Evaluation model for the design of distribution networks

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Abstract
An extensive literature review of models on the design of distribution networks is presented in this article. The literature review elaborates on the classification of Aikens (1985) and Vidal and Goetschalckx (1997). In our classification we emphasis on approximations used for the underlying tactical decisions and compare it to state of art literature in the specific area’s. We conclude that the main drawback of the existing models is that when modeling the impact of strategic decisions the changes in the underlying tactical control and their cost implications are not taken into account and we propose a new model for the evaluation of distribution networks. The main contribution of our proposed model is that it correctly models the trade-off’s between customer service levels and inventory levels and between transportation costs and inventory costs.

1 Introduction
This paper proposes a new model for the design of distribution networks. The design of distribution networks is a critical aspect for a broad spectrum of firms. Trends in markets, emergence of e-commerce, changes in labor costs, mergers, increasing competition are some of the aspects which create the necessity to redesign the distribution network regularly. In a typical distribution network the end-products are shipped from the plants to warehouses for intermediate storage, and then shipped to retailers or customers (see figure 1). Alternative distribution networks are compared by means of costs or profits and customer service levels. The goal is to find the distribution structure which minimizes the costs or maximizes the profits and fulfills the customer service requirements.

When redesigning the distribution network, three types of decisions can be distinguished depending on the time horizon, namely, strategic, tactical and
operational, see Ballou (1992) p32-35. The strategic decisions consider time horizons of more than one year. The tactical decisions consider time horizons between one day and a year and the operational decisions consider time horizons of less than one day.

Typical strategic decisions that need to be considered are:
- How many distribution centers are needed and where should they be located?
- Should the distribution centers hold inventories or should they merely be transition points?
- What should be the capacity of the distribution centre?
- Which transportation mode should be used?

Beside those strategic decisions, to be able to calculate the costs or the revenues and compare different alternative distribution structures, underlying tactical and operational decisions need to be addressed, such as:
- Which customer (or customer order) should be delivered from which distribution centre?
- How should the inventories be controlled?
- How frequently should the customers be distributed and the distribution centres be delivered?
- Which service levels and leadtimes are appropriate?
- What is the appropriate order size between warehouses?
- What is the appropriate packing unit?

Considerable research has been carried out to develop models on the design of distribution networks, see for references the reviews of Vidal and Goetschalckx (1997), Aikens (1985), Dasci and Verter (1998) and Owen and Daskin (2001). This paper focuses on discrete facility locations. In these models a given set of possible facility locations is given, for continuous facility location problems we refer to the review of Dasci and Verter (1998). The main problem with the existing models is that when modeling the impact of strategic decisions the change in underlying tactical control is neglected or included but with simple models, thereby ignoring the state of the art literature on tactical control. For example, in inventory management, the literature makes a trade-off between inventory costs and performance measures. When the demand is stochastic, safety stocks are kept to allow for the uncertainty in demand. The level of safety stock is directly related to the desired level of customer service. Two of those measures are the non-stockout probability per replenishment cycle and the fill rate, for references see Silver, Pyke and Peterson (1998). In literature on the design of distribution networks only a few models include stochastic demand and hence the relation between customer service and stock levels. However when the distribution network changes, the demand towards the warehouses changes and the safety stock should be re-evaluated to keep comparable customer service levels.

The few models that include the change in safety stocks due to a modification of the distribution network use linear or quadratic approximations, for references Nozick and Turnquist (2001) or Eppen (1979). These approximations hold under very strict assumptions such as a single echelon distribution network, a large number of parallel warehouses and identical demand at each warehouse. An exception to this is the model of Gross (1981). It evaluates beforehand for a given
number of alternative distribution structures the safety stocks with the model of Scarf (1958) and the inventory costs are directly included in the model, but the modeling of the transportation costs and facility costs is simplistic in this model. Currently more advanced models than Scarf (1958) are used in inventory management to model multi-echelon inventory systems. For recent surveys on multi-echelon inventory systems we refer to Federgruen (1993) and Axsäter (1993).

This paper elaborates the classifications given by Aikens (1985) and Vidal and Goetschalckx (1997) with an emphasis on approximations used on the tactical level and compare it with state of the art literature in these area’s. Furthermore, we will state the assumptions under which the used approximations hold. From this classification we will comment on the drawbacks of the current models and propose a new model for the design of distribution networks.

