The design of a combustion heated thermionic energy converter
van Kemenade, H.P.

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The Design of a Combustion Heated Thermionic Energy Converter

proefschrift

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de Rector Magnificus, prof.dr.J.H. van Lint, voor een commissie aangewezen door het College van Dekanen in het openbaar te verdedigen op donderdag 8 juni 1995 om 14.00 uur

door

Erik van Kemenade

geboren te Purmerend
Dit proefschrift is goedgekeurd door de promotoren

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prof.dr.ir. J.A. Klostermann
copromotor
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**Subscripts**

- \(a\) at the edge
- \(a\) neutral particle
- \(adia\) adiabatic
- \(amb\) ambient
- \(ar\) after recuperator
- \(av\) average
- \(B\) Boltzmann
- \(b\) boundary
- \(b\) bubble
- \(boi\) boiling
- \(C\) collector
- \(col\) between emitter and collector
- \(cond\) conduction
- \(cr\) critical
- \(cs\) cesium
- \(E\) emitter
- \(elastic\)
- \(capillary\)
- \(condenser\)
- \(E\) elastic
- \(e\) electron
- \(e\) equivalent
- \(el\) electron
- \(ent\) entrainment
- \(flue\) flue gases
- \(gap\) interelectrode gap
- \(hs\) hot shell
- \(i\) ion
- \(i\) inner
- \(in\) incompressible
- \(L\) limit
- \(l\) lattice
- \(lc\) laminar, compressible
- \(li\) laminar, incompressible
- \(lv\) liquid vapour interface
- \(m\) maximum
\( m \) mean
\( m \) metal
\( n \) normal
\( n \) neutral
\( o \) outer
\( ohm \) ohmic
\( p \) plasma
\( pc \) condenser pipe wall
\( pe \) evaporator pipe wall
\( pw \) pipe-wick interface
\( pla \) plasma
\( R \) at the critical point
\( r \) random
\( r \) reservoir
\( r \) recuperator
\( ref \) reference
\( S \) saturation
\( s \) sheath
\( s \) shear
\( s \) surface
\( s \) stochiometric
\( son \) sonic
\( sur \) surroundings
\( T \) tangent
\( tc \) turbulent, compressible
\( ti \) turbulent, incompressible
\( tot \) total
\( v \) lattice
\( v \) vapour
\( vl \) vapour - liquid interface
\( vis \) viscous
\( w \) wick
\( wv \) wick-vapour interface

\( 0 \) stagnation state
\( 0 \) uncesiated

coordinates

\((x,y,z)\) Cartesian
\((r,z,\theta)\) cylindrical
\((r,\phi,\theta)\) spherical
\((z,\phi,\theta)\) spherical
1 Introduction

1.1 Thermionic Energy Conversion

A Thermionic Energy Converter (TEC) is essentially a diode; the basic components of a TEC are shown in figure 1.1. One of the electrodes, the emitter, is supplied with heat of high temperature (1500–2000K). Electrons are evaporated thermionically into the evacuated, or rarefied-vapour filled interelectrode space. The other electrode, the collector (800–1000 K), is kept at a lower temperature and the electrons condense on it. Part of the heat, that is removed from the emitter by evaporating the electrons, is transported to the collector by the electrons. The remaining part is converted into electrical energy.

![Diagram of a thermionic converter](image)

*figure 1.1 diagram of a thermionic converter*

In practical applications the performance of the converter is limited by the space charge effect: the electrons travelling from the emitter to the collector push newly emitted electrons back to the emitter. Thermionic converters can be divided roughly in two groups by the manner in which the space charge is suppressed: a) close spaced converters and b) gas-filled converters. In a close spaced type converter, the space charge is reduced by making the space between emitter and collector small, or by applying auxiliary electrodes to speed-up the electrons. In the gas filled type of converter, the interelectrode space is filled with a rarefied vapour. The space charge is suppressed by positive ions produced by the ionisation of the vapour. Cesium vapour is generally used for this purpose, as it is the easiest ionisable of all stable gases.

Thermodynamically the thermionic converter can be treated as any other heat engine. The path from point 1 to 2 in the T-S diagram (figure 1.2) represents the heating of the electrons and ohmic losses. The electrons are emitted at a constant temperature (2-3). In a close spaced diode the electrons travel through the interelectrode space at a constant temperature (3-4). In the case that an ignited plasma is present, the electrons will experience a sharp rise and decrease in temperature (3-4'). The path from point 4 to 1 represents the condensing of the electrons on the collector, where electrical energy is produced.
The Design of a Combustion Heated Thermionic Energy Converter

1.1.1 Historical review

The start of the study of thermionics coincides with the invention of the electrical lamp. Because Edison had troubles during his experiments with the life time of the filaments in his electrical lamp, he used more filaments in one evacuated glass enclosure. During his experiments it was observed that an electric current could be made to flow between two electrodes in vacuum (the filaments) if one of them was heated. Although the effect was of no interest to Edison and his group, the effect was duly reported (Preece 1885).

Thirty years went by before emission of electrons was analysed and investigated by Richardson (1912). Schlichter (1915) recognised that this process could be used to convert heat to electricity and a patent was issued in 1923. Though Langmuir (1923) and his co-workers formulated the fundamentals for the understanding of the process in 1923, no further interest was shown in this conversion technique for many years. In 1941 Gurtovoy published experimental data demonstrating the conversion of heat into electricity by means of a thermionic converter. In 1957 the threshold of practical thermionic conversion was reached by several groups.

The following research effort was focused on the development of space applications. In that application, the high heat reject temperature is advantageous as in space only radiant heat rejection is possible. Since the power of the radiator depends to the fourth power on the absolute temperature, the heat rejection temperature is of decisive importance.

One of the earliest combustion heated thermionic converters is reported by Martini (1963). This work started in 1959, but since it was aimed at military applications, information about this work is scarce. Most of the early work on small scale thermionics was carried out in the USA and France. In 1969 Shefsiek published a realistic design of a TEC, already featuring a heat pipe collector and a SiC-CVD hot shell. A survey of practical TEC’s being developed is given by Wolff (1989).

The designs of the Thermo Electric Corporation (TECO) served as an example for the early Eindhoven University of Technology (EUT) designs. An overview of the work done by TECO is presented in an article of Goodale (1982). The converters feature a tri-layer hot shell, consisting of a graphite shell coated on the outside with SiC and on the inside with tungsten, or molybdenum. Both a bell-shaped and a cylindrical converter were constructed (figure 1.3). With the TECO converters power- and life tests were performed. Unfortunately, the TECO combustion heated converter program was stopped years ago to devote all their effort to space nuclear thermionics under president Reagan’s Star Wars project.
1.1.2 Terrestrial applications of Thermionic Energy Conversion

Thermionic systems are best suited where a low weight, silent operation, high heat rejection temperature, no moving parts, and no maintenance are desirable. Potential applications of terrestrial thermionics include:
- power plant topping
- cogeneration (heat and electricity) in industrial processes
- solar power plants
- cogeneration for domestic application

This study is primarily concerned with the last objective. A comprehensive survey on the feasibility of terrestrial thermionic applications is given in Veltkamp (1989a).

1.1.3 Thermionic research at the EUT

Research at the Eindhoven University of Technology (EUT) was initiated in the seventies by the search at the Chemistry Department for suitable emitter materials. The objective was to raise the performance of the TEC by materials development. A new emitter material was developed, based on a directionally solidified metal-ceramic eutectic (Wolff 1982). Beside the emitter material, the confinement which serves to protect the TEC from the combustion environment, the hot shell and the electrical insulation between emitter and collector, the ceramic seal, were studied. This study evolved in the design and construction of a water-cooled TEC (figure 1.4). The brazed cooling system was unpressurized. Consequently, the maximum water temperature is 100 °C. Several problems occurred during the operation of the water-cooled TEC. After start-up, the collector head became too hot, causing explosive boiling of the cooling water. Furthermore, the sapphire spacers crushed the brittle hot shell.

Research was taken over by the Department of Mechanical Engineering. The objective was to redesign the water cooled TEC to an air-cooled TEC (figure 1.5) with a heat pipe collector for measurement purposes (Veltkamp 1989b). Air cooling has the advantage over water-cooling that the rejected heat is available at a higher temperature level, thus increasing the Carnot efficiency. Due to the lower heat capacity and consequently lower heat exchange
capacity as compared to water cooling, air cooling requires a larger heat exchange area and flowrate. A geometry that allows for the necessary heat exchange area in the bell shaped collector is rather complex. A heat pipe collector eliminates this disadvantage as this passive isothermal element can amplify the heat exchange area at a more convenient location outside the TEC. The experimental results with this converter were not satisfactory. Though the hot shell was in good condition after long term thermal balance measurements (Rademacher 1991), a clip spalled off while cleaning the joint. The quality of the ceramic coating together with the joining of the hot shell and the bellows were a main problem in the design of the TEC as it existed at the start of this study.

1.2 A thermionic converter in a heating system

The aim of this study is to redesign the thermionic energy converter for use in a domestic heating system for an average Dutch family house. The existence of a list of demands for the thermionic converter is vital for the design process and hence the thermionic cogeneration system must be defined and investigated first. This subject is extensively covered by Veltkamp (1993). In chapter 5 the choice of a system to design a TEC for is made. In this paragraph the possible elements of a thermionic cogeneration system are introduced.
In thermionic cogeneration systems, the heat demand has the priority, while the electricity produced is a bonus. The heat produced by cooling the converter is not dumped as in space applications, but used to heat the house. In its simplest form the thermionic cogeneration system consists of a burner, heating the TEC which in turn heats the boiler. Such a system operates at very low efficiencies as the exhaust gases cannot be lower in temperature than the emitter, leading to a low burner efficiency. Furthermore, it takes time to heat the TEC and therefore electricity is not immediately produced when starting the burner. Heating systems without thermal storage switch on and off frequently and consequently electricity is produced during a fraction of the operating period of the burner only. Therefore two subsystems have to be considered: a recuperator to regenerate heat from the flue gases and a thermal storage to increase the operating time of the thermionic converter. An auxiliary burner can be included in the system as it is economically more attractive to design the TEC for the basic heat demand and not for the required peak demand. A diagram of a possible cogeneration system is given in figure 1.6.

Figure 1.6 diagram of a thermionic cogeneration system.
The thermionic cogeneration system replaces the conventional domestic boiler and consists of a recuperative burner module, a storage vessel, an auxiliary burner and a gas-water heat exchanger.
1.2.1 The recuperative burner

Only a small part of the energy present in the fuel is converted to work because the temperature of the flue gases cannot be lower than the emitter temperature. By using the heat in the flue gases to preheat the air or air-fuel mixture, the efficiency can be increased. The heat can be transferred by recuperation or regeneration. A counterflow recuperator is the best solution thermodynamically, but even in that case not all the heat can be recuperated due to the difference in heat capacity between the flue gases and air. Furthermore, the fuel (natural gas) can only be preheated to about 900 K to prevent carbonisation. An example of a recuperative burner system is given by Veltkamp (1993) (figure 1.7).

Adding a recuperator has another important advantage apart from raising the system efficiency. Being a feedback system, the recuperator has a stabilising effect on several aspects of the system. For instance, a decrease due to a change of the emission coefficient because of quartz forming on a SiC hot shell causes a higher burner temperature leading to an increase in the radiative heat transfer.

\[\text{fluegas } \sim 700 \text{ K} \quad \text{gas} \quad \text{air } \sim 300 \text{ K}\]

\[\text{burner } \sim 2000 \text{ K} \quad \text{preheated air } \sim 1900 \text{ K}\]

\[\text{flue gas } \sim 2500 \text{ K} \quad \text{hot shell } 1725 \text{ K}\]

\textbf{figure 1.7} burner structure with recuperator (Veltkamp 1993)

Air streams through a recuperator where it is heated by the flue gases. Natural gas is injected in the air stream and the mixture is burned in a vortex burner. The flue gases heat one of the headers of the recuperator which radiates to the hot shell.
1.2.2 The thermionic converter

The space charge effect is the most important limiting factor in obtaining good conversion efficiencies. The space charge effect is caused by the repulsive force of the electrons in transit between emitter and collector. Two fundamentally different approaches exist to enhance the power output of the TEC. One way is to reduce the interelectrode distance between emitter and collector, thus reducing the number of electrons in transit. The other way is by introducing a rarefied cesium vapour, which is done in most practical converters. The cesium vapour can be ionised, due to the high temperature and collisions in the interelectrode space, and a plasma can be maintained between the emitter and the collector. As a plasma is essentially neutral, the space charge is largely reduced. Besides the formation of a plasma, cesium has also a favourable effect on the work functions of the emitter and collector as will be explained in chapter 3. For that reason, cesium is also introduced in close spaced thermionic converters.

Close spaced thermionic converters did not prove to be competitive with ignited mode TEC's and they have been abandoned as practical devices since 1960 except for the work of Gary Fitzpatrick (Dick 1983). Recently however, Fitzpatrick (1993) demonstrated the possibilities of a close spaced thermionic converter. In his experiment the emitter was rigidly mounted to the heating element. The collector was fastened to a flexible mount. An interelectrode distance below 10 μm was maintained using three ceramic spacers mounted to the collector. The collector is pressed by helium pressure against the emitter to control the stresses. Efficiencies up till 13% were shown at relatively low emitter temperatures. The choice between ignited mode thermionic converters and close spaced thermionic converters must therefore be included in this study.

The goal of this study is to design a TEC for a domestic heating system in a systematic way, the objective is not to develop completely new materials, technologies, and materials. Therefore advanced, but largely unproven conversion techniques, like using triodes or pulsed converters are not considered. For the emitter a refractory metal is used to withstand the high temperatures. It also possesses good emitting properties. Both in the TECO designs and in the EUT designs, SiC is used as a protective coating against oxidation. As developing a new coating material is worth a separate study no alternatives are considered. For the collector a lower temperature metal can be used, like nickel-alloy or stainless steel.

The collector has to be cooled to a temperature of about 950 K. Direct cooling with system water is difficult because of the high heat flux and wall temperatures, as is proven by the water cooled design. Hot spots, boiling, and flow stagnation are likely to occur. In the case of a close spaced converter, water cooling may be feasible as the temperatures and heat flux density are lower, but in the case of ignited mode operation, heat flux densities of 200 - 400 kWm⁻² can be expected. To transfer 300 kWm⁻² from 950 K to 360 K, a heat transfer coefficient in excess of 500 Wm⁻²K⁻¹ is necessary. Such a heat transfer coefficient is attainable, but only at the cost of a large pumping power. These problems can be avoided by using a gas as intermediate medium. The heat transfer coefficient in the case of a gas however is considerably lower than that in the case of a liquid, which implies that the surface area must be enlarged. Enlargement of the surface can be effected without significant losses by applying a heat pipe. The temperature drop between the evaporator and the condenser is small and the surface area can be increased outside the emitter-collector area, accommodating the requirements of the heating system. Only thermionic converters using a heat pipe are considered in this study (figure 1.5).
1.2.3 Thermal storage

Thermal storage is commonly used in Dutch heating systems. It is a means of delivering hot tap water instantly at large quantities. The size varies from 10 litre in systems where the storage is used only to overcome the starting of the heating systems, to 120 litre in water storage systems. The thermionic converter has a certain delay time before it starts producing electricity. Heating systems without thermal storage start in the order of 10,000 times yearly, causing a low electrical performance. Using a thermal storage leads to an elongation of the operating period and reduction in the number of starts and stops.

The thermal storage can be realised in the boiler circuit, the domestic hot water and in any other thermal mass. We restrict ourselves to storage in water as other options are not common in the heating society. Obviously it is advantageous for the electrical power production to start with a cold storage and run the system until it is filled. The storage has to be kept at a temperature above 65 °C to avoid growth of legionella and for use in the kitchen. For the bathroom a water flow of 0.2 ls⁻¹ at a minimal temperature of 45 °C is required. Both functions can be combined by using a thermally stratified storage; the medium with the required load temperature is stored on top of medium with a lower temperature. The heat is stored in the domestic water as it requires the largest momentary power. Using this approach a smaller heat exchanger can be used than in the case of storing the heat in the boiler circuit.

1.2.4 House and electrical system

For the modelling of the house, the data for an average Dutch house are taken. In general the installed heating power amounts to 14 kW and the daily hot water drawl is around 120 l at an averaged temperature of 50 °C. The required room temperature amounts to 20 °C.

The performance of the TEC is rather sensitive to the load and as it is a high current–low voltage device, a power conditioner and inverter have to be added. It is assumed that the electric power produced can be delivered to the grid when it is not used in the house.

1.3 Problem definition

The aim of this study is: to design a thermionic energy converter which can be used in a domestic heating system. The aim may sound superfluous as more or less working prototypes were built already by TECO (Goodale 1984), the EUT (Rademacher 1991), and ECS b.v. (Klepikov 1992). These prototypes are essentially improvisations however. Due to the largely unknown behaviour of the construction elements under the severe working conditions, the construction problems were tackled component by component. Though this mode of operation is understandable and probably unavoidable considering the completely new technology involved, it is not the way to a coherent design. Furthermore, an application was sought for the device after designing it and in general it is better to look at the system characteristics first. Hence the need was felt to go through the design process again, using a more methodical approach.
As in every design process, the list of demands for the TEC must be formulated first. To quantify the requested requirements, answers to the following questions must be provided:

- the optimisation criterion
- the required power level
- the required heat/electricity ratio
- the allowed dimensions
- the environment in which the TEC must operate

The answers to these questions serve as a framework to the design process. In order to clarify these issues, the characteristics of the electrical and thermal load must be ascertained. As the heating system is not defined yet, the first step in the design process consists of designing one in so far as needed to state the list of demands (chapter 5). An economical criterion, the annual net profit, is used to validate the alternatives. When the system is optimised with regard to this criterion, the list of demands for the thermionic converter is known. The next step is to ascertain the shape and overall dimensions of the thermionic energy converter. The last step in the process is the design of the new converter which is described in chapter 6.

It is necessary for all stages of the design process that analysing tools are available for the thermionic process, the temperatures, and the structural integrity of the converter. For the system analysis it is sufficient to couple the existing models for thermionic emission to the models for the other system components. For the design stage a more detailed model is required in which the temperature and the voltage are allowed to vary over the emitter. One of the results of the system analysis was that the start-up time of thermionic converter has a major influence on the system efficiency. The start-up time of the converter is mainly governed by the start-up time of the heat pipe. Stationary heat pipe behaviour is well known, but some extra work had to be done in the field of dynamic behaviour. In chapter 2 the models which are connected to the energy flows, including thermionic emission, are described.

In the previous research, the main effort was finding suitable construction materials and material combinations. Moderate attention was given to the analysis of the mechanical behaviour of the construction. When matching the dimensions of the converter to the system requirements however, it is necessary to have tools describing the structural integrity of the construction, especially in the field of creep and stresses in material joints. The models describing the mechanical behaviour of the construction are given in chapter 3.

A large part of the models which were used in the design process could be checked with literature. Experimental validation was necessary to validate the model describing the energy flows through the converter and the start-up behaviour of the heat pipe (chapter 4). The models are used in a simple form when assessing the list of demands (chapter 5). More detailed calculations were performed to design the TEC (chapter 6). In chapter 7 the results of this investigation are discussed.
2 Energy models

The energy models used throughout the design process are described in this chapter. Models of the thermionic process in the ignited and the close spaced mode are introduced and evaluated in §2.1. In §2.2 the model incorporating the other energy transfer mechanisms in the TEC is treated. This model is used in a simple form for the system analysis and more detailed during the dimensioning and designing stage (chapter 6). Knowledge of the static and dynamic behaviour of the heat pipe is essential for the design of a thermionic converter and is discussed in paragraph 2.3.

2.1 Thermionic emission

The basic phenomena characteristic for thermionic energy conversion can be divided into two classes, emission phenomena, and transport phenomena. Emission phenomena are related to processes by which particles are emitted from, or collected at the surfaces of the TEC. Transport phenomena are related to the processes of migration of- and interaction between the particles in the interelectrode space. In this study the widely used phenomenological model of Rasor (1982) and the model of Sidelnicov (1993) are used. In Appendix A.1 the behaviour of the thermionic converter is analysed under idealised conditions, the results are summarised in § 2.1.1. Subsequently the behaviour of the charged particles in transit between the interelectrode space are superimposed using Rasor’s more phenomenological approach.

2.1.1 Ideal thermionic converter performance

Derivation of expressions for the electron emission of a metal, starts with a model for the behaviour of electrons in a metal. When a metal is at temperature $T$, the free electrons may be thought of as moving in all directions with a large variety of speeds and kinetic energies (the Sommerfeld gas approximation). The average number of electrons having a total energy $\varepsilon$ is described by the rules of quantum mechanics. For the present discussion it suffices that the number of electrons having an energy between $\varepsilon$ and $\varepsilon + d\varepsilon$ is proportional to the Fermi factor $u(\varepsilon)$. Apart from the temperature and the electron energy, $u$ is a function of the Fermi energy $\mu$ of the material. $\mu$ is a quantity characteristic of the material and describes the highest energy of the electrons in the metal. In appendix A it is shown that the electron saturation current $I_{es}$, the current found at the points just outside the electrode in the absence of a strong electrical field, can be described as

$$I_{es} = A T^2 \exp \left( -\frac{\phi}{kT} \right)$$

where

$$A = \frac{4\pi m_e k^2 e}{h^3}$$

$m_e$ is the mass of an electron, $k$ the Boltzmann constant, $e$ the elementary charge, $A$ the Richardson constant, $h$ the Planck constant, $T$ the temperature, and $\phi$ the work function of the surface. The work function is defined as $\phi = \psi - \mu$, where $\mu$ is the Fermi level and $\psi$ the
The interelectrode motive. The interelectrode motive is defined as a scalar whose negative gradient equals the force exerted on an electron.

The net current exchanged between two electrodes equals the emitter-collector current \( J_{EC} \) minus the collector-emitter current \( J_{CE} \) minus the ion current \( J_{iEC} \). In first approximation we suppose that the collector-emitter and the ion current can be neglected. If the emitter motive is larger than the collector motive, which means that we can write for the output voltage: \( eV < \phi_E - \phi_C \), all electrons emitted from the emitter will reach the collector and the current is given by equation 2.1. In the case that the output voltage exceeds the difference in motive between emitter and collector, \( eV > \phi_E - \phi_C \), not all electrons emitted have sufficient energy to reach the collector and some will return to the emitter.

Electrons emitted from a hot surface have speeds that differ both in magnitude and direction. The distribution of the velocities is to a good approximation described by a half-Maxwellian distribution function. For a half-Maxwellian distribution the fraction of electrons that will reach the collector can be written as

\[
q = \exp \left( -\frac{\psi_C - \psi_E}{kT} \right) = \exp \left( -\frac{\phi_E - \phi_C + eV}{kT} \right) \tag{2.2}
\]

In appendix A.1 it is shown that the emitter current \( J_{EC} \) is given by the relation

\[
J_{EC} = J_{ES} = AT_E^2 \exp \left( -\frac{\phi_E}{kT} \right) \text{ for } eV < \phi_E - \phi_C \tag{2.3}
\]

and

\[
J_{EC} = \tilde{q} J_{ES} = AT_E^2 \exp \left( -\frac{\phi_E + eV}{kT} \right) \text{ for } eV > \phi_E - \phi_C \tag{2.4}
\]

and for the collector

\[
J_{CE} = \tau J_{CS} = AT_C^2 \exp \left( -\frac{\phi_C}{kT_C} \right) \text{ for } eV < \phi_E - \phi_C \tag{2.5}
\]

and

\[
J_{CE} = J_{CS} = AT_C^2 \exp \left( -\frac{\phi_C}{kT_C} \right) \text{ for } eV > \phi_E - \phi_C \tag{2.6}
\]

In the case that \( eV \geq \phi_E - \phi_C \), \( J \) is given by subtracting equation (2.5) from (2.3). In the case that \( eV \leq \phi_E - \phi_C \), \( J \) is given by subtracting equation (2.6) from (2.4). The combined effects of equation 2.2 to 2.6 are shown in figure 2.1.

The output current of a close spaced converter is, in general, smaller than that computed under idealised conditions of particle transport in the interelectrode space for given electrode properties and a given output voltage. The decrease is a result of interactions between electrons that are in transit. These electrons create decelerating forces that reduce the output current to a level smaller than its ideal value.

To enhance the performance of the TEC a rarefied cesium vapour is often introduced in the interelectrode space. The work function of a metal is primarily determined by its atomic roughness (due to the image force, see appendix A) and secondarily by the charge asymmetry of the top atomic layer. There is a fundamental relationship between thermionic emission from a surface and its rate of vaporisation, as metals with large atoms (high thermionic emission) tend to have a low evaporation temperature (Rasor 1964). By adding
cesium in the electrode gap, large cesium atoms absorb on the electrode surface, reducing the work function of the electrode (figure 2.2).

\[ J_{CS} \] (2.5) \[ J_{CE} \] (2.6) \[ eV = \Phi_E - \Phi_C \] (2.3) (2.4) (2.5) (2.6)

**Figure 2.1** Ideal output current for a thermionic energy converter without ion emission

The emitter-collector current \( J_{CE} \) has the same shape as the collector-emitter current \( J_{EC} \), but is smaller in magnitude and is mirrored in the voltage axis and the line \( eV = \Phi_E - \Phi_C \). In the left side of the figure \( J \) equals the emitter saturation current \( J_{EC} \) and in the right side \( J \) equals the collector saturation current \( J_{CE} \).

**Figure 2.2** Work functions of surfaces surrounded by a cesium vapour

The ratio \( T_{surface}/T_{cesium} \) determines the absorption ratio of the cesium monolayer on the surface. At very low coverage the work function equals the work function of the substrate material in vacuum, the bare work function \( \Phi_0 \). With increasing cesium coverage the work functions drops to a minimum. After the minimum the work function rises to a value equal to that of liquid cesium. Materials with a strong work function tend to adsorb cesium more strongly and require less cesium pressure to maintain a certain cesium coverage (Rasor 1964).
2.1.2 Close spaced thermionic converters

The distinguishing feature of close spaced thermionic converters is that their interelectrode space is sufficiently free of particles other than electrons so that the interactions of these particles with electrons are negligible. Hence the only important transport phenomenon is that due to the presence of electrons.

For given electrode properties and a given output voltage, the output current of a close spaced converter is, in general, smaller than that computed under idealised conditions of particle transport in the interelectrode space. In figure 2.3 the interelectrode motives for the two most important modes of operation for a thermionic converter are drawn, the retarding range and the space charge limited range. The curvatures are caused by the space charge effects. In the retarding mode, when the output potential is high, the interelectrode motive shows no maximum in the interelectrode space. The voltage at which the motive shows a maximum just outside the collector surface is called the critical voltage. When the output drops below the critical voltage \( V_R \), the shape of the motive exhibits a maximum in the interelectrode space. This range is called the space charge limited range.

\[
\begin{align*}
\text{retarding range} & : V_R, \\
\text{transition point} & : \phi_c, \\
\text{space charge limited range} & : \psi_{\text{max}}
\end{align*}
\]

In the retarding range an electron travelling from the emitter to the collector experiences a negative force throughout the interelectrode space. In the space charge limited force the electron experiences a positive force near the collector. \( \mu \) denotes the Fermi energy, \( \phi \) the work function, and \( \psi \) the interelectrode motive.

In all practical cases the dimensions of the emitter and collector are much larger than the interelectrode space. In that case the net electron flow is nearly uni-directional. The exact analysis of a diode operating in this region was performed by Langmuir (1923) and is summarised in appendix A.2. A typical output characteristic with the corresponding ideal characteristic is depicted in figure 2.4. In the retarding range the characteristic corresponds with the corresponding ideal characteristic because the interelectrode motive has no maximum in the interelectrode space. This region has no practical importance due to the low power output.

2.1.3 Ignited mode thermionic converters

In the ignited mode, the cesium vapour is ionised and forms a plasma. Though charged particles occur in a plasma, the net space charge over a volume that contains more than 10 to 100 particles is zero. This quasi neutrality occurs because deviations from neutrality leads to strong electrical fields. A plasma can be used in a TEC to suppress the space charge.
figure 2.4  typical output-current characteristic of a close spaced thermionic converter under the conditions that ion emission from the emitter and back emission from the collector are absent

Analysis of the behaviour of a TEC with an ionised gas between the electrodes involves simultaneous calculation of the emission phenomena as described in the previous paragraph and of the transport phenomena. The two types of phenomena are coupled because the emission properties of both electrodes are influenced by the electrical field (Schottky effect, see appendix A) caused by the electrons in transit.

The analysis of performance characteristics in the ignited mode is a tedious numerical task, that has not yet been achieved completely (Rasor 1993). Generally an analytical model is used for the emission properties of the surface in zero field and the interelectrode space is divided in three regions, two electrode sheaths and the plasma (Rasor 1983, Hatsopoulos 1979, Lawless 1986). Each sheath is subsequently described as collisionless and with this assumption the emission properties can be related to the plasma by boundary conditions only.

Part of the electric power generated by the converter in the ignited mode, is dissipated internally in the interelectrode gas by collision processes, heating the electrons in the gas to a sufficient high temperature so that they ionise the gas and maintain a neutral plasma. Electrons are retained in the plasma by sheath barriers $V_E$ and $V_C$ at the emitter and at the collector (figure 2.5), but ions diffuse freely to the electrodes and recombine on their surfaces. The potential drop across the interelectrode space, the arc drop $V_{d}$, sustains this plasma.

figure 2.5  interelectrode motive diagrams for the obstructed mode, the transition point and the saturation mode. $\mu$ denotes the Fermi energy, $\phi$ the work function $\psi$ the interelectrode motive, and $V_p$ the voltage drop across the plasma
The transition point divides the two major regions of ignited mode operation having significantly different phenomenology: the saturation region \((V < V')\) and the obstructed region \((V > V')\). In the saturation mode, the arc drop \(V_d\) over the plasma is greater than the arc drop \(V_d'\) required to produce the ions necessary to neutralise the plasma. The excess energy \(\Delta V\) cannot be transferred to the plasma to generate ions as this would violate quasi neutrality. Experimental data indicate that all excess energy is expended on the production of ions in the emitter sheath which are immediately swept to the emitter (Rasor 1991). In the obstructed mode, there is an insufficient voltage drop between the electrodes to maintain the plasma. The discharge can exist by erecting a space charge \(\Delta V\) at the emitter. The influence of these effects on the J-V characteristic is illustrated in figure 2.6.

Rasor proposed a phenomenological model for the thermionic converter which was refined by McVey (appendix A). The Rasor-McVey model was implemented in a computer program (appendix A.3). The emitter work function can be calculated according to the analytical method of Rasor (1964), or using the empirical relation of McVey (1992). The collector work function is only sensitive to the material involved and is approximated by an empirical function. The Richardson equation (2.1) provides the saturation current densities for the emitter and collector. The relation between the cesium reservoir temperature and the cesium pressure, necessary to calculate the plasma properties, is approximated using an empirical relation (McVey 1992). If the calculated saturation current is below the estimated output current, we have to deal with the obstructed mode.

In order to model the electron temperatures near the emitter and the collector, the mean free path, the ionisation energy and the ionisation factors of the plasma are calculated. The plasma density cannot exceed the density at which as many electrons locally combine as are produced. In this condition, known as local thermodynamic equilibrium (LTE), the plasma properties are equal to those that would exist in equilibrium with a hypothetical surface at the electron temperature and emitting a neutral plasma. If this restriction is met, the electron temperature and ionisation factor is adjusted according to the LTE constraint. In an actual converter, LTE occurs first at the collector side, where the electrons are coldest and spreads gradually through the interelectrode gap. This process can be approximated by using a separate LTE constraint at the collector side.

When the electron temperature is known, the potential barrier at the collector side can be calculated using an energy balance. In an iterative calculation the effective emitter current is
sought, using the fact that the energy of the electrons entering the interelectrode space should equal the energy of the electrons leaving the interelectrode space.

The results of the model as described here were checked with the original publications of Rasor (1964, 1982, 1983, 1991, 1992, 1993) and McVey (1989, 1992). In these publications the model is compared with experimental data and with a more fundamental model where the transport equations are integrated at every point in the plasma. The phenomenological model performs well in most respects except in the obstructed mode, where both models fail (figure 2.7). Several proposals have been made to explain this discrepancy but at present the issue is unresolved (Rasor 1993). In the region of interest for this study, around the transition point where maximum power is delivered, the accuracy is sufficient.

![Comparison of the phenomenological model with experimental data and a fundamental model for a converter with a Molybdenum hot shell and a Nickel collector (Rasor 1982)](image)

Besides the Rasor-McVey model, a simple model, made by Mironov and Sidelnicov (1990), was used (Appendix A.4). It is a semi-empirical model derived from the more analytical TOR model, developed at the Institute of Physics and Power Engineering at Obninsk. This model consists of a simple set of algebraic equations which allows fast calculation of the output voltage. Information about the model however is scarce and the way the fits are made, is not well documented. Whenever the Mironov model was used, the final result was checked using the Rasor model; figure 2.8 gives an example.

