Abstract—The fundamental objectives of patello-femoral joint biomechanics include the determination of its kinematics and of its dynamics, as a function of given control parameters like knee flexion or applied muscle forces. On the one hand, patellar tracking provides quantitative information about the joint’s stability under given loading conditions, whereas patellar force analyses can typically indicate pathological stress distributions associated for instance with abnormal tracking. The determination of this information becomes especially relevant when facing the problem of evaluating surgical procedures in terms of standard (i.e. non-pathological) knee functionality. Classical examples of such procedures include total knee replacement (TKR) and elevation of the tibial tubercle (Maquet’s procedure).

Following this perspective, the current study was oriented toward an accurate and reliable determination of the human patella biomechanics during passive knee flexion. To this end, a comprehensive three-dimensional computer model, based on the finite element method, was developed for analyzing articular biomechanics. Unlike previously published studies on patello-femoral biomechanics, this model simultaneously computed the joint’s kinematics, associated tendinous and ligamentous forces, articular contact pressures and stresses occurring in the joint during its motion. The components constituting the joint (i.e. bone, cartilage, tendons) were modeled using objective forms of non-linear elastic materials laws. A unilateral contact law allowing for large slip between the patella and the femur was implemented using an augmented Lagrangian formulation.

Patellar kinematics computed for two knee specimens were close to equivalent experimental ones (average deviations below 0.5° for the rotations and below 0.5 mm for the translations) and provided validation of the model on a specimen by specimen basis. The ratio between the quadriceps pulling force and the patellar tendon force was less than unity throughout the considered knee flexion range (30-150°), with a minimum near 90° of flexion for both specimens. The contact patterns evolved from the distal part of the retropatellar articular surface to the proximal pole during progressive flexion. The lateral facet bore more pressure than the medial one, with corresponding higher stresses (hydrostatic) in the lateral compartment of the patella. The forces acting on the patella were part of the problem unknowns, thus leading to more realistic loadings for the stress analysis, which was especially important when considering the wide range of variations of the contact pressure acting on the patella during knee flexion.

Keywords: Patella; Knee; Kinematics; Stress; Large slip contact.

INTRODUCTION

The main functions of the patella (knee-cap) are to improve the efficiency of the extensor forces through the entire knee flexion range (Ahmed et al., 1987; Kaufer, 1971), to centralize the forces of the different quadriceps muscle bellies and to provide a smooth sliding mechanism for the quadriceps muscle with little friction due to its cartilage cover (Ficat, 1970). It also indirectly contributes to the global stability of the knee (Bonnel, 1988).

Finally according to some authors (Fick, 1904; Freehafer, 1962) it provides the anterior aspect of the knee with a protecting shield. The patella represents thus an important element of the extensor apparatus; its removal (patellec- tomy) leads to quadriceps atrophy and loss of extensor force in proportions and this can greatly vary in the view of various authors (Stougard, 1970; Sutton, 1976). Because of the very high mechanical stresses to which the patello-femoral joint is subjected, it possesses the thickest articular cartilage in the human body, but at the same time is the site of the greatest frequency of degenerative changes (Ficat and Hungerford, 1977). These pathologies have motivated a number of studies, in order to understand the causes of patellar degeneracies better and eventually to recommend corrective treatments [detailed reviews on these studies can be found in Hefty and Hirokawa (1993)]. These studies have further delineated a number of fundamental topics, the set of which can be generically referred to as patello-femoral biomechanics. More specifically, patello-femoral biomechanics typically include analyses of

(1) patellar kinematics (tracking),
(2) extensor forces,
(3) patello-femoral contact pressure,
(4) stresses in the patella.
The common approach to these studies has been to assess only one particular aspect of the above list, and in some cases to compare related parameters between healthy and pathological knees.