The sequel of this paper is organized as follows. In the second section we will elaborate on the classification of Aikens (1985) and Vidal and Goetschalckx (1997). In the third section the drawbacks of the existing models will be discussed. In the fourth section a new model is proposed and the fifth section gives the conclusions and thoughts for further research.

Figure 1: Typical distribution structure

2 Classification of the models on the design of distribution network

The example mentioned in the introduction shows that the models on the design of distribution networks neglect or use simple approximations to model the change in the underlying tactical level due to a change at the strategic level. In this section we will elaborate on the classifications of Aikens (1985) and Goetschalckx (1997) with an emphasis on approximations used for the underlying tactical decisions and compare it to the state of the art literature in the specific area’s. Models on the design of the distribution network can be classified according to:

1. Production-distribution or distribution models

   We distinguish between production-distribution models and distribution models. A production model takes the location of the factories into account. These models redesign the network from the suppliers to the customers, whereas in the distribution model the location of factories is given. The review of Vidal and Goetschalckx (1997) focuses on production-distribution models. In production-distribution models the flow of goods is expressed as a bill of material (BOM). The BOM constraints express all the products as combinations of raw materials (for references see Arntzen et al. (1995)). To decide between different production facilities the production costs are included. Arntzen et al. (1995) assumes
fixed costs to open a facility at a location and product variable production costs. Cohen and Moon (1991) assume economies of scale in the production costs by a piece-wise linear function. In the production literature, the production costs are dependent on the size and the location of the facility, the number of needed machines and the needed workforce (for references see chapter 5 of Silver et al. (1998)). The requirements for needed machines and workforce depend on the planning and scheduling which can be extremely complex, for example with product sequence dependent setup times, for references see Lawler et al. (1993). Therefore it is difficult to determine the production costs of a item at a location independently of the distribution structure.

2. Objectives

Two types of objectives are possible. The most common objective is the minimization of costs. The other is the maximization of profits. The maximization of profits is used in models where the revenues are dependent on the chosen distribution network. This is the case when international factors such as taxes, duties or offset requirements are included (for references see Cohen and Lee (1989)).

3. Number of echelons

Aikens (1985) defines an echelon as the number of warehouses between the factories and the customers. A zero-echelon model, as the model of Khuen and Hamburger (1963), is a model without warehouses, the problem is to allocate the customers to the factories. In a multi-echelon model, we will define the first tier warehouse as the warehouse which only delivers to customers, second tier warehouses as the ones that deliver to first tier warehouses and customers, and so on until the last tier warehouses.

4. Demand pattern

Vidal and Goetschalckx (1997) distinguish between three types of demand deterministic demand, stochastic and dynamic. We will extend this classification to different types of stochastic demand. We will distinguish between three types of stochastic demand. Type 1, the demand size per time unit is described by a distribution function. For example the demand per week is normal distributed. Type 2, the demand size is one and the inter-arrival time between customers is described by a distribution function. Type 3, the inter-arrival time and the demand size of a customer are described both by distribution functions. The latter is called a compound renewal process.

5. Multi-product versus single product model

Warszawski (1973) was one of the first to address multi-product aspects. In a multi-product model an extra dimension is added to the decision variable to be able to model multiple products.

6. Capacitated versus uncapacitated warehouse location

The possible size of the facilities can be capacitated at a location. Aikens (1985) makes a distinction between capacitated models and uncapacitated models. We will extend this classification by different types of capacitated facilities at a location. The model of Arntzen et al. (1995) and Goetschalckx et al. (1995) have different possible types of facilities at one location. The different types of facilities have different capacities and different costs. The different types of ca-
pacitated warehouses are included because the authors assume that there are economies of scale in the size of the facility.

7. Single sourced constraints

Arntzen et al. (1995) distinguish between single sourced customers and non-single sourced customers. Single sourced customers are customers that are only delivered by a single warehouse, whereas non-single sourced customers are delivered from more warehouses. We will extend this classification with single sourced products. Single sourced products are products that customers or warehouses receive from only one warehouse. A distribution network where the products are single sourced is also called a divergent distribution network and this is an important property in inventory management.

