Fully abstract denotational semantics
for concurrent PROLOG
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Abstract. We develop a denotational, hence, compositional semantics for a subset of Concurrent Prolog and relate it to an operational one, that makes divergence and the resultant substitutions of finite computations together with the termination mode — success, failure or deadlock — observable. Relative to this notion of observation we prove that the denotational semantics is fully abstract in the sense that it records the minimal amount of extra information beyond the observables to make it compositional. Full abstraction is an important property because it quantifies the information that one needs in order to reason about individual program-parts independently. It is the first such result in the area of concurrent logic programming.

1. Introduction

Logic programming is based on the idea that first order logic can be used as a programming language. Its origins lie in Robinson’s resolution principle [Rob65], an inference rule that is eminently suitable for mechanization, and in Kowalski’s and Colmerauer’s [Kow74, Co73] realization that logical deduction has a procedural interpretation that makes it effective as a programming language.

A typical logic programming clause, \( a \leftarrow b_1, \ldots, b_n \), has as logical or declarative meaning “\( a \) is satisfied whenever all \( b_i \)'s are satisfied”. Such a clause also has a procedural reading: “to solve the goal \( a \) (i.e., to satisfy \( a \)) solve the goals \( b_1, \ldots, b_n \).” Together with resolution — or rather unification — as the computational mechanism, this procedural interpretation yields efficient programming languages, as the various Prolog dialects show.

A third interpretation of a logic clause is possible: the process reading. The atom, \( a \) is now interpreted as a process that may spawn off (instances of) the body-atoms \( b_1, \ldots, b_n \) with which it is replaced. Clearly, this interpretation is geared towards parallel and distributed implementations.

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For “pure” logic programs the declarative, procedural and process readings all coincide; see, e.g., [Llo84]. Most concurrent logic programming languages, like Prolog itself, introduce extra-logical constructs for efficiency and expressiveness reasons. Specifically, concurrent logic programming languages all have constructs that allow control information to flow between goals or processes so that the reduction of goals can be synchronized or suspended. Such constructs, on the one hand, are essential for reducing the otherwise intractable computational effort of reducing all processes in parallel, but on the other hand, destroy the correspondence between the declarative meaning of a concurrent logic program and its operational or process meaning. There is an analogy here with the introduction of the cut-operator in Prolog [Llo84].

Strictly speaking, such languages no longer belong to the realm of logic programming. They correspond more closely to imperative languages, although the basic computation step still is the resolution of clauses, and a successful computation provides a proof of the goal statement from the clauses of the program.

As with imperative languages, most of the concurrent logic languages have had there meaning defined by an operational semantics. There are good reasons not to be satisfied with this state of affairs and ask for denotational semantics as well. Apart from the theoretician’s argument — that they are obviously good to have — we state the following practical reasons

— to localize the debugging [Lic87], analysis and transformation of programs [GCS88], and
— to aid distributed implementations [TSS87].

In either case, it is necessary to consider program parts independently from the other parts and, hence, to determine in what way such parts can be influenced (and can influence). It is precisely the compositional nature of a denotational semantics that makes it important in this context.

We develop in this paper a denotational semantics for a subset of Concurrent Prolog [Sha86], Theoretical Flat Concurrent Prolog, TFCP. The starting point is a decision on what one wants to observe or know about program executions. As usual, we codify this in an operational semantics, \( O \). Our notion of observation is a minimal one: we observe the resultant substitutions of finite computations with their “type”, success, failure or deadlock, and whether divergence occurs. Moreover, in this paper we limit ourselves to uniprocessor implementations and, hence, allow at most one reduction step at a time.

The next step is to construct a denotational semantics from which not only the operational semantics can be reconstructed, but which also gives independent meaning to the syntactic building blocks of programs. In general this will entail extending the recorded behaviour of programs beyond what we want to observe.

It is not only a theoretical nicely to ask for the minimum amount of extra information about programs, upon which a compositional semantics can be based. This is of obvious importance, too, if the semantics forms the theoretical basis for debugging tools and distributed implementations. In other words, we want a denotational semantics that is fully abstract with respect to \( O \).

The fully abstract semantics developed here is based on the divergence set semantics of Brookes

\[3\] Full abstraction is usually formulated differently. The present characterization as the “smallest morphism above \( O \)” is an equivalent one; see e.g., [HGR87].
et al. for TCSP [BHR84]. Hence, the meaning of a program is expressed in terms of the sequences of interactions, i.e., substitutions, that programs participate in, including both the substitutions that a program produces and the substitutions that it assumes its environment produces. So-called divergence traces, if any, express at which points a divergent computation can start. Moreover, because at some points, progress in a program-part may depend on the environment, the meaning of a program also quantifies the dependencies on the environment. These take the form of failure sets which collect substitutions that will either suspend the program or will not release it from suspension.

In fact, we treat a TFCP program as an ordinary parallel program: substitutions are treated as assignments to variables that are shared between the goal processes and which may cause different processes to synchronize.

The only other work that we know of in this area is a recent paper by Kok [Kok88]. He gives a denotational semantics for a larger set of Concurrent Prolog than we do, using a Banach space of trees as domain. Unlike us, he does not relate his model to any operational semantics and his model is far from being fully abstract relative to our operational semantics.

Section 2 of the paper introduces the operational semantics. The domain and the denotational semantics is the subject of Section 3. Section 4 shows full abstraction and Section 5 contains some final remarks.

For reasons of space, both the operational and the denotational semantics as presented in this abstract ignore divergences of programs. However, we have proven these results for the more general case. The results and proofs will be included in the full paper.

2. Concurrent Prolog and its Operational Semantics

2.1 Syntax

The notions of signature, \( \Sigma \), terms, \( Tm(Var, \Sigma) \), atoms, \( At(Var, \Sigma', \Sigma') \), substitutions, re-namings, most general unifiers, etcetera, are assumed to be understood. Their definitions and other non-standard notation can be found in the appendix.