The sensitivity of the $J-V$ characteristic to variation of the process parameters is illustrated in the figures 2.9 to 2.12. In all cases there exists an optimum. The cesium reservoir temperature $T_r$ has a major influence on the shape of the $J-V$ characteristic of a TEC. In figure 2.9 it can be seen that the current increases with increasing cesium reservoir temperature (lower work function) until the $J-V$ characteristic has the Boltzmann shape. A further increase in the cesium reservoir temperature shifts the $J-V$ characteristic to the left due to the increasing losses caused by collisions of the electrons with the dense plasma.

Variation of the interelectrode space $d_{gap}$ has a comparable influence on the $J-V$ characteristics. The maximum in the electrode voltage is explained by the competing processes of ion generation and elastic scattering (figure 2.10). When the interelectrode distance is in the order of the mean free path of the plasma, hardly any collisions occur and hence ion production is low. Enlarging the interelectrode distance increases the number of collisions that produce ions but more electrons are scattered back to the emitter.
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The influence of the emitter temperature $T_E$ on the $J-V$ characteristics is shown in the figures 2.11 and 2.12. When the emitter temperature increases, the emitter work function increases (figure 2.2) and hence the electrode output voltage. The increase in emitter work function enhances the generation of ions (equation A.8), reducing the space charge. The emitter saturation current however decreases (equation 2.3) in spite of the raise in temperature due to the increase in workfunction. As the work function hardly changes any more (see figure 2.2), at higher values of the cesium reservoir temperature the increase in thermionic emission is more noticeable than the increase in the emitter work function. The bend in the line disappears and the I-V characteristic follows the Boltzmann curve.

At low temperatures of the collector ($T_C/T_r<1.8$), increasing the collector temperature leads to a higher output voltage because of a decrease in the collector work function (equation A.56). Above the optimum however, a further increase in collector temperature results in enhanced electron emission from the collector (equation 2.6), reducing the voltage (figure 2.13).
Summarising, raising the emitter temperature only results in enhanced performance of the thermionic energy converter when all other parameters are optimised with respect to the new emitter temperature.

2.2 Modelling the thermionic converter

The emitter of an axis-symmetrical thermionic energy converter is divided into elements facing a collector with a uniform temperature. The elements are coupled by the heat conduction and the electrical current. Each element is characterised by its:

- temperature $T_E$
- voltage $V$
- length $l$
- surface $A$
- cross section $A_d$
- hot shell thickness $d_{hs}$
- interelectrode distance $d_{gap}$
- heat conduction coefficient $\lambda_{hs}$
- specific resistance \( \rho_{hs} \)
- emissivity to the surroundings \( \varepsilon_{hs} \)
- emissivity to the collector \( \varepsilon_E \)

The temperature of the collector, cesium reservoir, and the emissivity of the collector are taken constant. The temperature and voltage field are calculated by solving the energy balance for each element. The energy balance consists of:

- \( Q_{sur} \) the heatflux from the surroundings to the element; it is supposed to consist of radiation only
- \( Q_{cond} \) heat conduction from an element to its neighbours
- \( Q_{ohm} \) the current produced by the thermionic process is partly consumed by ohmic heating
- \( Q_e \) heat loss caused by the fact that the kinetic energy of the electrons thermionically emitted is higher than that of the electrons supplied by the electrical lead, which results in a cooling of the emitter
- \( Q_{col} \) heat exchange by radiation between the emitter and the collector
- \( Q_{cs} \) heat conduction through the cesium vapour

All heat fluxes to the element are taken positive. By each element the temperature and voltage are updated by solving the energy balance until convergence is reached. The calculation is essentially one dimensional, which is allowable due to the small interelectrode distance.

### 2.2.1 Heat fluxes

Both the radiative heat exchange between the surroundings and the hot shell and the heat exchange between emitter and collector are calculated using the assumption of two plan-parallel planes. This approximation is valid between the emitter and collector and might be valid between the surface and the surroundings, depending on the situation.

\[
Q_{\text{radiation}} = A \sigma \frac{1}{\varepsilon_1 + \frac{1}{\varepsilon_2} - 1} \left( T_1^4 - T_2^4 \right)
\]  

(2.7)

\( \varepsilon_1 \) represents the emission coefficient of the surroundings, or the inner side of the hot shell, \( \varepsilon_2 \) is the emission coefficient of the outer surface of the hot shell, or the collector and \( \sigma \) is the Stefan-Boltzmann constant.

As the TEC is supposed to be axis-symmetric, conduction can be described by the one-dimensional Fourier equation

\[
Q_{\text{cond}} = \lambda_{hs} \frac{A_d}{d} \left( T_{\text{element1}} - T_{\text{element2}} \right)
\]

(2.8)

The conduction coefficient is supposed to be constant. The length is taken as the mean of the length of two neighbouring segments. This induces a minor error when calculating with few elements for a circular emitter.

The ohmic losses are given by the well known equation

\[
Q_{\text{ohm}} = \rho_{hs} \frac{1}{A_d} I^2
\]

(2.9)
For the current $I$ the mean of the current at both boundaries of the element is taken. The specific resistance of the hot shell $\rho_{hs}$ is supposed to be constant.

The heat flux associated with the electron transport is given by (McVey 1992)

$$Q_e = A \left( \phi_e + \Delta V + 2kT_{eE} \right) - 2kT_{eE} - 2kT_{eE}$$

$$Q_e = A \left( \phi_e + \Delta V + 2kT_{eE} \right) - 2kT_{eE} - 2kT_{eE}$$

where $T_{eE}$ is the electron temperature in the emitter region.

At very low pressures the heat conduction of cesium is strongly dependent on the cesium pressure. An empirical method is used to account for this effect (Hatsopoulos 1973)

$$Q_{cs} = A \frac{A_{m}(T_E - T_C)}{d_{gap} + 1.51 \times 10^{-2} \frac{T_E + T_C}{p_{cs}}}$$

When the electron temperature is known (equation A.45-46), the plasma radiation, which can be of importance at low current densities, can be calculated according to the following relation, derived by Norcross (1967)

$$Q_{pla} = 1.26 \times 10^6 \frac{\rho_c d_{gap}}{T_e} \exp \left( \frac{2}{kT_e} \right) \left( 1 + 0.69 \exp \left( \frac{0.58}{kT_e} \right) \right) \left( \frac{\varepsilon_k}{d_{gap}} + \frac{1}{2} \right)$$

$\varepsilon_k$ is the spectral emissivity of the electrodes at the resonance wavelength of cesium and $p$ the cesium pressure. When the Sidelnicov model is used, the plasma radiation can not be calculated.

### 2.2.2 Model results

In figure 2.14 the $J-V$ characteristic of a practical converter heated by radiation from the surroundings is compared to the $J-V$ characteristic of a converter with a constant emitter temperature. The temperature of the surroundings is chosen such that the maximum emitter temperature at an output voltage of 0.5 V is 1723 K.

At high values of the voltage and low currents, the difference is small. When the voltage decreases and the current increases, the power output of the non uniform converter decreases. The increase in electron cooling causes the emitter temperature to drop. This implies that when running a thermionic converter, the output voltage must be controlled to keep the output voltage in hand. When the electrical circuit is broken, the emitter temperature may rise to an intolerable high value.

The influence of the number of elements used in the calculations can hardly be seen in the plots for the temperature and the voltage over the hot shell (figure 2.15 and 2.16). The temperature at the top of the hot shell is fairly constant.
**Figure 2.14** I-V characteristics for a constant temperature of the emitter and a constant temperature of the surroundings as a function of the output voltage. For a surroundings temperature of 2000 K, the maximum emitter temperature is 1723 K at an output voltage of 0.5 V.

**Figure 2.15** Temperature distribution over the hot shell as a function of the dimensionless coordinate $\xi$ along the surface.

**Figure 2.16** Voltage distribution over the hot shell as a function of the dimensionless coordinate $\xi$ along the surface.
That the varying emitter temperature and voltage have a major influence on the efficiency can be seen in figure 2.17, where the efficiency is plotted against the number of elements. The relative importance of the heat fluxes described in §2.2.1 is illustrated in figure 2.18.

The shapes of the temperature and potential profiles are mainly governed by the processes of heat conduction to the seal and ohmic heating in the hot shell. As the hot shell thickness and the geometric lay out of the hot shell directly influence both processes, it can be expected that optimising the geometry of the hot shell is one of the main tasks when designing a thermionic energy converter.

In figure 2.19 the results are given for a time dependent calculation. It is assumed that the temperature raise of the heat pipe is linear during the first 500 s and constant thereafter. From the plot of the heat fluxes against the time (figure 2.20) it can be deduced that thermionic emission becomes noticeable after 200 s. After this period the electron cooling becomes noticeable. The conclusion is that starting the thermionic converter will be limited by two effects: thermoshock of the hot shell material and the start-up of the heat pipe. Furthermore, these results indicate that there is the danger of a temperature overshoot in the period that electron cooling is absent. The TEC must be carefully started indeed.

**figure 2.17**  efficiency as a function of the number of elements

**figure 2.18**  heat fluxes as a function of the number of elements

**figure 2.19**  temperature profiles over the hot shell during start-up

- temperature of the surroundings = 2000 K
- radius = 0.02 m
- length = 0.02 m
- thickness SiC = 0.3 mm
- thickness Mo = 0.2 mm
- interelectrode distance = 0.19 mm
- collector temperature (after 500 s) = 984 K
- cesium reservoir temperature = 590 K
- output voltage = 0.42 V
2.3 Heat pipe

In the introduction it is argued that using a heat pipe is an elegant way to remove the heat from the collector. The stationary behaviour of a heat pipe is treated in §2.3.1. Although the heat pipe is almost isothermal, there exists a small temperature difference between the evaporator section and the condenser section. This temperature difference is not important for the design process, but it is useful to have a model describing the temperature drop over the heat pipe to distinguish anomalies from normal phenomena in the experiments (§2.3.2). In §2.2.2 it is already indicated that the start-up behaviour has a major influence on the start-up time for the thermionic process. As will become evident in the system analysis (chapter5), the annual profit of the thermionic converter depends heavily on the start-up time of the converter. In §2.3.3 and 2.3.4 simple models for the start-up time of the heat pipe are proposed.

2.3.1 Stationary heat pipe operation

A heat pipe is a tube at low pressure (figure 2.21) with a small amount of fluid inside. If one end of the tube is heated and the other end is cooled, the working fluid will evaporate at one side and condense on the other side, transporting heat at a nearly uniform temperature. Liquid transport from the condenser to the evaporator can be established by gravity or by the capillary pressure of a wick. The theory of stationary heat pipe operation as presented in appendix B.1 is adapted from the work of Chi (1976), Dunn (1978), v.d. Weide (1988), and Biemans (1992).

The maximum heat transfer of a heat pipe is limited by the capillary-, the sonic-, the entrainment-, the viscous-, and the boiling limitation.

In order to operate the heat pipe, the maximum capillary pressure must be larger than the pressure drop in the heat pipe caused by the liquid return, vapour flow, and gravitation (capillary limitation). If this condition is not met, the wick will dry out due to insufficient liquid transport to the evaporation section.

At low temperatures the viscous forces are dominant in the vapour flow down the heat pipe. The viscous limit is reached when the temperature at the condenser section has reached its lowest possible value.
At somewhat higher temperatures, when both viscous and inertia forces play a role, but the last are predominant, choking of the vapour flow may occur as the vapour flow at the end of the evaporator reaches the sonic velocity. At that moment, a further increase in the heat transfer is not possible and the sonic limitation is met.

Since liquid and vapour flow are counter-current, a shear force exists over the liquid vapour interface. At a certain velocity and shear force, liquid droplets will be entrained in the vapour flow. Less liquid will reach the evaporator, reducing the heat rate of the heat pipe. This effect is called the entrainment limit.

When the heat flux to the evaporation section is continuously increased, the liquid is increasingly superheated, until nucleate boiling will occur in the wick, the boiling limit. The formation of vapour bubbles in the wick is undesirable because it can cause hot spots and obstruct the liquid flow. In figure 2.22 the limits are given for the heat pipe of the prototype which is specified in table 2.1.

**table 2.1 heat pipe specification**

<table>
<thead>
<tr>
<th>Wick type</th>
<th>Screen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working fluid</td>
<td>Sodium</td>
</tr>
<tr>
<td>Shape of the evaporator</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Length of the evaporation section</td>
<td>20 mm</td>
</tr>
<tr>
<td>Length of the condenser section</td>
<td>40 mm</td>
</tr>
<tr>
<td>Length of the adiabatic section</td>
<td>20 mm</td>
</tr>
<tr>
<td>Outer heat pipe radius</td>
<td>18 mm</td>
</tr>
<tr>
<td>Inner heat pipe radius</td>
<td>17.8 mm</td>
</tr>
<tr>
<td>Vapour core radius</td>
<td>17.5 mm</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Inclination angle</td>
<td>0.5° (against gravity)</td>
</tr>
<tr>
<td>Screen mesh number</td>
<td>3.9 mm⁻¹</td>
</tr>
<tr>
<td>Screen wire diameter</td>
<td>0.085 mm</td>
</tr>
<tr>
<td>Thermal conductivity of the wick</td>
<td>20 Wm⁻¹K⁻¹</td>
</tr>
<tr>
<td>Thermal conductivity of the heat pipe</td>
<td>20 Wm⁻¹K⁻¹</td>
</tr>
</tbody>
</table>
The Design of a Combustion Heated Thermionic Energy Converter

The capillary limitation restricts the amount of heat which can be transported, but a heat duty of around 10 kW is quite sufficient for most practical situations. The stationary characteristics of the heat pipe can be easily accommodated to the requirements of the thermionic process. Hence it can be concluded that the decision about the shape and the dimensions can be made independently of the requirements for the heat pipe.

2.3.2 Temperature drop over the heat pipe

Though the temperature of the heat pipe is nearly uniform, there exists a temperature drop over the heat pipe. The total heat resistance of the heat pipe is composed of:

- \( R_{pe} \) the thermal resistance of the evaporator pipe wall
- \( R_{we} \) the thermal resistance of the liquid saturated wick at the evaporation section
- \( R_{lv} \) thermal resistance of the liquid vapour interface at the evaporation section
- \( R_{v} \) thermal resistance of the vapour column
- \( R_{vl} \) thermal resistance of the vapour liquid interface at the condenser section
- \( R_{wc} \) thermal resistance of the liquid saturated wick at the condenser section
- \( R_{pc} \) thermal resistance of the heat pipe wall at the condenser section

In appendix B.2 expressions for the resistances are derived. As an example the results of the thermal resistance for the heat pipe of table 2.1 is given in table 2.2.

Not enough experimental data are available to validate the model, but the total temperature differences predicted with this model are in the same order as those reported by Biemans (1992). Though there exists a temperature gradient over the heat pipe, the temperature difference is small and it is justified to treat the heat pipe as an isothermal element during stationary operation.
table 2.2  Results of a thermal resistance calculation for the heat pipe specified in table 2.1 for an evaporator temperature of 873 K and a heat duty of 1200 W

<table>
<thead>
<tr>
<th></th>
<th>Re</th>
<th>Ma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number of the vapour flow</td>
<td>534</td>
<td></td>
</tr>
<tr>
<td>Mach number of the vapour flow</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>evaporator:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature difference across the pipe wall $\Delta T_{pe}$</td>
<td>5.5 K</td>
<td></td>
</tr>
<tr>
<td>temperature difference across the wick $\Delta T_{we}$</td>
<td>4.0 K</td>
<td></td>
</tr>
<tr>
<td>temperature difference across the liquid-vapour interface $\Delta T_{lv}$</td>
<td>4.3 K</td>
<td></td>
</tr>
<tr>
<td>vapour column:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature difference $\Delta T_v$</td>
<td>0.01 K</td>
<td></td>
</tr>
<tr>
<td>condenser:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature difference across the liquid-vapour interface $\Delta T_{vl}$</td>
<td>2.2 K</td>
<td></td>
</tr>
<tr>
<td>temperature difference across the wick $\Delta T_{wc}$</td>
<td>1.9 K</td>
<td></td>
</tr>
<tr>
<td>temperature difference across the pipe wall $\Delta T_{pc}$</td>
<td>2.7 K</td>
<td></td>
</tr>
<tr>
<td>condenser temperature</td>
<td>853 K</td>
<td></td>
</tr>
</tbody>
</table>

2.3.3 One dimensional start-up model

A literature study (Biemans 1992) showed that study of the dynamical behaviour of a heat pipe is a relatively new field of research. Most authors agree however that the following sequence of events occurs during heat pipe start-up;

Initially the working substance is in solid state and the vapour density is extremely low. Free molecular flow conditions prevail throughout the vapour space. The heat flux to the evaporator raises the temperature and the frozen medium in this region starts to melt. Heat transport from the evaporation section to the condensation section proceeds slowly via conduction through the heat pipe wall, the wick, and the frozen medium. The heat transport through the vapour is negligible.

At the moment that the frozen medium is melted, evaporation starts at the evaporator, increasing the vapour density in this region. The molecular mean free path becomes small and continuous flow is established. The cool region of the collector is still in free molecular flow. Energy is transported from the high pressure evaporation section to the condensation section by evaporation and condensation processes. The temperature in the evaporation section is almost constant and the temperature gradient moves gradually to the condensation section until the medium is completely melted and continuous flow is established. At this time the liquid return to the evaporator should be sufficient for normal transient operation. Eventually the heat pipe reaches a steady state condition.

Up till now, models have only been developed for specific constructions. Nevertheless, these methods require much calculation time and are not well verified even for specific applications. This led to the conviction that extensive numerical modelling is not useful within the context of our research. To be able to compare different designs considering our main criterion, the system efficiency, another description was sought.
It is postulated that in order to provide a safe start of the heat pipe, the demands must be met that the time consumed by transporting an amount of working medium to the evaporator must be less than the time in which the same amount evaporates. Furthermore, a continuous flow in the heat pipe must be guaranteed, which means that there must be a liquid throughout the heat pipe. Using the results of this one dimensional approach, a simple two dimensional model was developed to describe the main processes during start-up (§2.3.4).

In first approximation, the unsteady heat conduction through the heat pipe is described by the well-known expressions for an one-dimensional semi-infinite solid, for which analytical solutions can be obtained (Biemans 1993). The simplifications made in the one dimensional model are in the first place that the radial heatflux over the evaporation section to the adiabatic section is neglected during the calculation of the time necessary to heat the evaporator to a certain temperature. Secondly, though the semi-infinite solid approximation may be appropriate for the adiabatic section, this is certainly not the case for the evaporation section.

The criterion for establishing liquid is that the condensation section must be at the melting temperature of the medium, for sodium 371 K. Convective heat transfer is non-existing during start-up, at least until evaporation starts. As radiation is neglected, we can use the conduction equation to calculate the time necessary to reach the melting temperature at the condensation section for a certain heatflux $q_{w}$ through the wall, $t_{2}$.

Evaporation is supposed to start when the working medium reaches its evaporation temperature after a time $t_{1}$. It is supposed that the heatflux to the evaporator area $q_{ev}$ is only used for evaporation when the vapour side of the screen wick reaches the evaporation temperature of the medium. The medium is evaporated in a time $t_{3} = M\frac{L}{q_{ev}A_{ev}}$ in which $M$ is the mass of the medium, $L$ the latent heat, $q_{ev}$ the heatflux to the evaporator, and $A_{ev}$ the surface of the evaporator.

When evaporation starts, sodium is withdrawn from the screen wick of the evaporation section. To find a value for the time being consumed by transporting liquid to the evaporator which guarantees a safe start-up, it is supposed that all liquid must be supplied from the collector. The capillary pressure is the driving force for the liquid flow and can be derived from the capillary limit (appendix B)

\[
M = \frac{\rho_{l} \sigma_{l} L}{\eta_{l}} \frac{\lambda A_{w}}{l_{eff}} \left( \frac{2}{r_{eff}} \frac{\rho_{l} g l_{eff} \cos \beta}{\sigma_{l}} \right)
\]  

(2.13)

where $M$ denotes the mass flow, $\rho_{l}$ the liquid density, $\sigma_{l}$ the surface tension, $\eta_{l}$ the dynamic viscosity, $\lambda$ the thermal conductivity, $l_{eff}$ the distance between the evaporator and the condenser section, $L$ the length of the heat pipe, $r_{eff}$ the effective radius, and $\beta$ the inclination with the horizontal. The characteristic time for liquid return is given by

\[
\tau_{4} = \frac{M}{M(T)}
\]  

(2.14)

if $M$ is the mass of sodium being evaporated.

In figure 2.23 the time scales for conduction through wall and wick, $\tau_{1}$, at the evaporation section, through the wall, $\tau_{2}$, and for evaporation, $\tau_{4}$, are plotted against the incoming heatflux for the heat pipe specified in table 2.3, which was used for the experiments. The time scale for liquid return $\tau_{3}$ is not depicted as it is below 2 s. For safe start-up the condition should be fulfilled that the time constant for establishing liquid state plus liquid return
should be less than the time constants for reaching the evaporation temperature plus evaporation \((\tau_2 + \tau_4 < \tau_1 + \tau_3)\). At a heat flux density of \(1.80 \cdot 10^5 \text{ Wm}^{-2}\) the critical condition is met. In order to prevent burnout of the evaporator, the heat flux during start-up must be below this value.

Experimental validation (chapter 4) showed that the model is valid for gravity assisted operation only. But even in that case the start-up of the heat pipe is faster than predicted by the model due to evaporation and condensation processes. The one dimensional model provides safe guidelines for starting the heat pipe, but for a more accurate prediction it is necessary to extend the model into two dimensions and to include phase transformations.

![Graph showing the time scales during heat pipe start-up for the heat pipe specified in table 2.3](figure 2.23)

**Table 2.3 Heat pipe specification**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wick type</td>
<td>Screen</td>
</tr>
<tr>
<td>Working fluid</td>
<td>Sodium</td>
</tr>
<tr>
<td>Shape of the evaporator</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>Length of the evaporation section</td>
<td>10 mm</td>
</tr>
<tr>
<td>Length of the condenser section</td>
<td>9 mm</td>
</tr>
<tr>
<td>Length of the adiabatic section</td>
<td>9 mm</td>
</tr>
<tr>
<td>Outer heat pipe radius</td>
<td>28.3 mm</td>
</tr>
<tr>
<td>Inner heat pipe radius</td>
<td>26.3 mm</td>
</tr>
<tr>
<td>Vapour core radius</td>
<td>26 mm</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>5 mm</td>
</tr>
<tr>
<td>Inclination angle</td>
<td>0.5 (\pi) (against gravity)</td>
</tr>
<tr>
<td>Screen mesh number</td>
<td>3900 mm(^{-1})</td>
</tr>
<tr>
<td>Screen wire diameter</td>
<td>0.85 mm</td>
</tr>
<tr>
<td>Thermal conductivity of the wick</td>
<td>20 Wm(^{-1})K(^{-1})</td>
</tr>
<tr>
<td>Thermal conductivity of the heat pipe</td>
<td>20 Wm(^{-1})K(^{-1})</td>
</tr>
</tbody>
</table>
2.3.4 Two dimensional start-up model

For the numerical model the heat pipe is stylised to a cylinder which is divided into elements. The conduction through the wall is calculated using a central finite difference scheme (de Vries 1994). The outer boundary conditions are radiative heating on the evaporator side of the heat pipe and convective cooling at the condenser section. The inner boundaries are imposed by condensation and evaporation processes, radiation is neglected. In Ivanovskii (1982) it is argued that the maximum evaporation rate of a fluid into vacuum can be described by

\[ Q_{\text{evap max}} = f_L \frac{1000 M}{2\pi R} \frac{p_f}{\sqrt{T_f}} \]  

(2.15)

where \( L \) denotes the latent heat, \( M \) the molecular weight, \( R \) the universal gas constant, \( p_f \) the fluid pressure, and \( T_f \) the fluid temperature. The factor \( f \) is empirically found. Equation (2.20) describes the situation well in situations close to equilibrium. According to Collier (1972) condensation can be easily included in equation (2.15)

\[ Q = f L \sqrt{\frac{1000 M}{2\pi R}} \left( \frac{p_v}{\sqrt{T_v}} \pm \frac{p_f}{\sqrt{T_f}} \right) \]  

(2.16)

The vapour pressure \( p_g \) is calculated from the difference between evaporation and condensation in each time step according to the ideal gas law.

In figure 2.24 a typical result is given. The top of the heat pipe raises sharply in temperature during the first seconds but the condenser side stays at a low temperature level until the heat transported by evaporation and condensation processes becomes significant. For this reason the heat pipe starts faster than predicted by the one dimensional model. These results are consistent with the experimental results (chapter 4).

![Figure 2.24](image-url) Predicted temperatures of the evaporator and the condenser during start-up with a heat flux of 120 kW m\(^{-2}\) for the heat pipe specified in Table 2.3.
2.4 Closure

Two models from literature describing thermionic emission are presented in this chapter. Both models are adequate in the region of interest for this study. The first model, developed by Rasor and McVey, is well documented and tested. The second model, derived by Mironov and Sidelnicov, has the advantage that it is simple and allows for straightforward calculation of the $J-V$ characteristic. The disadvantage is that information about the model is scarce.

Both thermionic emission models can be used in combination with a computer program describing the energy fluxes through the converter. Actually two computer models were made. For use in the system analysis a simple model was made (appendix C.1) which describes the sleeve only as a heat and electrical resistance and assumes that the emitter is at constant temperature. Thermionic emission is calculated according to the Mironov-Sidelnicov model. The results of the system analysis (chapter 5) were checked using a more detailed model which uses the Rasor-McVey method and allows for a varying temperature and voltage field over the hot shell. This model is also used in the design stage (chapter 6).

A simple time dependent calculation of converter start-up, supposing a linearly rising collector temperature, indicated that the start-up behaviour of the heat pipe has a major influence on converter start-up. Two models were developed to describe heat pipe start-up. The first method describes the time scales of the important processes during start-up and provides safe guide lines but overestimates the start-up time. Hence for use in an economical analysis a more detailed two dimensional model was developed which provides a more accurate prediction of the start-up time.

Using the models presented in this chapter the optimum dimensions of the hot shell and the heat pipe can be assessed when the limitations set to the optimisation problem by the mechanical behaviour of the construction are known.
3 Mechanical models

The TEC operates under atmospheric pressure at cycling temperatures up to 1700 K, which implies that stresses are induced by the pressure difference between the interelectrode space and the surroundings, the temperature gradients, and the mismatch in thermal expansion at the joints between different materials. It is necessary to model the critical components of the thermionic energy converter in order to assess the boundaries to the optimisation problem (Chapter 6) set by the structural integrity.

First some methods are introduced to calculate the elastic stresses due to the atmospheric pressure on the hot shell. Subsequently the stresses in the hot shell and the ceramic seal due to the difference in thermal expansion are investigated.

Structures operating at high temperatures are subjected to creep. In §3.2 expressions are proposed to describe the creep rate of the construction materials used. To get an impression of the creep rate of the structures, the creep rate equations are applied to a thin walled sphere.

Thin walled structures may buckle. Buckling of a shell starts after a critical deflection, which can be caused by elastic deformation, or by creep. Methods to predict both phenomena are investigated in §3.3.

3.1 Stresses

A large safety margin must be invoked while designing the hot shell because there are uncertainties both in the properties of the materials used and in the exact circumstances during operation. From a simple calculation of a thin walled shell (§3.1.1) it can be seen that we need a finite element method (FEM) to describe the elastic and plastic stresses in the hot shell. The stresses due to the difference in thermal expansion coefficient are treated in §3.1.2. The stresses in the ceramic seal are described in §3.1.3.

3.1.1 Elastic stresses in the hot shell

The first approximation for the stresses in the hot shell is made by regarding the case of a spherical shell with a clamped edge which is submitted to a uniform pressure on the outer surface (figure 3.1 and 3.2).

According to Timoshenko (1975), the bending moments on a thin walled spherical shell can be described by

\[ M_\phi = -\sqrt{M_\alpha} \exp[(\alpha - \phi)\lambda] \sin[(\alpha - \phi)\lambda + \frac{1}{4}\pi] \]  

(3.1)

where the leanness \( \lambda \) is defined as

\[ \lambda = \left( \frac{3(1-\nu^2)R^2}{d^4} \right)^{1/4} \]  

(3.2)
\[ \nu \text{ is the Poisson’s ratio. The bending moment at the edges is given by} \]
\[
M_a = -\frac{p R^2 (1-\nu)}{4 \lambda^2} \quad (3.3)
\]

\( p \) is the pressure applied on the outer surface. Using this moment, we can calculate the membrane force due to bending
\[
N_\theta = -\lambda \sqrt{2} C \cdot e^{-\lambda \psi} \sin(\lambda \psi - \frac{\pi}{4}) \quad (3.4)
\]

the integration constant \( C \) being
\[
C = \frac{M_a 2\lambda}{R} \quad (3.5)
\]

The problem of calculating the stresses in a hot shell with a similar geometry was treated numerically by Bushnell (1985). The maximum von Mises equivalent stress occurred in the transition from the spherical to the cylindrical part. As this part is not described by the membrane theory, a Finite Element analysis must be used. In this study the finite element package ABAQUS was used.

The calculated von Mises stress profile over a molybdenum hot shell without coating is depicted in figure 3.3. The deviation of the stress \( \sigma_\theta \) in the top of the hot shell to the values predicted by the membrane analysis is small, so it may be assumed that these values are not far from reality. In order to compare the results of expression (3.4) with the finite element calculations, the line force is divided by the thickness of the hot shell. The material parameters used, can be found in appendix D.

The calculated von Mises stress in the hot shell is low, but that is also the case with the yield strength of molybdenum at 1700 K (in the order of 10 MPa). This exercise indicates that it is probably the SiC coating which takes the mechanical load and not the molybdenum.

### 3.1.2 Elasto–plastic stresses in the hot shell

The hot shell consists of a molybdenum layer, a thin TiN diffusion barrier, and a SiC layer. In the calculations the influence of the TiN diffusion barrier is neglected. As molybdenum and SiC have a different thermal expansion, thermal stresses will develop under working conditions. In figure 3.4 the results are shown of a FEM calculation for a hot shell with a length of 25 mm and a radius of 25 mm which is raised in temperature from ambient conditions to working conditions, while an external pressure of 10^5 Pa is applied. The temperature profile, calculated with the numerical model described in §2.2, is given in
The SiC layer is supposed to behave elastic and the Mo part is modelled as an elastic-plastic material. The shell is supposed to be stress free under ambient conditions. The largest stresses occur in the cylindrical part where the temperature gradient is high.

For SiC a von Mises stress in the order of 100 MPa is dangerously high when under tension. Fortunately the SiC layer is deposited onto the molybdenum shell at 1450 K and consequently the SiC layer is under pressure at room temperature.

![Figure 3.3](image1.png)

*Figure 3.3* Stresses in a molybdenum hot shell under atmospheric pressure at a temperature of 1723 K calculated according to the membrane theory and with the finite element method ABAQUS

![Figure 3.4](image2.png)

*Figure 3.4* Stresses in a molybdenum-SiC hot shell under atmospheric pressure under working conditions
3.1.3 Elasto–plastic stresses in the ceramic seal

The most severe stresses in the joint between the aluminium oxide isolation ring and the stainless steel flanges occur while cooling down from the brazing temperature of 1450 K and do not occur under working conditions. In Vogels (1994) an elastic–plastic ABAQUS calculation is described for the brazing of the prototype TEC. In this construction an intermediate ring of VACON-70 is used. The VACON ring is connected to the aluminium oxide by a thin layer of brazing material. According to the results of this calculation (figure 3.6), the construction is critical: the Al₂O₃ is subjected to a von Mises stress of 170 MPa while under tension which is high compared to the tensile strength of 250 MPa. This result is in agreement with practical experiments: the ceramic seal often fails. In a new design the stresses imposed on the ceramic part must be reduced. The easiest way to accomplish this is by reducing the diameter of the ceramic seal.

3.2 Creep

The strength behaviour of materials depends on strain, strain rate, and temperature. Generally materials show a rate dependent plasticity. At low strain rates ($k < 10^{-4}$) this phenomenon is called creep. Above temperatures of 0.3 times the melting point $T_m$ for pure metals and 0.4×$T_m$ for ceramics creep can be caused by the following deformation mechanisms:

- power law creep by dislocation glide or glide and climb
- diffusional flow (limited by lattice diffusion, Nabarro-Herring creep, or grain boundary diffusion, Coble creep)

In §3.2.1 expressions are derived for the creep rates resulting from these mechanisms. The formulations for the creep rates are applied to molybdenum in §3.2.2 and to SiC in §3.2.3. In §3.3 order estimations are given for the creep of a cylinder and a sphere.
Figure 3.6 von Mises stress distribution in an Al₂O₃-VACON-RVS-316 brazing joint after cooling down from the brazing temperature of 1450 K.
3.2.1 Expressions for the creep rates

At high temperatures dislocations can glide as well as climb. The climb is lattice-diffusion controlled. Most expressions describing power law creep are of the following form (Frost 1983):

$$\dot{\varepsilon} = A_{1} \frac{D_{l} \mu b}{kT} \left( \frac{\sigma}{\mu} \right)^{n}$$  \hspace{1cm} (3.6)

where $\dot{\varepsilon}$ denotes the creep rate, $A_{1}$ is the Dorn constant, $D_{l}$ the lattice diffusion coefficient, $\mu$ the shear modulus, $b$ the Burgers vector, and $\sigma$ the applied stress. According to theory the value of the stress exponent $n$ should be 3, but most materials do not obey (Frost 1983). At present no satisfactory explanation for this behaviour is available. The power law is valid up to stresses of about $\sigma/\mu = 10^{-3}$. At higher stresses a transition occurs from climb controlled to glide controlled flow.