8. Customer service

The customer service requirements include three important elements:
- **Delivery reliability**: This means that the order is delivered at the right time, in the right quantity and at the right place.
- **Response time**: It is the time elapsed from the moment that the customer places an order until he receives it.
- **Order consolidation**: The customers prefer to receive all products within an order at the same moment and not to receive different products from different warehouses at different moments in time. If a customer receives all its orders from one warehouse the customer is said to be single sourced.

In the classification of Vidal and Goetschalckx (1997) the different customer service features are customer demand satisfaction, maximum time or distance to a customer and percentage of orders satisfied from shelf. We will generalize the last feature to inventory performance measures because some models like Nozick and Turnquist (2001) use the non-stockout probability or Gross (1981) uses penalty costs for the amount of units short. For the different inventory performance measures we refer to Silver, Pyke and Peterson (1998). In the model of Gross (1981) the desired service level is not expressed by a constraint but by including in the objective the costs of having units short. In practical situation it is often difficult to determine the value of the shortage costs.

9. Transportation

Early models on the design of distribution networks include only fixed distribution costs for supplying the entire demand of a customer from one facility, see for references Kuehn and Hamburger (1963). If the customer is not single sourced the distribution costs per link is relative to the demand coming from each link. A link is defined as the connection between two elements in the distribution network. Later on in the model of Kaufman et al. (1977) the transportation costs where expressed in costs per transported unit. Arntzen et al. (1995) and Goetschalckx et al. (1995) allow different transportation modes. An extra dimension is added to the decision variables indicating which mode of transportation is used on a link. A transportation mode can be a train or a truck or different transportation options like full truck load (FTL) or less than truck load (LTL). The transportation costs are composed of fixed costs of using a mode and product variable costs. In practice, when the LTL mode is used the costs are dependent on utilization degree of the truck. In the model
of Fleischmann (1993) a probability distribution function of the utilization degree is used to calculate the transportation costs. In this model the probability distribution function of the utilization degree is independent of the distribution structure. In practice, the probability distribution function of the utilization degree is dependent on the flows of goods which in turn depends on the distribution structure. We will therefore extend the classification to a probability distribution function of the utilization degree which is dependent on the distribution structure.

10. Facility costs

Early models on the design of distribution networks include only fixed costs to open a facility, for references see Kuehn and Hamburger (1963). Cohen and Moon (1991) presented a model with concave piece-wise linear production costs, this technique is used in the models of Arntzen et al. (1995) and Goetschalckx et al. (1995) to model piece-wise concave facility costs. At each location different types of warehouses can be located and each type of warehouse has its own product dependent variable costs and fixed costs. In nearly all the models the costs are dependent on the total throughput. But the costs of the warehouse can also be dependent on size of the warehouse which on its turn is dependent on the maximum inventory level, therefore we will elaborate the classification to facility costs which are dependent on the maximum inventory level, for references see the model of Goetschalckx et al. (1995).

11. Inventories

Only a few papers on the design of distribution networks include inventories. In the model of Arntzen et al. (1995) the demand is assumed to be dynamic. The inventories are calculated as follows; the inventories at the end of period t are equal to the inventories at the end of period (t-1) plus the production during t minus the demand during t. In the model no indication is given about the length of t, if t is small and reflect all the changes in inventories then the above calculation of inventories are correct but only for this case, but if t is large like once a month and the demand arrives every day, the method above can lead to large errors. If the period t is small the model will be complex to solve because of the large number of variables. Also no indication is given about the starting inventory levels. The starting inventory levels need to change if the number of warehouse changes, to maintain customer performance measure at a required level.

Dogan and Goetschalckx (1999) assume dynamic demand. The inventories are calculated by estimating the batchsize transported to the warehouse from the transport frequency and the continuous production rate. It is assumed that production and demand are continuous. The model does not include safety stocks and the shipping frequency is independent of the distribution structure.