Concurrent Prolog extends logic programming with the notion of read-only variables as a synchronization primitive and the commit operator which distinguishes between guard atoms and the proper body atoms of a clause. Concurrent Prolog distinguishes between the writable occurrence of a variable, \( X \), and its read-only occurrence, \( X? \). The intension is that a program that needs \( X? \) to be instantiated will suspend until its environment will instantiate a writable occurrence of \( X \).

We restrict ourselves to the so-called flat subset of Concurrent Prolog in which the guard atoms are constructed from a fixed set of test predicates, \( T \).

Definition:

- For any set \( Var \) of variables, \( Var? = \{ X? \mid X \in Var \} \) and \( Var_\gamma = Var \cup Var? \)
- \( \mathcal{S} = \{ \sigma \mid \sigma \) is a substitution on \( Var_\gamma \) and \( \text{dom}(\sigma) \subseteq Var \} \)
• \( B = \{ \sigma \in S \mid \sigma \text{ is idempotent} \} \)
• \( B^1 = \{ \sigma \in S \mid \forall X \in \text{dom}(\sigma) \ d(\sigma(X)) \leq 1 \} \)
• \( \mathcal{R} = \{ \rho \in B \mid \rho \text{ is a renaming} \} \)
• if \( \sigma \in B \) and \( t \in Tm(Var, \Sigma) \) then \( t\sigma \) is defined as usual except that
\[
(X?)\sigma = \begin{cases} 
\sigma(X)? & \text{if } \sigma(X) \in \text{Var} \\
\sigma(X) & \text{otherwise}
\end{cases}
\]

**Definition:** Flat Concurrent Prolog, \( \text{FCPT}_T,\Sigma; \) initialized programs, \( \text{iFCPT}_T,\Sigma \)

- A (flat) guarded \((T, \Sigma)\)-clause has the form \( a \leftarrow g_1, \ldots, g_m \mid b_1, \ldots, b_n, \) where \( \{a, b_1, \ldots, b_n\} \subseteq \text{At}(Var, \Sigma) \) and \( \{g_1, \ldots, g_m\} \subseteq \text{At}(Var, \Sigma', T) \). We call \( a \) the head, \( g_1, \ldots, g_m \) the guard and \( b_1, \ldots, b_n \) the body of the clause. If \( n = 0 \), we take the body to be true.

- An \( \text{FCPT}_T,\Sigma \) program is a finite set of flat guarded \((T, \Sigma)\)-clauses.

- A program \( P \in \text{FCPT}_T,\Sigma \) is called closed if every predicate in \( \Sigma^p \) that occurs in \( P \) is also defined in \( P \), i.e., also occurs in the head of a clause in \( P \).

- \( X \in \text{iFCPT}_T,\Sigma \) iff \( \exists P \in \text{FCPT}_T,\Sigma, a \in \text{At}(Var, \Sigma)^+ \quad X \equiv P; a \)

Such an initialized program is usually written as \( P, \leftarrow a \).

In theoretical FCP, we give syntactic structure to programs and view them as the parallel composition of a number of closed, independent sub-programs. In addition, a goal-list (or resolvent as it is usually called), \( a_1, \ldots, a_n \), is interpreted in accordance with the process-reading of clauses and, hence, is viewed as the parallel composition of processes. We do not make this explicit in the syntax of the language.

**Definition:** Theoretical Flat Concurrent Prolog, \( \text{TFCPT}_T,\Sigma; \) initialized programs, \( \text{iTFCPT}_T,\Sigma \)

- \( \text{TFCPT}_T,\Sigma \) is the smallest set \( X \) such that
  - \( P \in X \) if \( P \in \text{FCPT}_T,\Sigma \), \( P \) is closed and every guarded clause in \( P \) is uniquely determined by its head and guard\(^4\)
  - \( P_1 \parallel P_2 \in X \) if \( P_1, P_2 \in X \) and \( P_1 \) and \( P_2 \) have no predicate symbols in common.

- \( \text{iTFCPT}_T,\Sigma \) is the smallest set \( X \) such that
  - \( P; a \in X \) if \( P \in \text{FCPT}_T,\Sigma \cap \text{TFCPT}_T,\Sigma \) and \( P; a \in \text{iFCPT}_T,\Sigma \)
  - \( P_1 \parallel P_2 \in X \) if \( P_1, P_2 \in X \) and \( P_1 \) and \( P_2 \) have no predicate symbols in common.

Observe that any predicate in a TFCP-program, \( P \), inherits its definition from one of the (closed) FCP sub-programs of \( P \). As we will see, this means that parallel components can only influence each other via the variable bindings they produce. Likewise for iTFCP-programs. We feel that at this stage, it is a reasonable assumption to make.

We usually write TFCP instead of \( \text{TFCPT}_T,\Sigma \), write \( C \equiv H \leftarrow G \mid B \) for a guarded clause \( C \) and often interpret a TFCP-program as just a collection of clauses.

\(^4\) This unique determination is a non-essential assumption that makes the denotational semantics somewhat easier to formulate.
To define the operational semantics we have to define read-only unification. Although \( p(X) \) and \( p(f(a)) \) will unify in the classical sense, they should not unify here, because \( X \) can only become instantiated through a writable occurrence of \( X \). Such unification attempts will become suspended. Moreover, successful unification depends on satisfaction of guards.

**Definition**: admissible substitutions, read-only \( \text{mgu} \), \( \text{mgu}^\delta \).

- A substitution \( \theta \in \mathcal{B} \) is admissible for a term \( t \in \text{Tm}(\text{Var}, \Sigma) \) if \( X \in \text{vars}(t) \Rightarrow X \in \text{vars}(t) \) holds for any \( X \in \text{dom}(\theta) \). A substitution is admissible for an atom \( p(t_1, \ldots, t_n) \) if it is admissible for every \( t_i \).