The other creep mechanism which may be encountered is diffusional flow. A stress field with a deviatoric component changes the chemical potential $f$ on some grain surfaces more than on others, introducing a potential gradient $\Delta \phi$. At high temperatures this gradient induces a diffusion flow of matter through or around the surfaces of the grains. This type of creep was first discussed by Nabarro and Herring.

A crystal in thermal equilibrium at a temperature $T$ contains a concentration of vacancies, $C_{0} \exp(-Qf/RT)$. Each time a vacancy is produced at the boundaries of the grain, experiencing a tensile stress $\sigma$, the work done by the stress is $W_{s}$ where $W$ denotes the volume of a vacancy. The energy needed to create a vacancy at a right angle to the tensile axis is consequently reduced from $Qf$ to $Qf - W_{s}$. As a result the local vacancy concentration $C_{+}$ is larger than the equilibrium value $C_{0}$ as $C_{+} \exp(-W_{s}/RT)$. Similarly, the vacancy concentration near a boundary under a compressive stress, is lower than the equilibrium value $C_{-} \exp(-W_{s}/RT)$. The vacancy concentration gradient $\partial C/\partial x$ can be written as

$$\left( \frac{C_{+} \pm C_{-}}{d'} \right) = \frac{2C_{0}}{d} \sinh\left( \frac{Qf}{RT} \right) = \frac{2C_{0}}{d} \frac{Qf}{RT}$$ \hspace{1cm} (3.7)

where $d'$ is the average path length which is related to the average grain diameter $d$. Along this path, the net flux $J_{v}$ of vacancies will be given by Fick’s law $J_{v} = -D_{v} \partial C/\partial x$ where $D_{v}$ is the vacancy diffusion coefficient. For diffusion through the crystal lattice $D_{v}$ is related to $D_{l}$ according to $D_{l} = D_{v} C_{0} \Omega$. With this the steady state creep rate can be written as

$$\dot{\varepsilon} = \frac{J_{v}}{d} = K_{1} \frac{D_{l}}{d^{2}} \frac{\Omega \sigma}{RT}$$ \hspace{1cm} (3.8)

the constant $K_{1}$ depends on the precise grain geometry, but is usually in the order of 10. In the original theory developed by Nabarro and Herring, diffusion was considered to occur only through the crystal lattice. However, material can also be transferred along the grain boundaries. Under these circumstances the area where diffusion occurs is reduced by $\delta/d$ where $\delta$ is the effective thickness of the boundary.

Most models result in a rate equation for the combined effects of both types of diffusion controlled flow of the form

$$\dot{\varepsilon} = \frac{K_{2} \sigma \Omega}{kTd^{2}} D_{\text{eff}}$$ \hspace{1cm} (3.9)
where \( d \) is the grain size and \( D_{\text{eff}} = D_v(1 + \frac{\pi \delta}{d} \frac{D_b}{D_v}) \) the effective diffusion coefficient. The constant \( K_2 \) usually has a value around 40 (Frost 1983).

**3.2.2 Creep of Molybdenum**

Figure 3.7 gives a stress-temperature deformation map for pure molybdenum with a grain size of 100 \( \mu \text{m} \) (Frost 1983). The data used can be found in appendix D.5.

![Stress-temperature map for molybdenum](image)

In figure 3.8 the result of equation (3.6) is compared with data from literature. Both TZM-Mo and Mo-0.5 HiC are dispersion strengthened molybdenum types originally developed for space nuclear reactor applications. From figure 3.8 it can be seen that the performance of these dispersion strengthened metals is only several orders better than that of pure molybdenum. The molybdenum tested by Luo (1993) is fabricated by Westinghouse Electric Corp. and is the usual commercial brand. The materials could possibly be optimised as to their creep behaviour by several orders of magnitude, but even in that case the creep rate is very high.

![Comparison of calculated creep rate with literature data](image)
The transition between diffusional flow and power law creep is dependent on the stress and the temperature. The region in which the transition occurs (figure 3.9) can be found by equalling expressions (3.6) and (3.7).

The calculated values for the elastic stresses in the hot shell are in the order of 10 MPa, which means that for practical cases the power law can be used.

![Figure 3.9: Transition between diffusional- and power law creep as a function of the temperature and the stress for pure molybdenum with a grain size of 100 μm](image)

### 3.2.3 Creep of Silicon Carbide

For SiC, not enough data were found to calculate the creep rate theoretically. Consequently, experimental relations were used.

Yarahmadi (1985) performed creep tests on pressureless sintered SiC, Si infiltrated SiC (Si-SiC) and reaction bonded SiC. Only in the case of Si-SiC a stationary creep rate was obtained. The stress ranged from 45 to 190 MPa and the temperature from 1500-1900 K. The activation energy was found to be 126 kJmol\(^{-1}\) and the stress exponent varied from 1 to 3.75. For stresses below 100 MPa the stress exponent stayed close to 1. Creep of this material is mainly caused by creep of the Si particles.

In Lane (1988) the creep behaviour of polycrystalline SiC is described. The stress exponent ranged from 1.44 to 1.71 as a function of the temperature (1670-2020 K) and was found to be stress-independent. The curves of \(\ln(\varepsilon)\) against \(1/T\) showed a change in slope at 1920 K. The activation energy amounts below this knee to 400 kJmol\(^{-1}\) and above to 850 kJmol\(^{-1}\).

Nixon (1988) investigated creep of sintered SiC. He too found a jump in the activation energy at 1920 K (from 630 to 910 kJmol\(^{-1}\)). Between 1770 K and 2020 K a stress exponent is reported of 2.

Davis (1989) published, amongst others, creep experiments with SiC produced by the Chemical Vapour Deposition (CVD) technique as is the outer layer of the prototype hot shell. The temperatures at which the tests were performed, ranged from 1800 K to 2020 K. Below 1923 K the stress exponent was found to be 2.3. The mean value of the activation energy was 175 kJmol\(^{-1}\).

In figure 3.10 the literature values are extrapolated to 1723 K. CVD-SiC clearly performs better than the other SiC types. The creep rate of SiC is about an order of 10\(^{20}\) less than the creep rate of molybdenum, so it is a good choice for the construction material of the hot shell.
Creep of SiC is an order 10 to 20 slower than the creep of molybdenum and hence the SiC part of the hot shell will take most of the mechanical load. For estimating the creep rate of the hot shell it is sufficient to consider the SiC part of the hot shell only.

**Figure 3.10** calculated creep rate of several SiC types at 1723 K as a function of the stress

### 3.2.4 Creep of a sphere

To get an impression of the creep rate of an axi-symmetrical geometry creep of a spherical membrane shell is analysed. For this shape an analytical solution can be obtained (appendix E.1). The following assumptions are consistent with the thin shell approximation (figure 3.11):

1. the stress $\sigma_z$ is negligible
2. a line originally normal to the shell midsurface will remain normal during deformation

In membrane shells it is assumed that the strains and consequently the stresses are constant through the thickness.

**Figure 3.11** definitions for a membrane shell

Assuming that the power law holds, we can write for the displacement at the top of the sphere (appendix E.1)

$$w = \frac{1}{2} B \left( \frac{pR}{d} \right)^n$$

(3.10)
\( R \) is the radius of the sphere, \( p \) the externally applied pressure, \( d \) the thickness, and \( n \) the stress exponent. The constant \( B \) comprises the constant part of the power law creep equation (3.6)

\[
B = A_1 \frac{D_1 \mu b}{kT \mu^n}
\]  
(3.11)

As can be seen in figure 3.12, creep of SiC calculated for a thin sphere has a low order of magnitude.

\[ 
\text{figure 3.12 creep rate of a CVD-SiC sphere at a temperature of 1723 K as a function of the radius and the thickness} 
\]

### 3.3 Buckling

Buckling or creep buckling may be one of the limitations in optimising the hot shell geometry. We therefore have to develop a simple design tool for the creep buckling criterion for use during the optimisation process.

When structures are subjected to a net compressive stress, they are susceptible to instability at a critical value of the compressive stress. If the material behaviour is elastic or elasto-plastic, the phenomenon is instantaneous. However if the structure operates at temperatures where creep is present, buckling may not be instantaneous, rather it may develop at some critical time for which the stress becomes critical. Creep buckling is a complicated non-linear problem. Given enough time, any net compressive stress could lead to buckling of the hot shell if creep is present.

As with all creep problems, exact analytical solutions are hard to come by. Several approximate techniques have been developed, but they can only be used when stress redistribution during creep is not considered. When we want to consider stress redistribution, numerical procedures must be followed. Nevertheless, it is useful to perform the preliminary analysis with an approximation technique, if possible associated with the elastic situation.

In this paragraph first elastic buckling is considered, where the prototype hot shell serves as an example (§3.3.1). Subsequently three approximations for elastic buckling are discussed (§3.3.2) and one of them is used to develop a criterion for creep-buckling (§3.3.3).
3.3.1 Elastic buckling

Using the results of an extensive test program Klöppel (1953) stated an empirical expression for buckling of spherical segments (figure 3.13):

\[ p_{cr} = k(\alpha)k\left(\frac{R}{d}\right)0.3E\frac{d^2}{R^2} \]  

(3.12)

with

\[ k(\alpha) = 1 - 0.175\frac{\alpha - \pi/9}{\pi/9} \]  

(3.13)

and

\[ k\left(\frac{R}{d}\right) = 1 - 0.07\frac{R}{400d} \]  

(3.14)

Between 20° ≤ \(\alpha\) ≤ 60° and 400 ≤ R/d ≤ 2000 the calculated results were proven correct within 20%. In Klöppel (1956) buckling of spherical segments without a clamped edge are investigated. The shells had the same shape as the hot shell. When the wall thickness is constant, expression (3.12) was proven to be valid in this case also.

With expression (3.12) it is possible to calculate the minimum wall thickness with regard to buckling, given the radius of the shell (figure 3.14). We have to introduce a safety factor however due to the following uncertainties:

- deviations in the quality of the material
- deviations in the loading
- deviations in the geometry

Ultimately the safety factor has to be found by testing. Furthermore, the ratio R/d for the hot shell is about 150, a region that was not investigated by Klöppel. For these reasons a small test series was performed. In literature a safety factor of 2 is being used in combination with the expressions presented here and the manufacturing process of deepdrawing. The safety factor of 2 is supported by a test series performed at the EUT (figure 3.15 and 3.16). The hot shell was placed on a rubber seal on a flange and pressured on the outside by water. The deformation of the hot shell was monitored through a wide hole in the flange. The pressures at which the hot shell failed were not below half the calculated critical pressure of 3 MPa.
3.3.2 Creep buckling

Several approximate techniques for the prediction of creep buckling have been developed, but they can only be used when stress redistribution during creep is ignored. Several methods exist to calculate analytically creep buckling of which the Reference Stress Method (RSM), the critical effective strain method, and the constant effective strain method were considered.

The aim of the RSM is to relate the deformation of structures to a simple creep test. The main assumption in this method is that the deformations of a creeping shell and the deformations in a reference creep test are similar. The reference stress $\sigma_R$ is given by the ratio between the actual load $P$ and the elastic collapse load $P_L$ times the yield stress $\sigma_y = \sigma_R P / P_L$. When the collapse load of the structure and the uni-axial creep rate as a function of the stress and the time are known, the deflection of the structure can be calculated using small time increments (appendix E.2). When the displacement rate begins to grow without bounds, the structure will buckle.

In the critical effective strain method, it is assumed that the critical strains are equal for elastic- and creep buckling. When the load applied to the structure is less than the critical
load $P_L$, creep strains will develop. If the critical strain at the onset of buckling can be estimated, the critical buckling time can also be estimated.

According to Chern (1978) the constant critical strain method is the easiest to use and provides the best accuracy. In the constant critical strain method, it is assumed that the critical buckling strains corresponding to the instantaneous buckling and the creep buckling are the same. Therefore the creep flow law and the elastic strain law can be equalled. Using a creep rate equation, the critical time is calculated (appendix E.3).

In Chern (1978) the above mentioned methods are compared for several loading situations. In the only available reference case where the RSM method was used, it provided the best (conservative) results. Preference however is given to the described constant critical strain method as it provided slightly better results than other critical strain methods and is simple to use. Furthermore, it can be used to develop a buckling criterion which is based on elastic calculations.

As an example, we treat the case of the prototype SiC shell at a temperature of 1723 K (table 3.1). The critical elastic load can be calculated according to equation (3.12). For simplicity we suppose that the effective stress in our case is given by

$$\sigma_e = \frac{p R}{2 d}$$

Using the values of table 3.1 we can subsequently calculate the critical strain.

The critical pressure in this case amounts to 1.4 MPa. A safety factor of 3 is used to allow for imperfections during the manufacturing process, reducing the critical pressure to 0.5 MPa. The critical effective stress is subsequently found to be 30 MPa, while the effective stress amounts to 5.8 MPa. With these values the critical strain can be calculated according to equation E.23) $\varepsilon_{cR} = 3.3 \times 10^{-4}$. Using the expression for power law creep (equation 3.6), the time in which this strain is reached can be found. It amounts to approximately 5 years, which is not tolerable.

### Table 3.1 Dimensions and Material Constants of the SiC Shell

<table>
<thead>
<tr>
<th>Material Constants</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>3.1 x 10^{11} Pa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Temperature</td>
<td>1723 K</td>
</tr>
<tr>
<td>Dimensions</td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>70 mm</td>
</tr>
<tr>
<td>Angle ($\alpha$)</td>
<td>0.5 rad</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.3 mm</td>
</tr>
</tbody>
</table>

#### 3.4 Closure

The most likely failure modes for the hot shell are excess thermal stresses, creep and buckling. Simple tools are derived to assess the restrictions set by these processes before optimising the geometry of the hot shell with respect to the system efficiency (chapter 6). The thermal stresses are calculated using the finite element ABAQUS. The creep rate equations as presented in this chapter are a first order approximation only, because creep is influenced by small changes in the material used. Fortunately SiC is extremely creep resistant.
4 Experimental validation

In the preceding two chapters, models were developed for the thermal and mechanical behaviour of the TEC. The major parts of these models were adapted from literature and were checked with their originals. What still needs verification is whether the combination of models gives trustworthy results, especially in the case of the heat fluxes through the TEC and the start-up time of the heat pipe. In §4.1 the thermionic emission experiments for the unignited mode and the ignited mode are described. The models for the heat pipe are tested in §4.2.

4.1 Thermionic emission experiments

The tests that were performed, consisted of a validation of the close spaced and ignited mode model (§4.1.2), but first the test rig itself is described. Although it is the aim of the project to design a combustion heated thermionic converter, it was decided to test the TEC electrically heated in a vacuum atmosphere (figure 4.1). The main reason for doing this was that in earlier experiments (de Vries 1990, Rademacher 1991) the hot shell often failed. The measurements were frustrated by the fact that the hot shell could not be replaced quickly because the coating process of the hot shell is costly and time consuming for single products. By testing the TEC in a vacuum environment it is not necessary to coat the hot shell and the molybdenum shell can be made in-house if the need arises. Furthermore, it is easier to install measuring equipment in a vacuum atmosphere at high temperatures.

figure 4.1 the test rig
4.1.1 Test rig and prototype

The vacuum vessel which was used, is depicted in figure 4.2. It was developed by the Central Engineering Group of the EUT. The construction drawings can be found in appendix F. The vessel features double stainless steel walls which are water cooled. The TEC is mounted at the top side of the vessel. The heating element is made of graphite with a SiC coating to prevent degassing under operating conditions. Provisions are made to install thermocouples. The top side of the heating element follows the curvature of the hot shell to achieve a controlled radiative heat transfer. The heating element itself is in turn heated by radiation from a molybdenum wire, isolated by ceramic aluminium oxide rings. The heating wire is connected to copper electrical terminals which are coated with nickel and cooled by water. The heating element is surrounded by five molybdenum radiation shields to reduce heat losses to the walls. Furthermore, the vessel is equipped with a sapphire window to allow optical measurements. The cooling elements are connected in series to allow for an easy measurement of the heat removed by the cooling system. To ascertain that there will be no electrical short through the cooling system, a closed demineralised water loop is used which is cooled through a heat exchanger.

Figure 4.2 vacuum vessel and heating element

The vacuum installation (figure 4.3) comprises an Edwards backing pump which is able to maintain a vacuum of 2 Pa. For the high vacuum an Edwards oil diffusion pump featuring a water cooled baffle is used. Although the vacuum system is of a simple and straightforward design, a vacuum below $10^{-3}$ Pa can be reached. The vacuum is measured after the diffusion pump and before the vacuum vessel by Balzers TPR 010 Pirani gauges, while a Penning gauge is installed just before the vacuum vessel for the high vacuum measurements.
During the first experiments, the power supply consisted of a transformer and a VARIAC which was able to deliver 100 A at 15 V. This was insufficient, however, to reach a temperature of 1700 K. Furthermore, the electrical field produced by the AC current severely interfered with the thermocouple measurements. Therefore a custom made power supply was built at the EUT which is capable to deliver 200 A at 15 V DC.

The measuring and controlling equipment consists of a Balzers TPG 300 total pressure controller, a West 3810 PID controller, a Fluke digital thermometer, a Brooks rotameter, a Strawberry tree data acquisition system, a digital Fluke multimeter, a Tektronix curve tracer and a computer. The position where the measurements were taken are given in figure 4.4.

The set up which can be used during experiments with forced heat pipe cooling is described by de Vries (1990). Provisions were made to measure the output current and voltage by the curve tracer or by simply using resistors and a multimeter.

The construction of the TEC prototype is depicted in figure 4.5. A large part of the engineering again stems from the Central Engineering Group of the EUT. A detailed description of the production process can be found in appendix F. The construction is essentially the same as described by Veltkamp (1989b), apart from the detachable hot shell and the brazing of the ceramic-metal insulation. The deepdrawing of the hot shell was performed at Philips PMF b.v.

The test series were mainly in the unignited mode because it was not possible to raise the temperature of the cesium reservoir above 473 K due to an insufficient isolation between the end of the heat pipe and the support. Furthermore, the hot shell temperature could not be raised above 1400K due to insufficient cooling of the seals. Although the KALREZ rubber O seals are able to withstand a temperature of 600 K, a vacuum leak occurred at a temperature of 450 K after some time due to creep of the material.

Nevertheless, the measurements were useful to validate the model of the unignited mode. Thermionic emission in the unignited mode is mainly influenced by the interelectrode distance and the work function of the materials. Checking the unignited mode model, the assumptions for the work functions can be checked in combination with the accuracy of the gap adjustment device.
In the unignited mode, the output current is low and a standard Tektronix curve tracer was used to measure the electrical output of the converter. The curve tracer applies an alternating voltage with a frequency of 50 Hz to the converter and measures the resulting current. The voltage and the current were made visible on a scope screen and recorded by a video camera. After the experiments, the images of interest were selected, digitised and converted to numerical information. To check if the fast voltage sweep affects the thermionic emission, some measurements were made using a constant voltage. No difference was noticed.
4.1.2 Experiments

In figure 4.6 the experimentally determined $I-V$ characteristics of the converter in the unignited mode are shown for an emitter temperature of 1352 K and a collector temperature of 694 K for various interelectrode distances. The interelectrode distance was varied by adjusting the nonius at the threaded fill tube. The distance adjustment was calibrated by short circuiting the converter before the measurement.

Any attempt to describe the $I-V$ characteristic in this situation with the model described in chapter 2 will fail beyond 0.5 V as ion emission is not included and because the retarding region is not included (figure 4.7). The ion saturation current for an inter electrode distance of 0.25 mm is 1.4 A (equation A.8). The collector electron emission saturation current (equation A.15) amounts to 0.008 A and can be neglected. In figure 4.8 the results are given for another situation where the emitter temperature is somewhat higher and the interelectrode distance is larger. In both cases the ion current becomes noticeable beyond 0.5 V. In the space charge range the correlation between the model and the experiments is good. At lower voltages, where the current is approaching the emitter saturation current, the predicted current is lower than in the experiment. This is due to an underestimation of the cesium temperature, which was measured on the outside of the cesium reservoir tube at the condenser side of the heat pipe. Experiments performed with a planar research diode (Hatsopoulos 1979) show that the effect of the cesium reservoir temperature in this range is very strong (figure 4.9)
The Design of a Combustion Heated Thermionic Energy Converter

**Figure 4.6** I-V characteristics of the thermionic converter in the unignited mode

**Figure 4.7** I-V characteristic of the converter in the unignited mode for an inter-electrode distance of 0.025 mm and an emitter temperature of 1352 K

**Figure 4.8** I-V characteristic of the converter in the unignited mode for an inter-electrode distance of 0.35 mm and an emitter temperature of 1358 K

**Figure 4.9** Dependence of an experimental I-V characteristic on the cesium reservoir temperature for a planar research diode (Hatsopoulos 1979)
4.1.3 Thermionic emission in the ignited mode

Although the thermionic converter could not get in the ignited mode under its own power, it was possible to start the plasma by applying a higher alternating current with the curve tracer (figure 4.10). The results were checked using a constant current source. At a voltage above 0.5 V electron emission from both the emitter and the collector is small and ion emission is predominant, leading to a negative current. On reducing the voltage, the retarding range is encountered (§2.1.2), where the electrons experience a negative force caused by the positive voltage all along the way from the emitter to the collector. The critical point where the transition from the retarding range to the space charge limited range occurs lies around 0.1 V. In the space charge limited range the interelectrode motive has a maximum in the interelectrode range, which means that the space charge is the main limiting factor. When the voltage is reduced to -2.5 V the electron current is high enough to create enough collisions to ignite the plasma and the current shoots out of the range which can be handled by the curve tracer. When the voltage is raising again, the current returns within the range of the curve tracer at -0.9 V. The thermionic process is in the obstructed mode which means that the plasma can exist by erecting space charge at the emitter (§2.1.3). When the voltage further increases, the plasma extinguishes at the same current as it was ignited.

![I-V characteristic of the converter at an emitter temperature of 1425 K](image)

Unfortunately, the large negative voltage range is not covered in the Rasor-McVey model and hence the results can only be compared with the Mironov-Sidelnicov model, which predicts the output current in the obstructed mode remarkably well.

It was not possible to test the models in the region where power is delivered in the ignited mode as the test rig broke down. The temperature of the cesium reservoir tube was too low at the point where it exits the heat pipe. When it was tried to reach a high enough temperature at this spot, the temperature of the heat pipe rose too fast for the ceramic seal and the seal failed. Manufacturing of the ceramic seal is a time consuming process and it was not repeated within this project as it was not the goal to validate the model for thermionic emission itself. The Rasor-McVey model is well documented and tested. What needed verification were the assumptions made for the work functions and whether it is allowable to use a one dimensional thermionic emission description in combination with a
two dimensional temperature field. The experiments performed in the unignited mode and in the ignited mode with a negative output voltage are sufficient for these purposes.

4.2 Validation of the start-up models

To gain insight in the start-up behaviour of the heat pipe, a series of tests were performed with the sodium heat pipe as it was used by Postel (1987). The experiments are described in detail in Biemans (1993).

In the experiments the evaporation section of a heat pipe, initially at ambient temperature, was suddenly exposed to the radiant heatflux of a furnace, while the remainder of the heat pipe was well isolated. By measuring the temperatures at several places on the surface of the heat pipe, the heat propagation was monitored. The thermocouples were attached to the outside of the heat pipe at the positions sketched in figure 4.11.

![Figure 4.11: The heat pipe and the position of the thermocouples](image)

The temperature of the elements was measured and translated to a heatflux using a radiation model of the furnace (Biemans 1993). The main flaw of this set-up is the temperature dependence of the heatflux, which means that the actual processes would take place at a slower pace than predicted. Typical results of these experiments are given in the figures 4.12 to 4.14.

In the one dimensional model for the heat pipe (§2.4.1), evaporation is supposed to start when the gradient of the temperature at the top of the heat pipe is zero, after 180 s. This indeed occurs when starting the heat pipe horizontally or gravity assisted (figure 4.12 and 4.13). After 240 s the temperature starts to rise again, which indicates that most sodium is evaporated. It takes about 320 seconds to reach the melting temperature of sodium at the condensing section which means that liquid sodium is established in the entire screen wick. After that, the temperature in the complete heat pipe starts to rise again as there is no cooling of the heat pipe.

Starting the heat pipe horizontally or gravity assisted presented no difficulty within our range of heat fluxes (up to 180 kWm⁻²). Starting the heat pipe against gravity however led to problems when applying a heatflux over 80 kWm⁻². The temperature profiles which occur during start-up against gravity differ significantly from the curves obtained while starting horizontally or gravity assisted (figure 4.14). The results indicate that part of the evaporation section does not take part in the heat transport process, which leads to an unacceptable overheating of the heat pipe surface.
The time scales for the start-up processes as described in chapter 2 are compared to the experimental results obtained by Biemans (1993). Comparison of the experimental results of figures 4.15 to 4.17 and the calculated values show the best agreement with the predicted values for gravity assisted starting. The starting time of evaporation is calculated within 15%. The same value was found for establishing liquid flow up to a heat flux of 90 kWm$^{-2}$. At the higher heat fluxes the experimental values are considerably lower. This implies that condensation of vaporised sodium at the condensation section has started, leading to a faster heat exchange between evaporator and condenser. The evaporation time is predicted well within 20%.
In figure 4.18 the results of the two dimensional model of de Vries (1994) described in §2.4.2 are compared with experimental results of Biemans (1993). The calculated evaporator temperature stabilises at a higher temperature than measured, which may be caused by contaminations in the heat pipe. Furthermore, the measured evaporator temperature starts to rise earlier than calculated, indicating that mass transport through the wick already exists. Nevertheless, both experiments and numerical simulations show that the evaporator temperature rises to a temperature level sufficient to sustain the thermionic process within 300 seconds.
4.3 Closure

The thermionic converter and the heat pipe tested, were constructed as prototypes and not especially for measurement purposes. The main conclusion drawn from the experiments is that they can function the way they are constructed.

The heat pipe was not difficult to manufacture and performed well. The only point of concern is starting the heatpipe against gravity. The simple models which were developed to describe heat pipe start-up are sufficiently accurate to be used during the design process.

The construction of the prototype converter was quite another matter. The ceramic seal was difficult to manufacture because the ceramic was strained nearly to the limit. Another design flaw was the positioning of the cesium reservoir tube. At the start it seemed to be an elegant solution to lead the tube through the heat pipe. The heat pipe itself should have a higher temperature than the cesium reservoir and therefore no problems were expected with the cesium pressure control. Unfortunately, the end of the heat pipe remains at low temperature level for a long period. Furthermore, the end of the heat pipe is connected to the distance adjustment support which is at a lower temperature. Consequently, it was not possible to reach a high cesium pressure. Nevertheless, it was possible to validate the assumptions made for the work functions and the energy flows.
5 System analysis

The main components which the thermionic cogeneration system can comprise, are illustrated in the introduction. It will be evident that the choices for the system layout have a major influence on the demands for the thermionic energy converter. Unfortunately, the reverse also holds: the characteristics of the TEC largely dictate the necessary system elements, which implies that the decisions must be made in an iterative process. The analysis of thermionic cogeneration devices is treated extensively in Veltkamp (1993). In this chapter the results of the last iteration are summarised. Due to the iterative character of the design process, it is inevitable that some of the results obtained in the next chapter, the design of the TEC, are used in this chapter.

In §5.1 the global system and the functions of the components are introduced and discussed. An attempt is made to restrict the number of alternatives based on qualitative considerations. In §5.2 the remaining alternatives for the system components are combined in structures. In order to make a quantitative analysis, simple calculations for the overall system efficiency are made and the final system layout is chosen. The task of dimensioning the system components is described in §5.3. Finally the demands on the TEC, imposed by the system, are presented in §5.4.

5.1 Functions and methods

Thermionic energy conversion has been developed for space power and was intended for electricity production only. In this study terrestrial thermionic cogeneration is investigated. The main difference between both applications is that in cogeneration systems the size is dictated by the heat load. In order to obtain reasonable system efficiencies, additional system components might be necessary to recover the heat from the flue gases and the collector.

The functions of a complete domestic cogeneration system can be stated as follows:
- producing the required heat for the heating system
- producing the required heat for domestic hot water
- producing electricity

In order to accomplish demands as a direct delivery of heat plus electricity and economic feasibility, several derived functions may be added if they raise the overall efficiency:
- storing heat
- storing electricity
- recovering heat from flue gases

To provide the required functions the following sub systems are considered:
- a heating system (necessary)
- heat exchanger to recover the heat from the flue gases (option)
- a burner for the thermionic converter (necessary)
- a thermionic converter (assumption)
- an auxiliary burner (option)
- thermal storage (option)
- electrical storage (option)
- invertor to alter DC to usable AC (necessary)
- power conditioner to maximise output (option)
A diagram of an example domestic cogeneration system is shown in figure 5.1. Both hot tap water and hot water for the heating system are extracted from a thermal storage vessel. When the thermal storage is depleted, the burner system starts. Part of the fresh air is heated by the collector of the TEC and another part is heated by the flue gases in the recuperative burner. In the recuperative burner natural gas is injected and the combustion mixture is fired. The heat is convectively transferred to a porous ceramic burner wall that in turn radiates to the emitter of the TEC. The flue gases flow through the recuperator heating the incoming fresh air. The remaining heat is extracted by the gas-water heat exchanger. In case the thermal power of the recuperative burner and the TEC is sufficient, the air stream from the collector is led directly to the gas-water heat exchanger. Otherwise the auxiliary burner can be applied. In this chapter a choice is made for the lay-out of the sub-systems and their necessity is discussed.

**figure 5.1 diagram of a domestic thermionic cogeneration system**

As the thermionic cogeneration system replaces the conventional heating system, it has to fulfil the same demands to be competitive. A standard enhanced efficiency boiler, as is used widely in the Netherlands satisfies the following demands (Veltkamp 1993):

- gross heating system efficiency >88%
- heating power >14 kW
- storage temperature intermittently >65°C, to prevent legionella growth
- stability of the hot water temperature: peak/peak < 2°C, no delay and a threshold of 0°C
- a closed device
- NOx production < 40 ppm
- CO production <100 ppm
- longevity > 15 year
- size < 1.8 * 0.7 m, weight < 50 kg
- the device has to be safe

The next step in the design process is to evaluate methods for the subsystems mentioned, which will be done in the following paragraphs. Whenever a method can be ruled out for qualitative reasons it will be done. The remaining options are combined in structures and analysed quantitatively in §5.2.
5.1.1 The domestic heating system

A domestic heating system is so small that other cogeneration systems like gas engines and turbines need too much maintenance. The power production is synchronous with the peak heat load. The functions of the domestic heating system are to heat a normal single family house within comfort level and to deliver the hot water demand. Demands on the heating system are:

- a room temperature of 20°C
- variation in room temperature less than 1°C
- heating power 14 kW
- yearly demand of 2000 m³ natural gas for heating
- a yearly demand of 500 m³ natural gas for hot water

The demands can be met using an air- or a water heating system. Water heating is the more common option in the Netherlands and is selected for this system analysis. The standardised water temperatures are 90°C for the supply and 70°C for the return temperature at maximum heat demand. The heating system is on/off controlled by the room temperature and the (maximum) water temperature in the boiler. This implies frequent on/off switching of the system which decreases the efficiency of the thermionic conversion system considerably. The switching frequency can be reduced by applying a thermal storage.

5.1.2 Heat exchanger for the flue gases

The thermionic converter, contrary to an expansion machine, is not able to cool the combustion gases to a lower temperature than that of the emitter. The high temperature of the flue gases implies that only a small part of the heat released in burning a fuel is converted to electricity. The flue gases can be used for the heating system, or they can be used to preheat the combustion air plus the fuel. In that case an extra heat exchanger is necessary.

The radiative heat transfer to the emitter may change during time as the emissivity of the hot shell and the burner surface can change, for instance by the forming of quartz on the SiC of the hot shell. A heat exchanger, being a feedback system, may stabilise the heat transfer. Furthermore, the heat exchanger has to prevent the fuel from self ignition or carbonisation. These functions can be achieved by exchanging the heat from the flue gases to the combustion gas by conduction through a wall (recuperation) or by exchanging the heat through an intermediate thermal mass (regeneration). Without a recuperator the system efficiency drops to a very low value. For both heat exchanging methods, the flow of the combustion gas and the flue gases can be arranged as parallel flow, cross flow, and counterflow.