Goetschalckx et al. (1995) assume deterministic demand, the inventories are calculated by including the safety stock and the batch size. The batch size is calculated by using the total flow and the delivery frequency. The delivery frequency is independent of the distribution network. No indication is given about the calculation of the safety stocks, it is exogenous to the model. A way to calculate the safety stocks is to use the methods given in Silver, Pyke and

Eppen (1979) derives a square root relation between the amount of parallel warehouses and the amount of safety stocks. It is assumed that the underlying inventory problem is a type of newsboy problem. This approximation can only be used in a single echelon case with identical demand at each warehouse.

Nozick and Turnquist (2001) extends this approximation by showing a linear relation between the safety stock level and the amount of parallel warehouses. An extra assumption to this model is that the number of parallel warehouses should be high (15 to 70). This is a special case of Eppen (1979) because if the number of warehouses is large the square root function converges to a linear function.

Balanchandran and Jain (1976) present a model with a general type of stochastic demand. For the determination of the inventory levels the same model as in Silver et al. (1998), p 257, is used. The underlying model is a single echelon (s,Q) model with holding costs and penalty costs. The penalty costs are expressed as the cost per unit short. Balanchandran and Jain (1976) and LeBlanc (1977) have proposed heuristics to solve this model.

Gross et al (1981) present a multi-echelon model with a general probability distribution function for the order size per unit time. In this model beforehand different alternative designs are selected. For the different alternative designs the average inventory levels and the amount of units short are calculated with the model of Clark (1958). After this a linear programming model is constructed, using the calculated inventory levels and amount of units short per alternative design, to decide between the different alternatives. This model concentrates only on the inventory costs, the modeling of the transportation and facility costs are simple.

Currently more advanced models than Scarf (1958) are used in inventory management to model multi-echelon inventory systems, for recent surveys we refer to Federgruen (1993) Axsäter (1993) and the models below.

Diks and De Kok (1999) present fast, accurate and easy-to-implement algorithms that generate close-to-optimal echelon order-up-to-policies for divergent N-echelon networks with stochastic lead times.

Axsäter (1998) presents a two-echelon inventory system with (s,Q)-installation stock policies, Poisson demand and fixed lead times.

Andersson, Axsäter and Marklund (1998) present a two-echelon inventory system with (s,Q)-installation stock policies and fixed lead times but assume a more general distribution for the demand (Normal distribution).

Smits et al. (2000) present a multi-echelon inventory (s,nQ) model with compound renewal demand. I.e., customer orders for an item arrive according to a renewal process and the demand per customer has some arbitrary distribution function. In this model, the lead time includes also the waiting time due to a lack of stock at the higher warehouse.

12. Handling

In most models on the design of distribution networks the handling costs are not explicitly included. The handling costs are included in the variable facility costs which are dependent on the total flow through the facility. In the model of
Goetschalckx et al. (1995) the handling costs are included and are dependent on the average throughflow. In cost accounting a system is develop to calculate the costs accurately. This system is called activity-based costing, see for reference Pirttilä and Hautaniemi (1995). More accurate calculations of the costs are obtained using cost drivers to trace the costs of activities to the products that consume the resources used in those activities. Used cost drivers can be pallets, boxes and order-lines.

Due to emerge of e-commerce the demand pattern changes, instead of delivering a large order to a retailer a customer places directly an order at the warehouse. So the warehouse experience more frequent and smaller orders. The administration of order-lines is important, to include this in the model, we introduce clerical handling costs.

Clerical handling is defined as all the handling done by people which have to do with the order flow as a whole and with the administrative management of the whole goods flow, such as: location registration, order planning, transport planning, stock administration, order administration, order issue, vehicle loading lists, waybills, etc.

13. Taxes, duties and offset requirements

In international models the inclusion of different taxes and duties, differential exchange rates, trade barriers, transfer prices, and duty drawbacks is fundamental for a model to represent accurately practice. Cohen and Lee (1989) were the first to include those features in their model.

