possible market structures. Hotelling (1931) is of course to be mentioned here as the founder of resource economics but he has had some predecessors (e.g. Gray (1913, 1914), Jevons (1866)), who have also addressed this issue. Hotelling considers the cases of perfect competition, monopoly and duopoly. The duopoly case has been extended to the more general case of an arbitrary number of oligopolists by Lewis and Schmalensee (1980a, 1980b). Salant (1976) studies the cartel versus fringe model using the Nash equilibrium concept, whereas Gilbert (1978), Newbery (1980) and Ulph (1982) employ the open-loop von Stackelberg equilibrium concept. Rather recently Groot (1990) has solved the cartel versus fringe model for the feedback von Stackelberg equilibrium concept. This list is not exhaustive but serves to indicate that quite some work has been done in this area. For recent surveys the reader is referred to Withagen (1990) and Karp and Newbery (1989). Typically these theories depart from a given demand schedule and investigate the features of a partial market equilibrium, assuming that each trader maximizes his total discounted profits subject to the conditions prevailing on the market and the physical constraints imposed by the exhaustibility of his resource. Here discounting takes place at a given and constant discount rate, which, according to Hotelling’s rule or an analogue, always turns out to play a crucial role. Clearly the partial equilibrium analysis suffers from some weaknesses. By definition it is not able to capture possible spill-overs to other markets. Related to this problem is the fact that the demand schedule is assumed given. An important drawback is of course also that the rate of discount or the interest rate is exogenous; it is hard to believe that the interest rate is moving independently of the market price of the raw material. In atemporal models of price formation this drawback can be deemed innocuous, but the horizon agents face in resource economics is generally indeterminate or infinite and this makes the above-mentioned interaction nonnegligible. Finally, a general equilibrium analysis is preferable, if only from a methodological point of view.

In spite of these arguments, attention paid to a general equilibrium analysis of trade in raw materials is rather modest, at least on the micro economic level in the line of the overview given above (on the “macro” level there are the well-known studies of Marion and Svensson (1984), Van Wijnbergen (1985) and some others). For a survey we refer to Withagen (1990). Let us summarize. In view of real world phenomena all the work in this field is concerned
with international trade. Kemp and Long (1980a) analyse the interaction between resource-rich and resource-poor countries. They model a two country world. One of the countries is resource-rich. Its resource can be exploited at no cost. The raw material is exported to the resource-poor country, which possesses the technology to convert it into a consumer good which is used for own consumption and which is traded with the resource-rich country. The analysis is carried out for Cobb-Douglas type specification of the functions involved and this yields a complete characterization of the equilibrium price trajectory. Elbers and Withagen (1984) introduce bilateral ownership of the resource and extraction costs. Finally, Chiarella (1980) studies a two country model with unilateral ownership of the resource and the non-resource technology and with physical capital and labour as means of production. For Cobb-Douglas specifications he is able to characterize the general equilibrium for the case where there exists a perfect world market for "financial" capital as well as the case where such a market does not exist. Clearly these models can and should be generalized in several respects and this is the aim of the present paper. Our generalization lies in the fact that the number of agents on the consumption side as well as on the production side is arbitrary and, more importantly, we are able to derive interesting results without imposing severe restrictions on the functional forms. It will turn out that the equilibrium interest rate can be a constant (this occurs when the resources are in some sense abundant) and that otherwise the interest rate decreases monotonically. It will furthermore be shown that the capital intensity is non-decreasing and that consumption will eventually decrease. The plan of the paper is as follows. In section 2 the model is presented and the assumptions will be introduced and discussed. Section 3 goes into the problem of the existence of a general equilibrium. Section 4 derives the features of the general equilibrium. Finally, section 5 concludes.

2. THE MODEL

The model is presented here as describing international trade between a given number of countries, although this is by no means the only possible interpretation. One can also think of a closed economic system with several resources and production sectors. Let there be \( n \) countries. The initial endowments of country \( i (i = 1, 2, ..., n) \) consist of a stock of an exhaustible natural resource, denoted by \( S_i \), and a stock of a composite commodity, denoted by \( K_i \), with \( K_i > 0 \). The resource stocks can be distinguished according to the costs that have to be made in order to extract the raw material. However, the raw material once extracted is homogeneous, so that there is only one price. Also the composite commodity is homogeneous. As a stock it is a factor of production and will be called capital. As a flow it can be used for consumption purposes as well as for investments. Finally the composite commodity is a store of value. This will be clarified below.

Preferences of each country are given by a utilitarian social welfare functional having only the consumption profile as an argument; no direct welfare is derived from holding stocks or having raw materials. Welfare of economy \( i \) is given by

\[
U_i(C_i) := \int_0^\infty e^{-\rho_i t} u_i(C_i(s))ds, 
\]

where \( \rho_i \) is the positive constant rate of pure time preference, \( C_i(t) \) is the rate of consumption at instant of time \( t \) and \( u_i \) is the instantaneous utility function. About \( u_i \) the following assumptions are made:

2
$U^1$ $u_i$ is continuous on $\mathbb{R}^+_+$ and continuously differentiable on $\mathbb{R}^{++}_+$, $u_i(0) = 0$.