- \( \pi : \text{At}(\text{Var}, \Sigma, T)^* \rightarrow \{\text{true, false, suspend}\} \) is some fixed interpretation of guard atoms such that \( \pi(e) = \text{true} \).

- For a clause \( C \equiv H \leftarrow G \mid B \) and an atom \( a \)
  - \( \text{mgu}^\delta(a, H) = \{ \theta \in \mathcal{B} \mid \theta \) is an mgu for \( \{a = H\} \), and \( \text{ran}(\theta) \cap \text{vars}(a) = \emptyset \} \)

  Our definition of hiding, in section 3, requires the goal variables in \( \text{mgu}'s \) to be in the domain.

- \( \theta \in \text{mgu}^\delta(a, H) \) is admissible if \( \theta \) is admissible for \( a \) and for \( H \).

- \( \text{mgu}^\delta(a, C) \equiv \begin{cases} \theta & \text{if } \theta \in \text{mgu}^\delta(a, H) \text{ is admissible and } \pi(G\theta) = \text{true} \\ \text{fail} & \text{if } \text{mgu}^\delta(a, H) = \emptyset \text{ or} \\ \text{suspend} & \text{for all admissible } \theta \in \text{mgu}^\delta(a, H), \pi(G\theta) = \text{false} \\ \text{otherwise} \end{cases} \)

Note that for \( t \in \{\text{fail, suspend}\} \), if \( t \in \text{mgu}^\delta(a, C) \) then \( \text{mgu}^\delta(a, C) \) is a singleton set.

The results of this paper are actually quite independent of any particular variant of (read-only) unification. The only assumptions that we need to make, is that unification yields a substitution, \( \text{fail} \) or \( \text{suspend} \), that its yield only depends on \( \pi \), a clause and a head and that it yields \( \text{suspend} \) if a read-only instance needs to be bound.

### 2.2 Operational Semantics

The operational semantics gives only minimal information about program executions. For any finite computation it records the resultant substitution \( \theta \) together with the termination mode — success, \((\theta, d)\), deadlock, \((\theta, dl)\), or fail, \((\theta, \text{ff})\) — . This seems to be the minimal amount of information that one would like to know about a program. With every program, \( P \), we associate a transition system, \( \Pi_P \) as follows

**Definition**: Transition system, \( \Pi_P \)

Given a program \( P \in \text{TFCP}, \Pi_P = (Q, \rightarrow) \), where

- \( Q = \{\{A; \theta\} \mid A \) is a multiset of atoms or \( A \in \{t, dl, ff\}, \theta \in \mathcal{B}\} \).

- \( \rightarrow \subseteq Q \times \mathcal{B} \times Q \) is defined as the smallest relation such that:

  - \( \{\{A_1, \ldots, A_i, \ldots, A_n\}; \theta\} \rightarrow \{\{A_1, \ldots, B\rho, \ldots, A_n\}; \theta \circ \sigma\} \)

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5 These definitions differ slightly from the ones used in FCP [Sha86]
for any \( C \equiv H \leftarrow G \mid B \in P \) and renaming \( \rho \in \mathcal{R} \) such that \( \text{vars}(C) \subseteq \text{dom}(\rho) \), \( \text{ran}(\rho) \cap (\text{vars}(A_1, \ldots, A_n) \cup \text{dom}(\theta)) = \emptyset \) and \( \sigma \in \text{mgu}(A, C\rho) \in \mathcal{B} \):

- \( \{A_1, \ldots, A_i, \ldots, A_n; \theta\} \vdash (\theta; \theta) \)
  - if \( A_i \neq \text{true} \) and for every clause \( C \in P \) fail \( \in \text{mgu}(A_i, C) \).
- \( \langle A; \theta\rangle \vdash (\theta; \theta) \)
  - if \( \exists A_i \in C A_i \neq \text{true} \) and \( \forall A_i \in A \forall C \in P, A_i \neq \text{true} \Rightarrow \text{suspend} \in \text{mgu}(A_i, C) \).
- \( \langle \text{true}, \ldots, \text{true}; \theta\rangle \vdash (\theta; \theta) \)
- \( \text{Seq}(\Pi P, q) = \{ (q_i, q_{i+1})_{i<\omega} \mid \alpha < \omega, q_0 = q, q_0 = (R; \theta), R \in \{\alpha, \beta, \gamma\} \} \)

**Definition:** *Operational Semantics* \(-\circ-\)

For \( \text{Prog} = P_1; a_1 || \ldots || P_n; a_n \in \iTPFCP\), let \( P = P_1 \cup \ldots \cup P_n \) and \( a = a_1, \ldots, a_n \). Then

- \(-\circ\text{Prog} = \{ (\theta \mid \text{vars}(a), R) \mid \exists (q_i, q_{i+1})_{i<\omega} \in \text{Seq}(\Pi P, (a; \tau)), \alpha < \omega, q_0 = (R; \theta) \} \)

3. Denotational Semantics

A compositional semantics has to give meaning to each individual program part, independent from the environment consisting of the other parts. For TFCP this means that we have to give meaning to the individual clauses of a program and hence to anticipate in these meanings the possible substitutions that any environment may produce. In this sense, there is a close correspondence with ordinary CSP-like, concurrent languages. Here, too, meanings or denotations must be expressed in terms of sequences of interactions — i.e., substitutions — differentiating between input-substitutions, \( \theta^I \), offered by the environment, and output-substitutions, \( \theta^O \), offered by the program part. This correspondence goes even further as we, too, have to describe the ways in which a program depends on its environment in order to proceed — e.g., because a read-only variable needs to be bound —.