Tracking studies have provided quantitative information about the patella motion during knee flexion and about the joint's stability under given loading conditions (Brossmann et al., 1993; Fujikawa et al., 1983; Heegaard et al., 1994; Reider et al., 1981; Sikorski et al., 1979; van Kampen, 1987; Veress et al., 1979). Goldstein et al. (1986) reported an analysis of patellar surface strains (which from a mechanical point of view should more correctly be considered as a kinematics study rather than a stress analysis). Force analyses have highlighted the double role played by the patella (as a spacer and as a lever), have thrown light on the effectiveness of the quadriceps extensor forces (Ellis et al., 1980; Huberti et al., 1984, Reilly and Martens, 1972), and have further invalidated earlier concepts in which the patello-femoral joint was considered as a pulley mechanism. Furthermore, successful attempts to relate the extensor forces to the patello-femoral contact forces were reported (Ahmed et al., 1987). Patellar cartilage degeneration has generally been assumed to be directly correlated to some abnormal contact pressure distribution in the patello-femoral joint (Ficat and Hungerford, 1977; Outerbridge and Dunlop, 1975). Accordingly, several studies have reported patello-femoral contact characteristics (Ahmed et al., 1983; Fujikawa et al., 1983; Goodfellow et al., 1976; Goymann and Mueller, 1974; Hille et al., 1985; Huberti and Hayes, 1984; Matthews et al., 1977; Seedhom et al., 1979; Shoji, 1974; Sadowsky et al., 1990): near extension there is virtually no contact between the patella and femur but during progressive flexion a bean-shaped contact area increases over both the lateral and medial facets and migrates proximally. Contact pressures were generally deduced from calculated forces and measured contact areas (Goodfellow et al., 1976; Matthews et al., 1977). However, in a few studies, contact forces (Bandi, 1970) or contact stresses (Ferguson et al., 1979) were directly measured at discrete locations of the articular surface, using adequate transducers. Finally, the few published stress analyses of the patella typically attempted to correlate stress trajectories to cancellous bone architecture in a way to characterize further the pathogenesis of articular lesions (Da Silva and Bratt, 1970; Haasters, 1974; Hayes et al., 1982; Maquet, 1984; Minns et al., 1979; Muller et al., 1980).

Assessing these various aspects of patellar biomechanics becomes especially relevant when facing the problem of evaluating surgical procedures in terms of standard (i.e. non-pathological) knee functionality (classical examples of such procedures include total knee replacement (TKR) and elevation of the tibial tubercle, Maquet's procedure.) It must be noted, however, that in the real patello-femoral joint, unlike any currently available analysis, many variables used to describe its biomechanics cannot be considered as independent but are in fact related to each other in a way which is still not fully understood. For instance, it is clear that the stresses occurring in the patella will largely depend (among other things) upon the loads to which the patella is submitted during knee flexion. These loads are in turn expressed in terms of quadriceps and patellar tendon tensions, and patello-femoral contact pressures which are all intimately related to the tracking pattern of the patella. Despite its importance, the independence of these parameters has not yet been investigated: all the available studies on the patello-femoral joint did only consider one set of biomechanics parameters (e.g. kinematics or stress analysis) at a time. Hence, the purpose of the present contribution is to provide a global description of the patello-femoral joint biomechanics, in which its kinematics and dynamics (including analysis of patellar tendon tension, contact pressure and patellar stresses) are assessed simultaneously, i.e. taking their interdependence into account.

To obtain such a global description, a comprehensive computer model was constructed, based on an experimental setup for providing input data and model validation. The essential features characterizing this model were to consider three-dimensional deformable continuum solids undergoing large displacements and deformation, and rigorous treatment of the large slip contact problem typical of joint mechanics.

**METHODS**

*Formulation of the model*

The adopted formulation for constructing the three-dimensional computer model included:

- a material description for the patellar kinematics,
- a weak formulation for the linear momentum balance,
- elastic constitutive laws for the different media constituting the joint,
- a unilateral contact formulation allowing for large slips.

Inertial and gravitational effects were neglected in the present model reflecting its *quasi-static* nature. Consistently, time did not appear explicitly in the formulation.