$U^2$ $u_i$ is strictly increasing and strictly concave.

$U^3$ $u_i'(0) = \infty; 0 < \eta_i = \frac{-cu_i''(c)}{u_i'(c)} = \eta_i(c) < \infty$ for all $c > 0$ and some constant $\eta_i$.

These assumptions are standard in the growth literature and they facilitate the analysis to a large extent.

The technological capabilities of the economies can be described as follows. The resources are not replenishable. So, if $E_i(t)$ is the rate of exploitation of resource stock $i$ at instant of time $t$, it is required that

2.2 $\int_0^\infty E_i(s)ds \leq S_{i0},$

2.3 $E_i(t) \geq 0$ for all $t \geq 0$.

Exploitation is not costless. In order to exploit one has to use capital as an input. Following Heal (1976), Kay and Mirlees (1975) and Kemp and Long (1980b) we postulate an extraction technology of the fixed proportions type:

2.4 $K_i(t) = a_i E_i(t),$

where $a_i$ is a positive constant and $K_i(t)$ denotes the amount of capital used at $t$ by economy $i$ for extraction purposes.

Capital can, together with the homogeneous raw material, also be allocated to the production of the composite commodity. Let $R_i(t), K_i(t), Y_i(t)$ and $F_i$ denote the rate of use of the raw material at $t$, capital input at $t$, composite commodity output at $t$ and the production function respectively, then

2.5 $Y_i(t) = F_i(K_i(t), R_i(t)).$

About $F_i$ the following assumptions are made.

$F^1$ $F_i$ is continuous on $\mathbb{R}^+_+^2$, continuously differentiable on $\mathbb{R}^+^{++}_+, concave and homogeneous of order 1.

$F^2$ $F_i$ is strictly increasing on $\mathbb{R}^+_+^2$.

$F^3$ $F_i(K,0) = F_i(0,R) = 0$ for all $K \in \mathbb{R}_+$ and all $R \in \mathbb{R}_+$.

$F^4$ $\lim_{K \to 0} F_i(K,R)/K = 0$ for all $R$; $\lim_{K \to 0} F_i(K,R)/K = \infty$ for all $R > 0$.

Some comments are in order. For the characterization of the equilibrium the constant returns to scale assumption is obviously important. It is commonly made in resource economics (see e.g. Dasgupta and Heal (1974)). It implies that in a competitive environment profits will be zero, from which nicely behaving factor price frontiers can be obtained. If one does not make the CRS assumption this is impossible and, moreover, the existence of an equilibrium becomes problematic. Necessity of both inputs seems to be a plausible assumption. In $F^4$ one recognizes elements of the well-known Inada conditions.

There is a world market for raw materials. The prevailing spot price in terms of the composite commodity will be denoted by $p(t)$ at time $t$. It is also assumed that there exists a perfect world market for capital services with spot price $r(t)$ (also in terms of the composite
commodity). This implies that capital is perfectly mobile and can instantaneously and costlessly be transferred from one country to another. In the context of an infinite horizon model this assumption makes sense as a first approximation. We finally postulate the existence of a perfect world market for "financial" capital. So no permanent equilibrium on the current accounts is required: an economy can borrow and lend at will provided its total discounted expenditures do not exceed its total discounted income. Define

\[ \pi(t) := e^{-\int_0^t r(\tau) d\tau}. \]

Then the budget constraint of economy \( i \) can be written as

\[ \int_0^\infty \pi(s)\{p(s)(E_i(s) - R_i(s)) + Y_i(s) - C_i(s) - r(s)(K^r_i(s) + K^y_i(s))\}ds \geq K_{10}. \]

The interpretation of this condition is straightforward. \( E - R \) are the net exports of the raw material. \( Y - C \) gives net exports of the produced composite commodity. \( r(K^r_i + K^y_i) \) are the capital costs for production and extraction. In a general competitive equilibrium with perfect foresight each economy maximizes its social welfare (2.1) subject to the technological constraints (2.2-2.5) and its budget condition (2.7), taking the price trajectories \( p \) and \( r \) as given. This yields demand and supply schedules for the composite commodity and the raw material. In equilibrium supply meets demand:

\[ \sum_i R_i(t) \leq \sum_i E_i(t) \text{ for all } t, \]

\[ \sum_i K^y_i(t) + \sum_i K^r_i(t) \leq \sum_i K_i(t) \text{ for all } t, \]

where \( K_i(t) \) denotes the capital holdings of economy \( i \) at time \( t \). Formally

\[ K_i(t) = K_{10} + \int_0^t \pi(s)\{p(s)(E_i(s) - R_i(s)) + Y_i(s) - C_i(s) - r(s)(K^r_i(s) + K^y_i(s))\}ds. \]

In addition, clearly \( p(t) = 0 \) if (2.8) holds with strict inequality and \( r(t) = 0 \) if (2.9) holds with strict inequality.