3.1 The Domain

In fact, our semantics will be based on the *failure set semantics* for TCSP [BHR84]. So, a program denotation is a set, \( F \), of suspensions, \((c, s)\). Each \((c, s)\) records a computation sequence, \( c \), of input and output substitutions together with a set of substitutions, \( s \), input of none of which will allow the program to produce any other output substitution.

**Definition:** *Sequences, Suspensions*

- \( \mathcal{B}^O = \{ \theta^O \mid \theta \in \mathcal{B} \}, \alpha \in \{I, O\} \) for \( \alpha = I (\alpha = O) \) \( \mathcal{B}^O \) is called the set of input (output) bindings.
- Let \( A \subseteq \mathcal{B}^I \cup \mathcal{B}^O \)

\[ ^6 \text{As } \text{dom}(\theta\sigma) \cap \text{ran}(\theta\sigma) = \emptyset, \theta \circ \sigma \text{ exists and is equal to } \theta\sigma; \text{ see [LMM86].} \]
\[ SEQ(A) = \left\{ \theta_0^{a_0} \theta_1^{a_1} \cdots \theta_n^{a_n} \in A^* \mid \forall i, j = 1..n \ i \neq j \Rightarrow \text{dom}(\theta_i) \cap \text{dom}(\theta_j) = \emptyset, \right\} \]

- \[ SEQ = SEQ(B^I \cup B^O) \]

- If \( c = \theta_0^{a_0} \cdots \theta_n^{a_n} \in SEQ \) then \( \theta_c = \theta_0 \circ \cdots \circ \theta_n \)

- \( SUSP = \{(c, s) \mid c \in SEQ, s \subseteq B^I \text{ or } s \in \{J, U\} \} \)

Before we can define the denotational domain, we have to introduce renamings of suspensions.

**Definition: renaming of suspensions**

Let \( (c, s) \in SUSP, c = \theta_0^{a_0} \cdots \theta_n^{a_n} \). Let \( \rho \in R \) be such that \( \text{vars}(\rho) \cap (\bigcup_{i=1}^n \text{vars}(\theta_i) \cup \text{dom}(\theta)) = \emptyset \). Then \( (c, s)\rho \) is a renaming of \( (c, s) \) and defined as \( (c\rho, s\rho) \)

Since \( \theta^O \) is the first substitution produced by a unification of the program that \( (c, s) \) is a behaviour of, this definition mirrors the way that variables are renamed during actual computations.

Program denotations will be sets of suspensions. However, not every such set will be a program denotation. In the definition below we will impose a number of closure conditions on these sets. Their purpose is not so much to characterize precisely those sets that do obtain as program denotations, but rather to express the closure conditions that we need in the sequel. E.g., conditions L1 and L2 allow us to use set inclusion as the ordering on the domain.

**Definition: Denotational Domain, \( \mathcal{DEN} \)**

- \( \mathcal{DEN} \subseteq 2^{SUSP} \) and \( F \in \mathcal{DEN} \) iff:

  - (L1) \( (c, \emptyset) \in F \)
  - (L2) \( (c', \emptyset) \in F \Rightarrow (c, \emptyset) \in F \)
  - (L3) \( \forall s' \subseteq s \ (c, s) \in F \Rightarrow (c, s') \in F \)
  - (L4) \( \forall s' \subseteq_{fin} s \ (c, s') \in F \Rightarrow (c, s) \in F \)
  - (L5) \( \text{Let } (c, s) \in F, (c, \emptyset) \notin F \text{ and } \emptyset \in B^I. \text{ Then } \forall \emptyset \in \mathcal{B} \ (c, \emptyset) \notin F \Rightarrow (c, \emptyset) \in F \)
  - (L6) \( \forall \emptyset \in \mathcal{B} \ (c, \emptyset) \in F \Rightarrow (c, \emptyset) \in F \text{ for any renaming } (\tilde{c}, \tilde{s}) \text{ of } (c, s) \).
  - (L7) \( (c, \emptyset) \in F \Leftrightarrow (c(\emptyset^I, \emptyset)) \in F \text{ for any } \emptyset \in \mathcal{B} \text{ and some renaming, } \rho, \text{ such that } \text{vars}(c\tilde{c}) \cap \text{dom}(\emptyset, \rho) = \emptyset \)
  - (L8) \( (c, \emptyset) \in F \Rightarrow (c(\emptyset^I, \emptyset)) \in F \text{ for any } \emptyset \in \mathcal{B} \text{ and some renaming, } \rho, \text{ such that } \text{vars}(c\tilde{c}) \cap \text{dom}(\emptyset, \rho) = \emptyset \)

The closure conditions L1, ..., L4 are the standard ones, corresponding to the conditions N1, ..., N4 of [HGR84]). L5 adapts condition N5 to the asynchronous nature of "communication" in TFCP. L6 is a compactness condition analogous to L4, and is needed to ensure continuity of the restrict operator. It expresses the arbitrariness of renamings. L7 and L8 function in the full abstraction proof. Bounded nondeterminism, finally, is expressed in condition L9. We take the quotient with respect to \( \approx \) so as to ignore the effect of renamings of variables.

**Theorem:**

\( D = (\mathcal{DEN}, \subseteq, \perp) \) with \( F \subseteq F' \) iff \( F \supseteq F' \) and \( \perp = SUSP \) is a complete partial order.
3.2 The Equations

Technically speaking, the semantic equations pose fairly standard problems. As stated earlier, computations are modelled after the failure set model for TCSP [HGR84]. The “recursion skeleton” of the semantics is analogous to, e.g., the one used by Joost Kok [Kok88] or by Neil Jones for PROLOG in [Jon87]. There are two exceptions.