*Material description.* A material description was adopted to describe the kinematics of the different components of the joint as it proved to be more suitable for cohesive solids undergoing moderate deformations (Truesdell, 1977; Gurtin, 1981): the actual position of a material particle $x$ was expressed by the map $y = y(x)$ called the motion or the deformation. The deformation of an infinitesimal material fiber was captured by the (unsymmetric) deformation gradient $F$ (i.e. $dy = F dx$). The (right Cauchy–Green) material metric tensor

$$ C = C(x) = F^T F $$

and the (Green–Lagrange) material strain tensor

$$ E = E(x) = \frac{1}{2} (C - I) = \frac{1}{2} (F^T F - I) $$

were used as alternative objective strain measures (indifferent to rigid body motion).

The only forces assumed to act on a material particle were the contact forces per unit of reference surface, represented by the nominal stress vector $p = p(x)$. The state of stress at a material point was recorded by the (unsymmetric first Piola–Kirchhoff) *nominal stress tensor*...
Kirchhoff-St. Venant's (objective form of Hooke's law) had to be imperatively employed. The linear elastic law of tautations) during knee flexion, an objective constitutive law to be elastic, isotropic, and homogeneous. Moreover, the different as bones and tendons. The patella was assumed needed to characterize the behavior of specific media as less of their constitutive materials. Constitutive laws were governed the equilibrium of all deformable bodies, regardless of their strain E over the deformation gradient F.

General principles. Since conservation of mass and balance of angular momentum are satisfied a priori in the material description, the balance of linear momentum was the only principle of mechanics which remained to be satisfied. A weak form of this principle, suitable for discretization, is the principle of virtual work:

\[ \int_V \text{tr} (\nabla w^T \mathbf{P}) \, dV = \int_{\partial V} w^T \mathbf{p} \, dA, \quad \forall \mathbf{w}, \quad (5) \]

where \( \mathbf{w} = \mathbf{w}(x) \) is an arbitrary test function satisfying \( \mathbf{w}(x) = 0, \forall x \in \partial \Omega_e \), best interpreted as a virtual displacement, and whose material gradient is \( \nabla \mathbf{w} \), represents the part of the boundary where displacements are prescribed and \( \partial \Omega_e \), the complementary part where stresses are applied.

Constitutive laws. The principle of virtual work (5) governs the equilibrium of all deformable bodies, regardless of their constitutive materials. Constitutive laws were needed to characterize the behavior of specific media as different as bones and tendons. The patella was assumed to be elastic, isotropic, and homogeneous. Moreover, the patella being subjected to large displacements (finite rotations) during knee flexion, an objective constitutive law had to be imperatively employed. The linear elastic law of Kirchhoff-St. Venant (objective form of Hook's law) was used to simulate the elastic behavior of the cortical and cancellous bones:

\[ \mathbf{S} = \mathbf{S}(\mathbf{E}) = \lambda \mathbf{I} \text{tr} \mathbf{E} + 2\mu \mathbf{E} \quad (6) \]

where \( \lambda \) and \( \mu \) stand for the Lame's coefficients. This formulation was adequate to filter the large rotations of the patella while keeping track of its deformations.

The major drawback of Kirchhoff-St. Venant's law is its lack of monotonicity under axial or triaxial compression. This 'instability' can become a serious limitation in the modeling of the articular cartilage which works primarily in compression. Most recent models of the cartilage (Suh et al., 1991) took both its incompressible liquid and viscoelastic solid phases into account. In the current model an alternative law (Rajaonah, 1990) was adopted to model an alternative law (Rajaonah, 1990) was adopted to represent the cartilage as an elastic nearly incompressible neo-Hookean material:

\[ \mathbf{S} = \mathbf{S}(\mathbf{C}) = \lambda (J - 1) \mathbf{C}^{-1} + \mu (\mathbf{C} - \mathbf{I}) \quad (7) \]