This ends the description of the model. Note that, compared with the models mentioned in the previous section, a number of generalizations is introduced: extraction costs, arbitrary composite commodity production functions etc.

3. EXISTENCE OF A GENERAL EQUILIBRIUM

The model outlined in the previous section does not allow for the standard Arrow/Debreu approach to establish the existence of a general equilibrium. In this section we go into the causes of this problem and discuss how existence can nevertheless be dealt with. The treatment here will be rather informal because a thorough analysis would go beyond the scope
of our intentions with regard to the present paper. The interested reader is referred to van Geldrop et al. (1989a) and van Geldrop and Withagen (1990a and 1990b).

The Arrow/Debreu approach fails for the following reason. It would require to work with dated commodities because commodities are now to be distinguished according to the moment at which they become available. In view of the infinite horizon this necessitates working in an infinite-dimensional commodity space. At this point the traditional approach breaks down, since there the dimension of the commodity space is finite. So one has to look for an alternative. Evidently the literature on infinite-dimensional commodity spaces, developed by Bewley (1972), elaborated upon by Richard (1986), MasColell (1986), Zame (1987) and others, comes into mind. However their methods are not conclusive in the model at hand either. This has to do with the fact that the initial endowments of the agents do not lie in the interior of their consumption sets and, more importantly, that the economies' production possibilities need not be bounded. However, due to the structure of our model, the method originating from Bewley can be pursued, at least to some extent. The procedure amounts to showing the existence of a general equilibrium for the truncated (finite horizon) economy, proving the uniform boundedness of the equilibrium allocations, the application of a theorem of Alaoglu yielding limits and, finally, to ascertaining that the limiting allocations constitute an equilibrium in the infinite horizon economy. Crucial in our approach is that it enables us to say a good deal about the mathematical properties of the functions describing the equilibrium. Even if the Bewley approach would work, prices for example would lie in a function space which, by itself, would have rather unattractive properties. Our way of tackling the problem employs optimal control theorems (in the step where the existence of a finite horizon equilibrium is proven), from which it is relatively easy to see that prices and the equilibrium allocations of consumption are continuous functions. The results are summarized in

**Theorem 3.1**
An economy satisfying $U^1 - U^3$ and $F^1 - F^4$ possesses a general competitive equilibrium. In the equilibrium the interest rate $r : [0, \infty] \rightarrow R_+$, the price of the raw material $p : [0, \infty] \rightarrow R_+$ and the rates of consumption $C_t : [0, \infty] \rightarrow R_+$ are continuous. Moreover there is always (i.e. for all $t$) production of the composite commodity.

It is instructive to give the flavour of the underlying economic arguments that yield the continuity results and positive production of the composite commodity. This gives some insight into how the assumptions are used. It can be shown that the first theorem of welfare economics applies so that the general equilibrium is Pareto efficient. This is mainly due to non-satiation in consumption. It follows from $u_t(0) = \infty$ that consumption is always positive. Continuity is then arrived at by the strict concavity of the instantaneous utility functions. Since the marginal product of capital is infinite at zero input, it is not efficient to consume only by eating-up the capital stock, even in the presence of exhaustible resources. Therefore there will always be production of the composite commodity. Continuity of the price trajectories then follows from the absence, in equilibrium, of arbitrage opportunities. Plausible as these results may seem, they are not trivial at all. Again, the interested reader is referred to van Geldrop and Withagen (o.c.)

4. GENERAL COMPETITIVE EQUILIBRIUM: CHARACTERIZATION

In this section a characterization of the general equilibrium is given. This is greatly facilitated by two special features of the model, which are discussed first.
The existence of a perfect world market for financial capital allows for the application of Fisher's separation theorem so that the maximization of social welfare requires the maximization of total discounted profits from productive activities. So, in equilibrium, we have for all $i$, all $t$ and all $(K, R) \in \mathbb{R}_+^2$:

$$F_i(K^i_t(t), R_i(t)) - r(t)K^i_t(t) - p(t)R_i(t) \geq F_i(K, R) - r(t)K - p(t)R,$$

on the part of the production of the composite commodity. Furthermore there exist nonnegative constants $\lambda_i$ such that

$$\pi(t)(p(t) - a_i r(t)) \leq \lambda_i, \quad E_i(t)[\pi(t)(p(t) - a_i r(t)) - \lambda_i] = 0.$$

This is the Hotelling rule, which essentially says that in equilibrium there are no arbitrage possibilities.

The fact that each economy's production set is a cone, makes it possible to work with factor price frontiers, so that much of the subsequent analysis of price behaviour can conveniently be illustrated in $(r, p)$ space. The factor price frontier corresponding to a production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ ($fpf(F)$) is defined as follows:

$$fpf(F) := bndV(F)$$

where

$$V(F) := \{(r, p) | F(K, R) - rK - pR \leq 0 \text{ for all } (K, R) \geq 0\}.$$

Hence the factor price frontier is the locus of input prices which yield at most zero profits. Some well-known properties of factor price frontiers are listed below.

Lemma 4.1

Suppose that $F$ is a continuous, concave, non-decreasing production function and that both inputs are necessary.