If both heat capacity flows are equal, the effectivity in parallel flow is low (50% maximum), in cross flow mediocre (75% maximum), and in counter flow a maximum of 100% can be obtained. To extract all heat from the flue gases, the combustion gases need to have the same heat capacity flow [WK⁻¹]. The mass flows [kgs⁻¹] being the same, the heat capacities [Jkg⁻¹K⁻¹] must be equal, which is not the case (appendix D). In order to reach good system efficiencies it might be necessary to apply this extra heat exchanger for the flue gases.
5.1.3 The thermionic process

The thermionic energy converter has the following functions:
— keeping the thermionic process in required conditions
— allowing for sufficient heat flows
— transporting electricity

We can state the following demands on the thermionic process:
— a constant characteristic to allow control
— a low sensitivity of efficiency on variables
— a “high” voltage for low electrical losses

In achieving these demands we can choose for an ignited mode thermionic converter, a close spaced converter and advanced converters (i.e. triodes, pulsed converters etc.). As is discussed in §1.2.2 the last category is not considered here.

Unfortunately, it is characteristic for the thermionic process that the efficiency, defined as the electrical power produced divided by the heat input, is rather dependent on many variables and furthermore that it is rather sensitive to changes in some of them. The efficiency of the thermionic process and other key variables as a function of the emitter temperature are illustrated in figure 5.2 for an ignited mode converter.

*figure 5.2 optimised parameters of the thermionic process as function of the emitter temperature (molybdenum emitter, nickel collector, other variables optimised, (Houston 1965)*
5.1.4 Collector cooling

If water is selected as the medium for distributing heat, it may seem to be the least complex solution to cool the collector directly with water, but even in that case a heat exchanger may be necessary to use the excess heat in the flue gases. The same heat exchanger can easily be used when the collector is cooled by air. As the collector has a relatively high temperature, both in the case of ignited and close spaced mode operation, exchanging the heat to a gas flow seems to be a good solution if we choose to remove the excess heat in the flue gases by water-cooling.

5.1.5 Burner

The burner must be capable of transferring about 300 kWm\(^{-2}\) to the emitter at 1723 K for the ignited mode (table 6.1), in the close spaced mode the heat flow is an order 10 smaller. The NO\(_x\) production should be under 40 ppm with a CO content lower than 100 ppm to comply with present regulations. To allow for a fast system behaviour, the temperature response of the burner should be fast.

Preheating the combustion gases might raise the system efficiency, but is limited by self-ignition of the mixture if the preheat temperature exceeds 900 K. If the heat exchanging area of the hot shell is too small for direct convective heating, a porous body with sufficient heat exchanging area can be installed which radiates to the hot shell. In figure 5.3 an example is given (Veltkamp 1993). The air flows through a recuperator and is heated by the flue gases. Gas is injected through a separate, insulated channel to prevent carbonisation. The combustion mixture is fired and leaves the combustion area through a porous radiative body. The flue gases pass through the recuperator, heating the incoming air.

In the case of an ignited mode thermionic converter a radiative burner integrated with a recuperator is a good option (Veltkamp 1993). In the case of a close spaced thermionic converter the choice between convective and radiative heat transfer is still open.

![Diagram of a burner integrated with a recuperator (Veltkamp 1993)](image-url)
5.1.6 Auxiliary burner

If a thermal storage is added, the status of the thermionic system can be: at maximum efficiency or off. The hot domestic tap water load is important in dimensioning the thermionic system and the storage, as it is the only load for a large part of the year (appendix C). An optimisation will be necessary to balance storage size and thermionic power with respect to efficiency and costs. Installing a thermionic power equal to the maximum heating power demand may not be very cost effective. In case the installed thermionic system exceeds about one third of the maximum power required, the electrical output does not increase anymore due to start-up and storage losses (§5.2.2). It might be a cheaper solution to use an auxiliary burner to supply the peak heat demand instead of enlarging the thermionic converter.

5.1.7 Thermal storage

The thermionic converter has a certain inertia. During start-up the converter delivers no electricity. Using the model of appendix C.2 for a heating system with a very small thermal storage (1 litre), the number of starts and stops was calculated to be in the order of 10,000 times yearly. This may cause a very low electrical performance. Elongation of the duration of the on period decreases the number of start-ups and herewith increases the electrical performance. The functions of a thermal storage in a TEC-system are:

- decreasing the number of starts of the thermionic converter relative to the duration of operation
- eliminating the wait period of delivering hot water
- increasing the power to the hot tap water or the heating load

To fulfil these functions, the heat content in the thermal storage should at least be able to span the start-up time of the heating system. Secondly, the temperature in the storage has to be higher than the hot water demand of 60°C.

The thermal storage can be realised by storing the heat in:

- water
- any other thermal mass

Storing the heat in hot water is in this case the most logical thing to do. The method to serve the function of decreasing the number of start-ups is to start with a cold storage and run the converter continuously until the storage is filled. In the off period of thermionic conversion, the storage has to be emptied to allow for an as long as possible uninterrupted on period. To serve the other functions, it is necessary to keep the storage at a high temperature level. Both functions however can be combined in one storage by using part of the storage for each function. In a thermally stratified storage it is possible to store water with the required load temperature on top of water with a lower temperature without mixing. This alternative is selected for the quantitative system analysis.
5.1.8 Invertor

The thermionic converter is a low voltage high current device. Conversion to the more common 230 (or 115)V AC is required to avoid excessive losses and to apply with devices using electrical energy. Conversion can be achieved by the tacitron invertor, which essentially is a thermionic converter used as switching element. With this type of invertor high efficiencies (>95\%) are expected (El-Genk 1992). The device, however, is still in the development phase.

The efficiency of existing solid state invertors is very low. For a potential of 3V (6 TEC’s in series) the efficiency can be raised to about 80\%. For better efficiencies a switching chip with lower resistance has to be produced. Technically the efficiency can be high at the expense of high development costs. Electro-mechanical inversion is an alternative which introduces a moving part, but promises high efficiencies at acceptable development costs. In this study it is rather arbitrarily assumed that an invertor can be built with a threshold voltage of 0.5 V and an internal resistance of 0.0003 Ω (Veltkamp 1993).

5.1.9 Electric storage

It is supposed that the electric power produced can be delivered to the grid whenever it is not used in the house, as local electrical storage is considered to be too expensive.

5.1.10 Power conditioner

The power conditioning is necessary as the position of the maximum power output of a thermionic converter is rather sensitive to the external imposed voltage or current. Furthermore, in a thermionic cogeneration system, it is not the emitter temperature which is kept constant, but the gas supply to the system. If the imposed voltage on the converter changes, the thermionic current and hence the electron cooling changes. If the voltage is raised, while the heating power remains constant, the emitter temperature can rise to an intolerable high level.

5.2 Structures

In the preceding paragraph some general considerations about the system components and their behaviour are presented. To keep track of the design process the options reviewed are sketched in figure 5.4. The bold alternatives are chosen and the italic alternatives are discarded. The other options must still be decided upon. Whenever a subsystem is not mentioned it is assumed that it is not necessary to detail the subsystem in order to get the list of demands for the TEC, the goal of this system analysis.

The next step is to decide if it is useful to use a flue gases recuperator, a thermal storage and an auxiliary burner. Here we encounter a problem as the mode of operation for the TEC is undecided upon. As is discussed in chapter 6, however, a close spaced converter has severe constructional drawbacks and hence only ignited mode operation is selected for this system analysis.
In §5.2.1 it is assessed whether the recuperator is necessary by looking at the recuperator, burner and thermionic converter only using an analytical model (appendix C.1). The need for a thermal storage and an auxiliary burner is established in §5.2.2, using a numerical model for the exploitation of the TEC (appendix C.2).

For all calculations in this paragraph the thermionic cogeneration process was modelled as a set of coupled heat exchangers. The thermionic process is described using the Sidelnicov model. By this approach, the set of equations can be solved using a symbolic algebra program (appendix C). The accuracy of the model is less than that of the numerical model because of the supposed uniformity of the electrodes, but good enough for assessing the best system layout. The optimisation, if it can be called an optimisation, was done visually by plotting the required output as a function of all parameters and iteratively changing the parameters to the best value.

5.2.1 The combustion module

The first sub-system under consideration consists of the burner, the TEC and optionally the recuperator. The structures which are considered are schematically drawn in figure 5.5. First a system is investigated with no preheating and where the collector heat and the heat in the flue gases are used for the heating system. The second system is almost the same but oxygen is used to get a higher flame temperature. In the third system only the enthalpy in the flue gases is used for the heating system and the collector heat is used to preheat the combustion gases. In the last system both the collector heat and a part of the heat from the flue gases is...
used for the heating system. The main part of the heat in the flue gases is used to preheat the combustion gases. For the cooling of the collector only air is used for the calculations as it is shown that an auxiliary burner is necessary and the difference between air and water cooling in the system analysis will be small due to the supposed high effectivity of the heat exchanger (>90%).

\[
Q_{ohm} = \frac{\pi R^3 J^2 \rho}{2 d} \left( \frac{\pi R}{4} + L \right)
\]  
(5.1)

where \( J \) is the current density and \( \rho \) the specific electrical resistance of the hot shell. The conduction losses through the sleeve are calculated according to

\[
Q_{cond} = \frac{2\pi R d}{L} (T_E - T_{seal})
\]  
(5.2)

the temperature of the seal \( T_{seal} \) is taken equal to the collector temperature. The other heat losses are calculated as is described in §2.2.1. The required natural gas flow can be found from an enthalpy balance when assuming that the flue gases can be cooled to the emitter temperature \( T_E \)

\[
C_{p_{\text{air}}} T_{\text{amb}} M_{\text{gas}} + C_{p_{\text{gas}}} T_{\text{amb}} M_{\text{gas}} + H_{\text{gas}} M_{\text{gas}} = C_{p_{\text{flue}}} T_E (1 + \phi_{\text{seal}}) M_{\text{gas}} + Q_{\text{TEC}}
\]  
(5.3)

*Figure 5.5  diagram of the considered combustion module structures*
where $M_{\text{gas}}$ denotes the natural gas flow, $H_{\text{gas}}$ the enthalpy of natural gas without condensation (38.106 Jkg$^{-1}$), $\phi_s$ the mass ratio at stochiometric combustion (13.08), QTEC the heat flux to the TEC and $\phi$ the air ratio (taken at 1.1). The electrical system efficiency is defined as

$$\eta_{el} = \frac{\eta_{TEC} Q_{TEC}}{M_{\text{gas}} H_{\text{gas}}}$$

and amounts to a mere 1% for the parameter values of table 5.1.

**Table 5.1 System parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter temperature $T_E$</td>
<td>1723 K</td>
</tr>
<tr>
<td>Collector temperature $T_C$</td>
<td>980 K</td>
</tr>
<tr>
<td>Cesium reservoir temperature $T_r$</td>
<td>570 K</td>
</tr>
<tr>
<td>Interelectrode distance $d_{\text{gap}}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Hot shell radius $R$</td>
<td>20 mm</td>
</tr>
<tr>
<td>Sleeve length $L$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Mo layer thickness $d$</td>
<td>0.2 mm</td>
</tr>
</tbody>
</table>

Two options are left to increase the system efficiency: preheating the air or enriching the air with oxygen. As enriching air does not seem very practical, we have the air must be preheated using the collector cooling or recuperation. Preheating the air by collector cooling leads to maximum preheat temperatures of around 900 K. The maximum electrical efficiency can be found by evaluating equation (5.3) and (5.4) for an ambient temperature of 900 K and amounts to 3% which is still very low. The only way open to preheat the combustion gases to a higher temperature is by recuperating the heat from the flue gases.

A counterflow heat exchanger in combination with a compact burner can be used in combination with a porous body to increase the heat exchanging area of the burner. In Sampers (1993) several possibilities for heating the TEC are reviewed and a radiative burner emerged as the best option. Natural gas can be preheated to about 900 K before carbonisation starts. In this study a mixture of air and gas in the recuperator is modelled and the maximum temperature of the actual separate gas flow is accounted for by accommodating the heat capacity flow in the model. A diagram of the system investigated is given in figure 5.6. The recuperator is a counterflow heat exchanger. The physical properties of the gases are averaged with respect to the temperature, which allows for an analytical relation for the temperatures (VDI 1994). With the Theorist program of appendix C the efficiency was calculated and it amounted to 8%, indicating that recuperating the flue gases is necessary indeed.

### 5.2.2 Thermal storage and auxiliary burner

Using the numerical model for exploitation of appendix C we can calculate the yearly electrical output for a given storage size and start-up time of the TEC as a function of the installed TEC power. The main assumptions in this model are listed in table 5.1. The house is modelled as a static heat resistance and for each time step the heat loss of the house is calculated using the hourly data of the shortened reference year. In the shortened reference year the data of a whole year are reduced to 56 reference days. Irradiation of the sun through the windows is also taken into account. For calculating the heat required by
domestic hot water, the hourly data of Wolferen (1990) were used. In the simulation the
thermionic cogeneration was ran at full power until the thermal storage was filled and then
switched off. If the heat demand could not be met by the heat flow through the converter, the
auxiliary burner was switched on.

\[
\text{figure 5.6 diagram of a system comprising a recuperator and a radiative porous body}
\]

\[
\text{table 5.2 parameter values for the system analysis}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>heat capacity of water ( C_w )</td>
<td>4170 J/kg·K⁻¹</td>
</tr>
<tr>
<td>specific mass of water ( \rho_w )</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>temperature of the mains water ( T_{tapin} )</td>
<td>10 °C</td>
</tr>
<tr>
<td>set temperature of the hot domestic water ( T_{tapset} )</td>
<td>50 °C</td>
</tr>
<tr>
<td>set temperature of the house ( Stet )</td>
<td>21 °C</td>
</tr>
<tr>
<td>heat loss of the house ( 1 ) ( U_{house} )</td>
<td>200 W/K · m²</td>
</tr>
<tr>
<td>windows surface ( 2 ) ( A_{window} )</td>
<td>5 m²</td>
</tr>
<tr>
<td>heat conductivity of the storage insulation ( \lambda )</td>
<td>0.035 W/m·K⁻¹</td>
</tr>
<tr>
<td>thickness of the storage insulation ( d_{store} )</td>
<td>0.05 m</td>
</tr>
<tr>
<td>temperature difference in the storage ( \Delta T )</td>
<td>60 °C</td>
</tr>
<tr>
<td>time step (one hour) ( dt )</td>
<td>3600 s</td>
</tr>
<tr>
<td>start-up time of the TEC ( t_{start} )</td>
<td>300 s</td>
</tr>
<tr>
<td>TEC efficiency ( \eta_{TEC} )</td>
<td>0.065 [-]</td>
</tr>
<tr>
<td>efficiency of the heating system ( \eta_{CV} )</td>
<td>0.95 [-]</td>
</tr>
<tr>
<td>daily hot water demand (50 °C)</td>
<td>120 [l]</td>
</tr>
<tr>
<td>natural gas price</td>
<td>0.57 [f/m³]</td>
</tr>
<tr>
<td>electricity price</td>
<td>0.185 [f/kWh⁻¹]</td>
</tr>
<tr>
<td>variable costs of the thermionic converter</td>
<td>0.75 [f/W⁻¹]</td>
</tr>
<tr>
<td>variable costs of the storage</td>
<td>5000 [f/m³]</td>
</tr>
<tr>
<td>fixed costs</td>
<td>100 [f]</td>
</tr>
</tbody>
</table>

1) value based on a yearly heat loss equivalent of 2000 m³ natural gas
2) surface windows projected to average sun direction, multiplied by the transmission
3) \( f \) = Dutch guilders
In figure 5.7 the yearly annual output is given for a storage of 10 l which practically means that there is no extra thermal storage at all. Except for the unrealistic start-up time of 0 s, the output shows a distinct maximum between 3.5 and 4 kW as a large installed thermionic power causes frequent starts and stops of the system. The same effect, though less strong because the number of starts decreases, can be seen in figure 5.8 where a storage of 100 l is assumed. When all the heat has to flow through the thermionic converters, large storages and large converters must be applied. Consequently, it is economically attractive to use an auxiliary burner instead, as enlarging the thermionic capacity does not lead to a higher yearly electrical output.

The presence and size of a thermal storage also influences the operating time of a TEC as is illustrated in figure 5.9. For a low power and a large thermal storage the operating time is nearly maximal. At large thermal storages the yearly electrical output stabilises. The increasing yearly output with increasing thermal storage is caused by a larger heat loss of the storage. The conclusion is that a thermal storage is useful.

5.3 List of demands for the TEC

The considerations in the previous paragraph lead to a heating system featuring an auxiliary burner, a storage vessel and a radiative burner with a recuperator to heat the combustion gases. The domestic heating water is heated by the flue gases after the recuperator and by the cooling air from the collector. Now that the system is defined, the dimensioning of the TEC can start. As is shown in the preceding paragraph, an economical criterion must be used for dimensioning the TEC.

Decentral generation of electrical energy by thermionic heating systems coincides with the peak energy demand. For this reason it seems realistic to assume that the electricity can be supplied to the grid for the same price as extracting. In this study an electricity price of £0.185 per kWh and a natural gas price of £ 0.57 per m$^3$ is used.
The electrical losses in the leads and the inverter reduce the electrical output about 20%, reducing the profit to €411.11 for 1 GJ\,year$^{-1}$ of installed thermionic power. The boiler is supposed to operate at an efficiency of 90% which means that 1 GJ of heat will cost €17.5. Using these figures, the annual net value amounts to €95 at a TEC power of 4000 W and a storage of 100 l. Subtracting the capital costs of the extra investment in the thermionic system, the annual net value shows an optimum of €45 at a thermal TEC power of 3500 W and a storage of somewhat more than 10 l (figure 5.10).

Summarising, we can state that the start-up time is important considering the performance, that thermal storage may double the output and that the maximum value is reached in all cases around 3500 W of thermal power.

**figure 5.9** influence of the storage size on the yearly electrical output for a start-up time of 300 s

**figure 5.10** yearly annual net profit as a function of the installed TEC power
5.4 Closure

Two major assumptions are made in transforming the information presented in the previous paragraphs into a list of demands for the thermionic energy converter. The first one is that we need about 3 V for the inversion to 220 V 50 Hz AC current, which implies that we need 6 converters in series. The second one is that the maximum emitter temperature is rather arbitrarily restricted to 1720 K to keep the materials intact. Consequently, the maximum electrode efficiency is around 10%. With these assumptions the list of demands for a single TEC is:

- thermal power 580 W
- operating time > 80,000 hour
- number of start and stops > 19,000
- start-up time < 300 s
- voltage > 0.5 V
6 Design of the TEC

In this chapter first a comparison is made between operation in the ignited mode and in the close spaced mode. Subsequently the best emitter shape is chosen and the dimensions of the system are assessed by optimising the efficiency. The last step is to design the heat pipe and the cooling system.

6.1 Comparison between ignited mode and close spaced TEC's

In figure 6.1 and 6.2 the power density plots for a close spaced- and an ignited mode TEC are shown. The power density plot for the close spaced converter is easily understood. According to the Richardson-Dushman equation, the electron current increases when the temperature increases and increasing the interelectrode distance leads to a higher space charge thus reducing the output current.

The plot for the ignited mode converter is made using the McVey model. The ignited mode converter operates at a higher temperature level as the plasma must be sustained. It is not immediately clear, however, why the performance decreases when the interelectrode gap is reduced. An adequate explanation is provided by Hatsopoulos (1979). Here it is stated that the maximum in electrode output as a function of the interelectrode distance is the result of the competition between the processes of ion generation and elastic scattering of electrons back to the emitter. The ion current produced in a gas with emitter temperature $T_E$ according to Rasor (1982) is

$$J_i = K_1 J_r \frac{d_{\text{gap}}}{\lambda} \exp \left( -\frac{V_{\text{eff}}}{kT_E} \right)$$

(6.1)

$K_1$ being a dimensionless constant weakly dependent on the electron temperature, $V_{\text{eff}}$ the effective ionisation energy, $\lambda$ the mean free path and $J_r$ the random electron flux.
Hence when the interelectrode gap \( d_{\text{gap}} \) decreases, less ions are generated, eventually limiting the flow of electrons. At large spacings, ion generation is ample, but elastic scattering occurs so frequently that it predominates and limits the flow of electrons. As is explained in §12.2 only thermionic converters operating in the ignited mode have been used up till now, but the results of Fitzpatrick (1993) indicate that close spaced operation can be competitive for terrestrial applications. The main advantage of a close spaced mode converter is the reasonable performance in a temperature region where proven solutions for material problems exist, which can not be said of the ignited mode thermionic energy converter. In choosing for a close spaced converter however, the material problem is traded for the constructional problem of accurately maintaining a small interelectrode distance (in the order of micrometers).

The heat loss of a thermionic system consists of radiation between emitter and collector, conduction through the vapour in the interelectrode space and conductive losses through the leads. Beside heat loss, there is an electrical loss due to ohmic heating of the leads. The losses in the interelectrode space can be compared easily for both systems as they are only dependent on the emitter temperature, collector temperature and in the case of cesium conduction, the reservoir temperature and interelectrode gap. For the thermionic converters which have been under scrutinisation up till now, the electron cooling \( q_{el} \) and radiative heat transfer \( q_{\text{rad}} \) are an order higher than the other heat flows. It is therefore reasonable to suppose that the efficiency of both systems, close spaced and ignited mode, can be compared taking into account the interelectrode losses only. With these assumptions the efficiency can be defined as

\[
\eta = \frac{V_{\text{out}}J_{\text{out}}}{q_{\text{in}}} = \frac{V_{\text{out}}J_{\text{out}}}{V_{\text{out}}J_{\text{out}} + q_{el} + q_{\text{rad}} + q_{\text{cs}}}
\]

(6.2)

where \( q_{\text{cs}} \) denotes the conduction through the cesium vapour. This exercise provides the figures 6.3 and 6.4. In order to reduce the calculation time, the Sidelnicov model has been used for the calculations concerning the ignited mode thermionic converter. The results were spot-checked using the McVey model. Looking at the efficiency, the characteristic of the close spaced converter is more favourable than in the case of the power density plots, due to the reduction in radiation losses at the lower temperatures.

By looking at the physical process of thermionic energy conversion only, a well founded decision between close spaced and ignited mode converters can not be made. Both types could theoretically fulfil the list of demands for a thermionic converter.

The functions which have to be fulfilled by a TEC are:

- maintaining of the desired operating point
- maintaining the optimum electrode distance
- maintaining the cesium pressure
- maintaining the emitter temperature
- maintaining the collector temperature
- allowing for a sufficient heat flux to the emitter
- cooling the collector
- removal of the electrical power produced

These functions are specified for an ignited mode and a closed spaced converter in table 6.1.
From table 6.1 it can be deduced that manufacturing a close spaced converter has to be done with an extremely small tolerance and that almost no creep can be allowed. Furthermore, the apparatus will be ten times as large, compared to an ignited mode converter. Though the choice for a close spaced converter may be fruitful in certain circumstances, the option is discarded for use in a domestic heating system.

**table 6.1** functions of a close spaced and an ignited TEC

<table>
<thead>
<tr>
<th></th>
<th>ignited</th>
<th>close spaced</th>
</tr>
</thead>
<tbody>
<tr>
<td>maintaining the operating point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>interelectrode distance</td>
<td>0.2\text{±}0.1 mm</td>
<td>15\text{±}5 \text{µm}</td>
</tr>
<tr>
<td>cesium reservoir temperature</td>
<td>590 K</td>
<td>550 K</td>
</tr>
<tr>
<td>emitter temperature</td>
<td>1723 K</td>
<td>1400 K</td>
</tr>
<tr>
<td>collector temperature</td>
<td>980 K</td>
<td>870 K</td>
</tr>
<tr>
<td>heat flux to the emitter</td>
<td>290\text{×}10^3 Wm^2</td>
<td>30\text{×}10^3 Wm^2</td>
</tr>
<tr>
<td>emitter surface</td>
<td>17\text{×}10^{-4} m^2</td>
<td>200\text{×}10^{-4} m^2</td>
</tr>
<tr>
<td>cooling of the collector</td>
<td>260\text{×}10^3 Wm^2</td>
<td>26\text{×}10^3 Wm^2</td>
</tr>
<tr>
<td>removal of electrical power</td>
<td>25\text{×}10^3 Wm^2</td>
<td>3.5\text{×}10^3 Wm^2</td>
</tr>
</tbody>
</table>
6.2 Emitter shape

In its simplest form, the TEC consists of the following components:
- emitter
- collector
- collector cooling
- vacuum container
- cesium reservoir
- electrical/heat insulation between emitter and collector

These components must be combined in such a way that the functions mentioned in chapter 5 will be fulfilled.

In research diodes, planar surfaces have been used, but in practical situations cylindrical and spherical forms have been applied (figure 6.5). The emitter must guarantee the required interelectrode distance with regard to the required emitter temperature and emitter area. This means that we have to compare the three forms considered on criteria such as: alignment to the collector, deformation, creep and stability. The second important criterion is that it must be possible to supply sufficient heat to the emitter.

![figure 6.5 the three shapes of the emitter investigated](image)

6.2.1 A cylindrical emitter

Concentric cylinders can be rotated and translated along their common axis without affecting the interelectrode distance. Misalignment occurs when the axis of the cylinder is shifted or tilted. An estimation is made of the influence on the efficiency when the axis is shifted (figure 6.6) using the Sidelnicov model (van Kemenade 1993). The emitter temperature is supposed to be constant at 1723 K and the cylinder has been divided circumferentially in 30 segments (see figure 6.5). For each segment the difference in the interelectrode distance due to misalignment is calculated and subsequently the output current is assessed using the numerical model. The efficiency hardly changes until the misalignment amounts to more than half the interelectrode distance. Even at a large misalignment, just before the converter would be short circuited, the efficiency has only dropped by 1.6%.

The maximal pressure induced radial deflection of a thin infinitely long cylinder in case of small elastic deformations is given by (Seide 1988)

$$w = \frac{pR^2}{Ed} \left(1 \pm \frac{L}{2}ight)$$

(6.3)
being the radius, \( p \) the pressure applied on the outside wall, \( d \) the thickness of the cylinder and \( v \) the Poisson’s ratio.

\[
\begin{align*}
R & = 0.02 \text{ m} \\
TC & = 980 \text{ K} \\
Tr & = 590 \text{ K} \\
dmean & = 0.2 \text{ mm} \\
R & = 0.02 \text{ m}
\end{align*}
\]

figure 6.6 influence of misalignment on the efficiency of a cylindrical TEC

The cylindrical emitter exhibits a slight elastic deflection as can be seen in figure 6.7. In practical cases the thickness of the SiC layer is more than 0.1 mm and the deflection is in the order of 1 \( \mu \text{m} \). The influence of the elastic deflection on the efficiency for a layer thickness above 0.1 mm is minimal (figure 6.8).

In figure 3.12, the creep rate of a SiC sphere at 1700 K is given. According to table 6.1, a creep of 0.1 mm can be allowed in a time span of 80,000 hours, which means that a creep rate of \( 3 \times 10^{-13} \) is acceptable. The creep rates calculated for radii from 5 to 50 mm and a thickness of 0.1 mm and 1 mm are at least an order \( 10^8 \) lower than this critical creep rate. Assuming that creep of a cylinder and creep of a sphere are in the same order, it is unlikely that creep will present major difficulties.

\[
\begin{align*}
T_E & = 1723 \text{ K} \\
T_C & = 980 \text{ K} \\
T_r & = 590 \text{ K} \\
dmean & = 0.2 \text{ mm} \\
R & = 0.02 \text{ m}
\end{align*}
\]

figure 6.7 calculated elastic deflection of a cylindrical CVD-SiC hot shell with a radius of 0.02 m at 1723 K

figure 6.8 calculated influence of the elastic radial deflection of a CVD-SiC emitter on the efficiency of a TEC
In figure 6.9 elastic buckling of a clamped cylinder is given, calculated according to Klöppel (1953)

\[
p_{cr} = E \frac{0.89 \frac{d}{l} \sqrt{\frac{d}{R}}}{1 \pm 1.18 \sqrt{\frac{Rd}{l}}}
\]  

(6.4)

\( p_{cr} \) is the critical pressure on the outside wall, \( E \) the Young’s modulus, \( d \) the thickness, \( R \) the radius and \( l \) the length of the cylinder. Creep buckling has not been calculated as it is supposed that the comparison between the shapes regarding buckling is not influenced by the added effect of creep. The necessary thickness of the SiC layer in order to prevent buckling is in the order of \( \mu m \). If the ends were free, the critical thickness is in the same order as in the case of a sphere (§6.2.2), which means that in case of a cylinder, elastic buckling can also be prevented by constructional means.

![Figure 6.9](image)

**Figure 6.9** calculated critical thickness with respect to buckling of a SiC cylinder at 1723 K as a function of the radius and the length

### 6.2.2 A spherical emitter

Concentric spheres can be rotated around their common centre without change in the interelectrode distance. Misalignment can be caused by a shift in the centres of both spheres (figure 6.10). To assess the influence of such a misalignment, a sphere with a radius of 20 mm was divided in 100 segments. For each segment the interelectrode distance is calculated and subsequently the system is optimised with respect to the cesium pressure and collector temperature for the case of a converter operating in the ignited mode. Aligning with a tolerance of 0.1 mm is easily achievable for small axi-symmetrical forms and as can be seen from figure 6.10 such a misalignment hardly affects the electrode efficiency. The results are comparable with those for a cylinder.
The maximal radial elastic deflection of a spherical cap with clamped edges is given by (Timoshenko 1975)

\[ w = \frac{pR^2(1 \pm \nu)}{2Ed} \sin(\alpha) \quad (6.5) \]

where \( p \) denotes the pressure on the outside, \( R \) the radius, \( \nu \) the Poisson’s ratio, \( E \) the Young’s modulus, \( d \) the wall thickness and \( \alpha \) the angle of the spherical segment (see figure 3.13). The deflection is in the same order as in the case of a cylinder and is not plotted separately.

A sphere exhibits a good creep behaviour. When using a ceramic material, a sufficiently low creep rate can be obtained. The creep rates calculated for radii from 5 to 50 mm and a thickness of 0.1 mm and 1 mm (figure 3.12) are at least an order 10^8 lower than this critical creep rate.

In the case of pressurised thin shells, buckling can be the failure mode. When establishing the minimum hot shell thickness, we have to bear in mind that buckling is very sensitive to geometrical imperfections. Furthermore, the geometry will change during the lifetime of the converter, which may lead to creep buckling. The critical thickness regarding elastic buckling and creep buckling of a CVD-SiC hot shell is depicted in figure 6.11. The elastic buckling is calculated according to equation 3.12. For the estimation of the critical wall thickness in case of creep buckling, first the critical creep is assessed using the method described in §3.3.2. Subsequently the critical creep rate is found by dividing the critical creep by the required lifetime of 80,000 hours. When the critical creep rate is known, the relation between the radius of the spherical segment and the thickness can be calculated in an iterative process. In our case creep buckling is the limiting process.
6.2.3 Flat plate emitter

For the calculation of the influence of a spherical plate on the system efficiency the same procedure was followed as in the preceding paragraphs. The plate has been subdivided in 30 circular segments. Figure 6.12 shows that the influence is marginal, as it is in the other cases regarded.

\[
\frac{3}{16} \frac{(R_o^2 \pm r^2)^2(1 \pm \nu)p}{E d^3}
\]

(6.6)

\(R_o\) is the outer radius, \(\nu\) the Poisson’s ratio, \(p\) the pressure, \(E\) the Young’s modulus and \(d\) the thickness of the plate. Of course, compared to the cylindrical and the spherical shape, the deflection is very large.
The creep rate of a flat plate was not calculated but is expected to be as large as the elastic deformation. In order to obtain the required emitter area, spacers have to be used to keep the emitter in shape. By using spacers, however, other problems are introduced. When using for example ZrO₂ spacers, which have a low heat conduction coefficient of 2 Wm⁻¹K⁻¹, at a temperature difference of 600 K between the emitter and collector, the resulting heat flux through the spacers will be 6 MWm⁻². If only 1 % of the emitter area is occupied by the spacers, an additional heat flux of 60 kWm⁻² is introduced, reducing the electrode efficiency with 2.5 % to 9 % which cannot be tolerated. Consequently, a flat plate emitter is not a feasible option.

6.2.4 Cylinder or sphere

Considering the shape of the emitter, a cylinder and a sphere perform comparably considering the structural demands. A flat plate is not a viable option. The choice between a cylinder and a sphere should be made with the rest of the system in mind. In figure 6.14 a comparison is made between a spherical and a cylindrical shape with the same emitter area and thermal power. The temperature profiles over the hot shell in both cases were calculated using the numerical model. A cylinder might have a higher efficiency as the maximum temperature occurs over a circle instead of a point, the difference however is small. According to Veltkamp (1993) a spherical emitter is preferable as it facilitates the design of a radiative burner and for that reason a spherical emitter is chosen.
6.3 Dimensioning of the hot shell and ceramic seal

In this paragraph the dimensions of the hot shell are assessed by optimising the geometrical variables to the efficiency defined as the electrical power-out divided by the total heat flux in (§6.3.1). Subsequently the production technique in combination with the structural integrity is discussed (§6.3.2). The last point is the construction of the ceramic seal (§6.3.3).

6.3.1 Thickness and sleeve length

In figure 6.15 the efficiency at the sleeve is plotted as a function of the sleeve length for several thicknesses of the molybdenum layer. The efficiency at the sleeve is defined as the electrical current at the sleeve divided by the amount of heat entering the hot shell. The radius and emitter temperature are taken constant and all other parameters are optimised. If two of the three geometrical variables, \( R \), \( d \) and \( L_{sleeve} \) are given, we can find a value for the third variable which leads to the same efficiency.