In the operational semantics, every unification induces a renaming of the spawned-off body-clauses. Since we aspire at full abstractness, the denotational semantics must mimic this. Indeed, the equations are parametrized by three additional arguments: two renamings, used to rename the goal and the clause it unifies with, and a set of fresh variables that newly constructed renamings can rename to. An alternative would have been to use in both semantics a fixed renaming scheme. We felt that it was preferable to keep the operational semantics as simple and clean as possible.

The second exception concerns the hiding of variables. The variables in the initial goal are always visible, but a new variable becomes visible because it occurs in the binding to an already visible variable (see the definition of closure below). This dynamic character makes the hide-operator more difficult to define.

We need some notation and auxiliary functions.

Definition: Let $P$ be a TFCP program and $F \subseteq SUS;P$; let $c = \theta^V\bar{c} \in SEQ$ and let $g \in At(Var, \Sigma)^*$. 

- the closure of $g$ with respect to $c$: 
  \[
  \text{closure}(g, c) = \bigcup \{ v' \in \text{vars}(\nu_{\bar{c} \theta}) \mid v \in \text{vars}(g), c' \leq c \}
  \]

- input variables: $\text{input}(c) = \bigcup \{ \text{dom}(\theta) \mid \exists c_1, c_2 \; c = c_1 \theta^V c_2 \}$

- $c \upharpoonright V$ is defined by a recursion on the length of $c$:
  
  \[
  c \upharpoonright V = c, \quad (\theta^V \bar{c}) \upharpoonright V = \theta^V(\bar{c} \upharpoonright V), \quad (\theta^O \bar{c}) \upharpoonright V = \begin{cases} \theta \upharpoonright V \theta^O \upharpoonright V \upharpoonright V & \text{if } \theta \neq \tau \\ \bar{c} \upharpoonright V & \text{otherwise} \end{cases}
  \]

- suspended($g, P) = \{ s \subseteq B^1 \mid \forall \sigma \in s \; \forall C \in P \; \forall a \in g \; \text{suspend} \in \text{mgu}(\sigma, C) \}$

- $c^\upharpoonright F = \{(c^\bar{c}, s) \mid (\bar{c}, s) \in F\}$ for $c \in SEQ$

- PFC($F$) is the smallest set $X \supseteq F$ satisfying L1 and L2.

Now, we can define the auxiliary functions, restrict($V, F$), respectively, $F_1 \parallel F_2$, that hide variables in the suspension, $F$, respectively, parallelly compose the suspensions $F_1$ and $F_2$.

Definition: restrict and $\parallel$

- restrict : $2^{\text{Var}} \times \text{DEN} \rightarrow \text{DEN}$

  \[
  \text{restrict}(V, F) = \text{PFC} \left( \bigcup \left\{ \text{hide}(V, (c, s)) \mid (c, s) \in F \; \text{and} \; \exists \bar{c}, (c_i)_{i \leq w} \in SEQ \; \forall i < w \; c_i \leq c_{i+1} \& (c^\bar{c} c_i) \in F \; \& \; \text{hide}(V, c^\bar{c} c_i) = \text{hide}(V, c^\bar{c}) \right\} \right),
  \]

Proof: standard [BHR84].
\[
\text{hide}(V, (c, s)) = \begin{cases} 
\{ (c \upharpoonright \bar{V}, s \upharpoonright \bar{V}) \} & \text{where } \bar{V} = \text{closure}(V, c) \\
\emptyset & \text{if } \text{input}(c) \cap \text{ivar}(V, c) = \emptyset \\
\text{Var} \setminus \text{ivars}(V, c) & \text{otherwise}
\end{cases}
\]

\[
\text{ivars}(V, c) = \begin{cases} 
\exists c_1, \theta, c_2 \quad c = c_1 \theta^c c_2 \\
\nu \in \text{ran}(\theta) \setminus \text{closure}(V, c) & \text{local variables}
\end{cases}
\]

\[
\text{\(|\) : } \text{DEN} \times \text{DEN} \rightarrow \text{DEN}
\]

\[
F_1 \parallel F_2 = \{(c_1|c_2, s_1|s_2) | (c_1, s_1) \in F_i, i = 1, 2, \}
\]

\[
|c_0|c_1 = c_1|c_0, \varepsilon|\varepsilon = \varepsilon, \quad \text{for } l \in \{I, O\} \quad \theta^l c_0|\theta^l c_1 = \theta^l(c_0|c_1)
\]

**Lemma:**

The functions \(\text{restrict}(V, \cdot)\) and \(\|\cdot\|\) are well-defined and continuous. Also, \(\|\cdot\|\) is commutative and associative.

**Proof:** See the full paper.

The semantics of a TFCP-program, \(P\), will be given by \(P(P)\). It will be defined as a fixed point involving auxiliary functions \(A\) and \(B\) that give meaning to atom-lists and individual atoms relative to a program. We make use of program environments, \(\mathcal{E}\mathcal{N}\mathcal{V}\), that record the meaning of the program clauses (remember that they are identified by their head and guard).

\[
\mathcal{E}\mathcal{N}\mathcal{V} : \text{At}(\text{Var}, \Sigma) \times \text{At}(\text{Var}, \Sigma', T)^* \rightarrow 2^{\text{Var}} \times \mathcal{R} \times \mathcal{R} \rightarrow B \rightarrow \text{DEN}
\]

The types of the functions involved, are as follows:

- \(P : \text{FCP} \rightarrow \mathcal{E}\mathcal{N}\mathcal{V}\) program meaning
- \(A : \text{At}(\text{Var}, \Sigma)^* \rightarrow \mathcal{E}\mathcal{N}\mathcal{V} \rightarrow 2^{\text{Var}} \times \mathcal{R} \times \mathcal{R} \rightarrow B \rightarrow \text{DEN}\) atom list
- \(B : \text{At}(\text{Var}, \Sigma) \rightarrow \mathcal{E}\mathcal{N}\mathcal{V} \rightarrow 2^{\text{Var}} \times \mathcal{R} \times \mathcal{R} \rightarrow B \rightarrow \text{DEN}\) single atom

**Definition: FCP-program meaning**

Take \(P \in \text{FCP}_T, \Sigma \cap \text{FCP}_T, \Sigma\). Let \(P = C_1, \ldots, C_n\) and \(P \text{Var} = \text{vars}(P)\), where \(C_i \equiv H_i \mid B_i\) (and \(H_i \equiv a_i \mid g_{i1}, \ldots, g_{i m_i}, B_i \equiv b_{i1}, \ldots, b_{i n_i}\)).