Then

1) $V(F)$ is closed and convex.

2) If $(r, p) \in V(F)$ then $(r, p) \geq 0$ and $(r, p) \neq 0$.

3) $(r_1, p_1) \in f pf(F), (r_2, p_2) \in f pf(F)$ and $p_2 > p_1$ implies $r_2 \leq r_1$ and, if $r_2 = r_1$ then $r_1 = 0$ and $p_2 > p_1$.

4) $(r_1, p_1) \in f pf(F), (r_2, p_2) \in f pf(F)$ and $r_2 > r_1$ implies $p_2 \leq p_1$ and, if $p_2 = p_1$ then $p_1 = 0$ and $r_2 > r_1$.

5) The graph of $fpf(F)$ has no positive asymptotes.
Proof
This is straightforward and is omitted here.

Since all production functions $F_i$ satisfy the conditions of the lemma and since the extraction technology is linear, profit maximization on the part of every economy implies that at each moment of time where there is production of the composite commodity, prices are on the $fpf$ of some production function and, in addition, allow for nonnegative instantaneous profits in resource extraction. Therefore the set of prices which are candidate for an equilibrium is easily sketched. See figure 1 below (for the case $n = 2$).

Here the half lines originating in 0 describe the prices for which there is zero instantaneous profit in extraction for each of the resources, whereas the curves display the $fpf$'s. The thick curve gives the outer envelope of the $fpf$'s ($\text{bnd} \cap V(F_i)$). This curve contains the feasible equilibrium prices. Henceforth it will be called $fpf$ for short. Note that there are no strictly positive asymptotes (see lemma 4.1.), but also that $fpf$ may hit one or both of the axes.

For the sake of clarity of exposition the following additional assumptions are made:

$C_1$ The set of points $(r, p)$ for which $(r, p) \gg 0$ and $(r, p) \in \cap_i fpf(F_i)$ is finite.

$C_2$ $0 < a_1 < a_2 < \ldots < a_n$.

These assumptions amount to saying that essentially the technologies differ among countries.

One of the consequences of the fact that there is always production of the composite commodity (see theorem 3.1) is that it is relatively easy to show that equilibrium prices are positive.

Theorem 4.1
$r(t) > 0$ and $p(t) > 0$ for all $t$. 
Proof
Fix $t$. Since $Y_i(t) > 0$ for some $i$, $K_i^p(t) > 0$ and $R_i(t) > 0$ for this $i$. If $r(t) = 0$ then $K_i^p(t)$ is not profit maximizing because $F_i$ is an increasing function of $K_i^p$. So a contradiction is obtained. The same argument applies to show that $p(t) > 0$. 

Let $(r', p')$ be defined by $p' = a_1 r'$ and $(r', p') \in \text{fpf}$. In view of the assumptions made $(r', p')$ exists. In fact $r'$ is the maximal feasible $r$. The next theorem states that there are only two different possible price trajectories. Either $(r, p)$ is constantly at the $(r', p')$ level or prices are moving in the south-east direction of $\text{fpf}$, as indicated in figure 1.

Theorem 4.2
i) If, for some $t_1$, $(r(t_1), p(t_1)) = (r', p')$ then $(r(t), p(t)) = (r', p')$ for all $t$.
ii) If $r(0) \neq r'$ then
   a) $r(t_1) > r(t_2)$ and $p(t_1) < p(t_2)$ for all $t_2 > t_1 \geq 0$
   b) $r(t) \to 0$ as $t \to \infty$

Proof
i) $r(t) \leq r'$ for all $t$. For suppose that for some $t_1 \geq 0$, $r(t_1) > r'$. Since there exists $i$ such that $Y_i(t_1) > 0$, we must have $p(t_1) \leq p'$ (lemma 4.1). This implies from (4.2) and $C_2$ that $E_i(t_1) = 0$ for all $i$. But then $R_i(t_1) = 0$ for all $i$ and therefore $Y_i(t_1) = 0$ for all $i$, a contradiction.

So $r(t) \leq r'$ for all $t$. Suppose $(r(t_1), p(t_1)) = (r', p')$ for some $t_1$. Then $E_i(t_1) = 0$ for all $i > 1$ because of (4.2) (exploitation of resources more expensive than the first one gives a loss). $E_i(t_1) > 0$ because $Y_i(t_1) > 0$ for some $i$. Hence $\lambda_i = 0$ and $p(t) - a_1 r(t) \leq 0$ for all $t$. If, for some $t_2$, $p(t_2) - a_1 r(t_2) < 0$ then $E_i(t_2) = 0$. But then $Y_i(t_2) = 0$ for all $i$, a contradiction. This proves the first part of the theorem.

ii) Suppose there exist $t_1$ and $t_2$ with $t_2 > t_1 \geq 0$ such that $r(t_1) \leq r(t_2)$. Then $p(t_1) \geq p(t_2)$. Moreover $r(t_1) = r(t_2)$ if and only if $p(t_1) = p(t_2)$. Suppose $r(t_1) = r(t_2)$. Since $\pi$ is strictly monotonically decreasing $E_i(t_2) = 0$ for all $i$, implying that $R_i(t_2) = 0$ for all $i$. Hence $Y_i(t_2) = 0$ for all $i$, a contradiction. If $r(t_1) < r(t_2)$ then $p(t_1) > p(t_2)$ and $Y_i(t_2) = 0$ for all $i$ a fortiori, which is again a contradiction. This proves ii a).