14. Lead times

In the design of distribution network literature, only the simple model of Vidal and Goetschalckx (2000) include stochastic lead times. They show how uncertainties in the lead time affects the design of the production-distribution network. All other models assume deterministic lead times. The lead time is composed of the transportation time, handling time, the waiting time due to a lack of stock at the higher echelon warehouse and the waiting time due to shipment consolidation. The waiting time due to a lack of stock at the higher echelon warehouse and the waiting time due to shipment consolidation are dependent on the distribution network. When the distribution network is re-designed the changes in the waiting time due to a lack of stock at the higher echelon warehouse and the waiting time due to shipment consolidation need to be reflected in the lead time to be able to correctly estimate the new inventory levels for given inventory performance measures. In Smits et al. (2000), the lead time includes the waiting time due to a lack of stock and the transportation time. In Smits and de Kok (2001), approximations for the waiting time due to transportation consolidation are derived. These are also dependent on the distribution structure. The classification of the lead time will be elaborated to include endogenous waiting time due to a lack of stock and endogenous waiting time due to order consolidation. In this way for different alternative distribution structures the trade-off between transportation management and inventory management can be included.

15. Pipeline costs

Pipeline inventories include goods in transit (for example, in physical pipeline,
truck, trains or boats) between levels of a distribution network. The pipeline costs are the costs of the inventories hold in the pipeline, for reference see Goetschalckx et al. (1995).

Tables 1 summarize the classification and in table 2 the main models found in the literature are classified. By observing the tables we observe that some features are still missing in the present models.
Table 1: Features to classify existing production distribution models

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<th>Features</th>
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- 01: **Production-distribution model**: 0 distribution model, 1 production distribution model
- 02: **Objective**: 0 minimization of costs, 1 maximization of profits
- 03: **Number of echelons**: 0 no echelons, 1 1 echelon, 2 2 echelons, 3 and more echelons
- 04: **Demand pattern**: 0 Deterministic demand, 1 type 1 stochastic demand (demand sizes are stochastic), 2 type 2 stochastic demand (inter-arrival times are stochastic), 3 type 3 stochastic demand (both are stochastic), 4 Dynamic demand
- 05: **Single or multi-commodity**: 0 1 product, 1 more than 1 product
- 06: **CAPSACitated warehouses**: 0 uncapsicitated warehouse, 1 capacitated warehouse, 2 different capacitated warehouse
- 07: **Single sourced**: 0 else, 1 customer, 2 customer and product
- 08: **Customer service**: 0 not included, 1 only inventory performance measure, 2 only distance (time), 3 inventory performance measure and distance
- 09: **Transportation**: 0 not included, 1 fixed costs, 2 linear costs per transported unit, 3 different modes, 4 distribution of the utilization degree, 5 distribution of the utilization degree dependent on distribution structure
- 10: **Facility costs**: 0 not included, 1 fixed costs, total throughput, 2 linear costs per unit throughput, 3 concave costs per unit throughput, 4 concave costs, maximum inventory level
- 11: **Inventories**: 0 not included, 1 dynamic, control of inventory not included (Arnsten(1995)), 2 dynamic, control of inventory included (Dogan and Goetschalckx (1999)), 3 stochastic, single echelon, exogenous safety stocks (Goetschalckx (1999)), 4 stochastic, single echelon, linear approximation for safety stocks (Novick (1998)), 5 stochastic, single echelon, quadratic approximation for safety stocks (Eppe (1979)), 6 stochastic, single echelon (Balanuch and Jain (1976)), 7 stochastic, multi-echelon (Grose (1981)), 8 stochastic, multi-echelon and fill rate at higher echelon (Smits et al. (2000)), 9 stochastic, single echelon (Balanchandran and Jain) 0 included
- 12: **Handling**: 0 not included, 1 physical handling, 2 clerical and physical handling
- 13: **International features**: 0 not included, 1 included
- 14: **Lead time**: 0 not included, 1 deterministic, 2 stochastic, 3 stochastic and endogeneously dependent
- 15: **Pipeline costs**: 0 not included, 1 included
Inventory levels and this can lead to an incorrect estimation of the average end-product produced at a site by the variable production costs. The main constraints. The production costs are calculated by multiplying the amount of situation. Below some of the drawbacks of existing models are illustrated:

3 Drawbacks of existing models

The main problem with existing models is that when modeling the impact of strategic decisions the changes in the underlying tactical control and their cost implications are not taken into account or only by simplifying approximations. These approximations for operational and tactical costs are derived under strong assumptions and it is unknown how well the approximations work for a given situation. Below some of the drawbacks of existing models are illustrated:

1. Production distribution model

Very few models take into account the entire production distribution problem and when they do, they include the production process by so-called BOM constraints. The production costs are calculated by multiplying the amount of end-product produced at a site by the variable production costs. The main problem is that it is difficult to determine independently from the distribution structure the variable production costs of an item at a location.