- \(P(P) = \Omega\{C_1/H_1, \ldots, C_n/H_n\}\), where \((C_1, \ldots, C_n) = \mu C_1 \ldots C_n \[A(B_1(P(P)), \ldots, A(B_n(P(P)))]\)

- \(A(a_1, \ldots, a_n)\eta V \rho \bar{\rho} = B(a_1)\eta V_1 \rho_1 \bar{\rho} \quad \| \parallel B(a_2)\eta V_2 \rho_2 \bar{\rho} \parallel \ldots \parallel B(a_n)\eta V_n \rho_n \bar{\rho} \), where \(\text{ran}(\rho_i) \cap \text{ran}(\rho_j) \neq \emptyset \Rightarrow i = j, \rho_1, \ldots, \rho_n \in \mathcal{R}, P \text{Var} \cup \text{vars}(a_1, \ldots, a_n) \subseteq \bigcup_i \text{dom}(\rho_i), \bigcup_i \text{ran}(\rho_i) \subseteq V \) and \(V_1, \ldots, V_n\) is a partition of \(V \setminus \bigcup_i \text{ran}(\rho_i)\) such that each \(V_i\) is infinite.

- \(B(a)\eta V \rho \bar{\rho} = \text{restrict}(\text{vars}(a \rho \bar{\rho}), F)\) where

\[
F = \left\{ (\bar{\varepsilon}, s) \mid s \in \text{suspended}(a \rho \bar{\rho}, \text{dom}(\eta)), \forall \bar{\theta} \in s : \text{dom}(\bar{\theta}) \cap \text{dom}(\theta) = \emptyset, \text{vars}(\bar{\theta}) \cap V = \emptyset \right\} 
\]

\[
\cup 
\left\{ (\bar{\varepsilon}, \emptyset) \mid a \equiv \text{true} \right\} 
\cup 
\left\{ (\bar{\varepsilon}, \emptyset) \mid a \neq \text{true}, \forall H \in \text{dom}(\eta) \text{ fail } \in \text{mgv}(a \rho \bar{\rho}, H) \right\} 
\cup 
\left\{ \bar{\theta}^l : B(a)\eta V \rho \bar{\rho} \bar{\theta} \quad | \quad \bar{\theta} \in B, \text{dom}(\bar{\theta}) \cap \text{dom}(\theta) = \emptyset, \text{vars}(\bar{\theta}) \cap V = \emptyset \right\}
\]
Now, the semantics of a TFCP-program, \( P_1 \parallel P_2 \), is straightforwardly defined:

**Definition:** TFCP-program meaning

\[
\begin{align*}
\text{P}(P_1 \parallel P_2) &= \text{P}(P_1) \cup \text{P}(P_2). \\
\text{(Note that P}(P_1) \text{ and P}(P_2) \text{ have disjoint domains.)}
\end{align*}
\]

**Definition:** Denotational semantics, \([\cdot]\) : iTFCP_T,E \rightarrow \mathcal{DEN}

\[
\begin{align*}
\text{[P_1 \parallel P_2]} &= \text{[P_1]} \parallel \text{[P_2]} \\
\text{[P; a]} &= A(a)(\text{P}(P)) \text{Var } t r t r
\end{align*}
\]

**Theorem:** well-definedness and continuity

The functions \( \text{P} \), \( A \), \( B \) and \( [\cdot] \) are well-defined and continuous

**Proof:** Included in the full paper.

Finally, we can relate \( \varnothing \cdot \varnothing \) and \([\cdot]\) as follows

**Theorem:** \( \varnothing \cdot \varnothing = \alpha \circ [\cdot] \) where \( \alpha : \mathcal{DEN} \rightarrow B \times \{t_l, d_l, \mathcal{f}f\} \) is defined by

\[
\alpha(F) = \left\{ (\theta, t) \mid (c, s) \in F, \text{ input}(c) = \emptyset, t = \begin{cases} s & \text{if } s \in \{t_l, \mathcal{f}f\} \\
\text{dl} & \text{if } s \subseteq B \text{ and } s \neq \emptyset \end{cases} \right\}
\]

4. Full Abstraction

The last theorem of the previous section shows that the observables can be retrieved from the program denotations: \( \varnothing \cdot \varnothing = \alpha \circ [\cdot] \). But what is the status of those parts of the denotations that \( \alpha \) abstracts away from? We intend to prove that those parts are really necessary. Specifically, any other denotational semantics from which the observables can be retrieved will have at least as much information in its denotations as \([\cdot]\) has: \( [\cdot] = \beta \circ [\cdot] \) for some \( \beta \), where \( [\cdot] \) is any other denotational semantics such that \( \varnothing \cdot \varnothing = \gamma \circ [\cdot] \) for some \( \gamma \).

There is a (folk?) theorem that links such an abstract semantics to the discriminatory power of contexts: whenever \([\cdot]\) differentiates between two programs, \( P \) and \( Q \), there must be another program with a "hole", \( E(x) \), such that plugging in \( P \) and \( Q \) will result in an observable difference: \( \varnothing E(P) \varnothing \neq \varnothing E(Q) \varnothing \). See, e.g., [HGR87].