Suppose there exists $\hat{r} > 0$ such that $r(t) \to \hat{r}$ if $t \to \infty$. Then $\pi / \pi \to -\hat{r}$. So $\pi(t) \to 0$ as $t \to \infty$. Furthermore there exists $\hat{p} > 0$ such that $p(t) \to \hat{p}$ as $t \to \infty$. If $\lambda_i = 0$ for some $i$ then $E_i(t) = 0$ for this $i$ and all $t$. This follows from (4.2) and the previous proof. If $\lambda_i > 0$ for some $i$ then there exists $t_1$ such that for all $t \leq t_1$ $E_i(t) = 0$ because $\lambda_i / \pi(t) \to -\hat{r}$. Therefore $E_i(t) = 0$ for all $i$ and $t$ large enough so that $Y_i(t) = 0$ for all $i$ and $t$ large enough, a contradiction. 

If in an equilibrium the rate of interest is constant at its maximal level, only the cheapest resource will be exploited along the entire equilibrium trajectory. Intuitively speaking one would expect that for a constant rate of interest to prevail it is necessary that the cheapest resource is in some sense abundant. This is true but it is not a sufficient condition. In addition the rates of time preference need to be sufficiently large, the reason being that otherwise capital becomes negative, as is seen in the following theorem.
Theorem 4.3

\((r(t), p(t)) = (r', p')\) implies \(\min_i (\rho_i) > r'\).

**Proof**

If \((r(t), p(t)) = (r', p')\) then (for all \(t \geq 0\)) \(0 = p(t) - a_i r(t) > p(t) - a_i r(t)\) for all \(i \neq 1\). Hence \(E_i(t) = 0\) for all \(t \geq 0\) and all \(i \neq 1\). In an equilibrium consumption in each economy satisfies the Ramsey-Keynes rule, saying that for all \(i\) and all \(t\)

\[ u'_i(C_i(t)) = \phi_i e^{(\rho_i - r)t} \]

where \(\phi_i\) is a positive constant. Furthermore, for \(R := \sum_i R_i\), we have \(K = \sum_i K_i^g + a_1 R\), so that

\[ K/R = a_1 + \sum_i K_i^g/R. \]

\(K_i^g/R = 0\) if \(Y_i = 0\) and \(K_i^g/R\) is a constant if \(Y_i > 0\), due to constant returns to scale. So there are constants \(b_i, i = 1, 2, \ldots, n\), such that

\[ K/R < \sum_i b_i + a_1. \]

Hence, in view of the exhaustibility of the resources,

\[ \int_0^\infty K(s) ds < \infty. \]

Then the homogeneity of \(F_i\) implies that \(F_i(K_i^g, R_i) = r'K_i^g(t) + p'R_i\) so that

\[ \dot{K} = r'K - \sum_i C_i. \]

Therefore

\[ K(t) = K_0 e^{r't} - \int_0^t e^{-r'(t-s)} \sum_i C_i(s) ds. \]

It follows from \(\dot{K} < r'K, K \geq 0\) and \(\int_0^\infty K(s) ds < \infty\) that \(K(t) \to 0\). To see this note that

\[ \int_0^T (\dot{K} - r'K) ds = K(T) - K_0 - r' \int_0^T K ds. \]
So
\[ r' \int_0^T K ds + K_0 = K(T) + \int_0^T (r'K - \dot{K}) ds. \]

The left hand side of this expression is bounded uniformly in \( T \). The second term of the right hand side is monotonically increasing. Therefore \( K(t) \to 0 \) as \( t \to \infty \), for otherwise the integral would diverge.

Now if \( \rho_i \leq r' \) for some \( i \), then \( C_i(t) > C > 0 \) for some \( i \), some \( C \) and for all \( t \) large enough.

But this would imply that \( K \) becomes negative, which cannot occur in a general equilibrium.

Specialization is a well-known phenomenon in models of international trade exhibiting constant returns to scale in production. To this observation the model at hand is no exception. This holds with respect to the production of the composite commodity as well as with respect to resource extraction. But in addition it can be shown that, analogously to production, the cheapest resource will be exploited first. These statements are proven in the following theorems.

**Theorem 4.5**

Suppose there are no \( i \) and \( j (j \neq i) \) such that \( (r, a_1 r) \in V_i \cap V_j \). Suppose furthermore that there exist \( t_2 \geq t_1 \geq 0 \) and \( i \) and \( j (j \neq i) \) such that \( Y_i(t) > 0 \) and \( Y_j(t) > 0 \) for all \( t \in [t_1, t_2] \), then \( t_1 = t_2 \).