2. Stochastic aspects

A very few models include stochastic or dynamic customer demand. In practice, the demand is usually not deterministic and safety stocks are kept to allow for uncertainties in the demand.

The model of Vidal and Goetschalckx (2000) is the only model which includes stochastic lead times and it demonstrates how the design of the distribution network is affected by stochastic lead times. The models with deterministic demand and lead times do not include the effects of uncertainty on the average inventory levels and this can lead to an incorrect estimation of the average

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inventory levels and related to that to incorrect inventory performance measures and cost estimates.

3. Customer service requirements
In the classification, we explained that three customer service elements are important: the delivery reliability, the response time and the consolidation of orders. In a few models an approximation for the response time is introduced by setting restrictions on the maximum distance between warehouses and customers, but they don’t take into account the relation between inventory levels and inventory performance measures, like stockout probability or fill rate.

5. Inventory control
When the distribution network changes, the demand towards the warehouses changes, due to this the inventory control policies must be re-evaluated to keep comparable customer service. A few models explicitly include the inventory control policies but use approximations, which hold under strict assumptions, like a single echelon distribution network and identical warehouses, to calculate the inventory levels when the distribution network changes.

6. Transportation control
In most existing models the transportation costs are dependent on the average flow and not on the utilization degree of the trucks. Usually an average truckload is given which is assumed to be independent of the distribution structure. However, in the model of Fleischmann (1993) the transportation costs are dependent on the shipped quantity, the probability distribution function of the utilization degree should be given by user in advance independently of the distribution structure. Generally, the probability distribution function of the utilization degree of the trucks is dependent on the distribution network and therefore it is difficult to estimate the probability distribution function of the utilization degree in advance.

7. Clerical and physical handling
In many existing models the handling costs are not explicitly included and in none of the models the clerical handling costs are include.

8. Facility costs
In many existing models the facility costs are dependent on the average flow through the facility, whereas in practice the costs usually are dependent on the size of the facility.

In the next section a new model for the evaluation of distribution networks will be proposed. The new model will eliminate some of the above mentioned drawbacks.

4 A model for the evaluation of distribution structures

The objective of our proposed model is to evaluate the costs of a given distribution network. The costs are the transportation costs, pipeline costs, inventory costs, facility costs and handling costs. The customer service requirements are
expressed in terms of fulfillment of a target fill rate (percentage of orders which are directly delivered from stock) and a maximal distance between the facility and the customer. To be able to calculate the costs, assumptions are made about the control of the distribution network. So besides the main decisions there are some underlying tactical decisions that need to be taken on issues like the allocation of customers to the facilities, inventory policies, transportation policies, batchsizes of the items.

We assume in our proposed model:
- only one actor in the distribution network
- a set of possible locations for facilities is given.
- a divergent multi-echelon network. Divergent means that the facilities and the customers in the distribution network are single sourced on product level. A customer can be allocated to any facility in the distribution network.
- no transshipments between facilities at a same level.
- customer demand is distributed according to compound renewal processes, i.e. the arrival process of the customers follows a renewal process and the demand for the different items has a known distribution. The customer demands are independent of each other.
- the inter-arrival time and demand are stationary, independently distributed and demand processes of different items are independent from each other.
- the inventories are controlled by (s,nQ) installation stock policies
- two types of transportation consolidation policies. The time policy ships the goods when a shipping date is reached and the quantity policy ships the goods when a predetermined quantity is reached (full truck load).

Due to the assumptions above, our evaluation model has some restrictions. What our model doesn’t include:
- the locations of plants
- correlated customer demand
- non-stationary demand
- dynamic models with more than one central actor
- international features, like taxes and duties.
- routing at the customers
- optimization of the entire distribution network

The data our model needs to evaluate the costs:
- structure of the entire distribution network. The locations of the warehouses, customers and factories and connections between the different elements.
- The average and the variance of the inter-arrival time of the customers
- The average and the variance of the demand size of customer orders
- The customer service requirements expressed in target fill rate
- The batchsizes of the items at each warehouse in the distribution network
- transportation policy and methods
- tariffs of the transportation methods
- handling costs, per location, per item, per unit time.
- area costs, per location, per $m^3$, per unit time

In Section 4.1, we will clarify how the flow of goods through the entire distribution network, the transportation, the inventories and the handling is modeled in our evaluation model. In Section 4.2, we explain and justify the used methodology.