In our case, the syntax of iTFCP suggests the following notion of contexts: \( E(x) \in \text{Cont}_{T,E} \iff \exists P_x \in \text{iTFCP}_{T,E} \ E(x) = P_x \parallel x \) and \( \Sigma' \cap \Sigma = \emptyset \).

**Theorem:** Abstractness of \([\cdot]\) w.r.t. \( \varnothing \cdot \varnothing \)

Let \( \text{var}(\cdot) \in T \) be a unary predicate symbol such that \( \pi(\text{var}(\cdot)) \) is the characteristic predicate of \( \text{Var}_T \). Then, for every \( P, Q \in \text{iTFCP}_{T,\Sigma} \):

\[
\begin{align*}
\text{[P]} \neq \text{[Q]} \Rightarrow \exists E(x) \in \text{Cont}_{T,\Sigma} \ \varnothing E(P) \varnothing \neq \varnothing E(Q) \varnothing \ \text{and} \ E(P), E(Q) \in \text{iTFCP}_{T,\Sigma}.
\end{align*}
\]

Note that we prove abstraction under an assumption about the available test-predicates, \( T \). We have no such result yet for arbitrary \( T \).
Proof: sketch

Take some \((c, s) \in [P] \setminus [Q]\). We only sketch the proof for the most complicated case, where \(s \neq \emptyset\). The context, \(E(x)\), must somehow make the behaviour \((c, s)\) possible — by producing the necessary substitutions — and must be able to sense and make observable the fact whether \((c, s)\) has occurred or not. Assume for the moment that \(E(x)\) can sense when \(x\) has behaved like \((c, \emptyset)\). If, after having sensed that, \(E(x)\) would bind, say, true to the fresh variable success and after that produce, non-deterministically, any of the substitutions in \(s\), then we would have \((\theta, d)l \in \searrow E(P)\arrow for some \(\theta \in \mathcal{B}\) with \(\text{success}\theta = \text{true}\). Strictly speaking, the context will sense whether \(x\) behaves like \((\tilde{c}, \emptyset)\) or not, with \(\tilde{c} \approx c (\approx \mathcal{B} \times \mathcal{B} \text{ is extended pointwise to sequences}). This is because a context cannot sense the renamings of variables as, e.g., in \(\text{Proof}\).

Let \(c = \tilde{c}l^{i} \cdots \tilde{c}l_{n}^{\ast}, s = \{\sigma_{1}, \ldots, \sigma_{k}\}\) and \(B \approx = \{[\phi_{1}], \ldots, [\phi_{m}]\}\). Let \(V\) be the variables in the goals of both \(P\) and \(Q\). Define \(V_{1}\) as \(V\) and for \(i > 0\), \(V_{i+1} = \text{vars}(V_{i} \tilde{c}l_{i}^{1} \cdots \tilde{c}l_{i}^{\ast})\). I.e., \(V_{i}\) are the variables that could be affected “in step \(i\)”. With each \(\tilde{c}l_{i}^{j}\) we associate a set of clauses, \(P_{i}^{j}\). With \(s\) and \(B\) we associate sets of clauses \(P_{s}\) and \(P_{B}\). Basically, if \(l_{i} = O\) then \(P_{i}^{O}\) will fail if anything but \(\tilde{c}l_{i}\) is output. If \(l_{i} = I\) then \(P_{i}^{I}\) will produce the required input substitution. Similarly, \(P_{s}\) will produce any of the \(\sigma_{j}\) and \(P_{B}\) will await any of the \(\phi_{j}\) and will then fail.

In the pseudo code below, “;” stands for sequentialization, which can be achieved by the standard short circuit technique of A. Takeuchi [Sha86]. Moreover, \(\text{await}(\theta)\) waits until the variables in \(\text{dom}(\theta)\) are bound and fails if they are bound differently than in some \(\tilde{\theta} \in [\theta]\); \(\text{produce}(\theta)\) produces some \(\tilde{\theta} \in [\theta]\); \(\text{fail}\) is a clause that fails; \(\text{test}(V)\) checks whether the variables in \(V\) are bound to non-variables. The first three pseudo commands are FCP programs, while \(\text{test}((x_{1}, \ldots, x_{i}))\) stands for the guard \(\text{var}(x_{1}), \ldots, \text{var}(x_{i})\).

The environment is \(E(x) \equiv \bigcup_{i=1}^{n} P_{i}^{O} \cup P_{s} \cup P_{B} ; e_{i}(V_{i}, \text{success}) || x\), where

- \(P_{i}^{O} \equiv e_{i}(V_{i}, \text{success}) \leftarrow \text{await}(\tilde{c}l_{i}) ; e_{i}(V_{i+1}, \text{success}) \)
  \(e_{i}(V_{i+1}, \text{success}) \leftarrow \text{test}(V_{i+1}) \mid e_{i+1}(V_{i+1}, \text{success})\)
- \(P_{i}^{I} \equiv e_{i}(V_{i}, \text{success}) \leftarrow \text{test}(V_{i}) \mid \text{produce}(\tilde{c}l_{i}) ; e_{i+1}(V_{i+1}, \text{success})\)

\(^{7}\) By L3 and L4 we may assume that \(s\) is finite.
$P_1 \equiv e_{n+1}(V_{n+1}, true) \leftarrow produce(\sigma_1); e_{n+2}$

$\ldots$

$e_{n+1}(V_{n+1}, true) \leftarrow produce(\sigma_k); e_{n+2}$

$P_B \equiv e_{n+2} \leftarrow await(\phi_1); fail$

$\ldots$

$e_{n+2} \leftarrow await(\phi_m); fail$

As can be seen from the code, executing any of the fragments of $E(x)$ will result in more than one unification and, hence, will produce substitutions that do not appear in $(c, s)$. The closure conditions L7 and L8 ensure that we can “expand” $c$ to make room for the additional resolutions.