The reason for the drop in efficiency at very low values for the wall thickness in figure 6.15 is that here only the molybdenum layer is varied. Compared to molybdenum, SiC is a better...
heat conductor than an electricity conductor and consequently the ohmic losses become relatively more important.

Repeating the exercise using the numerical model, varying the sleeve length and the Mo layer thickness leads to a shift to smaller sleeve lengths because radiation from the sleeve to the collector is neglected in the simple model. If the thickness of the hot shell is matched to the sleeve length, the efficiency profiles are almost constant, indicating that there is much leeway in choosing the dimensions of the hot shell.

To use the smallest amount of expensive material, while keeping a good efficiency, the wall thickness of the hot shell is set at 0.2 mm. The corresponding sleeve length according to the numerical model is about 20 mm and the radius required to produce 580 W thermally is 25mm. An indication of the stresses in a hot shell of these dimensions under working conditions is given in figure 3.4.

6.3.2 Manufacturing of the hot shell

The hot shell of the prototype TEC was made by first deepdrawing the molybdenum shell and subsequently coating it with a layer of TiN to act as a diffusion barrier and a SiC layer. For the new dimensions of the hot shell, it must be checked if it is still possible to deepdraw the molybdenum shell. In (Biemans 1991) it is shown that the two major points are the isostatic tension during the drawing process and the critical bending radius.

The isostatic component of the stress in the hot shell can be calculated using the method as described in Biemans (1991) when the initial blank radius is known. The maximum strain and the isostatic stress during the drawing process are depicted in figure 6.16 and 6.17. The failure strength of molybdenum amounts to about 600 MPa at room temperature and hence the isostatic stress will not present difficulties (unless it is below the ductile to brittle transformation temperature).

The second important point is the critical bending radius. On bending sheet material, micro cracks are induced below a critical bending radius \( \rho_k \). In Kals (1983) it is argued, based on stability considerations that in practical cases the critical bending radius is given by

\[
\frac{\rho_{cr}}{d} = \frac{1}{2n}
\]  

(6.7)
where $d$ is the wall thickness and $n$ the hardening exponent. A factor is used to describe the effect of the subsequent strain and squeezing during the deep drawing process. In our case, using a factor of 2.5 (Kals 1983), the critical bending radius amounts to 2.5 mm.

![Effective Strain During the Deep Draw Process at the Critical Spot](image1)

**Figure 6.16** Effective strain during the deep draw process at the critical spot

![Isostatic Stress During the Deep Draw Process at the Critical Spot](image2)

**Figure 6.17** Isostatic stress during the deep draw process at the critical spot

### 6.3.3 Design of the ceramic seal

The reduction in diameter compared to the prototype facilitates the construction of the ceramic seal because the stresses due to the difference in expansion are reduced. In the prototype the aluminium oxide ring is brazed to flexible bellows to allow for adjustments in the interelectrode distance, which is not necessary in a thermionic converter meant for production.

The functions of the ceramic seal are to connect the hot shell vacuum tight to the collector, to isolate the hot shell from the collector and to keep the required electrode distance. The most simple solution is to braze the aluminium oxide ring directly to the hot shell and a rigid ring connected with the heat pipe (figure 6.18a). The drawback of this construction is that the stainless steel heat pipe has a large expansion and will force the VACON ring outwards which introduces shear stresses in the brazing and the aluminium oxide. Furthermore the brazing surface of the hot shell is small.

It might be useful to uncouple the functions of making a vacuum tight connection and keeping the required interelectrode distance. In that case the aluminium oxide isolation ring can be attached to thin VACON rings which are allowed to deform (figure 6.18b), reducing the stresses. Though both alternatives might not fail, the construction 6.18b is the safest to choose.

Another point of attention while constructing the ceramic seal is the brazing of the hot shell. SiC is not wetted by copper based brazing materials and consequently oxygen can diffuse to the molybdenum, causing it to oxidise. Silver based brazing materials cannot be used as they are not cesium resistant. In figure 6.19 three constructions are depicted that were tried.

The solution to this problem was sought in decoupling the functions of making a vacuum tight connection and preventing the oxygen from diffusing to the molybdenum. First the molybdenum is brazed to the VACON using a copper brazing material and subsequently a layer of silver based brazing material is added on the outside.
6.4 Heat pipe

In v.d. Berg (1995) it is shown that sodium is the best working medium for application in the heat pipe for the TEC. In this paragraph the dimensions of the sodium heat pipe container are assessed (§6.4.1). Subsequently a good solution for the wick is sought (§6.4.2). The last issue in this paragraph covers the merits of several methods to enhance heat pipe start-up (§6.4.3).

6.4.1 Container

The evaporator area of the heat pipe is already heated close to the working temperature during the first minutes after switching on. Though the heat pipe is not started in this period, the collector temperature is high enough to sustain the thermionic process (figure 4.18). The faster the collector temperature is at the working level, the better. The speed at which the collector heats up is mainly governed by the heat capacity of the evaporator head and hence it is useful to construct the heat pipe as thin as possible. The maximum pressure which occurs during normal operation according to the two dimensional model of §2.4.2 amounts to about $3\times10^5$ Pa. There is no reason to change the heat pipe material (stainless steel AISI 316). This quality steel has a good chemical resistivity, can be welded and is usable...
The Design of a Combustion Heated Thermionic Energy Converter

at temperatures of 1250 K. The minimum wall thickness can be found using the well known formula's for pressure vessels (figure 6.20).

\[ d = \frac{R_1 R_2}{R_2 - R_1} \]

\[ h = \frac{d}{2} \]

(6.8)

and

\[ \sigma_\theta = \frac{p R_1}{d} \quad \sigma_z = \frac{p R_1}{2d} \]

(6.9)

respectively, where \( p \) is the internal pressure applied. For the maximum meridian stress in the transition from the cylindrical part to the spherical part an empirical relation was used (Dubbel 1987)

\[ \sigma_\theta = C \frac{p R_1}{2d} \]

(6.10)

the constant \( C \) is given by table 6.2

<table>
<thead>
<tr>
<th>( h/R_1 )</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>6.7</td>
<td>3.8</td>
<td>2.0</td>
<td>1.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The radius \( R_1 \) was set at 22 mm. The radius \( R_2 \) must match the radius of the hot shell, which is not set yet. In order to reduce the chance on misalignments, the radius \( R_2 \) can be chosen such that the rotating points of the heat pipe and the hot shell coincide. Allowing 10 mm for the seal construction, the radius \( R_2 \) becomes about 35 mm and consequently \( h \approx 10 \) mm. Choosing a radius \( R_1 = 22 \) mm, leads to \( C=3.4 \). If a von Mises stress of 50 MPa is allowed in the wall, which is a factor 3 below the yield stress at 900 K, a wall thickness of only 75 \( \mu \)m is necessary. To keep the construction workable, a wall thickness of 0.5 mm is selected. The heat pipe can be made of a standard stainless steel tube welded to a formed evaporation head.

Having no strength problems with the heat pipe, it can be considered to reduce the diameter of the heat pipe after the evaporation head (figure 6.21). In this construction a radiation shield can be added between the sleeve and the collector, reducing the radiation losses. Furthermore the thermal stresses in the ceramic seal can be reduced. When supposing that the radiation from the emitter to collector is halved due to the presence of a radiation shield, the calculations only indicated an increase of the efficiency from 8.16 % to 8.39 %. Though the total radiation losses were reduced by 20 W on a total of 158 W, the conduction losses increased by 10 % due to the higher temperature of the seal (figure 6.22). Hence it is not very
useful to apply a radiation shield as it complicates the construction and especially the positioning of the wick in the evaporation head.

6.4.2 Performance of the heat pipe

In v.d. Berg (1995) it is shown that spot welded screen wicks may give rise to non uniformities in the liquid transport. Furthermore experiments indicated that the positioning of the screen wick is critical. The suggestion was to produce the wick by winding metal filaments on a spring element. As the heat pipe is almost iso-thermal the clamping pressure can be well controlled. Being an unproved method however, in the design of the heat pipe the “old” spot welded screen wick is still used. Using the method described in §2.3.1, the stationary performance of the proposed new heat pipe can be calculated (table 6.3). The values are given for operation against gravity for an estimated heat pipe length of 90 mm. A Sankey diagram for the energy flows through the TEC in stationary operation is given in figure 6.23.

![Diagram of heatpipe and radiation shield](image)

**figure 6.21** construction with a radiation shield along the sleeve

**figure 6.22** the difference in the temperature profile over the hot shell for a TEC with and without radiation shield along the sleeve

**table 6.3** stationary performance of the new heat pipe design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>evaporator temperature</td>
<td>980 K</td>
</tr>
<tr>
<td>condenser temperature</td>
<td>973 K</td>
</tr>
<tr>
<td>capillary limitation</td>
<td>10 kW</td>
</tr>
<tr>
<td>sonic limitation</td>
<td>7.2 kW</td>
</tr>
<tr>
<td>viscosity limitation</td>
<td>192378 kW</td>
</tr>
<tr>
<td>entrainment limitation</td>
<td>18 kW</td>
</tr>
<tr>
<td>boiling limitation</td>
<td>1300 kW</td>
</tr>
</tbody>
</table>
6.5 Cooling of the heat pipe

The function of the heat pipe is to transport the heat from the collector area to an area with enough space for realising the required heat exchanging surface necessary to transfer the heat to the air. While designing the prototype heat pipe, no attention was paid to the effect of radiation on the heat transfer characteristics of the heat pipe-cooling system. De Vries (1990) tried to study the effect of radiation, but the influence of radiative heat transfer on the performance of the fins was not considered. Kivits (1992) estimated the maximal heat transfer by radiation, assuming that the surrounding pipe is at ambient temperature and that the heat transfer coefficient is constant over the entire fin system.

In figure 6.24 the fin efficiency, defined as the ratio between the heat transferred by the fin and the amount of heat which would be transferred if the entire fin was at its foot temperature, is given with and without radiation. The increase in fin efficiency is between 3% and 5%. The most striking result is that under these circumstances, radiation may account for up to 20% of the total heat transferred. Nevertheless, convective heat transfer amounts to at least 80% of the total heat transfer so it will be considered as the main heat transfer mechanism.

Another result of this exercise is that the heat transfer coefficients calculated are considerably lower than those found by Postel (1987), who designed the first heat pipe. This difference is caused by the fact that Postel based the average velocity on the flow area surrounding the heat pipe instead of the minimum flow area, which is lower. The minimum flow velocity to achieve a heat rate of 1.5 kW, the design criterion of the prototype heat pipe, then amounts to considerably more than 2 ms⁻¹ (figure 6.25) found by Postel, which is also experimentally observed by Rademacher (1991). This leads to a severe drop in the end temperature of the air which can be reached, which consequently may affect the efficiency of the total system.
6.5.1 Fin patterns

To develop tools for comparing cooling systems and to enhance understanding of the main parameters, several alternatives were investigated. The most familiar possibility to enlarge the heat transfer area is the application of fins. The amount of literature on this subject is huge. The majority of articles however only consider the heat transfer of a single fin and leaves out the interaction between the fins. Another limitation is that the heat transfer is commonly considered to be constant over the fin, which is essentially not the case.

Non uniformity over the circumference of the heat pipe may induce severe problems during start-up. For this reason cooling systems which are radially symmetrical are considered only. Furthermore we restrict ourselves to fin-types which can easily be manufactured from standard material, being fins with a rectangular shape and fins with a cylindrical profile.

In order to balance heat duty versus pumping power, the performance ranking method of Soland (1978) is used. This method involves the use of a graph of the pumping power per volume against the heat duty per volume. This plot allows an easy comparison for the constraints of fixed heat exchanger volume and pumping power, fixed pumping power and heat duty and fixed volume and heat duty. The best solution performs best on all three constraints.

In order to make a fair comparison between fins and spines, the heat rates of the systems are calculated for the same heat transfer area, mass flow and an equal height and thickness (Kivits 1992). The spine system emerges as a good option, the heat transfer coefficient is a factor 4 higher compared with fin systems. Unfortunately, the total heat rate of the system is too low, which implies that we should use more spines. To meet the requirements over 1000 spines should be used with a diameter of 1.5 mm. Considering the manufacturing problems, this option is not further pursued.

The heat transfer coefficient for a spine system is high because there is no opportunity to develop a thick boundary layer. By using interrupted fins the same can be achieved in a fin system (figure 6.26), but the advantage must be balanced against the increased pumping power. A possibility is using single axial spacing, but Patankar (1981) showed that the heat transfer can be enhanced further by using double axial spacing. Baken (1982) proved that...
changing the formation of the plates, which leads to larger distances between the successive fins, results in heat transfer coefficients which are even larger than those of Patankar.

<table>
<thead>
<tr>
<th>single axial spacing</th>
<th>double axial spacing (Patankar 1981)</th>
<th>double axial spacing (Baken 1982)</th>
</tr>
</thead>
</table>

*figure 6.26  interrupted plate patterns*

The empirical correlations used by Kivits in the first calculations can be used under the assumption that the different rows of fins have no interactions with neighbouring rows. In reality the presence of neighbouring fins influences the flow pattern, which causes a significant change of the heat transfer characteristics and pressure drop characteristics. Especially when applying interrupted fins, the interactions between neighbouring rows cannot be neglected. Furthermore, the influence of a fin on a downstream fin is commonly left out of consideration. For this reasons the detailed calculations of the chosen interrupted fin geometry is performed using the numerical package FLUENT.

Detailed information about the calculations can be found in Kivits (1992). Excessive computer resources would be necessary if all three geometries had to be fully modelled. Fortunately the flow attains a periodic fully developed behaviour after a short entrance region. Cyclic boundary conditions provide a good option to obtain a fully developed velocity profile for one geometrical module, but proved useless for the temperature field. The solution is to use two successive geometrical modules (figure 6.26). The first module has a flat temperature and velocity inlet while the second senses a developed profile, at least in our case. Even then we have to limit ourselves to a two-dimensional case, averaging the properties in the z-direction, which means that the height of the fin does not influence the calculations.

The first step was to compare continuous fins with single- and double spaced patterns. For constructive reasons, the fin thickness was in all cases restricted to 1 mm. No quantitative analysis has been found in literature to determine the optimal fin spacing. To achieve a compact system, the fin spacing should be as small as possible. A decrease in plate spacing however enhances the pressure drop over the spacing. Patankar showed that the thickness ratio \( d/s \) has a large influence on the flow pattern for \( d/s > 0.1 \) while for \( d/s < 0.1 \) the influence is rather small. For comparison we arbitrarily choose a thickness ratio of 0.1, leading to a spacing of 10 mm. The fin height is in all cases 15 mm which is equal to the present system. The calculation was performed for constant fin length. The performance ranking of the three systems is given in figure 6.27. The double spaced system has the best overall performance.
6.5.2 Dimensions of the cooling system

The smaller the fin length, the better the cooling system performs. To keep the system workable the fin length is (rather arbitrarily) set at 5 mm. After setting the fin length the optimum spacing was assessed. As can be seen in figure 6.28 the Nusselt number hardly changes for a thickness ratio smaller than 0.05. Selecting a fin thickness of 1 mm, the fin spacing is 20 mm.

The results of the calculations with two units for a spacing of 20 mm and a fin length of 5 mm were used to construct correlations for the heat transfer coefficient and the pressure drop. With these relations subsequently a complete cooling unit was designed.

The correlation for the Nusselt number is generally written as

$$\nu = C_1 Re^a Pr^b$$  \hspace{1cm} (6.11)

the factor $b$ is commonly taken as 1/3. The FLUENT data for the Nusselt number were shown to be lying on a straight line when plotted in a double logarithmic graph against Reynolds, leading to

$$\log(\nu) = \log(C_1) + a \log(Re) + 1/3 \log(Pr)$$  \hspace{1cm} (6.12)

Since the Prandtl number of air is almost constant for the temperature range, it is assumed that $Pr^{1/3}$ is constant. Consequently, the average Nusselt number only depends on the Reynolds number. The slope of the line provides the constant $a$ and subsequently $C_1$ can be calculated. For laminar flow this results in (Kivits 1992)

$$\nu = 2.05 Re^{0.36} Pr^{1/3}$$  \hspace{1cm} (6.13)

and for turbulent flow

$$\nu = 1.78 Re^{0.41} Pr^{1/3}$$  \hspace{1cm} (6.14)

The friction number is also expected to be a function of Reynolds only, which dependence can be written as

$$\xi = D_1 Re^d$$  \hspace{1cm} (6.15)
using the same procedure as in the case of the Reynolds number this leads to
\[
\xi = 8.46 \, \text{Re}^{-0.23}
\] (6.16)
for laminar flow and
\[
\xi = 9.42 \, \text{Re}^{-0.25}
\] (6.17)
for turbulent flow.
The Nusselt numbers correlations deviate about 6% for laminar flow and about 3% for turbulent flow, compared to the numerical results. The correlations for the friction factor have a maximum deviation of 15%. These results were used to calculate the performance of the complete cooling unit. Since the TEC cooler is built of fins in longitudinal direction, the fin spacing is not constant but depends on the radial position. To translate the results for the two-dimensional modules, it is supposed that the performance of the system can be calculated by taking the average conditions at \( r = 1/2 \, (d_{\text{foot}}+d_{\text{top}}) \).

In figure 6.29 it is shown that the length of the system has only a marginal influence on the system performance. In Kivits (1992) a design is made for a cooling system for the prototype thermionic converter. The heat pipe in this prototype is nearly that proposed for the new heat pipe, but it was designed for a thermal power of 1500 W. Kivits proposed a cooling unit length of 90 mm, 6 modules in row with a fin height of 15 mm. According to his calculations a 1500 W can be transferred at a mass flow of 6.75 \times 10^{-3} \, \text{kgs}^{-1}, resulting in an outlet temperature of 521 K (table 6.4). For the new design, it is supposed that 5 modules in row, a length of 45 mm, are sufficient. A sketch of the layout is given in figure 6.30.

### Table 6.4: Performance of the Cooling Unit Calculated by Kivits (1992)

<table>
<thead>
<tr>
<th>( M ) [kgs(^{-1})]</th>
<th>( Q ) [W]</th>
<th>( T_{\text{out}} ) [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00 \times 10^{-3}</td>
<td>1086</td>
<td>645.1</td>
</tr>
<tr>
<td>4.50 \times 10^{-3}</td>
<td>1321</td>
<td>580</td>
</tr>
<tr>
<td>5.25 \times 10^{-3}</td>
<td>1414</td>
<td>557</td>
</tr>
<tr>
<td>6.00 \times 10^{-3}</td>
<td>1496</td>
<td>538</td>
</tr>
<tr>
<td>6.75 \times 10^{-3}</td>
<td>1570</td>
<td>522</td>
</tr>
<tr>
<td>7.50 \times 10^{-3}</td>
<td>1636</td>
<td>507</td>
</tr>
</tbody>
</table>

**Figure 6.29:** Performance ranking for systems with a spacing of 20 mm, a fin thickness of 1 mm, a fin length of 5 mm and a fin height of 15 mm and a length of 60, 75 and 105 mm.
figure 6.30 sketch of the new TEC-design
7 Discussion

The TEC design presented in the previous chapter can be regarded as the main conclusion of this study. In this chapter the methods used are reviewed and commented upon.

7.1 The design process

In general a design process can be split in the elements sketched in figure 7.1. In situations as encountered in this study, when there is no experience with predecessors and competing designs, a large effort must be made during the pre-investigation to state the list of demands correctly. Fortunately it was possible to use the tools which were developed to state the list of demands also in the design stage itself.

<table>
<thead>
<tr>
<th>actions</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>question</td>
<td>list of demands</td>
</tr>
<tr>
<td>pre-investigation</td>
<td>function structure</td>
</tr>
<tr>
<td>assessing the functions needed</td>
<td>principle</td>
</tr>
<tr>
<td>generation of solutions for the functions</td>
<td>modular structure</td>
</tr>
<tr>
<td>division of the functions in modules</td>
<td>final design</td>
</tr>
<tr>
<td>design of the complete product</td>
<td></td>
</tr>
</tbody>
</table>

figure 7.1 diagram of the design process

About the manner in which the design process should be approached no consensus exists; the two extremes being the intuitive- and the methodical design school. In this study a methodical style is adopted, within a frame work, however, that is based on more intuitive or practical considerations. Often these considerations can be retraced to time and resources, which are an integral part of every design process. The main choices that are made this way are that only thermionic processes that have been practically used are considered and that new construction materials are not developed or investigated (§1.2.2).

Apart from these limitations, further restrictions were placed to the search area which were not investigated into detail, but are supposed to be plausible. Most examples are encountered in the system analysis:
- a high temperature recuperative burner that satisfies the legal norms can be built (§5.1.5)
- in order to convert direct current to alternating current, a minimum voltage of 3 V is necessary (§5.1.8)
- the electrical grid can be used as electrical storage (§5.1.9)
- only rotationally symmetric emitter shapes are considered (§6.1)
- a radiative burner for a spherical TEC is considered easier to construct than for a cylindrical TEC(§6.2)

Nevertheless, it is an objective that the general approach of this study can be used within other frame works than this specific one.

7.2 Design tools

Characteristic of a design study is that the model development goes as far as necessary to answer a specific question in the design process. In a system analysis, for instance, a more simple model for the energy flows can be used than during the final design stage. The design tools can be classified into two groups. The first group consists of models used to optimise the design, in this case the models describing the system efficiency. The second group is formed by the models that set limitations to the search area. The models describing the mechanical behaviour of the construction belong to the second group.

The Rasor-McVey model used to describe the thermionic emission is widely used and well checked for the operation area of the thermionic energy converter. The same cannot be said of the Mironov-Sidelnicov model, but where applied during this study, the deviations between both models were small. Both models were used to describe the total electron emission of an emitter which is not at a constant temperature. As the process is essentially one dimensional, this is not expected to generate problems. The experiments performed with the prototype hot shell caused no reason to distrust this assumption.

The models describing the other heat flows are all standard. The points where errors may be introduced are in the coupling of the models into the computer program for the thermionic energy converter and in the assumptions for the workfunctions. Fortunately models based on the same principles were developed by Veltkamp (1993) and Roemen (1995) allowing that the computer programs could be debugged by comparing the results. The results of the model agree with the experiments which were performed. Unfortunately, power production in the ignited mode could not be tested.

It is a different matter with the dynamical description of the heat pipe. Literature about the dynamical behaviour uses detailed descriptions of the vapour and liquid flow. The effort it would take to develop such a model is not in balance with the importance and desired accuracy of the start-up model. For this reason it was tried to develop design rules for the thermionic converter in a less complex fashion. The nature of the assumptions invoked is such that these models had to be compared to experiments. Fortunately this is relatively simple in the case of a heat pipe.

The models describing the mechanical behaviour of the construction are not as elaborate as are the energy models. Further investigations were not useful because either material data were insufficient to describe a process in detail, as in the case of the creep calculations, or because a detailed model was not necessary to make a particular choice, as in the case of the elastic deformation of the three emitter shapes considered. The stresses which can occur in the hot shell and the ceramic seal were investigated in more detail, using a finite element method, as they were known to be important for the integrity of the construction. The major flaw in the set of mechanical energy models is that the effect of thermocycling is not considered.
7.3 The thermionic energy converter

One of the main conclusions of this work emerged naturally while dimensioning the system: it is very important to design the system for the required load. If this is not done, the resulting system efficiency will be very low.

When the demands on the thermionic converter are known, the design of the hot shell should be based on the strength of the ceramic part of the hot shell. The thickness of the ceramic layer must be chosen minimal in order to reduce conduction of heat through the ceramic layer. When the thickness of the ceramic layer is assessed, the other geometrical variables can be optimised with respect to the system efficiency.

A system based on a TEC operating in the close spaced mode, will be a factor 10 larger compared to a system based on a TEC operating in the ignited mode for the same power output. The advantage of the lower emitter temperature is relatively small as the creep behaviour of metals is already intolerable at 1400 K. Consequently, the ceramic layer of the hot shell must be regarded as the structural part and the behaviour of ceramics like SiC does not differ greatly between 1400 and 1723 K. A TEC operating in the ignited mode is the better choice.

The CVD-SiC as being used in the current hot shells is a good choice for the construction material, though it was not originally selected for that reason. There may be competitors, like pyrolytic graphite, but information about their behaviour and processing possibilities is scarce. Considering the experience we have with CVD-SiC, it is the obvious choice within the frame of this project.

By regarding the ceramic as the structural element, the demands on the metal part of the hot shell are lessened, which means that other materials can be used than tungsten or molybdenum. The essential electrode surface requirements are satisfied by a wide range of materials. This versatility arises because the work functions tend to be determined by the adsorbed cesium layer. Surfaces with a bare work function between 4.5 and 5.5 eV can satisfy the requirements of the ignited mode converter (Rasor 1993), which means that we are restricted to refractory metals or expensive metals like platinum. Advanced emission materials are not considered here as they are still in the development phase. There is no reason to change from molybdenum as the emitter material, as it is relatively cheap and can be formed.

Concerning alignment, strength and creep, a flat plate performs considerably less as compared to a cylindrically or spherically shaped emitter. The choice between a cylinder and a sphere depends mainly on the construction of the burner, which is not considered here. It is supposed that it is easier to radiate to six spherical converters than to six cylindrical converters.

For cooling of the collector a heat pipe is used. The only point of concern is the time needed to start the heat pipe. Theoretical and experimental investigations learned that, contrary to the expectations at the start of the work, the start-up of the heat pipe is governed by evaporation and condensation processes even at the very low vapour pressures during start-up. The design of the heat pipe construction should therefore be based on strength criteria, minimising the heat capacity. The heat pipe takes a relatively long time to start, but fortunately the converter can already produce electricity before the heat pipe is started completely.

The last step in the cooling process is transferring the heat to an air stream. In designing the fins of the heat pipe, an optimum must be found between raising the temperature of the air...
stream, which mainly depends on the available heat transferring surface and the pumping power required. In order to balance heat duty versus pumping power, the performance ranking method developed by Soland (1978) was used.

Summarising, it can be stated that the main flaw of the old TEC design is the dimensioning. The process, shape and materials are generally well chosen.

The calculated electrical efficiencies of about 8% for the thermionic converter are not high. Though a thermionic converter may give a yearly profit, it is probably too low for commercialisation of the apparatus. The best way to enhance the economical profit is by achieving a higher electrode efficiency. In Rasor (1993) it is stated that the way to achieve a better efficiency is by reducing the collector work function. Nevertheless, attempts to improve the back voltage by lowering the collector work function have been unsuccessful so far. The back voltage governing the performance of practical converters is still 0.5 eV higher than what ought to be achieved. It is suggested that negative cesium ion emission is responsible for the unexplained process that prevents the improvement of converter performance. Considering the importance to progress in terrestrial thermionic conversion, investigation of this problem should have a high priority.

7.4 Conclusions

The main conclusion of this work consists of the design of the new TEC itself. Apart from the design, the design process led to the following conclusions:

- It is insufficient to look at thermionic process only in evaluating the performance of a thermionic converter. The heat fluxes through the complete thermionic converter must be taken into account.
- Dimensioning the thermionic converter should be done by evaluating the efficiency of the complete system. The start-up time of the converter and the number of starts and stops of the system have a large influence on the amount of thermionic power that can be applied.
- The assumptions made for the heating demand as a function of the time have a large influence on the design of the converter.
- The thermionic heating system must comprise a heat storage and a recuperative burner in order to reach reasonable efficiencies.
- The system efficiency is rather sensitive to the emitter temperature, collector temperature, cesium reservoir temperature, interelectrode distance, dimensions of the hot shell, size of the thermal storage and the start-up time of the thermionic converter.
- The main flaw of the old TEC design is the dimensioning. The process, shape and materials are generally well chosen.
- Assessing the structural integrity of the hot shell can be done by looking at the ceramic coating only. The creep rate of molybdenum is too high to rely on this material.
- Ignited mode operation is the most viable option considering the system dimensions.
- A spherical and a cylindrical TEC perform comparable, a choice between these shapes must be based on the system performance.
7.5 Recommendations

Terrestrial thermionic energy conversion can only become economically attractive when the efficiency of the thermionic process can be raised by at least a factor 1.5. According to Rasor (1993) the first problem which must be solved is reducing the collector work function. This is proven difficult and is so even more because there are strong indications that either our basic understanding of the role of the collector or the well established methods to measure the collector work function are in error, or both. These problems however are more in the field of physics than in the field of mechanical engineering.

If the efficiency problem is solved, the general approach to a new apparatus can be as is outlined in this study. It would be advantageous to have more information about the behaviour of the materials used, especially for the creep calculations. Furthermore the stress calculations for the hot shell can be incorporated in the model calculating the energy flows. The procedure followed in this study where data were transferred between two programs is rather cumbersome. As is mentioned earlier, the behaviour of the construction under thermocycling is not investigated in this report, but it might be a failure mode.

It was quite amazing to establish that no good design tools exist for systems and technologies such as finned cooling systems, heat pipes and brazing which are nevertheless widely used in engineering. A lot of useful work can be done in these areas.

Heat exchangers are encountered everywhere in energy technology and in quite a number of situations these heat exchangers contain fins of some sort. Nevertheless, the models describing the influence of fins on the performance of the heat exchanger, both in heat duty and pressure drop are very primitive. It would facilitate the design of a heat exchanger enormously if a model would exist which can assess the influence of the fin pattern and geometry and which can balance the pressure drop against the heat duty. For high temperature applications it is a great assessment if a model incorporating radiative heat transfer is developed.

Heat pipes are applied in a large variety of systems, ranging from electronics cooling to heat exchangers to solar energy systems and measuring equipment. In quite a number of applications the dynamic start-up behaviour is important. Of course the transient behaviour of a heat pipe can be modelled using advanced numerical modelling, but that is hardly of use for the designer unless the heat pipe is his main concern. Generalisation of simple guidelines as being used here are a valuable asset when designing a heat pipe for a specific application.

High temperature brazing is an advanced joining technology which results in joints of high quality. With the advance of super alloys and ceramic materials this technology is used more frequently. While assessing the structural qualities of a joint produced by brazing the same problem is encountered as in the case of the heat pipe, it must be done either by experiments or by expensive computer models. General guide lines are welcome.

With the present state of the art, thermionic energy conversion is not a very convincing means of energy conversion. Nevertheless, it is still a promising technology if the claims which are made by several groups can be accredited. Work on the mechanical problems which occur in the construction of a TEC will continue anyway as they are common for most high temperature applications. In the past century thermionic energy conversion has been on and off the research hot list several times and has had a large off spring.
A Thermionic emission

A.1 Under idealised conditions

A metal surface at a temperature above 0 K will emit charged particles, like electrons or ions, when surrounded by a rarefied vapour. Normal metals hardly emit ions, except when they are covered by a liquid metal like cesium. If no provisions are made, the particles will return to the substrate. By collecting them on a different electrode, the net current may be different from zero. In first approximation (the Sommerveld gas approximation), we regard the metal lattice as a box in which the electrons can move freely. According to the rules of quantum mechanics, the number of electrons with an energy between \( \varepsilon \) and \( \varepsilon + d\varepsilon \) is given by Fermi-Dirac statistics and is proportional to the Fermi factor \( v(\varepsilon) \):

\[
v(\varepsilon) = \frac{1}{1 + \exp \frac{\varepsilon - \mu}{kT}}
\]

(A.1)

where \( k \) is Boltzmann’s constant and \( \mu \) is a quantity characteristic of the material which has the dimensions of energy and is numerically equal to the value of \( \varepsilon \) for which the \( v(\varepsilon) = 0.5 \). For \( T = 0 \), \( \mu \) coincides with the highest energy of the electrons, called the Fermi energy. In this report \( \mu \) will be called the Fermi energy regardless of the temperature. The number of electrons incident on the surface of material per unit time \( N(\varepsilon)d\varepsilon \) can be calculated using an approximate equation derived by Fowler (1955):

\[
N(\varepsilon)d\varepsilon = \frac{4\pi m_e kT}{h^3} \exp \left( -\frac{\varepsilon - \mu}{kT} \right)
\]

(A.2)

where \( m_e \) is the mass of an electron and \( h \) is Planck’s constant. Integration of expression (A.2) over all energies yields the total number of electrons that arrive at the metal-vapour interface. At this interface an electron experiences a decelerating force, determining whether the electron will be emitted or not. The force on an electron in the interelectrode space can be determined by the electron motive, a quantity introduced by Langmuir (1923), defined as a scalar \( \psi \) whose negative gradient equals the force exerted on an electron. In figure A.1 the interelectrode motive of an electron in the vicinity of an interface is depicted. In the metal the motive can be regarded constant and the dipole forces, due to the charge asymmetry at the top layer of the surface, lead to an increase near the border. When the electron is about 10 Å away from the surface, the dipole forces cease to influence the electron. Beyond the scale of atomic roughness, the electron interacts with the whole surface and experiences a so-called image force (figure A.2). Theoretically the electron will always experience the image force, but practically the influence of the image force is very small at a distance over 500 Å. Beyond that point the interelectrode motive is nearly constant unless there is an external electrical field present.
The electron saturation current is the current found at the points just outside the electrode in the absence of a strong electrical field. We will consider the two electrodes of figure 1.1 in such a state the emitter and collector are isothermal and that the applied electrical field is such that the force on an electron is zero just outside the emitter, the zero field steady state. The value of the electron saturation current can be found by integrating \( N(e) \) from \( \psi_1 \) to \( \infty \)

\[
J_{es} = \frac{4\pi m k^2 e}{h^3} T^2 \exp \left( -\frac{\psi_1 - \mu}{kT} \right)
\]

\( e \) being the charge of an electron. This equation is known as the Richardson-Dushman equation. Introducing the work function, defined as \( \phi = \psi_1 - \mu \) and the Richardson constant \( A = \left(4\pi m k^2 e^2\right)/h^3 \) we can write equation (A.3) in its more familiar form

\[
J_{es} = A T^2 \exp \left( -\frac{\phi}{kT} \right)
\]

When applying an external accelerating electrical field, the thermionic emission will increase, which is known as the Schottky effect (figure A.3). We can see that the maximum interelectrode motive \( \psi_m \) is smaller than \( \psi_1 \) at a place \( x_m \) smaller than \( x_1 \). In the area between 0 and \( x_m \), the interelectrode motive is the sum of the motive in zero field state and the motive resulting from the external electrical field \( \psi_a \). \( \psi_a \) decreases along the distance according to
A Thermionic emission

\[ \psi_a = -eEx \]  

in which \( E \) is the electrical field. \( x_m \) is at the point where the gradient of \( \psi_a \) is equal but opposite to the gradient of \( \psi_1 \). In practical situations this point depends on the image force

\[ \frac{e^2}{16\pi\varepsilon_0 x_m^2} = eE \]  

where \( \varepsilon_0 \) is the energy of a particle at infinity. The following expression can be derived for the difference in the interelectrode motive

\[ \Delta \psi = \psi_1 - \psi_m = e\sqrt{\frac{eE}{16\pi\varepsilon_0}} \]  

The steady state current is given by equation (A.3) with \( \psi_1 \) replaced by \( \psi_m \).