4.1 Modeling of the different components in our proposed model

- Flow of goods
In this paragraph it is explained how the flow of goods is calculated through the whole distribution network. We have customers or groups of customers with known arrival processes. These customers are allocated to the facilities. For each first tier facility we can aggregate the arrival processes of the allocated customers. First tier facilities are the facilities to which only customers are allocated. This results in an approximation for the compound renewal process of an arbitrary customer at each first tier facility. Given the batchsizes of the items at the first tier warehouses the replenishment process towards the second tier warehouses can be approximated. The replenishment process is approximated by a compound renewal process. Since the first tier warehouse are customers of second tier warehouses, we can aggregate the arrival processes of orders at the second tier warehouse and we can continue this process until the last tier warehouses. This procedure allows us to approximate the flow of goods through the entire network by compound renewal processes. Those approximations were tested by discrete event simulations for mixed Erlang distributed inter-arrival times and mixed Erlang distributed demand sizes and up to three echelons (Smits, de Kok, van Laarhoven 2000). The simulation experiments show that the approximations perform well under most circumstances.

- Transportation
Since the batchsizes for each item and the transport policy are given and the flow of goods is approximated by compound renewal processes, we can derive approximations for the distribution functions of the shipment sizes and the amount of pallets and boxes per shipment.

- Inventory
Given the arrival process of orders, the lead time, the fill rate and batchsize we can precisely calculate the average inventory level (Smits, de Kok, van Laarhoven 2000). The lead time includes the waiting time due to shipment consolidation, waiting time due to a lack of stock at the higher echelon warehouse and the shipping and handling time.

- Handling
Since the batchsizes for each item and the transport policy are given and the flow of goods is approximated by compound renewal processes, we can derive approximations for the amount of orderlines and the amount of different kind of batchsizes (pallet or box) arriving at and leaving from the facility.
4.2 Methodology

Our proposed model is a decision support model. The approximations in the model have been tested by discrete event simulations. We derived some approximations to be able to approximate the flow of goods as compound renewal processes. When the model is validated, we want to test the performance of the model in practice. This is done with the help of a case study. The case consists of historical data for which the distribution costs are known and the idea is to compare the real costs of that case with the costs obtained in our model and the costs obtained in the existing models. Our model is more complicated than the existing models and therefore difficult to optimize. We want to investigate in which situations it is correct to use the less complicated approximations of the existing models and in which situations we have to use more a complicated model. Finally we want to find some good methods and/or heuristics which results in close to optimal solutions for the problem.

5 Conclusions and further research

From the classification presented in table 1 and 2, it is easy to conclude that there are still features missing in the existing models on the design of distribution networks. The main drawback of the existing models is that when modeling the impact of strategic decisions the changes in the underlying tactical control and their cost implications are taken into account by simplistic approximations. In our proposed model the modeling of the underlying tactical control is included, this permits to model the trade-offs in the distribution network, like the trade-off between inventory performance measures and inventory levels or the trade-off between transportation management and inventory management. The proposed model is more complex than the existing models and therefore it will also be more complicated to optimize it. However, a good model must represent the real system in sufficient detail to make the results useful in improving decision making and developing insight but, at the same time, the model must be manageable, so that it can be solved.

In our model we assume a certain type of underlying control, further research could be done in the extension of this model for specific practical situations or in developing new models which include a different underlying control. Another line of research could be to measure, on sketches or on real data, the gain in performance accuracy brought by the modeling of the stochastic dimension of the flows. Finally, we could look for strategies based on search algorithms like statistical annealing, that could be designed on top of the evaluation tool for the optimization of a distribution network.

References

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[46] Smits, S.R., De Kok, A.G. (2001) Approximations for the waiting time in $(s,nQ)$-inventory models for different types of consolidation policies. Accepted for publication in lecture notes Economics and Mathematical Systems