**Proof**

If \( (r(t), p(t)) \neq (r', p') \) the result is immediate. Otherwise \( r \) is strictly monotonically decreasing and the desired result is obtained from assumption \( C_1 \).

**Theorem 4.6**

i) Suppose that there exist \( t_2 \geq t_1 \geq 0 \) and \( i \) and \( j (i \neq j) \) such that \( E_i(t) > 0 \) and \( E_j(t) > 0 \) for all \( t \in [t_1, t_2] \) then \( t_1 = t_2 \).

ii) If \( i < j \) then \( E_i(t) > 0 \) implies \( \int_{0}^{t} E_j(s) ds = S_{o_j} \)

**Proof**

i) Suppose there exist \( t_2 > t_1 > 0 \) and \( i \) and \( j (i \neq j) \) such that \( E_i(t) > 0 \) and \( E_j(t) > 0 \) for all \( t \in [t_1, t_2] \). According to (4.2), \( \pi(t)(p(t) - a_i r(t)) \) and \( \pi(t)(p(t) - a_j r(t)) \) are constant for all \( t \in [t_1, t_2] \). However, \( \pi(t_2) < \pi(t_1) \) since \( r(t) > 0 \) for all \( t \), which implies that \( (r(t_2), p(t_2)) > (r(t_1), p(t_1)) \) contradicting that \( (r(t_1), p(t_1)) \in fpf \).

ii) Suppose there exist \( t_1, i \) and \( j \) such that \( i < j \), \( E_j(t_1) > 0 \) and

\[ \int_{0}^{t_1} E_i(s) ds < S_{o_0}. \]
Distinguish between two cases:

a) \( \int_0^\infty E_i(s) ds < S_{id} \).

Then \( \lambda_i = 0 \) because if \( \lambda_i > 0 \) it follows from (4.2) that profits are not maximized without fully exhausting the resource, which would yield a contradiction. So \( \lambda_i = 0 \). But since \( a_i < a_j \) it follows from (4.2), with \( E_j(t_1) > 0 \), that \( \lambda_j < 0 \), a contradiction.

b) \( \int_0^\infty E_i(s) ds = S_{id} \).

There exists an interval \([\tau_1, \tau_2], t_1 \leq \tau_1 < \tau_2 \) with \( E_i(t) > 0 \) and continuous, whereas along the interval \( E_j(t) = 0 \) (see part i)). Take \( \tau_1 < t_2 < \tau_2 \).

\( E_i(t_1) \geq 0, E_j(t_1) > 0 \). Hence

\[ \lambda_i \geq \pi(t_1)(p(t_1) - a_i r(t_1)), \lambda_j = \pi(t_1)(p(t_1) - a_j r(t_1)) \]

\( E_i(t_2) > 0, E_j(t_2) = 0 \). Hence

\[ \lambda_i = \pi(t_2)(p(t_2) - a_i r(t_2)), \lambda_j \geq \pi(t_2)(p(t_2) - a_j r(t_2)) \]

So

\[ \pi(t_2)(p(t_2) - a_i r(t_2)) \geq \pi(t_1)(p(t_1) - a_i r(t_1)) \]

\[ \pi(t_1)(p(t_1) - a_j r(t_1)) \geq \pi(t_2)(p(t_2) - a_j r(t_2)). \]

Multiplication of the left and right hand side of the first inequality by \( a_j \) and of the second inequality by \( a_i \) and adding yields

\[ (a_j - a_i)(\pi(t_2)p(t_2) - \pi(t_1)p(t_1)) \geq 0, \]

implying \( p(t_2) > p(t_1) \). Just addition of the inequalities gives

\[ (a_j - a_i)(\pi(t_2)r(t_2) - \pi(t_1)r(t_1)) \geq 0, \]

implying \( r(t_2) > r(t_1) \). Therefore \( (r(t_2), p(t_2)) > (r(t_1), p(t_1)) \), contradicting lemma 4.1. \( \square \)

Finally we turn to a characterization of consumption and capital intensities. Initially the rates of consumption may rise, but as a consequence of the positive rate of time preference and the limited availability of the resources they eventually decrease. The decline in consumption is monotonic and consumption goes to zero as \( t \) goes to infinity. Asymptotically the relative shares of consumption in total consumption depend on the rates of time preference in relation to the elasticity of intertemporal substitution. As far as the capital intensity is concerned, it is easily seen that it is increasing.