Normal metals hardly emit any ions. Ion emission can be enhanced however in the case of a rarefied metal vapour in contact with a refractory metal surface. The analysis of ion emission is similar to that of electron emission. The ion saturation current, defined analogous to the electron saturation current is given by (Rasor 1982)

\[ I_{IS} = \frac{e\mu}{1+2\exp\left(\frac{V_{eff}-\phi_H}{kT}\right)} \]  

\( V_{eff} \) is the effective ionisation energy of the gas under consideration and \( \phi_H \) the neutral work function. \( \mu \) is the rate of arrival of the atoms from the vapour to the surface.

The electron motive in a thermionic energy converter under ideal circumstances is governed by the Laplace equation

\[ \frac{d^2\psi}{dx^2} = 0 \]  

the solution for the boundary conditions \( \psi(0)=\psi_E \) and \( \psi(d_{gap})=\psi_C \) is

\[ \psi(x) = \frac{\psi_E + \psi_C}{2} + \frac{\psi_E - \psi_C}{2} \left(1 - \frac{\cosh\left(\frac{x}{d_{gap}}\right)}{\cosh(1)}\right) \]
\[ \psi = \psi_C + \left( \psi_E - \psi_C \frac{1-x}{d} \right) \]  

(A.10)

The difference in electron motive at the emitter- and at the collector side, divided by the elementary charge is the internal voltage drop over the converter. The net current delivered equals the emitter-collector current \( I_{EC} \) minus the collector-emitter current \( I_{CE} \) minus the ion current \( I_{iEC} \). In first approximation we suppose that the collector-emitter and the ion current can be neglected. If the emitter motive is larger than the collector motive, which means that we can write for the output voltage \( eV \leq \phi_E - \phi_C \), all electrons emitted from the emitter will reach the collector and the current is given by equation A.4. In the case that the output voltage exceeds the difference in work function between emitter and collector, \( eV \geq \phi_E - \phi_C \), not all electrons emitted have sufficient energy to reach the collector and some will return to the emitter.

Electrons emitted from a hot surface have a variety of speeds that differ both in magnitude and direction. It follows from Fermi-Dirac statistics that the distribution of the velocities is to a good approximation described by a half-Maxwellian distribution function. For a half-Maxwellian distribution the fraction of electrons that will reach the collector can be written as

\[ \zeta = \exp \left( - \frac{\psi_C - \psi_E}{kT} \right) = \exp \left( - \frac{\phi_E - \phi_C + eV}{kT} \right) \]  

(A.11)

It follows that the emitter current \( I_{EC} \) is given by the relation

\[ I_{EC} = I_{ES} = AT_E^2 \exp \left( - \frac{\phi_E}{kT} \right) \text{ for } eV \leq \phi_E - \phi_C \]  

(A.12)

and

\[ I_{EC} = \zeta I_{ES} = AT_E^2 \exp \left( - \frac{\phi_E + eV}{kT} \right) \text{ for } eV \geq \phi_E - \phi_C \]  

(A.13)

The same kind of reasoning can be applied to the collector-emitter current which leads to the following set of relations for the idealised output current of a TEC

\[ I_{CE} = f I_{CS} = AT_C^2 \exp \left( - \frac{\phi_C + eV}{kT_C} \right) \text{ for } eV \leq \phi_E - \phi_C \]  

(A.14)

and

\[ I_{CE} = I_{CS} = AT_C^2 \exp \left( - \frac{\phi_C}{kT_C} \right) \text{ for } eV \geq \phi_E - \phi_C \]  

(A.15)

The combined effects of equation A.12 to A.15 are shown in figure 2.1
A.2 In the close spaced mode

An approximation for the critical current, dividing the space charge mode and the retarding mode is (Hatsopoulos 1973)

\[ I_R = 9.664 \times 10^{-2} \frac{(kT_e)^{3/2}}{d^2} \]  

(A.16)

the analysis used is valid for currents in the space charge limited range, between the emitter saturation current \( I_{ES} \) and \( I_R \). It can be seen from figure 2.3 that the interelectrode motive has a maximum, \( \psi_{max} \), somewhere between the two electrodes. For negligible back emission, the output current density, \( I \), is only a fraction \( f_E \) of the emitter saturation current density \( I_{ES} \). This fraction is related to the motive barrier, \( \psi_{max} - \psi_E \), by the expression:

\[ q_E = \exp \left( -\frac{\psi_{max} - \psi_E}{kT_E} \right) \]  

(A.17)

hence the output current density is given by

\[ I = q_E I_{ES} \]  

(A.18)

The emitter saturation current is determined by the Richardson equation

\[ I_{ES} = AT^2 \exp \left( -\frac{\phi_E}{kT} \right) \]  

(A.19)

The corresponding output voltage, \( V \), can be written as

\[ eV = (\phi_E + (\psi_{max} \pm \psi_E)) \pm (\phi_C + (\psi_{max} \pm \psi_C)) \]  

(A.20)

The output characteristic is determined if the \( \psi_{max} - \psi_E \) and \( \psi_{max} - \psi_C \) are known.

To achieve this, Langmuir defines the dimensionless motive barriers \( \gamma_E \) and \( \gamma_C \) and the dimensionless distances of the maximum motive \( \xi_E \) and \( \xi_C \) for the emitter and the collector respectively:

\[ \gamma_E = \frac{\psi_{max} - \psi_E}{kT_E} \]  

(A.21)

\[ \gamma_C = \frac{\psi_{max} - \psi_C}{kT_E} \]  

(A.22)

\[ \xi_E = \frac{x_E}{x_0} \]  

(A.23)

\[ \xi_C = \frac{x_C}{x_0} \]  

(A.24)

where \( x_E \) and \( x_C \) are the distance from the emitter, respectively the collector from the location of the maximum motive. \( x_0 \) is a normalisation distance defined as

\[ x_0 = 1.09 \times 10^{16} \frac{T_E}{I^{1/2}} \]  

(A.25)
Langmuir shows that $\gamma_E$ is related to $\xi_E$ and that $\gamma_C$ is related to $\xi_C$ through two universal relation which are graphically shown in the figures A.4 and A.5.

The output current characteristic of a close spaced diode can now be calculated according to the following procedure.

1. calculate the emitter saturation current according to equation (A.19)
2. calculate $J_R$ according to equation (A.16)
3. choose an output current density such that $J_{ES} > J > J_r$
4. compute $\gamma_E$ from expression (A.21)
5. find the value of $\xi_E$ from figure A.4
6. compute $\xi_C$ from expression (A.24) and (A.25)

$$\xi_C = \frac{d_{gap}}{x_0} \cdot \xi_E$$  \hspace{1cm} (A.26)

7. find the value of $\gamma_C$ from figure A.5
8. given the collector work function $\phi_C$ compute the output voltage according to expression (A.20)
A.3 In the ignited mode

A.3.1 Obstructed mode and transition point

In the obstructed mode, the potential drop over the interelectrode space is too small to maintain the plasma. The discharge can exist by erecting a space charge $\Delta V$ at the emitter. The output voltage in the obstructed region can be written as

$$V = \phi_E \pm \phi_C \pm V_d + \Delta V$$  \hspace{1cm} (A.27)

The electrons appear to be emitted by a virtual emitter with a work function of $\phi_E + \Delta V$. For every point in an equilibrium (Maxwellian) plasma, the electron velocity distribution is spherical about the origin for zero drift. The random current $J_r$ is defined as the integral of all current in one direction. If a drift occurs in the $x$ direction, the centre of the distribution is shifted to give an excess current of $J/2$ in the positive $x$ direction and of $-J/2$ in the negative $x$ direction. The emitter sheath $V_E$ reflects the electrons from the plasma. At the edge of the plasma, near the emitter, the current into the plasma $J_+$ is the sum of the emitted current $J_E$ and the fraction of the current in the plasma $J$ that is reflected by the sheath

$$J_+ = J_E + \frac{J}{2} = J_E + \left( J_{rE} \pm \frac{J}{2} \right) \left( 1 \pm \exp \left( \frac{\pm V_E}{kT_eE} \right) \right)$$  \hspace{1cm} (A.28)

where $T_{eE}$ is the electron temperature and $J_{rE}$ the random current near the emitter. Similar considerations for the collector yields

$$J_- = J_C \pm \frac{J}{2} = J_C + \left( J_{rC} \pm \frac{J}{2} \right) \left( 1 \pm \exp \left( \frac{\pm V_C}{kT_eC} \right) \right)$$  \hspace{1cm} (A.29)

where $T_{eC}$ is the electron temperature and $J_{rC}$ the random current near the emitter. Disregarding the electrical field due to the current $J$, the electron current through the plasma gives rise to a density gradient according to Fick’s law $J/e = \pm D \nabla n$

$$\frac{J}{e} = \frac{v \lambda_e}{3} \frac{\partial n}{\partial x}$$  \hspace{1cm} (A.30)

$n$ is the density, $v$ the mean velocity and $\lambda_e$ the mean free path of the electrons. Integration of (A.30) over the interelectrode distance $d$ yields

$$n_E - n_C = \frac{3J}{ev\lambda_e} d$$  \hspace{1cm} (A.31)

noting that $J_{rC} = en_{C}v/4$ and $J_{rE} = en_{E}v/4$ equations (A.28), (A.29) and (A.31) can be combined to

$$J_E - J = \exp \left( \frac{V_E}{kT_eE} \left( \frac{3}{4} \frac{d}{\lambda_e} J + R J \right) \right)$$  \hspace{1cm} (A.32)

where

$$R = \left( 1 + \frac{J}{I} \right) \exp \left( \frac{V_C}{kT_eC} \right) - 1$$

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The value of $R$ is constant and equal to 4.5 in absence of local thermal equilibrium constraints as will be discussed below. As a plasma is quasi-neutral, the rate at which ions are generated must equal the rate at which they are lost due to recombination and diffusion to the electrodes. The rate at which ions are produced is according to Rasor (1982)

$$I_i = K_1 J_r \frac{d}{T_e} \exp \left( -\frac{V_{\text{eff}}}{kT_e} \right)$$  \hspace{1cm} (A.33)

$K_1$ is a dimensionless constant including quantities like the ionisation cross section weakly dependent on the electron temperature $T_e$. $V_{\text{eff}}$ is the effective ionisation energy. $J_r$ is the mean electron random current. Ion production is strongly dependent on $T_e$, therefore it is stated that most of the ions are generated at the emitter side and that the effective electron temperature for ion production equals the emitter temperature. When volume ion production is negligible, we can write for the ion current that diffuses from such a plasma (Langmuir 1923)

$$I_i = K_d J_r \frac{\lambda_i}{d}$$  \hspace{1cm} (A.34)

$K_d$ is a dimensionless constant and $\lambda_i$ the mean free path for ion diffusion. Equations (A.33) and (A.34) can be combined to

$$T_{eE} = \frac{V_i}{2k} \ln \left( \frac{Bd}{\lambda_{e}} \right)$$  \hspace{1cm} (A.35)

with

$$B = \sqrt{\frac{I_{eC} K_1 \lambda_i}{I_{riC} K_d \lambda_e}}$$

$B$ is the dimensionless ionisability factor, a measure for ion production in the plasma and is essentially a constant. Quasi neutrality implies that the ratio between the electron random current and ion electron current is constant throughout the plasma. Equations (A.29) and (A.35) can therefore be combined into

$$V_C = kT_e \ln \left( \frac{H F + 1/2}{1 + I_C / J} \right)$$  \hspace{1cm} (A.36)

with $F = I_r \lambda_e / I$, $H = 0.5 K_D \beta \lambda_i / \lambda_e \approx 1.0 \lambda_i / \lambda_e$ The fraction of the total ion current which returns to the emitter is defined as $\beta = I_{iC} / I_i$.

Assuming that the total energy loss of the electrons to the plasma is negligible, the energy balance dictates that the energy of the electrons entering the interelectrode space equals the sum of the energy going out the interelectrode space by electrons returning to the emitter or reaching the collector, thus

$$JV_d + 2kT_e J_E + 2kT_e J_C = 2kT_e \{ J_e - J \} + 2kT_e \{ J_C + J \}$$  \hspace{1cm} (A.37)

Similarly the energy conducted through the plasma by the electrons must equal that removed on the collector side

$$2kT_e J \left( \frac{dT_e}{dx} \right)_{\text{mean}} = J \{ V_C + 2kT_e \} + 2k \{ T_e - T_C \} I_C$$  \hspace{1cm} (A.38)
$T_e$ being the mean electron temperature $(T_{eE} + T_{eC})/2$, $\lambda_t$ the mean thermal conduction coefficient of the vapour and $(dT_e / dx)_{\text{mean}} = (T_{eE} - T_{eC})/2$ the mean electron gradient. With these approximations we can write for (A.36)

$$V_C = k(T_{eE} - T_{eC})(1 + 2F) - 2k(T_{eC} - T_C)J_C/J$$

(A.39)

The total power $JV_d$ dissipated in the plasma is equal to the sum of the power dissipated in individual collisions. In a homogeneous plasma the total number of collisions on micro scale equals the number of collisions on macro scale

$$J_rV_d\lambda_e/d = J_r\lambda = F = 1$$

(A.40)

Equations (A.32) to (A.40) give for the obstructed region and transition point

$$J_E/J = 1 + \frac{3}{4} \frac{d_{\text{gph}} + R}{\lambda_e} \exp \left( - \frac{V_E}{kT_{eE}} \right)$$

(A.41)

$$V_d = 2k(T_{eE} \pm T_E) \left( \frac{J_E}{J} \pm 1 \right) + 2k(T_{eC} \pm T_E) \frac{J_C}{J}$$

(A.42)

$$V_C = 3k(T_{eE} \pm T_{eC}) \pm 2k(T_{eC} \pm T_C) \frac{J_C}{J}$$

(A.43)

$$V_E = V_d + V_C$$

(A.44)

$$T_{eC} = \ln \left( \frac{3T_{eE} + 2T_C J_C}{1 + J_C/2} \right) + 2J_C/3$$

(A.45)

$$T_{eE} = \frac{V_{\text{eff}}}{2k \ln \left( \frac{B \nu_{\text{eff}}}{\lambda_{\text{eff}}} \right)}$$

(A.46)

$\lambda_{\text{eq}}$ is the mean free path for electron-neutral collisions. A plasma cannot exceed the density at which as many electrons combine locally as are produced locally. In this condition, known as local thermodynamic equilibrium (LTE), the plasma properties are equal to those which would exist in equilibrium with a hypothetical surface at the electron temperature $T_e$ and emitting a neutral plasma. The electron emission current $J_n$ that can be neutralised by an ion emission current is

$$J_n = \alpha J_{is}$$

(A.47)

where $\alpha$ is defined as $\alpha = J_r / J_{is} = v_e / v_i = \sqrt{m_i / m_e}$, where $m_i$ is the ion mass. The ion emission current is given by the Saha-Langmuir equation (A.8) where $\phi_n$, the work function of an emitter emitting a neutral plasma is defined by the Richardson equation:

$$J_n = A T_{eE}^2 \exp \left( - \frac{\phi_n}{kT_{eE}} \right)$$

(A.48)
In an actual converter, LTE occurs first at the collector side, where the electrons are coldest and the spreads gradually through the interelectrode gap. This process can be approximated by using a separate LTE constraint at the collector side:

\[ J_{nc} = AT_{ec}^2 \exp \left( \frac{-\phi_{nc}}{kT_{sc}} \right) \]  

(A.49)

The temperature \( T_{sc} \) can be found by replacing \( H \) in equation (A.45) by

\[ H_s = \frac{J_{nc}}{J} \]  

(A.50)

\[ T_{sc} = \frac{3T_{eE} + 2T_{nc}}{ln \left( \frac{H_s + 1/2}{1 + J_{nc}} \right) + 2J_{nc} + 3} \]  

(A.51)

LTE requires that \( T_{ec} \geq T_{sc} \).

A.3.2 Saturation mode

In the saturation mode, the arc drop \( V_d \) over the plasma is greater than the arc drop \( V_d' \) required to produce the ions necessary to neutralise the plasma. The excess energy cannot be transferred to the plasma due to quasi neutrality. Experimental results (Hatsopoulos 1973) show that the excess energy \( \Delta V = V_d - V_d' \) is dissipated in the emitter sheath. The extra ions produced, \( J_{ix} \), immediately return to the emitter. Adding this term to the ion conservation terms leads to an equation for the emitter-side temperature

\[ T_{eE} = \frac{V_{ieff}}{2k \ln \left( B \frac{d}{2a} \right) \pm k \ln \left( 1 \pm B \frac{J_{ix}}{J} \right) } \]  

(A.52)

The parameter \( B' \) is a constant. Continuity of current gives

\[ \frac{J_E}{J} = 1 + \left( \frac{3}{4} \frac{d_{gap}}{d} + R \pm \frac{1}{2} \frac{J_{ix}}{J} \right) \exp \left( -\frac{V_{E}}{kT_{eE}} \right) \]  

(A.53)

where \( R \) is given by equation (A.32). Energy conservation leads to

\[ V_d = 2k(T_{ec} \pm T_E) \left( \frac{J_{Es}}{J} \pm 1 \right) + 2k(T_{ec} \pm T_E) \frac{J_{C}}{J} \pm (V_{C} + V_{ieff} + 2kT_{eE}) \frac{1}{1 + \frac{J_{ix}}{J}} \] \  

(A.54)

The large ion current at the emitter reduces the work function due to the Schottky effect. Hansen (1967) derived the following equation for the effective saturation current

\[ J_{ES} = J_{ES}' \left( 1 - \frac{\Delta V}{V_{ieff}} \right) \exp \left( \frac{G}{T_E} - \frac{\Delta V}{V_{ieff}} \right) \]  

(A.55)

\( G \) is a constant.
A.3.3 Implementation in the computer model

The computer model calculates the output voltage for a given current and starts with calculating the emitter work function according to Rasor (1964). The formulation of Rasor is not valid at the high cesium coverage characteristic of the collector. Experimental data indicate that the collector work function is only dependent on the ratio $T_C / T_r$, where $T_r$ denotes the cesium reservoir temperature (Hatsopoulos 1973).

$$\phi_C = 4.6 \pm 3.4 \frac{T_C}{T_r} + 0.94 \left(\frac{T_C}{T_r}\right)^2$$  \hspace{1cm} (A.56)

Subsequently the emitter and collector saturation current densities are calculated according to the Richardson equation (A.4). An empirical relation (Hatsopoulos 1973) is used for the cesium pressure and the mean free path of the electrons

$$p_{cs} = 2.45 \times 10^8 \exp \left(-\frac{-8910}{T_r}\right)$$  \hspace{1cm} \frac{1}{T_r^2} \hspace{1cm} (A.57)

$$\frac{\lambda_e}{d} = \frac{1}{127.8 p + 3.4 \times 10^8 f_0 / T_e^{-5/2}}$$  \hspace{1cm} (A.58)

Next, the collector back emission can be calculated according to equation (A.15). The average temperature of the heavy particles, the ions and the neutrals is $T_a = \frac{T_E + T_C}{2}$. If the calculated emitter saturation current is larger than the given current $J$, the calculation for the obstructed mode is started.

To calculate the ratio between the interelectrode gap and the electron-neutral mean free path McVey (1992) derived the following relation

$$\frac{d}{\lambda_{el}} = p_{cs} d \frac{5.26 \times 10^5}{T_a}$$  \hspace{1cm} (A.59)

for $p$ in Pa, $d_{gap}$ in m, $T$ in K and $f$ in Am$^{-2}$. The electron temperatures are subsequently calculated according to equations (A.45) and (A.46) with the values $H = 5$, $B = 30$ and $V_{eff} = 3.2$ eV for cesium (McVey 1992). For the rate of arrival of cesium atoms $\mu$ we can write (Rasor 1982)

$$\mu = D \exp \left(\frac{\pm h}{kT_e}\right)$$  \hspace{1cm} (A.60)

with $h=0.75$ eV and $D=10^{23}$ sm$^{-2}$. Combining equation (A.8) and (A.48) for $\phi_n \ll V_{eff}$

$$\phi_n = \frac{1}{2} \left( h \frac{T_E}{T_r} + V_{eff} \pm kT_E \ln \left( \frac{I_n E}{I_S} \right) \right)$$  \hspace{1cm} (A.61)

Since the logarithmic term varies less than 0.1 eV over the range of interest, it is allowable to approximate $\phi_n$ by a linear relation. With this estimation for $\phi_n$ the LTE current density $J_n$ can be calculated with equation (A.48). If $J_n / H < H$, the LTE limit must be imposed at the collector edge. In an iterative loop $T_eC$ is updated using equation (A.49) and (A.51). Subsequently the average electron temperature is calculated

$$T_e = T_{eE} + T_{eC}$$  \hspace{1cm} (A.62)
The ratio between the spacing and the electron-ion mean free path is given by (McVey 1992)

$$\frac{d_{gap}}{\lambda_{ei}} = 3.4 \times 10^{14} \frac{I d_{gap}}{T_{e}^{5/2}}$$ \hspace{1cm} (A.63)

The effective electron mean free path is given by

$$\frac{1}{\lambda} = \frac{1}{\lambda_{ea}} + \frac{1}{\lambda_{ei}}$$ \hspace{1cm} (A.64)

If the LTE limit was applied to the collector side, it is now checked that the average electron temperature is above the bulk LTE limit defined by equation (A.46) and (A.48). If so, the mean electron temperature and the mean free path are exchanged for their proper LTE values. $T_{eC}$ must then be updated as it depends on $T_{eE}$ which has changed to keep $T_{e}$ above the LTE limit.

Next the collector sheath can be found using equation (A.43) and the reflection factor $R$ can be calculated from equation (A.32). A guess is made for the value of $I_{e}/J$ and the arc drop is calculated from equation (A.42). The emitter sheath height is calculated from equation (A.45) and a new value for $I_{e}/J$ can be found from equation (A.41). This process is repeated until convergence is reached. The barrier height is given by

$$\Delta V = kT_{e}\ln\left(\frac{I_{eS}}{I_{e}}\right)$$ \hspace{1cm} (A.65)

if $I>I_{eS}$ or if $\Delta V<0$ we have to invoke the calculation for the saturation mode else we can compute the output voltage according to (A.27). The calculation for the saturation mode is similar to that for the obstructed mode, except that the modified equations (A.52) to (A.55) are used. $G$ is taken at 6150 K/(Am$^{-2}$)$^{-1/4}$ (Rasor 1982).

A.4 The Mironov-Sidelnicov model

As is discussed in §2.1.3., the Mironov-Sidelnicov (1990) model was used alongside the Rasor-McVey model, despite the lack of information on this model. For the sake of completeness, the equations used are given in this paragraph.

The effective output voltage is written as

$$V = \phi_{B} \pm \phi_{0} \pm V_{loss}$$ \hspace{1cm} (A.66)

where the Boltzmann work function $\phi_{B}$ is defined by the Richardson equation

$$\phi_{B} = \frac{k}{e} T_{E} \ln\left(\frac{T_{E}^{2}}{T_{B}}\right)$$ \hspace{1cm} (A.67)

the voltage losses are

$$V_{loss} = V_{0} + V_{1} + V_{2} + V_{3} \pm V_{4} + V_{5}$$ \hspace{1cm} (A.68)

$V_{0}$ is correlated from the TOR model $V_{0}=0.4$. The voltage loss $V_{1}$ is caused by a non optimal combination of the cesium reservoir temperature and interelectrode distance

$$V_{4} = a p_{cs} d_{gap} \left(1 + \frac{I}{80 \times 10^{4}}\right) + \left(\frac{0.27}{p_{cs} d_{gap}}\right)^{2}$$ \hspace{1cm} (A.69)

$a$ is a quantity depending on the uniformity of the emitter, in practical situations it has the value of 0.4.
The loss caused by emitter saturation is given by
\[ V = z + \sqrt{z^2 + 0.05^2} \]  
(A.70)
where \( z \) is defined as
\[ z = 0.1 \frac{1}{p_{cs} d_{gap}} + 0.4 \left( \sqrt{p_{cs} d_{gap} \ln[J_E + 1]} + 1 \frac{1}{J_E} \right) \]  
(A.71)
the emitter current is
\[ J_E = \exp \left( \frac{\phi_0}{0.2} \frac{T_E}{250} + \frac{\ln[p_{cs}]}{0.6} \pm 15.85 \times 10^4 \right) \]  
(A.72)
The plasma losses are given by \( V_3 \)
\[ V_3 = \frac{kT_E}{e} \ln \left( 1 + \frac{J_2}{J} \right) \]  
(A.73)
where \( J_2 \) is the so called knee current
\[ J_2 = \frac{10^{11}}{T_E \sqrt{p_{cs} d_{gap}}} \]  
(A.74)
a correction for the diffusion conditions is
\[ V_4 = \frac{kT_E \ln \left( 1 + \frac{J_2}{J_3} \right)}{1 + \left( \frac{J_2}{J_3} \right)} \]  
(A.75)
where the diffusion saturation current is defined as
\[ J_3 = \frac{5 \times 10^7 \sqrt{p_{cs} T_E^{3/4}} \exp \left( \frac{22500}{T_E} \right)}{1 + 20 p_{cs} d_{gap}} \]  
(A.76)
the voltage loss due to back emission from the collector is
\[ V_5 = \frac{kT_C}{e} \ln \left( 1 + \frac{J_2}{J} \left[ 1 + \frac{T_E + T_C}{p_{cs} d_{gap}} \frac{1}{T_C} \right] \right) \]  
(A.77)
in which the collector saturation current \( J_C \) is given by
\[ J_C = A T_C^2 \exp \left( \frac{e \phi_c}{kT_C} \right) \]  
(A.78)
B Stationary heat pipe behaviour

In §B.1 the calculation methods for the operating limitations of the heat pipe are discussed. The way the temperature drop over the heat pipe (§2.3.2) is calculated, is explained in §B.2.

B.1 The limitations

B.1.1 Capillary limitation

During steady state operation of a heat pipe, the vapour flows continuously from the evaporator to the condenser, creating a vapour pressure gradient along the axis of the heat pipe. Simultaneously the liquid flows from the condenser to the evaporator, creating a liquid pressure gradient. These pressures are only equal to each other at one point along the axis of the heat pipe. The pressure difference between the liquid and the vapour elsewhere in the heat pipe, the capillary pressure, is established by menisci that form at the interface where the liquid recedes into the wick. The pressure difference can be expressed as

\[ p_c(z) = p_c(z_{\text{ref}}) + \Delta p_v(z_{\text{ref}}) + \Delta p_l(z_{\text{ref}}) \]

where \( p_c(z) \) denotes the capillary pressure at point \( z \), \( z_{\text{ref}} \) a reference position, \( \Delta p_v \) the vapour pressure drop and \( \Delta p_l \) the liquid pressure drop. The liquid and vapour pressure drop can be found when choosing \( z_{\text{ref}} \) at the position where \( p_c \) equals zero. Equation (B.1) then reduces to

\[ p_c(z) = \Delta p_v(z) + \Delta p_l(z) \]

Integrating the liquid and vapour pressure drop leads to

\[ p_c(z) = \int_0^z \left( \frac{\text{dp}_v}{\text{dz}} \pm \frac{\text{dp}_l}{\text{dz}} \right) \text{dz} \]

and the maximum capillary pressure is

\[ p_{c,\text{max}} = \frac{2\sigma}{R_{\text{hs}}} \]

where \( \sigma \) is the surface tension and \( R_{\text{hs}} \) denotes the hydraulic radius of the wick surface pores. The effective maximum capillary pressure is reduced by gravitation according to

\[ p_{c,\text{max}} = \frac{2\sigma}{R_{\text{hs}}} \pm \rho_l g 2R_v \cos(\beta) \]

where \( \rho_l \) is the specific density of the fluid, \( g \) the gravitational acceleration, \( R_v \) the vapour core area, \( \rho_l \) the viscosity of the liquid and \( \beta \) the incline of the heat pipe to the horizontal. The wick will begin to dry out when the integral of equation (B.2) equals the maximum capillary pressure. For a conventional heat pipe the minimum capillary pressure occurs at the end of the condenser and the maximum at the end of the evaporator. The liquid pressure gradient of the fluid in steady state is determined by the frictional drag and the gravitational force. Darcy’s law gives for the liquid pressure drop

\[ \Delta p_l = F_l l_{\text{eff}} Q \]
$l_{\text{eff}}$ is the distance between the evaporation section and the condenser section, $Q$ the heat flux and the friction coefficient $F_l$ is given by

$$F_l = \frac{\eta_l}{K A_w L \rho_l}$$  \hspace{1cm} (B.7)

where $\eta_l$ is the dynamic viscosity of the liquid, $A_w$ the total cross sectional area of the wick, $L$ the latent heat and $\rho_l$ the specific mass of the liquid. The wick permeability $K$ is defined as

$$K = \frac{8 \varepsilon_w R_{hl}^2}{f_l \text{Re}_l}$$  \hspace{1cm} (B.8)

$\varepsilon_w$ denotes the porosity of the wick, $R_{hl}$ the hydraulic radius of the wick and $f_l$ is a friction factor. For a screen wick, the wick permeability can be written as

$$K = \frac{d^2 \omega \varepsilon_w^3}{122 (1+\varepsilon_w)^2}$$  \hspace{1cm} (B.9)

The vapour pressure gradient is given by

$$\frac{d p_v}{dz} = \pm F_v Q$$  \hspace{1cm} (B.10)

The vapour friction coefficient $F_v$ can be determined from the Hagen-Poiseuille equation, assuming a steady state laminar flow and an incompressible fluid through a circular cross section with radius $R_v$

$$F_v = \frac{f_v \text{Re}_v \eta_v}{2 \pi R_v^4 \rho_v L}$$  \hspace{1cm} (B.11)

where $L$ is the latent heat and $f_v$ the friction drag coefficient. Eq. B.11 holds when the flow is laminar ($\text{Re} < 2300$) and incompressible ($\text{Ma} < 0.2$). If the flow is laminar but $\text{Ma} > 0.2$ the ratio of the drag coefficient for compressible flow to that for incompressible flow can be correlated by (Dunn 1978)

$$\frac{f_{v,\text{ci}}}{f_{v,\text{li}}} = \left(1 + \frac{\text{Ma}_v}{2} \gamma_v \frac{\gamma_v^{1/2} - 1}{\gamma_v^{1/2} + 1} \right)^{1/2}$$  \hspace{1cm} (B.12)

where $\gamma_v$ is the vapour specific heat ratio. Dunn(1978) gives an empirical relation for the drag coefficient in turbulent incompressible flows

$$f_{v,ti} = \frac{0.0791}{\text{Re}_v^{0.25}} \quad \text{Re}>2300 \quad \text{Ma}<0.2$$  \hspace{1cm} (B.13)

the drag factor for turbulent incompressible flow is correlated to the drag coefficient for turbulent compressible flow by equation (B.12). The friction factors $F_l$ and $F_v$ are only constant when the flow is laminar and incompressible. In other cases the capillary limit has to be determined iteratively. Figure B.1 gives the capillary limit for the sodium heat pipe specified in table 2.1.
B.1.2 Viscous limitation

At low temperatures viscous forces are dominant in the vapour flow down the heat pipe. The heat flux increases as the pressure in the condenser is reduced, the maximum heat flux occurring when the pressure is reduced to zero. Poiseuille flow is assumed to determine the axial vapour pressure gradient and integration over the effective heat pipe length yields

\[ Q_{\text{max,visc}} = \frac{R_v^2 \rho_v p_{v,e} A_v}{16 \eta_v l_{\text{eff}}} \]  

(B.14)

where \( P_{v,e} \) denotes the vapour pressure at the evaporator, \( A_v \) the vapour core area, \( \eta_v \) the viscosity of the liquid, and \( \rho_v \) the specific weight of the vapour. The effective length \( l_{\text{eff}} \) is the length of the vapour column with the assumption that \( Q = Q_{\text{max}} \) along the entire vapour column. Assuming a linear increase of \( Q \) in the evaporation section, a constant \( Q \) at the adiabatic section and a linear decrease of \( Q \) at the condenser section yields

\[ l_{\text{eff}} = \frac{l_v \pm d_{\text{wall}} (R_v R_0)}{2} + l_{\text{adi}} + \frac{l_c \pm d_{\text{wall}} (R_v R_0)}{2} \]  

(B.15)

Figure B.2 shows the viscous limit for the heat pipe of table 2.1.
B.1.3 Sonic limit

When both viscous and inertia forces play a role, but the last are predominant, choking of the vapour flow may occur as the vapour flow at the end of the evaporator section reaches sonic velocity. At that moment a further increase in heat transfer is not possible. The axial vapour flow density is related to the axial flux by

\[ m_v = \frac{Q}{A_v L} \quad (B.16) \]

Invoking Bernouilli’s first law, the ideal gas law, conservation of energy, conservation of momentum and equation (B.16) yields

\[ Q = \frac{A_v \rho_0 L (\frac{\gamma v T_0 R}{2M} \sqrt{\frac{1+Ma^2 \gamma_v + 1}{1+Ma^2 \gamma_v}})}{1+Ma^2 \gamma_v} \quad (B.17) \]

where the subscript 0 refers to the stagnation state, \( R \) is the universal gas constant and \( M \) the molecular mass. The sonic limitation occurs when the vapour velocity at the evaporator exit is unity, hence the sonic limitation can be expressed by

\[ Q_{\text{max,son}} = \pi R_0^2 \rho_0 L (\frac{\gamma T_0 R^2}{2M(\gamma + 1)}) \quad (B.18) \]

Figure B.3 shows the sonic limit for the heat pipe of table 2.1.