5. Conclusions and Further Work

We have developed, here, the first fully abstract semantics for any concurrent logic programming language. In doing so, we have consciously ignored the logic programming origins of FCP and have treated it as just another concurrent language. It might come as a surprise to see that standard modeling techniques, developed for CSP, apply in this context, too. The asynchronous nature of the interaction of FCP processes does complicate things, as does unification as the basic computation step. This can be seen from the full abstraction proof. The construction of the context here is more cumbersome than for CSP, CCS or for DNP-R [HGR87].

We intend to extend our results to more general computational domains. In particular, we want to have denotational models that correspond to step- semantics and to partial order semantics for FCP; these semantics capture more of the behaviour of distributed implementations of FCP.

Finally, the resulting denotational semantics will be the starting point for proof systems and verification and debugging tools for FCP.

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6. References


Let $\text{Var}$ denote some (countable) set of variables and $\Sigma$ some first-order signature; $\Sigma = \Sigma^p \cup \Sigma^f$, where $\Sigma^p$ collects the predicate-symbols of $\Sigma$ and $\Sigma^f$ the function-symbols.

Terms and atoms over $\text{Var}$ and $\Sigma$ are defined as usual and are denoted by $Tm(\text{Var}, \Sigma)$ and $At(\text{Var}, \Sigma)$. Also, $At(\text{Var}, \Sigma_1, \Sigma_2)$ denotes the set of atoms that take their predicate-symbols solely from $\Sigma_2$. So, $At(\text{Var}, \Sigma) = At(\text{Var}, \Sigma^f, \Sigma^p)$.

**Definition:** depth of a term, $d(t)$

- $d(t) = 0$ for $t \in \text{Var} \cup \Sigma^f$,
- $d(t) = 1 + \max(d(t_1), \ldots, d(t_n))$ for $t = p(t_1, \ldots, t_n)$

For any syntactic object, $\psi$, $\text{vars}(\psi)$ stand for the set of variables occurring (free) in $\psi$.

We often use sequences of objects. Then, $\cdot\cdot\cdot$ denotes concatenation of two sequences, "$\varepsilon"$ stands for the empty sequence and "$c_1 \preceq c_2$" means that $c_1$ is a prefix of $c_2$: $\exists \bar{c}_1 \bar{c}_1 \bar{c}_1 = c_2$.

**Definition:** substitutions

- A substitution on $\text{Var}$ is a total function $\sigma: \text{Var} \rightarrow Tm(\text{Var}, \Sigma)$ such that the set $\text{dom}(\sigma) = \{X \in \text{Var} | \sigma(X) \neq X\}$ is finite. The set $\text{dom}(\sigma)$ is the domain of the substitution $\sigma$. We let $\text{ran}(\sigma) = \{\text{vars}(\sigma(X)) | X \in \text{dom}(\sigma)\}$. The identity function on $\text{Var}$ is called the empty substitution and is denoted by $\tau$.
- A substitution $\rho$ is a renaming if $\rho(X) \subseteq \text{Var}$, for every $X \in \text{Var}$, $\text{dom}(\rho) \cap \text{ran}(\rho) = \emptyset$, and $\rho$ is injective (on its domain $\text{dom}(\rho)$).
- For terms or atoms $t$, $t\sigma$ is defined inductively by $X\sigma = \sigma(X)$ and $f(t_1, \ldots, t_n)\sigma = f(t_1\sigma, \ldots, t_n\sigma)$. This generalizes to sets of terms or atoms in the obvious way.
- The composition of two substitutions \( \sigma \) and \( \varphi \) is defined by \( X(\sigma \varphi) = (X\sigma)\varphi \). A substitution \( \sigma \) is idempotent if \( \sigma \sigma = \sigma \). The idempotent composition of two substitutions is defined by \( \sigma \circ \varphi = \theta \) if \( \theta \) is idempotent and \( \exists n \geq 0 \ (\sigma \varphi)^n = \theta \), where \( \psi^n \) is defined by \( \psi^0 = \tau \) and \( \psi^{n+1} = \psi \psi^n \).

- A substitution, \( \alpha \), is more general than a substitution, \( \beta \) if there exists a substitution \( \gamma \) such that \( \beta = \alpha \gamma \).

- A substitution, \( \sigma \), is a unifier for a set of equations, \( E = \{a_1 = b_1, \ldots, a_n = b_n\} \), where the \( a_i, b_i \) are atoms or terms, if \( a_i \sigma = b_i \sigma \) for \( i = 1..n \). We call \( \sigma \) a most general unifier, mgu, for \( E \), if \( \sigma \) is more general than any other unifier for \( E \).

- If \( \sigma \) is a substitution and \( V' \) is a subset of \( V \), then the restriction \( \sigma \downarrow V' \) of \( \sigma \) to \( V' \) is defined by \( (\sigma \downarrow V')(X) = \sigma(X) \) for \( X \in V' \) and \( (\sigma \downarrow V')(X) = X \) for \( X \in V \setminus V' \). For a set of substitutions \( s \) the restriction \( s \downarrow V \) is defined by \( s \downarrow V = \{ \sigma \downarrow V \mid \sigma \in s \} \).

- If \( \sigma \) is a substitution and \( \rho \) is a renaming then the \( \rho \)-renaming of \( \sigma \) is the substitution \( \sigma \rho \) defined by \( \forall X \in \text{dom}(\sigma) \ (X \rho)\sigma \rho = X \sigma \rho \). This generalizes to sequences of substitutions in the obvious way.

- The equivalence relation \( \cong \subseteq B \times B \) is defined by \( \theta \cong \phi \) if \( \exists \rho \in \mathcal{R} \ \theta \rho = \phi \). Note that only the variables in the range of \( \theta \) are renamed.
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