These properties are proven in the final two theorems.
Theorem 4.7

i) There exists $t_1$ such that $\dot{C}_i(t) < 0$ for all $i$ and all $t > t_1$.

ii) $C_i(t) \to 0$ as $t \to \infty$, all $i$.

iii) $C_i(t)/\sum_i C_i(t) \to 0$ as $t \to \infty$ if and only if

$$\frac{\rho_i - \tau(\infty)}{-\eta_i(0)} > \max_{j \neq i} \frac{\rho_j - \tau(\infty)}{-\eta_j(0)}.$$

Proof

i) It follows from the Ramsey-Keynes rule that

$$\dot{C}_i/C_i = \frac{\rho_i - \tau}{-\eta_i(C_i)}.$$

If $r(t) = r'$ then $\rho_i > r'$ for all $i$ (theorem 4.3) so that $\dot{C}_i(t)/C_i(t) < 0$ for all $i$ and all $t$. If $r(0) < r'$ then $r(t) \to 0$ as $t \to \infty$.

ii) This follows immediately from i) and the fact that $\eta_i$ is bounded.

iii) The asymptotic growth rate of $C_i$ is $\frac{\rho_i - \tau(\infty)}{-\eta_i(0)}$.

\[ \square \]

Theorem 4.8

Suppose $r(0) < r'$. Then $Y_i(t) > 0$ for $t \in [t_1, t_2]$ with $t_2 > t_1$ implies $\partial(K_i^Y(t)/R_i(t))/\partial t < 0$ for $t \in (t_1, t_2)$.

Proof

This is immediate from the fact that $r$ is decreasing and the homogeneity of $F_i$.

\[ \square \]

5. CONCLUSIONS

The aim of the present paper has been to analyse trade in raw materials from natural exhaustible resources in the context of a general equilibrium model. We have been able to characterize prices and allocations along the general equilibrium trajectory, under assumptions which are usually made in models of international trade. Not surprisingly many of the by now classical results from the theory of international trade are shown to remain valid in the model extended with raw materials. Apart from this modest contribution, the merits of the analysis lie in the generalization of models from resource economics. The main result in this respect is that only in rather special circumstances constancy of the interest rate can be postulated in partial equilibrium models. In spite of the complexity of the model the analysis can be clarified in a rather simple diagram in the price space. It should be admitted that this is to a large extent due to the constant returns to scale assumption. Current research is directed towards characterization of equilibria when that assumption is relaxed.