![Figure B.3 sonic limit as a function of the evaporator temperature for the heat pipe specified in table 2.1](image)

B.1.4 Entrainment limit

As the liquid and the vapour move in opposite directions in the heat pipe, a shear force is exerted at the liquid-vapour interface. At a certain velocity and shear force, liquid droplets will be entrained in the vapour flow. Less liquid will reach the evaporator, reducing the heat rate of the heat pipe. The shear force \( F_s \) and the surface force \( F_t \) at the liquid-vapour interface can be written as

\[ F_s = \frac{K_1 \rho_s v_s^2 A_s}{2} \quad (B.19) \]

and

\[ F_t = K_2 \sigma C_s \quad (B.20) \]
As denotes the surface and \(C_s\) the wetted perimeter. \(K_1\) and \(K_2\) are proportionality constants. Entrainment begins when the Weber number reaches unity, \(\text{We} = \frac{F_s}{F_l} = 2\). In our case the value of \(K_1 / K_2\) can be taken as 8 (Dunn 1978). For screen wicks, the hydraulic diameter is defined as \(R_{hs} = \frac{2A_s}{C_s} = 1/(2\text{meshnr.})\). The mesh number is the number of pores per meter. Substituting the vapour velocity, obtained from (B.16) yields

\[
Q_{\text{max,ent}} = \pi R_v^2 L \frac{\alpha p_v}{2 R_{hs}}
\]  

(B.21)

Figure B.4 shows the entrainment limit for the heat pipe of table 2.1.

![Figure B.4: Entainment limit as a function of the evaporator temperature for the heat pipe specified in table 2.1](image)

B.1.5 Boiling limitation

When the heatflux to the evaporation section is continuously increased, the liquid is increasingly superheated, until nucleate boiling will occur in the wick. The formation of vapour bubbles in the wick is undesirable because it can cause hot spots and obstruct the liquid flow. For a spherical vapour bubble in the vicinity of the pipe-wick interface in equilibrium holds

\[
\pi R_b^2 (p_{pw} \pm p_l) = 2 \sigma \pi R_b
\]  

(B.22)

\(R_b\) is the bubble radius and \(p_{pw}\) the vapour saturation at the pipe-wick interface. The liquid pressure in the heatpipe equals the vapour pressure \(p_v\) at the liquid vapour interface minus the capillary pressure \(p_c\) at the same position. Using Clausius-Clapeyron under the assumption that the volume per unit mass of the liquid is much smaller than the volume per unit mass of the vapour \(V_v\) gives

\[
\frac{d p}{dT} = \frac{L}{TV_v}
\]  

(B.23)

The liquid pressure in the heat pipe equals the vapour pressure \(p_v\) at the liquid vapour interface minus the capillary pressure \(p_c\) at the same position. Since

\[
(p_{pw} \pm p_v) \gamma (T_{pw} \pm p_{wv}) dp / dT
\]

where \(T_{pw}\) is the temperature at the pipe-wick interface and \(T_{wv}\) at the wick-vapour interface. Equation (B.23) can be written as

\[
T_{pw} \pm T_{wv} = T_v \frac{2 \sigma R_b \pm p_c}{L \rho_v}
\]  

(B.24)
The temperature difference across the heat pipe wall is

$$T_{pw} - T_{w} = \frac{\ln \left( \frac{R_i}{R_f} \right)}{2\pi \lambda_0 \left( d_{wall} \left[ R_f - R_i \right] \right)}$$  \hspace{1cm} (B.25)

Thermal conductivities for liquid saturated wicks have been correlated by Gorring (1961)

$$\lambda_{eff} = \frac{\lambda_f \lambda_v + (1+\epsilon_w) \lambda_f \lambda_w}{\lambda_f + \lambda_w + (1+\epsilon_w) \lambda_f \lambda_w}$$

where $\lambda_f$ is the thermal conductivity of the fluid and $\lambda_w$ the thermal conductivity of the wick material. The wick porosity is defined as $\epsilon_w = 1 - \frac{\pi}{0.56} \frac{d_{wire}}{d_{mesh}}$. 1.05 is a factor to allow for crimping of the wick.

Combining equation (B.24) and (B.25) yields

$$Q = \frac{2\pi \lambda_0 \left[ e \pm d_{wall} \pm (R_f \pm R_i) \right] T_v \left( 2\sigma / R_i + P_i \right)}{L \rho_v \ln \left( \frac{R_i}{R_f} \right)}$$  \hspace{1cm} (B.26)

When replacing $R_i$ by $R_n$, the initial value of the vapour bubble for which a bubble does not collapse (≈2.54×10^{-7} m for a conventional heat pipe, Dunn 1978) in equation (B.25), the boiling limitation is found.

Figure B.5 shows the boiling limit for the heat pipe of table 2.1.

![Boiling Limit Chart](image)

**Figure B.5** Boiling limit as a function of the evaporator temperature for the heat pipe specified in table 2.1

**B.2 Heat resistance**

There is a temperature drop over the heat pipe, although it is small. The total heat resistance of the heat pipe is composed of:

- $R_{pe}$: the thermal resistance of the evaporator pipe wall
- $R_{we}$: the thermal resistance of the liquid saturated wick at the evaporation section
- $R_{lv}$: thermal resistance of the liquid vapour interface at the evaporation section
- $R_v$: thermal resistance of the vapour column
- $R_{vl}$: thermal resistance of the vapour liquid interface at the condenser section
- $R_{wc}$: thermal resistance of the liquid saturated wick at the condenser section
- $R_{pc}$: thermal resistance of the heat pipe wall at the condenser section
The thermal resistance of the heat pipe wall at the emitter and the condenser are calculated from

$$R_p = \frac{\ln\left(\frac{R_o}{R_i}\right)}{\pi \lambda_{eff} l}$$

(B.27)

The thermal resistance of the liquid saturated wick at the evaporator and the condenser can be deduced from equation (B.25)

$$R_{we} = \frac{\ln\left(\frac{R_i}{R_v}\right)}{\pi \lambda l - d_{wall} - (R_t - R_v)}$$

(B.28)

$$R_{wc} = \frac{\ln\left(\frac{R_i}{R_v}\right)}{\pi \lambda l - d_{wall} - (R_t - R_v)}$$

(B.29)

If a liquid is in equilibrium with a vapour, the net mass transport between the liquid and vapour phase is zero. When a surface is losing mass by evaporation, the temperature of the vapour must be less than its equilibrium value. At the condensation side the temperature of the vapour temperature must be higher than the equilibrium value. The average velocity $v_{av}$ in a vapour at temperature $T_v$ and with a molecular mass $M$ is given by

$$v_{av} = \sqrt{\frac{8kT_v}{\pi M}}$$

(B.30)

where $k$ is Boltzmann’s constant and $M$ is the molecular mass. The average flow of molecules in any given direction per unit area is $n v_{av} / 4$ in which $n$ is the molecule density. The corresponding heat flow per unit area is

$$q = \frac{1}{4} n v_{av} LM$$

(B.31)

The heat flux to the liquid surface for a perfect gas is

$$q_{vl} = p_v L \sqrt{\frac{M}{2\pi k T_v}}$$

(B.32)

and for the heatflux away from the liquid surface

$$q_{lv} = p_L L \sqrt{\frac{M}{2\pi k T_l}}$$

(B.33)

Using the Clausius-Clapeyron relation and the ideal gas relation, we can write for the net heatflux

$$Q = A_{eff} \frac{L^2 \rho_v}{2\pi T_k R} \frac{\Delta T}{M T_s}$$

(B.34)

$A_{eff}$ is the liquid vapour area and $T_s \approx T_v \approx T_l$ is the surface temperature. The thermal resistance of the liquid-vapour interface at the evaporator interface is given by
\[ R_{lv} = \frac{T_s \sqrt{\frac{2\pi T_s R}{M}}}{L^2 \rho_v \left[l_{c\text{vap}} - d_{\text{wall}} - (R_{\text{in}} - R_{\text{vap}})\varepsilon_{\text{sur}} \right]} \]  
(B.35)

and at the condenser interface

\[ R_{lv} = \frac{T_s \sqrt{\frac{2\pi T_s R}{M}}}{L^2 \rho_v \left[l_{\text{cond}} + d_{\text{wall}} + (R_{\text{in}} - R_{\text{vap}})\varepsilon_{\text{sur}} \right]} \]  
(B.36)

\( \varepsilon_{\text{sur}} \) is the liquid-wick material ratio at the liquid-vapour interface. For a screen wick

\[ \varepsilon_{\text{sur}} = 1 \pm \frac{1}{4} \text{meshnr} \cdot d_{\text{wire}} \]

Using expression (B.10), the total pressure drop over the vapour column can be written as

\[ \Delta p_{\text{tot}} = \Delta p_c + \Delta p_{\text{adia}} + \Delta p_e = F_v Q \left[l_c \pm d_{\text{wall}} \pm (R_{\text{in}} \pm R_v) \right] + l_{\text{adia}} \pm d_{\text{wall}} \pm (R_{\text{in}} \pm R_v) \]  
(B.37)

Using the ideal gas equation and (B.23) we can write

\[ \Delta T = \frac{T_v F_v Q \Delta p_{\text{tot}}}{L \rho_v} \]  
(B.38)

consequently the thermal resistance across the vapour column is

\[ R_v = \frac{T_v F_v \Delta p_{\text{tot}}}{L \rho_v} \]  
(B.39)

In table 2.2 the results of the thermal resistance calculation are given for the heat pipe specified in table 2.1 for an evaporator temperature of 873 K and a heat duty of 1200 W.
C System models

C.1 Analytical system model

The analytical system model consists of a set analytical algebraic equations which are solved with the symbolic algebra program Theorist. The equations used are listed below.

The following variables were “optimised” for each calculation. This was done by representing the desired output parameter graphically as a function of each variable. All variables were manually adjusted until the output variable had its maximum value.

\[d = 0.2\] interelectrode gap [mm]
\[T_E = 1700\] emitter temperature [K]
\[T_c = 973\] collector temperature [K]
\[T_R = 600\] cesium reservoir temperature [K]
\[r_{shell} = 0.02\] radius of the hot shell [m]
\[L_{sleeve} = 0.04\] length of the sleeve [m]
\[J = 6.58\] effective current density [A cm\(^{-2}\)]
\[d_{mo} = 1 \times 10^{-4}\] thickness of the Mo layer [m]
\[d_{SiC} = 3 \times 10^{-4}\] thickness of the SiC layer [m]
\[T_a = 293\] ambient temperature [K]
\[a = 0.1\] coefficient for ignition losses [-]
\[k = 8.62 \times 10^{-5}\]
\[A = 120.4\]
\[e = 1\]
\[\phi_0 = 13.08\]
\[\phi = 1.1\]

\[C_{p_{rc}} = \frac{1488 + 1194}{2} + \frac{1277 + 1011}{2} (\phi \pm 1)\]
\[C_p = \frac{1277 + 1011}{2\phi} + \frac{2342 + 1958}{2\phi + 1}\]
\[C_{p_{af}} = \frac{1137 + 1011}{\phi} (\phi \pm 1)\]
\[C_{p_{af}} = \frac{1490 + 1277}{\phi} (\phi \pm 1)\]
\[p_S = 0.833\]
\[H_{vg} = 31669000\]
\[V_g = 3950 / H_{vg}\]
\[Q_{gas} = H_{vg} V_g\]
\[Q_{gas} = H_{vg} / r_S\]
\[H_{mg} = H_{vg} / r_S\]
\[e_{bs} = 0.9\]

mean heat capacity flue gas (1273 K) [J kg\(^{-1}\) K\(^{-1}\)]
mean heat capacity fresh mixture (1273 K, gas up to 873 K) [J kg\(^{-1}\) K\(^{-1}\)]
mean heat capacity of the flue gas (293 K) [J kg\(^{-1}\) K\(^{-1}\)]
mean heat capacity of the flue gas (2273 K) [J kg\(^{-1}\) K\(^{-1}\)]
specific mass of natural gas (273 K) [kg m\(^{-3}\)]
tenhalpy of natural gas without condensation [J m\(^{-3}\)]
volume flow of natural gas for a TEC [m\(^3\) s\(^{-1}\)]
heating power gas of 3950 W [W]
tenhalpy of natural gas [J kg\(^{-1}\)]
enmissivity of the hot shell (burner side) [-]
\[ \lambda A = 10 \] 
heat exchanging capacity of the burner wall [WK\(^{-1}\)]

\[ \lambda A_c = 20 \] 
heat exchanging capacity of the burner wall [WK\(^{-1}\)]

\[ \eta_{CV} = 0.95 \] 
efficiency of the heating system [-]

\[ \Delta V = 0.5 \] 
voltage drop of the inverter [V]

\[ R_{\text{inverter}} = 0.0003 \] 
internal resistance of the inverter [\Omega]

\[ \rho_{Mo} = 0.45 \times 10^{-6} \] 
specific resistivity of molybdenum at 1723 K [\Omega m]

\[ \sigma = 5.67 \times 10^{-8} \] 
constant of Stefan Boltzmann [Wm\(^{-4}\)K\(^{-2}\)]

\[ e_b = 0.8 \] 
emissivity of the burner wall [-]

\[ e_{hs} = 0.8 \] 
emissivity of the hot shell outside [-]

\[ e_{es} = 0.24 \] 
emissivity of the hot shell inside [-]

\[ e_c = 0.24 \] 
emissivity of the collector [-]

\[ \lambda = 6 \times 10^{-5} \] 
heat conduction coefficient cesium [Wm\(^{-1}\)K\(^{-1}\)]

\[ \lambda_{mol} = 90 \] 
heat conduction coefficient Molybdenum [Wm\(^{-1}\)K\(^{-1}\)]

\[ \lambda_{SiC} = 16 \] 
heat conduction coefficient Molybdenum [Wm\(^{-1}\)K\(^{-1}\)]

\[ P_{\text{Tec}} = V_{\text{Tec}} I \] 
electrical power density of the converter [Wcm\(^{-2}\)]

\[ p = 2.4510^8 T_R^{-2} \exp \left( \frac{8910}{T_R} \right) \] 
ceesium pressure Hatsopoulos (1973) [torr]

\[ I_e = e^{\frac{0.2}{250} \ln(p)} + 15.85 \] 
extrigger current density [Acm\(^{-2}\)]

\[ V_0 = 0.4 \] 
evoltage threshold [V]

\[ \phi_T = 4.6 \pm 3.4 \frac{T_C}{T_R} + 0.94 \left( \frac{T_C}{T_R} \right)^2 \] 
emitter work function (McVey 1983) [eV]

The \( I-V \) characteristic is calculated according to the Sidelnikov model:

\[ V_1 = a p d \left( 1 + \frac{I}{80} \right) + \frac{(0.036)^2}{pd} \] 
voltage loss due to non optimum \( pd \) value [eV]

\[ z = \left( 0.1 \frac{1}{dp} + 0.4 \right) \left( \pm \frac{1}{\sqrt{dp} \ln \left( \sqrt{J_E} + 1 \right)} + 1 + \frac{I}{J_E} \right) \] 
dummy variable [-]

\[ V_2 = z + (z^2 + 0.055)^{\frac{1}{3}} \] 
voltage loss in emission saturation section [eV]

\[ I_2 = \frac{10^7}{T_E^2 \sqrt{pd}} \] 
[Acm\(^{-2}\)]

\[ V_3 = \frac{k}{e} T_E \ln \left( 1 + \frac{J_2}{J} \right) \] 
voltage loss due to arc losses [eV]

\[ I_3 = \frac{510^3 \sqrt{p} T_E^4 e^{22500}}{1 + 20 pd} \] 
[Acm\(^{-2}\)]

\[ V_4 = \frac{k}{e} T_C \ln \left( 1 + \frac{J_2}{J + J_3} \right) \] 
voltage loss for the transition to diffusion conditions [eV]
\[ I_C = ATEC \ln \left( \frac{T_E}{T_C} \right) \]

Collector current density \([\text{Acm}^{-2}]\)

\[ V_5 = \frac{E}{T_C} \ln \left( 1 + \frac{J_C}{J_T} \left[ 1 + \frac{T_E}{T_C} \right] \right) \]

Voltage loss due to emission from the collector \([\text{eV}]\)

\[ V_{loss} = V_0 + V_1 + V_2 + V_3 + V_4 + V_5 \]

Total voltage loss \([\text{eV}]\)

\[ \phi_B = \frac{k}{e} \ln \left( \frac{T_E^2}{T_c} \right) \]

Boltzmann work function \([\text{eV}]\)

\[ V_{TEC} = \phi_B - \phi_C - V_{loss} \]

Effective converter voltage \([\text{eV}]\)

\[ A_{TEC} = \pi r_{shell}^2 \]

Active surface area \([\text{em}^2]\)

\[ d_{shell} = d_{Mo} + d_{SiC} \]

Thickness of the hot shell \([\text{m}]\)

\[ R_{sleeve} = \frac{\rho_{Mo}}{2\pi d_{r_{shell}}} \left( \frac{\pi r_{shell}}{4} + L_{sleeve} \right) \]

Resistance of the hot shell \([\Omega]\)

\[ P_{brute} = A_{TEC} V_{TEC} \]

Electrical power without losses \([\text{W}]\)

\[ V_{sleeve} = V_{TEC} - A_{TEC} R_{sleeve} \]

Voltage after the sleeve \([\text{W}]\)

\[ p_{sleeve} = A_{TEC} V_{sleeve} \]

Electrical power after the sleeve \([\text{W}]\)

\[ P_{inverter} = A_{TEC} (V_{sleeve} + \Delta V_{TEC} + R_{inverter}) \]

Power after the inverter \([\text{W}]\)

\[ \lambda_{shell} = \frac{d_{Mo} \lambda_{Mo} + d_{SiC} \lambda_{SiC}}{d_{shell}} \]

Heat conduction coefficient shell \([\text{Wm}^{-1}\text{K}^{-1}]\)

\[ Q_{col} = 10^4 A_{TEC} \sigma \left( T_E^4 + T_C^4 \right) \]

Radiation from the emitter to the collector \([\text{W}]\)

\[ Q_{el} = A_{TEC} \phi_B \]

Electron cooling of the emitter \([\text{W}]\)

\[ Q_{Cs} = 10^4 A_{TEC} \lambda (T_E - T_C) \]

Conduction through the cesium \([\text{W}]\)

\[ Q_{sl} = 2\pi r_{shell} L_{sleeve} \]

Conduction through the shell \([\text{W}]\)

\[ Q_{tot} = Q_{col} + Q_{el} + Q_{Cs} + Q_{sl} \]

Total heat flow to the collector and sleeve loss \([\text{W}]\)

Heat balance for the rest of the system

\[ \eta_{burner} = \frac{Q_{tot}}{Q_{gas}} \]

Heat efficiency burner \([-]\)

\[ M_g = \rho_g V_g \]

Mass flow gas \([\text{kgs}^{-1}]\)

\[ M_I = \rho_I M_g \]

Mass flow air \([\text{kgs}^{-1}]\)

\[ M_r = M_g + M_I \]

Mass flow flue gas \([\text{kgs}^{-1}]\)

\[ \eta_{rec} = \frac{1 + \exp \left( \frac{\lambda A_{r.c} \left( 1 + \frac{C_{pc}}{C_{pm}} \right)}{C_{pc} M_r} \right)}{1 + \exp \left( \frac{\lambda A_{r.c} \left( 1 + \frac{C_{pc}}{C_{pm}} \right)}{C_{pc} M_r} \right)} \]

Effectivity of the recuperator \([-]\)

\[ T_1 = \eta_{rec} (T_{flue} - T_g) + T_a \]

Temperature of the preheated air-gas mixture \([\text{K}]\)

Heat recuperated \([\text{W}]\)

\[ Q_{m} = M_r C_p r \left( \frac{1 + \frac{T_a}{T_{flue} + T_a}}{T_{flue} + T_a \pm T_1} \right) \]

Heat efficiency burner \([-]\)

\[ Q_{m} = C_p (T_{flue} + T_{flue}) M_r \]
\[ T_{\text{flue}} = \frac{Q_m}{M_r C_p r} + T_{\text{flue}} \]

\[ \pm M_r C_p r \left( \frac{\sigma}{\varepsilon_e + \frac{1}{\varepsilon_e} + 1} \right) T_E^{\frac{1}{4}} + \frac{Q_{\text{tot}}}{M_r \eta_{\text{burner}} C_p r} + T_a \] \[ \pm H_{\text{mg}} M_g \]

\[ T_{\text{flue}} = \frac{M_r C_p r \left( \frac{\sigma}{\varepsilon_e + \frac{1}{\varepsilon_e} + 1} \right) T_E^{\frac{1}{4}} + \frac{Q_{\text{tot}}}{M_r \eta_{\text{burner}} C_p r} + T_a}{M Cp} + T_a \]

\[ Q_a = M_s H_{\text{mg}} \]

\[ Q_{\text{loss}} = 0 \]

\[ Q_i = M_r C_p r (T_{\text{flue}} + T_a) \]

\[ Q_m + Q_b = Q_{\text{loss}} + Q_{\text{gb}} + Q_r \]

\[ Q_{\text{gb}} = \frac{\sigma}{\varepsilon_e + \frac{1}{\varepsilon_e} + 1} \] \[ 10^{-4} A_{\text{TEC}} \]

\[ \pm 10^{-4} A_{\text{TEC}} \]

\[ \pm \exp \left( \pm \frac{\lambda A}{C_p r M_r} \right) \]

\[ + 1 \]

\[ \eta_{\text{burner}} = \pm \exp \left( \pm \frac{\lambda A}{C_p r M_r \eta_{\text{burner}}} \right) \]

\[ Q_{\text{gb}} = M_r C_p r \left( \frac{\sigma}{\varepsilon_e + \frac{1}{\varepsilon_e} + 1} \right) T_E^{\frac{1}{4}} + \frac{M_r C_p r (T_a + T_a + H_{\text{mg}} M_g)}{M_r \eta_{\text{burner}} C_p r} + T_a \] \[ \pm H_{\text{mg}} M_g \]

\[ T_i = \frac{M_r C_p r \left( \frac{\sigma}{\varepsilon_e + \frac{1}{\varepsilon_e} + 1} \right) T_E^{\frac{1}{4}} + \frac{Q_{\text{tot}}}{M_r \eta_{\text{burner}} C_p r} + T_a}{M Cp} \] \[ + T_a \]

\[ Q_{\text{gs}} = Q_{\text{go}} = Q_{\text{tot}} \]

\[ \eta_{\text{bruto}} = \frac{P_{\text{bruto}}}{Q_{\text{gas}}} \]

\[ \eta_{\text{sleeve}} = \frac{P_{\text{sleeve}}}{Q_{\text{gas}}} \]

\[ \eta_{\text{inverter}} = \frac{P_{\text{inverter}}}{Q_{\text{gas}}} \]

conversion efficiency without losses [-]

conversion efficiency after sleeve [-]

conversion efficiency after invertor [-]
C.2 Numerical model for exploitation costs

The parameters which are kept constant value are listen in table C.1

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>heat capacity of water $C_{pw}$</td>
<td>4170 Jkg⁻¹K⁻¹</td>
</tr>
<tr>
<td>specific mass of water $\rho_w$</td>
<td>1000 kgm⁻³</td>
</tr>
<tr>
<td>temperature of the mains water $T_{tapin}$</td>
<td>10 °C</td>
</tr>
<tr>
<td>set temperature of the hot domestic water $T_{tapout}$</td>
<td>50 °C</td>
</tr>
<tr>
<td>set temperature of the house $T_{set}$</td>
<td>21 °C</td>
</tr>
<tr>
<td>heat loss of the house ¹) $U_{house}$</td>
<td>200 Wkm⁻¹</td>
</tr>
<tr>
<td>windows surface ²) $A_{window}$</td>
<td>5 m²</td>
</tr>
<tr>
<td>heat conductivity of the storage insulation $\lambda$</td>
<td>0.035 Wm⁻¹K⁻¹</td>
</tr>
<tr>
<td>thickness of the storage insulation $d_{store}$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>temperature difference in the storage $\Delta T$</td>
<td>60 °C</td>
</tr>
<tr>
<td>time step (one hour) $dt$</td>
<td>3600 s</td>
</tr>
<tr>
<td>start-up time of the TEC $t_{start}$</td>
<td>300 s</td>
</tr>
<tr>
<td>TEC efficiency $\eta_{TEC}$</td>
<td>0.065 [-]</td>
</tr>
<tr>
<td>efficiency of the heating system $\eta_{CV}$</td>
<td>0.95 [-]</td>
</tr>
</tbody>
</table>

¹) value based on a yearly heat loss equivalent of 2000 m³ natural gas

²) surface windows projected to average sun direction, multiplied by the transmission

For each time step the following parameters are calculated, taking the irradiation of the sun and the ambient temperature from the hourly weather data of 1965 as supplied by the KNMI. To reduce the calculation time the shortened version was used, where the year is represented by 56 days (figure C.1).

![Ambient temperature according to the shortened reference year](image)
In van Wolferen (1990) a representative domestic hot water drawl (50 °C) is given (table C.2).

<table>
<thead>
<tr>
<th>hour</th>
<th>litres</th>
<th>energy [kJ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>12</td>
<td>2502</td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td>5004</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>2502</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>1251</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>2502</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>1251</td>
</tr>
<tr>
<td>17</td>
<td>12</td>
<td>2502</td>
</tr>
<tr>
<td>18</td>
<td>24</td>
<td>5004</td>
</tr>
<tr>
<td>19</td>
<td>12</td>
<td>2502</td>
</tr>
<tr>
<td>total</td>
<td>120</td>
<td>25020</td>
</tr>
</tbody>
</table>

The heat loss of the house is given by

\[ Q_{\text{heat}} = \int (U_{\text{house}} T_{\text{set}} - T_{\text{amb}} - I_{\text{window}}) \, dt \]  

(C.1)

where \( T_{\text{amb}} \) and \( I \) are the ambient temperature and the irradiation of the sun at that hour of the day. The hot water content of the storage is reduced according to

\[ i_{\text{house}} = \frac{Q_{\text{heat}}}{\Delta T \rho_w C_w} \]  

(C.2)

If at the time of day hot domestic water is used the hot water content of the storage is further reduced by

\[ i_{\text{tap}} = \frac{Q_{\text{tap}}}{\Delta T \rho_w C_w} \]  

(C.3)

The hourly heat demand caused by expression (C.1), (C.2) and (C.3) is represented in figure C.2.

Assuming that the storage is spherical, the outside surface area can be calculated from

\[ A_{\text{store}} = 4\pi (\frac{3}{4} I_{\text{store}})^{2/3} = 5 I_{\text{store}}^{2/3} \]

and to account for extra losses of a non-spherical shape, the storage is taken somewhat larger. \( I_{\text{store}} \) is the capacity of the storage in m³. The heat loss of the storage is subsequently given by

\[ Q_{\text{loss}} = \frac{6 I_{\text{store}}^{2/3} \Delta T}{d_{\text{store}} T_{\text{store}}^2} i_{\text{hot}} \, dt \]  

(C.4)

\( i_{\text{hot}} \) is the amount of water with a temperature of 60 °C above the temperature of the surroundings in the thermally stratified storage. Initially \( i_{\text{hot}} \) has the value zero. The hot water content of the storage is reduced according to

\[ i_{\text{loss}} = \frac{Q_{\text{loss}}}{\Delta T \rho_w C_w} \]  

(C.5)
In figure C.3 an example is given for the yearly electricity production as a function of the heater is supposed to run at the same thermal efficiency as the thermionic converter. TEC, the auxiliary heater is switched on for the appropriate time interval. The auxiliary produced. In this study the estimations of the costs are based on a feasibility study by Wolff that energy distribution companies will pay the same price for electricity delivered to the variable costs of the storage 5000 [\text{€m}^{-3}].

If the thermionic converter is running at the start of the time interval, the thermal storage is filled with

$$i_{TEC} = \frac{Q_{TEC}}{\Delta T} \rho_u C_w \eta_{CV}$$  \hspace{1cm} (C.6)

where $Q_{TEC}$ is the thermal power of the TEC system. If $i_{actual} dt = (i_{hot} + i_{TEC} - i_{loss} - i_{house} - i_{tap}) dt$ exceeds the hot water storage capacity $I_{store}$ within the time interval, the thermionic converter is switched off after the appropriate time interval.

In the case that $i_{actual} dt$ is negative and the TEC is not running, the thermionic energy converter is started for the part of the interval that is necessary. Electrical power is only produced after the given start-up time. When $i_{actual} dt$ remains negative despite a running TEC, the auxiliary heater is switched on for the appropriate time interval. The auxiliary heater is supposed to run at the same thermal efficiency as the thermionic converter.

In figure C.3 an example is given for the yearly electricity production as a function of the installed TEC-power for a system with a thermal storage of 100 l and a start-up time of 100 s. The exploitation calculations are based on the prices for a home user in Eindhoven in 1992 of gas (0.57 \text{ €m}^{-3}) and electricity (0.185 \text{ €kWh}^{-1}). As the generation of electricity with thermionic heating systems coincide with the peak power electricity demand it is supposed that energy distribution companies will pay the same price for electricity delivered to the grid. Stimulating subsidies and adjustments of the fixed costs are not considered.

The costs of thermionic converters is not very well known as they are not yet commercially produced. In this study the estimations of the costs are based on a feasibility study by Wolff (1990) and a simplified estimation of the fixed costs and the costs of the storage are used:

<table>
<thead>
<tr>
<th>Cost Category</th>
<th>Cost</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable costs of the thermionic converter</td>
<td>0.75</td>
<td>[\text{€W}^{-1}]</td>
</tr>
<tr>
<td>variable costs of accessories (burner, invertor)</td>
<td>0.25</td>
<td>[\text{€W}^{-1}]</td>
</tr>
<tr>
<td>variable costs of the storage</td>
<td>5000</td>
<td>\text{€m}^{-3}</td>
</tr>
<tr>
<td>fixed costs</td>
<td>100</td>
<td>[€]</td>
</tr>
<tr>
<td>interest</td>
<td>10</td>
<td>[%]</td>
</tr>
</tbody>
</table>

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Figure C.3  Yearly electrical power production as a function of the installed thermionic power for a system with a start-up time of 100 s and a thermal storage of 100 l
D Physical properties

In this appendix the physical properties used in this study are documented.

D.1 Air

The physical properties mentioned in this paragraph are taken from Nederlandse Gasunie N.V. (1980) unless stated otherwise. The pressure is supposed to be constant at 101.325 kPa.

The density can be determined according to

\[ \rho = \frac{T_0}{V_m z_T} \sum_{i=1}^{n} n_i M_i \]  

\[ \text{(D.1)} \]

where:

- \( \rho \): density at \( T_v \) [kgm\(^{-3}\)]
- \( T_0 \): normal temperature (273.15 K) [K]
- \( T_v \): temperature of the mixture [K]
- \( V_m \): normal molar volume [m\(^3\)kmol\(^{-1}\)]
- \( z \): compressibility factor [-]
- \( n \): number of components [-]
- \( n_i \): concentration of component \( i \) [-]
- \( M_i \): molar mass of component \( i \) [kg kmol\(^{-1}\)]

The composition of “wet” (50% humidity) is given in table D.1, the compressibility factor is taken to be 0.9994.