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# COSOR-memoranda - 1990

<table>
<thead>
<tr>
<th>Number</th>
<th>Month</th>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-02</td>
<td>January</td>
<td>D.A. Overdijk</td>
<td>Meetkundige aspecten van de productie van kroonwielen</td>
</tr>
<tr>
<td>90-03</td>
<td>February</td>
<td>I.J.B.F. Adan, J. Wessels, W.H.M. Zijm</td>
<td>Analysis of the asymmetric shortest queue problem Part II: Numerical analysis</td>
</tr>
<tr>
<td>90-04</td>
<td>March</td>
<td>P. van der Laan, L.R. Verdooren</td>
<td>Statistical selection procedures for selecting the best variety</td>
</tr>
<tr>
<td>90-05</td>
<td>March</td>
<td>W.H.M. Zijm, E.H.L.B. Nelissen</td>
<td>Scheduling a flexible machining centre</td>
</tr>
<tr>
<td>90-06</td>
<td>March</td>
<td>G. Schuller, W.H.M. Zijm</td>
<td>The design of mechanizations: reliability, efficiency and flexibility</td>
</tr>
<tr>
<td>90-07</td>
<td>March</td>
<td>W.H.M. Zijm</td>
<td>Capacity analysis of automatic transport systems in an assembly factory</td>
</tr>
<tr>
<td>90-08</td>
<td>March</td>
<td>G.J. v. Houtum, W.H.M. Zijm</td>
<td>Computational procedures for stochastic multi-echelon production systems</td>
</tr>
<tr>
<td>Number</td>
<td>Month</td>
<td>Author</td>
<td>Title</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>M 90-09</td>
<td>March</td>
<td>P.J.M. van Laarhoven</td>
<td>Production preparation and numerical control in PCB assembly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>W.H.M. Zijm</td>
<td></td>
</tr>
<tr>
<td>M 90-10</td>
<td>March</td>
<td>F.A.W. Wester</td>
<td>A hierarchical planning system versus a schedule oriented planning</td>
</tr>
<tr>
<td></td>
<td></td>
<td>J. Wijngaard</td>
<td>system</td>
</tr>
<tr>
<td></td>
<td></td>
<td>W.H.M. Zijm</td>
<td></td>
</tr>
<tr>
<td>M 90-11</td>
<td>April</td>
<td>A. Dekkers</td>
<td>Local Area Networks</td>
</tr>
<tr>
<td>M 90-12</td>
<td>April</td>
<td>P. v.d. Laan</td>
<td>On subset selection from Logistic populations</td>
</tr>
<tr>
<td>M 90-13</td>
<td>April</td>
<td>P. v.d. Laan</td>
<td>De Van Dantzig Prijs</td>
</tr>
<tr>
<td>M 90-14</td>
<td>June</td>
<td>P. v.d. Laan</td>
<td>Beslissen met statistische selectiemethoden</td>
</tr>
<tr>
<td>M 90-15</td>
<td>June</td>
<td>F.W. Steutel</td>
<td>Some recent characterizations of the exponential and geometric</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>distributions</td>
</tr>
<tr>
<td>M 90-16</td>
<td>June</td>
<td>J. van Geldrop</td>
<td>Existence of general equilibria in infinite horizon economies with</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C. Withagen</td>
<td>exhaustible resources. (the continuous time case)</td>
</tr>
<tr>
<td>M 90-17</td>
<td>June</td>
<td>P.C. Schuur</td>
<td>Simulated annealing as a tool to obtain new results in plane geometry</td>
</tr>
<tr>
<td>M 90-18</td>
<td>July</td>
<td>F.W. Steutel</td>
<td>Applications of probability in analysis</td>
</tr>
<tr>
<td>M 90-19</td>
<td>July</td>
<td>I.J.B.F. Adan</td>
<td>Analysis of the symmetric shortest queue problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td>J. Wessels</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>W.H.M. Zijm</td>
<td></td>
</tr>
<tr>
<td>M 90-20</td>
<td>July</td>
<td>I.J.B.F. Adan</td>
<td>Analysis of the asymmetric shortest queue problem with threshold</td>
</tr>
<tr>
<td></td>
<td></td>
<td>J. Wessels</td>
<td>jockeying</td>
</tr>
<tr>
<td></td>
<td></td>
<td>W.H.M. Zijm</td>
<td></td>
</tr>
<tr>
<td>M 90-21</td>
<td>July</td>
<td>K. van Ham</td>
<td>On a characterization of the exponential distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>F.W. Steutel</td>
<td></td>
</tr>
<tr>
<td>M 90-22</td>
<td>July</td>
<td>A. Dekkers</td>
<td>Performance analysis of a volume shadowing model</td>
</tr>
<tr>
<td></td>
<td></td>
<td>J. van der Wal</td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>Month</td>
<td>Author</td>
<td>Title</td>
</tr>
<tr>
<td>--------</td>
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<td>-------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>90-23</td>
<td>July</td>
<td>A. Dekkers, J. van der Wal</td>
<td>Mean value analysis of priority stations without preemption</td>
</tr>
<tr>
<td>90-24</td>
<td>July</td>
<td>D.A. Overdijk</td>
<td>Benadering van de kroonwielflank met behulp van regeloppervlakken in kroonwieloverbrengingen met grote overbrengverhouding</td>
</tr>
<tr>
<td>90-25</td>
<td>July</td>
<td>J. van Oorschot, A. Dekkers</td>
<td>Cake, a concurrent Make CASE tool</td>
</tr>
<tr>
<td>90-26</td>
<td>July</td>
<td>J. van Oorschot, A. Dekkers</td>
<td>Measuring and Simulating an 802.3 CSMA/CD LAN</td>
</tr>
<tr>
<td>90-27</td>
<td>August</td>
<td>D.A. Overdijk</td>
<td>Skew-symmetric matrices and the Euler equations of rotational motion for rigid systems</td>
</tr>
<tr>
<td>90-28</td>
<td>August</td>
<td>A.W.J. Kolen, J.K. Lenstra</td>
<td>Combinatorics in Operations Research</td>
</tr>
<tr>
<td>90-29</td>
<td>August</td>
<td>R. Doornbos</td>
<td>Verdeling en onafhankelijkheid van kwadratensommen in de variantie-analyse</td>
</tr>
<tr>
<td>90-30</td>
<td>August</td>
<td>M.W.I. van Kraaij, W.Z. Venema, J. Wessels</td>
<td>Support for problem solving in manpower planning problems</td>
</tr>
<tr>
<td>90-31</td>
<td>August</td>
<td>I. Adan, A. Dekkers</td>
<td>Mean value approximation for closed queueing networks with multi server stations</td>
</tr>
<tr>
<td>90-32</td>
<td>August</td>
<td>F.P.A. Coolen, P.R. Mertens, M.J. Newby</td>
<td>A Bayes-Competing Risk Model for the Use of Expert Judgment in Reliability Estimation</td>
</tr>
<tr>
<td>90-33</td>
<td>September</td>
<td>B. Veltman, B.J. Lageweg, J.K. Lenstra</td>
<td>Multiprocessor Scheduling with Communication Delays</td>
</tr>
<tr>
<td>90-34</td>
<td>September</td>
<td>I.J.B.F. Adan, J. Wessels, W.H.M. Zijm</td>
<td>Flexible assembly and shortest queue problems</td>
</tr>
<tr>
<td>Number</td>
<td>Month</td>
<td>Author</td>
<td>Title</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td>-------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>M 90-35</td>
<td>September</td>
<td>F.P.A. Coolen M.J. Newby</td>
<td>A note on the use of the product of spacings in Bayesian inference</td>
</tr>
<tr>
<td>M 90-36</td>
<td>September</td>
<td>A.A. Stoorvogel</td>
<td>Robust stabilization of systems with multiplicative perturbations</td>
</tr>
<tr>
<td>M 90-37</td>
<td>October</td>
<td>A.A. Stoorvogel</td>
<td>The singular minimum entropy $H_{\infty}$ control problem</td>
</tr>
<tr>
<td>M 90-38</td>
<td>October</td>
<td>Jan H. van Geldrop Cees A.A.M. Withagen</td>
<td>General equilibrium and international trade with natural exhaust...</td>
</tr>
</tbody>
</table>