<table>
<thead>
<tr>
<th>( n_i ) [%]</th>
<th>( N_2 )</th>
<th>( O_2 )</th>
<th>( Ar )</th>
<th>( CO_2 )</th>
<th>( H_2O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V[\text{m}^3\text{kmol}^{-1}] )</td>
<td>22.408</td>
<td>22.3914</td>
<td>22.3936</td>
<td>22.2569</td>
<td>21.629</td>
</tr>
<tr>
<td>( M_i[\text{kgkmol}^{-1}] )</td>
<td>28.014</td>
<td>31.9988</td>
<td>30.0061</td>
<td>46.0055</td>
<td>18.0152</td>
</tr>
</tbody>
</table>

Subsequently the specific density of air can be calculated. In fig. D.1 the calculated density is compared with data from the VDI Wärmeatlas (1994).
The heat capacity is calculated from the tabulated data (table D.2) of the components according to:

\[ c_p = \sum_{i=1}^{n} n_i c_{p_i} \]  

(D.2)

Table D.2  heat capacity of the components of air

<table>
<thead>
<tr>
<th>T [°C]</th>
<th>N₂</th>
<th>Ar</th>
<th>CO₂</th>
<th>H₂O</th>
<th>O₂</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kJm⁻³K⁻¹</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1.302</td>
<td>0.926</td>
<td>1.62</td>
<td>1.491</td>
<td>1.31</td>
</tr>
<tr>
<td>1001.306</td>
<td>0.926</td>
<td>1.821</td>
<td>1.516</td>
<td>1.331</td>
<td></td>
</tr>
<tr>
<td>2001.315</td>
<td>0.926</td>
<td>1.972</td>
<td>1.553</td>
<td>1.373</td>
<td></td>
</tr>
<tr>
<td>3001.336</td>
<td>0.926</td>
<td>2.114</td>
<td>1.599</td>
<td>1.424</td>
<td></td>
</tr>
<tr>
<td>4001.365</td>
<td>0.926</td>
<td>2.219</td>
<td>1.654</td>
<td>1.465</td>
<td></td>
</tr>
<tr>
<td>5001.398</td>
<td>0.926</td>
<td>2.303</td>
<td>1.708</td>
<td>1.503</td>
<td></td>
</tr>
<tr>
<td>6001.428</td>
<td>0.926</td>
<td>2.374</td>
<td>1.758</td>
<td>1.532</td>
<td></td>
</tr>
<tr>
<td>7001.457</td>
<td>0.926</td>
<td>2.428</td>
<td>1.817</td>
<td>1.557</td>
<td></td>
</tr>
<tr>
<td>8001.482</td>
<td>0.926</td>
<td>2.483</td>
<td>1.876</td>
<td>1.574</td>
<td></td>
</tr>
<tr>
<td>9001.503</td>
<td>0.926</td>
<td>2.52</td>
<td>1.93</td>
<td>1.591</td>
<td></td>
</tr>
<tr>
<td>10001.524</td>
<td>0.926</td>
<td>2.554</td>
<td>1.98</td>
<td>1.608</td>
<td></td>
</tr>
<tr>
<td>12001.557</td>
<td>0.926</td>
<td>2.604</td>
<td>2.072</td>
<td>1.633</td>
<td></td>
</tr>
<tr>
<td>14001.578</td>
<td>0.926</td>
<td>2.642</td>
<td>2.156</td>
<td>1.654</td>
<td></td>
</tr>
<tr>
<td>16001.595</td>
<td>0.926</td>
<td>2.675</td>
<td>2.223</td>
<td>1.675</td>
<td></td>
</tr>
<tr>
<td>18001.612</td>
<td>0.926</td>
<td>2.7</td>
<td>2.282</td>
<td>1.696</td>
<td></td>
</tr>
<tr>
<td>20001.629</td>
<td>0.926</td>
<td>2.713</td>
<td>2.332</td>
<td>1.717</td>
<td></td>
</tr>
<tr>
<td>22000.01631</td>
<td>0.926</td>
<td>2.726</td>
<td>2.374</td>
<td>1.733</td>
<td></td>
</tr>
<tr>
<td>24000.01645</td>
<td>0.926</td>
<td>2.734</td>
<td>2.41</td>
<td>1.75</td>
<td></td>
</tr>
</tbody>
</table>

Figure D.1  density of air as a function of the temperature
The calculation results in the plot of fig. D.2.

![Graph showing heat capacity vs. temperature]

*Figure D.2  heat capacity of air as a function of the temperature*

The thermal conductivity and the dynamic viscosity are taken from tabulated data.

![Graph showing thermal conductivity vs. temperature]

*Figure D.3  thermal conductivity of air as function of the temperature*  

![Graph showing dynamic viscosity vs. temperature]

*Figure D.4  dynamic viscosity of air as function of the temperature*
D.2 Natural gas

The values given in this paragraph are valid for Groningen natural gas of the average composition, which is given in table D.3. The physical properties mentioned in this paragraph are taken from Nederlandse Gasunie N.V. (1980) unless stated otherwise. The pressure is supposed to be constant at 101.325 kPa.

<table>
<thead>
<tr>
<th>component</th>
<th>$x_i$ % (vol)</th>
<th>$n_i$ % (mol)</th>
<th>$g_i$ % (mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>methane</td>
<td>81.30</td>
<td>81.29</td>
<td>69.97</td>
</tr>
<tr>
<td>ethane</td>
<td>2.85</td>
<td>2.87</td>
<td>4.63</td>
</tr>
<tr>
<td>propane</td>
<td>0.37</td>
<td>0.38</td>
<td>0.9</td>
</tr>
<tr>
<td>butane</td>
<td>0.14</td>
<td>0.15</td>
<td>0.47</td>
</tr>
<tr>
<td>pentane</td>
<td>0.04</td>
<td>0.04</td>
<td>0.16</td>
</tr>
<tr>
<td>hexane</td>
<td>0.05</td>
<td>0.05</td>
<td>0.23</td>
</tr>
<tr>
<td>nitrogen</td>
<td>14.35</td>
<td>14.32</td>
<td>21.52</td>
</tr>
<tr>
<td>oxygen</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>carbon dioxide</td>
<td>0.89</td>
<td>0.89</td>
<td>2.10</td>
</tr>
</tbody>
</table>

The density is calculated according to

$$\rho = \frac{p M}{z RT}$$

with the molar mass $M = 18.637$ kg kmol$^{-1}$ and the compressibility $z = 0.9977$.

The heat capacity is calculated using expression D.2 with the data from table D.3 and D.4.

<table>
<thead>
<tr>
<th>T [K]</th>
<th>methane</th>
<th>ethane</th>
<th>propane</th>
<th>butane</th>
<th>pentane</th>
<th>hexane</th>
<th>nitrogen</th>
<th>oxygen</th>
<th>carbon dioxide</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M[kg kmol$^{-1}$]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.043</td>
<td>30.069</td>
<td>44.096</td>
<td>58.123</td>
<td>72.150</td>
<td>86.177</td>
<td>28.013</td>
<td>31.999</td>
<td>44.010</td>
<td></td>
</tr>
<tr>
<td>253.15</td>
<td>34.49</td>
<td>47.768</td>
<td>65.627</td>
<td>88.014</td>
<td>104.99</td>
<td>126.84</td>
<td>29.052</td>
<td>29.085</td>
<td>35.263</td>
</tr>
<tr>
<td>273.15</td>
<td>35.08</td>
<td>49.912</td>
<td>69.257</td>
<td>93.151</td>
<td>112.13</td>
<td>133.92</td>
<td>29.057</td>
<td>29.172</td>
<td>36.166</td>
</tr>
<tr>
<td>288.15</td>
<td>35.581</td>
<td>57.575</td>
<td>72.02</td>
<td>96.986</td>
<td>117.37</td>
<td>139.36</td>
<td>29.067</td>
<td>29.25</td>
<td>36.824</td>
</tr>
<tr>
<td>298.15</td>
<td>35.942</td>
<td>52.706</td>
<td>73.88</td>
<td>99.533</td>
<td>120.82</td>
<td>143.04</td>
<td>29.077</td>
<td>29.307</td>
<td>37.254</td>
</tr>
<tr>
<td>373.15</td>
<td>39.212</td>
<td>61.636</td>
<td>88.073</td>
<td>118.28</td>
<td>145.39</td>
<td>171.32</td>
<td>29.212</td>
<td>29.851</td>
<td>40.261</td>
</tr>
<tr>
<td>473.15</td>
<td>44.674</td>
<td>74.093</td>
<td>106.90</td>
<td>141.94</td>
<td>174.81</td>
<td>208.13</td>
<td>29.549</td>
<td>30.785</td>
<td>43.72</td>
</tr>
<tr>
<td>573.15</td>
<td>50.725</td>
<td>86.241</td>
<td>124.56</td>
<td>163.54</td>
<td>200.73</td>
<td>240.74</td>
<td>30.024</td>
<td>31.805</td>
<td>46.619</td>
</tr>
<tr>
<td>673.15</td>
<td>56.792</td>
<td>97.326</td>
<td>140.20</td>
<td>182.72</td>
<td>223.43</td>
<td>267.71</td>
<td>30.597</td>
<td>32.781</td>
<td>49.031</td>
</tr>
</tbody>
</table>
The thermal conductivity and the dynamic viscosity are taken from tabulated data.

D.3 Combustion gases

The physical properties mentioned in this paragraph are taken from Nederlandse Gasunie N.V. (1980). The heat capacity of the combustion gases after stochiometric combustion is calculated using equation D.1 and the data of table D.2. The composition of the combustion gases is given in table D.5.
The Design of a Combustion Heated Thermionic Energy Converter

The heat capacity of the combustion gases after stochiometric combustion is calculated using equation D.2 and the data of table D.2. The composition of the combustion gases is given in table D5.

<table>
<thead>
<tr>
<th>composition of the combustion gases</th>
</tr>
</thead>
<tbody>
<tr>
<td>N2</td>
</tr>
<tr>
<td>x_i [vol %]</td>
</tr>
<tr>
<td>V [m3kmol⁻¹]</td>
</tr>
<tr>
<td>M_i [kgkmol⁻¹]</td>
</tr>
</tbody>
</table>

The thermal conductivity is calculated according to

\[
\lambda = 1 + \frac{x_p}{100} \left( \frac{x_p}{100} \right)^2 \sum_{i=1}^{n} \frac{x_i \lambda_i}{100}
\]  

(D.3)

The factor before the summation has the function of “refining” factor. \(x_p\) stands for the concentration of polar components. Of the components listed in table D.5 only water is polar.

The conductivity of the components of the flue gases are listed in table D.6.
table D.6  thermal conductivity of the components of the flue gases

<table>
<thead>
<tr>
<th>T [K]</th>
<th>CO₂</th>
<th>N₂</th>
<th>H₂O</th>
<th>Ar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ [Wm⁻¹K⁻¹]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>273.15</td>
<td>0.0145</td>
<td>0.0242</td>
<td>0.0159</td>
<td>0.0163</td>
</tr>
<tr>
<td>373.15</td>
<td>0.0223</td>
<td>0.0311</td>
<td>0.0248</td>
<td>0.0221</td>
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**figure D.11**  thermal conductivity of the combustion gases as function of the temperature

**figure D.12**  dynamic viscosity of combustion gases as function of the temperature
D.4 Stainless steel AISI-316

The data presented in this paragraph are taken from Peckner (1977). The specific mass is taken at 8000 kgm$^{-3}$.

\[\text{figure D.13 Young’s modulus of stainless steel AISI-316 as function of the temperature}\]

\[\text{figure D.14 Poisson’s ratio constant of stainless steel AISI-316 as function of the temperature}\]

\[\text{figure D.15 plastic strain, tensile strength and yield stress of stainless steel AISI 316 as function of the temperature}\]

\[\text{figure D.16 thermal expansion of stainless steel AISI-316 as function of the temperature}\]
D.5 Molybdenum

The data presented in this paragraph were taken from the Plansee documentation (1990) and Frost (1983)

- specific mass: 10200 kgm⁻³
- Poisson’s ratio: 0.27
The Design of a Combustion Heated Thermionic Energy Converter

figure D.21 thermal expansion of molybdenum as a function of the temperature

figure D.22 thermal conductivity of molybdenum as a function of the temperature

figure D.23 specific heat of molybdenum as a function of the temperature

figure D.24 stress/temperature map for molybdenum with a grain size of 100 μm
table D.7  molybdenum creep data

<table>
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<td>atomic volume, $\Omega$ [m$^3$]</td>
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<td>Burger’s vector, $b$ [m]</td>
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<td>melting temperature, $T_m$ [K]</td>
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**Modulus**

- shear modulus at 300 K, $\mu_0$ [MPa] 1.34 $10^5$
- temperature dependence of modulus $\frac{T_m}{\mu_0} \frac{d\mu}{dT}$ -0.42

**Lattice diffusion**

- pre-exponential, $D_v$ [m$^2$s$^{-1}$] 5 $10^{-5}$
- activation energy, $Q_v$ [kJ/mole] 405

**Boundary diffusion**

- pre-exponential$^1$, $D_b0$ [m$^2$s$^{-1}$] 5.5 $10^{-14}$
- activation energy, $Q_b$ [kJ/mole] 263

**Power law creep**

- exponent, $n$ 4.85
- Dorn constant, $A$ 1 $10^8$

$^1 D_b = D_{b0} \exp \left( \frac{Q_b}{RT} \right)$

D.6  VACON-70

The data presented in this paragraph is taken from the supplier

Young’s modulus 130 MPa

Poisson’s ratio  0.3

---

**figure D.25**  plastic strain, tensile strength and yield stress of VACON-70 as function of the temperature

**figure D.26**  expansion coefficient of VACON-70 as a function of the temperature
D.7 Silicon carbide

The data presented in this paragraph is taken from Richardson (1992)

specific mass $3170 \text{ kgm}^{-3}$

Poisson’s ratio $0.14$

---

![Graph of Young's modulus of SiC as a function of the temperature](image)

*Figure D.27 Young's modulus of SiC as a function of the temperature*

![Graph of thermal expansion of SiC as a function of the temperature](image)

*Figure D.28 thermal expansion of SiC as a function of the temperature*

![Graph of thermal conductivity of SiC as a function of the temperature](image)

*Figure D.29 thermal conductivity of SiC as a function of the temperature*
D.8 Aluminium oxide

The data presented in this paragraph is taken from Richardson (1992)

specific mass \(3950 \text{ kgm}^{-3}\)
Poisson’s ratio \(0.26\)

---

**figure D.30** Young’s modulus of \(\text{Al}_2\text{O}_3\) as a function of the temperature

**figure D.31** thermal expansion of \(\text{Al}_2\text{O}_3\) as a function of the temperature

**figure D.32** thermal conductivity of \(\text{Al}_2\text{O}_3\) as a function of the temperature
### D.9 Sodium

The data presented in this paragraph is taken from Dunn (1978).

The vapour specific heat is taken constant at $904 \text{ kJ kg}^{-1} \text{K}^{-1}$.

---

**Figure D.33** Latent heat of sodium as a function of temperature

**Figure D.34** Liquid density of sodium as a function of temperature

**Figure D.35** Vapour density of sodium as a function of temperature

**Figure D.36** Liquid thermal conductivity of sodium as a function of temperature

**Figure D.37** Liquid dynamic viscosity of sodium as a function of temperature

**Figure D.38** Vapour dynamic viscosity of sodium as a function of temperature
figure D.39  liquid surface tension of sodium as a function of temperature
figure D.40  vapour pressure of sodium as a function of temperature
E Creep calculations

E.1 Creep of a sphere

To describe the creep of a spherical segment, the coordinate system \((z, \phi, \theta)\) of figure 3.11 is used. This coordinate system may be taken as the principal axis for stress and strain. The stress and strain depend on \(\phi\) alone. The equilibrium equations can be obtained as

\[
RN_r \pm N_\phi R \cos \phi + q_\phi R^2
\]

\[
\frac{N_\phi + N_\theta}{R} + q_n = 0
\]

(E.1)

Where \(N_\phi\) and \(N_\theta\) are the resulting membrane forces, \(q_\phi\) and \(q_n\) are the applied forces in the meridional- and normal direction per unit area and \(R\) is the radius of the sphere. On eliminating \(N_\theta\) and integrating, there results

\[
N_\phi = \frac{1}{R \sin^2 \phi} \left( c_1 \pm \int \{q_\phi R^2 \sin^2 \phi + q_n R^2 \sin \phi \cos \phi \} \, d\phi \right)
\]

(E.2)

In principle we have determined the stresses for the problem of loading alone, the problem is statically determined. The deformations can subsequently be evaluated if we integrate the strain-displacement equations for a membrane shell

\[
\epsilon_\phi = \frac{1}{R} \left( \frac{du}{d\phi} + w \right)
\]

\[
\epsilon_\theta = \frac{1}{R} (u \cos \phi + w \sin \phi)
\]

(E.3)

where \(u\) is the displacement in \(x\) direction and \(w\) is the displacement in \(y\) direction. The solution is complete if the constitutive relations are given as

\[
\epsilon_\phi = \frac{f(N/d)}{N} \left( N_\phi + \frac{1}{2} N_\theta \right)
\]

\[
\epsilon_\theta = \frac{f(N/d)}{N} \left( N_\theta + \frac{1}{2} N_\phi \right)
\]

\[
\epsilon_z = \pm \frac{f(N/d)}{N} \left( \frac{N_\phi + N_\theta}{2} \right)
\]

\[
\overline{N}^2 = N_\phi^2 + N_\phi N_\theta + N_\theta^2
\]

(E.4)

\(d\) stands for the thickness of the shell. For our spherical segment we can write

\(q_n = p, \quad q_\phi = 0\)

\(N_\phi = N_\theta = pR/2, \quad \overline{N} = pR/2\)

From equation E.2. the solution for the creep rate at the top is calculated to be
\[ w = 2f \left( \frac{pR}{d} \right)^{1/2} \]  
(E.5)

in case of the power-law \( \epsilon = B \sigma^n \) we can write

\[ \dot{w} = \frac{1}{2} B \left( \frac{pR}{d} \right)^{1/2} \]  
(E.6)

### E.2 The reference stress method

The aim of the RSM is to relate the deformation of structures to a simple creep test. Sim (1968) proposed that the reference stress can be approximated by

\[ \sigma_R = \frac{p}{p_L} \sigma_y \]  
(E.7)

where \( p \) is the load on the structure, \( p_L \) is the limit value of the load and \( \sigma_y \) is the yield stress.

The application of the RSM to problems of this class was first made by Penny and Marriot (1969). They based their study on the similarity between deformations of a creeping shell and the strains in a reference stress creep test, which is the foundation of the RSM as discussed here. In that case

\[ \frac{\Delta[p,t]}{\epsilon(\sigma_R,t)} = \frac{\Delta[p]}{\epsilon_E(\sigma_R)} \]  
(E.8)

where

\[ \Delta[p] = \frac{p}{S_E} \]

\[ \epsilon_E(\sigma_R) = \frac{\sigma_R}{E} \]  
(E.9)

\( \epsilon \) is the total strain, \( p \) is the loading, \( S_E \) is the stiffness, \( \Delta[p,t] \) is the deformation rate of the shell and \( \dot{\epsilon}(\sigma_R,t) \) is the strain rate in a creep test at the reference stress \( \sigma_R \). Combining these equations gives

\[ \dot{\Delta}[p,t] = \frac{p}{\sigma_R} \frac{E}{S_E} \dot{\epsilon}(\sigma_R,t) \]  
(E.10)

involving expression (E.7) leads to

\[ \dot{\Delta}[p,t] = \frac{p_L}{\sigma_y} \frac{E}{S_E} \dot{\epsilon}(\sigma_R,t) \]  
(E.11)

Up till now the stiffness \( S_E \) has been an elastic stiffness. To make the expression apply to large deformations, the elastic stiffness is replaced by the stiffness at a given displacement \( \Delta \), obtained from the slope of a short time load-displacement curve of the structure thus

\[ \dot{\Delta}[p,t] = \frac{p_L}{\sigma_y} \frac{E}{S_L} \dot{\epsilon}(\sigma_R,t) \]  
(E.12)
where $S_T$ is the tangent stiffness. The critical time to creep buckling can now be established as follows:

1. find the collapse load $p_c$ of the structure (theoretically or experimentally)
2. calculate the reference stress from equation (E.7)
3. find the value of $\dot{\varepsilon}(\sigma_c, t)$ from uniaxial tensile creep test data
4. obtain a curve of tangent stiffness $S_T$ against displacement (experimentally or theoretically)
5. calculate the deformation rate using (E.12)
6. add a small time increment and calculate the new deflection from

$$\Delta[p(t+\Delta t)] = \Delta[p(t)] + \Delta[p(t)]\Delta[p(t)]$$

(E.13)

7. repeat 4-6 until the displacement rate begins to grow without bounds as indicated by a displacement-time plot.

### E.3 The critical effective strain method

The critical effective strain method was applied to the problem of creep buckling of shells as follows by Chern (1978).

We start with the instantaneous critical buckling load $p_{cr}$ with corresponding elastic stresses and strains which can include the effects of plastic deformation and initial imperfections. This critical load can be obtained by analysis and experiment. When the applied load $p$ is less than the critical load, creep strains will develop, thus

$$\varepsilon_{ij} = \dot{\varepsilon}_{ij} + \varepsilon_{ij}^C$$

(E.14)

If in an approximate analysis stress redistribution is ignored, the stresses $\sigma_{ij}$ can be determined from elastic analysis and the creep strains can be determined from the flow rule, associated to the von Mises yield criterion

$$\varepsilon_{ij}^C = \frac{3}{2} \frac{\dot{\varepsilon}^C}{\sigma_e^C} S_{ij}$$

(E.15)

$S_{ij}$ is the deviatoric stress tensor. The equivalent stress is defined as

$$\sigma_e = \sqrt{3J_2}$$

(E.16)

where $J_2$ is the second invariant of the deviatoric stress tensor. The effective creep strain is likewise defined

$$\dot{\varepsilon}^C = \sqrt{\frac{4I_2}{3}}$$

(E.17)

where $I_2$ is the second invariant of the strain tensor. The effective creep strain is correlated for constant effective stress and temperature by an equation of the form

$$\dot{\varepsilon}^C = f(t, \sigma_e T)$$

(E.18)

from which time can be extracted as

$$t = g(\dot{\varepsilon}^C, \sigma_e T)$$

(E.19)
If the critical strain \( \varepsilon_{CR} \) at the onset of creep buckling can be estimated, the critical buckling time can also be estimated. To estimate the critical creep strain, several methods are available. According to Chern (1978) the constant critical strain method is the easiest to use and provides the best accuracy.

In the constant critical strain method, it is assumed that the critical buckling strains corresponding to the instantaneous buckling and the creep buckling are the same. Therefore the creep flow law and the elastic strain law can be equalled:

\[
\frac{3}{2} \frac{\varepsilon^C}{\sigma^C} S_{ij} = 1 + \frac{1}{E} \left( \varepsilon_{ij}^{CR} - \sigma_{ij} \right) + \frac{1}{3E} \left( \varepsilon_{ji}^{CR} - \sigma_{ij} \right) \delta_{ij} \tag{E.20}
\]

The differences in equation (E.20) are there to indicate that the creep strains develops in going from the current stress \( \sigma_{ij} \) to the critical stress \( \sigma_{ij}^{CR} \). Equation (E.20) can be used to evaluate buckling caused by an individual stress component. If we proceed on the basis of the equality of deviators, we can replace (E.20) by:

\[
\frac{3}{2} \frac{\varepsilon^C}{\sigma^C} S_{ij} = 1 + \frac{1}{E} \left( \varepsilon_{ij}^{CR} - \sigma_{ij} \right) \tag{E.21}
\]

and after rearranging

\[
\left( \frac{3}{2} \frac{\varepsilon^C}{\sigma^C} + \frac{1}{E} \right) S_{ij} = \frac{1}{E} \varepsilon_{ij}^{CR} \tag{E.22}
\]

using the definition of \( \sigma^C \) and squaring both sides we get

\[
\varepsilon_{CR}^C = \frac{\sigma^C}{E} \left( 2(1 + \nu) \left( \frac{\sigma^F}{\sigma^C} - \frac{1}{3} \right) \right) \tag{E.23}
\]

where \( \sigma^C \) is the effective stress corresponding to the critical stress \( \sigma_{ij}^{CR} \). Using equation (E.20) and (E.23) in combination with a creep model of the material we can make an estimation of the critical time to buckling.
In this appendix a detailed description is given of the construction of the prototype TEC and the vacuum vessel as they are used in the test rig. Unless specified otherwise the parts are manufactured and assembled by the Central Engineering Group or the faculty of Mechanical Engineering of the EUT.

**F.1 The prototype TEC**

The construction of the hot shell is depicted in figure F.1. The molybdenum was deepdrawn at Philips PMF b.v.. The molybdenum is brazed to a standard stainless steel-316 conflet flange using a compensation ring and a copper based filler material. A VACON ring is brazed to the counter flange and this ring is in turn brazed to the aluminium oxide isolation ring.

For brazing the aluminium oxide to the VACON the passive brazing method was used, implying that the ceramic has to be metallised first. This was done using a Mo-Mn mixture that was metallised in a wet $H_2$ (30 %) - $N_2$ (70 %) environment at 1475 °C during half an hour. Subsequently an ordinary copper based filler material was used for the brazing.

**figure F.1 construction of the ceramic seal**

The heat pipe is schematically drawn in figure F.2. The cylindrical part of the heat pipe is made from a standard stainless steel-316 pipe $\varnothing 56 \times 52$. The evaporation head is made from sheet material by forming as are the fins. The head is adapted to the form of the hot shell by an extra nickel head. The evaporator head, the cesium supply tube and the condenser cap are welded together. The other parts are brazed using a copper based filler material. The wick consists of three layers of stainless steel - 316 wire gauze with a gauze width of 190 µm.
and a wire thickness of 85 μm which is spot welded to the cylinder, the condenser cap and the evaporator head. The radius at the evaporation side is made by forming.

![Diagram of the heat pipe](figure F.2)

The heat pipe is connected to the hot shell by a flexible bellow of stainless steel - 316 to allow for axial movement. The bellow is made of 0.5 mm sheet material by forming and is welded to the flange and the heat pipe.

### F.2 The vacuum vessel

The construction drawings of the vacuum vessel, the heating element and the electrical connections are given in the figures F.3-F.5.

![Construction drawing of the vacuum vessel](figure F.3)
**figure F.4**  construction drawing of the heating element

**figure F.5**  construction drawing of the electrical connection
Summary

A thermionic converter is a diode of which one electrode is heated to a sufficient high temperature (~1700 K) that electrons are thermionically emitted. The electrons are collected on a cooled counter electrode (~900 K) converting heat into electricity. The goal of the work is to design a thermionic converter for use in a domestic heating system. A systematic design process requires a list of demands and in order to state the list of demands on the thermionic energy converter, the thermionic system, including the house is modelled. Optimising the system with respect to the economical profit leads to the list of demands. The demands imposed by the system can be fulfilled by several concepts of the thermionic energy converter. The limitations that are set by the structural integrity of the construction are investigated, especially with regard to creep and creep buckling. One of the main conclusions is that only ceramics can serve as the constructional emitter material. Further restrictions are set by the specific heat flux which can be transferred from a burner to the emitter surface and by the heat which can be removed from the collector. As both the system analysis and the experiments show that the start-up time of the heat pipe, used as collector cooling, has a major influence on system efficiency, a model for start-up behaviour was developed. A sensitivity analysis revealed that the result of the design optimisation is sensitive to the assumptions made in the thermal models. A test rig was built to validate these models.

Flat plate, cylindrical and hemispherical converters were compared in the ignited mode and as close spaced converter. A model was developed which enables local description of the temperatures and the heat fluxes in the hot shell. The influence on the efficiency of the geometry, misalignment and mechanical deformation was evaluated. The dimensions of the most viable solutions are given and the critical components were built and tested. The conclusion to be drawn is that the converter must be designed with respect to the complete system in order to produce a system with a reasonable efficiency.
Samenvatting

Een thermionische energieomzetter is een diode waarvan één electrode wordt verhit tot een zodanig hoge temperatuur (~1700 K) dat thermisch electronen worden geëmitteerd. De electronen worden opgevangen door een electrode met een lagere temperatuur (~900 K), waarbij warmte in electriciteit wordt omgezet. Het doel van dit werk is om een thermionische energieomzetter te ontwerpen die kan worden toegepast in huishoudelijke verwarmingssystemen. Een systematisch ontwerp proces vereist een lijst van eisen en om deze lijst van eisen op te kunnen stellen, is het thermionische systeem, inclusief het huis, gemodelleerd. Optimalisatie van dit systeem naar de economische opbrengst geeft de lijst van eisen aan de omzetter. Verscheidene uitvoeringsvormen van de thermionische omzetter kunnen voldoen aan de lijst van eisen. De beperkingen die worden opgelegd door de sterkte van de constructie zijn onderzocht, in het bijzonder wat betreft kruip en kruip-knik. Eén van de belangrijkste conclusies is dat alleen keramieken kunnen worden gebruikt als emitter materiaal. Verdere beperkingen worden opgelegd door de warmtestroom van de brander naar het emitter oppervlak en door de warmtestroom die moet worden afgevoerd van de collector. Aangezien zowel de systeem analyse als de experimenten laten zien dat de opstarttijd van de heat pipe een grote invloed hebben op de systeem efficiency, is een model voor het opstartgedrag ontwikkeld. Een gevoeligheidsstudie wees uit dat het resultaat van de optimalisatie gevoelig is voor de aanname in de thermische modellen. Een testopstelling is gebouwd om deze modellen te kunnen valideren.

Vlakke, cilindrische en bolvormige omzetters zijn vergeleken in de ignited mode en als close spaced omzetter. Een model is ontwikkeld dat lokale beschrijving van de temperaturen en de warmtestromen mogelijk maakt. De invloed op het rendement van de geometrie, uitlijnfouten en vervorming is geëvalueerd. De afmetingen van de beste oplossingen zijn vastgesteld en de kritische componenten zijn gemaakt en getest. De conclusie uit dit werk is dat het hele systeem moet worden beschouwd om een omzetter te kunnen ontwerpen met een redelijk rendement.
Nawoord

Alweer vier jaar geleden ben ik begonnen aan dit projekt, waar ik toen overigens nog niet van vermoedde dat het tot een promotie zou leiden. Mijn motivering lag meer in de verwachting het terrein van de energietechniek en werktuigbouwkunde verder te kunnen verkennen. Dat is uitgekomen, het werk aan de TEO is enorm divers gebleken, zowel wat de aard van de werkzaamheden betreft als het aantal vakgebieden wat aan de orde is gekomen. Ik heb me hier met veel plezier op kunnen uitleven.

Een gevolg van de diversiteit is ook dat ik met mensen van verscheidene instanties en bedrijven heb mogen samenwerken. Buiten de TUE, zijn de bijdragen van NOVEM b.v., Philips PMF b.v., ECS b.v. en de TUD cruciaal geweest.

Aan de basis van dit projekt staan Bart Veltkamp en Toon Schoonen. Hoewel beiden hun eigen werkerrein inmiddels wat hebben verlegd, heb ik er nooit wat van gemerkt in hun enthousiaste ondersteuning.

De nauwe samenwerking met Frank Habraken, Jan Kuiken en Wil Sampers heeft ertoe geleid dat ik een veel groter scala aan modelleringstechnieken en vervaardigingsmethoden heb gebruikt dan ik alleen zou aandurven.

Anton Klostermann, Anton van Steenhoven, Rick de Lange en Bert van Lierop hebben zich veel moeite getroost om me op het goede spoor te houden en het werk in al zijn facetten te volgen. Hun frisse kijk op het geheel heeft veel verhelderd.

Afstuderen in de werktuigbouwkunde wil niet automatisch zeggen dat je ook goed werktuigen kunt bouwen. Gelukkig was er de kunde en ervaring van Jan van Bussel, Theo Maas, Rudi Dirikx, Frits van Veghel, Frans Filippini, Lambert van der Schoot, Ad van Huijgevoort en Peter Ewalts.

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Verder zijn er veel mensen geweest die, hoewel niet direct betrokken bij de uitvoering van het onderzoek, altijd klaar hebben gestaan met goede suggesties, oplossingen en hulp. Van hen wil ik speciaal noemen, in willekeurige volgorde: Hans Laurense, Jan Heino, Rein van de Velde, Rien van de Linden, Rian Tielemans, Jacob van Berkel, Anton Schaap, Camillo Rindt, Rob Bastiaans, Jos Sillekens, Theo Janssen, Wim van Helden, Peter Poolman, Piet Boot, Anton Lamers, René Vijgen, Frank Fey, Rob van de Berg, Rob König, Toon Hoeben, Hans Dautzenberg en Philip de Goey.
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