where \( J = \det \mathbf{F} \). The patella is connected to the tibia through the patellar tendon, which is composed of parallel, flexible collagen fibers bundles that can only support tensile stresses. This latter characteristic was taken into account through a unilateral Kirchhoff-St. Venant's material law which was applied to a uni-directional homogeneous material (Heegaard, 1993):

\[ \mathbf{S} = \mathbf{S}(\mathbf{F}) = \begin{cases} \mu (F^2 - 1) (\mathbf{I} \otimes \mathbf{I}) & \text{if } F > 0 \text{ (tension)}, \\ 0 & \text{if } F < 0 \text{ (compression)} \end{cases} \quad (8) \]

where \( F = l/l_0 \) is the stretch ratio between the fiber's initial length \( l_0 \) and its actual length \( l \) along the fiber's axis \( \mathbf{I} \).

Large slip contact. The contact problem between the patella (considered as a deformable striker body) and the femoral groove (considered as a smooth rigid target obstacle) was solved by using frictionless large slip contact elements: starting from a three-dimensional finite element discretization of the striker body into \( N \) nodes and of the interface into \( M \) node-on-surface contact elements, a non-linear gap vector and its first variation were derived in terms of the nodal displacements. The gap vector \( \mathbf{d} \) between a 'striker' point \( y^1 \) belonging to the patellar articular surface and a 'target' surface \( \mathcal{S} \) (Hermite bicubic patch), locally fitting the femoral groove, could be expressed by

\[ \mathbf{d} = \mathbf{d}(y^1, \xi) = (y^2 - y^1), \quad (9) \]

where \( \xi = (\xi_1, \xi_2) \) represent parametric coordinates of \( \mathcal{S} \) defined over a simply connected domain \( \mathcal{A} \) and \( y^2 = y(\xi) \) stands for the projection of \( y^1 \) on \( \mathcal{S} \), i.e.

\[ \xi_0 = \arg \min_{\xi \in \mathcal{A}} \| y(\xi) - y^1 \|. \quad (10) \]

The associated signed contact distance \( d_n \) could then be defined as:

\[ \mathbf{d}_n = \mathbf{d}_n(y^1, \xi_0) = \mathbf{d} \cdot \mathbf{n}, \quad (11) \]

where \( \mathbf{n} = \mathbf{n}(\xi_0) \) represent the inward normal vector to the target surface at \( y^2 \). In this way, \( d_n \) became negative whenever a striker point penetrated the target surface. The extremum \( (\xi_0^1, \xi_0^2) \) satisfying equation (10) was obtained by solving the following minimization problem:

\[ \min_{\xi \in \mathcal{A}} (\frac{1}{2} \mathbf{d} \cdot \mathbf{d}) = \frac{1}{2} \mathbf{d}_n^2(\xi_0). \quad (12) \]

The contact interaction is governed by the principle of action and reaction which took the discrete local form

\[ \mathbf{p} = p^e = - p^i, \quad (13) \]

where the contact force \( \mathbf{p} \) was defined as the force \( p^e \) exerted by the striker node \( y^1 \) on the rigid obstacle at \( y^2 \), and \( p^i \) the reaction at \( y^2 \). This force, which depended on \( \mathbf{d}_n \), was distributed on the striker node according to the discrete principle of virtual work. More specifically

\[ \delta W_c = \mathbf{p}(\delta \mathbf{d}(u)) \cdot (\delta \mathbf{d}(u) = f(u) \cdot \delta \mathbf{u} = \delta W, \quad (14) \]

i.e. the virtual work \( \delta W_c \) developed by the contact force \( \mathbf{p}(\delta \mathbf{d}(u)) \) through a variation of the contact distance \( \delta \mathbf{d}(u) \) must be equal to the virtual work \( \delta W \) developed by the element nodal force \( f^i \) through the corresponding nodal displacements variation \( \delta \mathbf{u} \). In the frictionless case, the contact force is directed along the target facet normal \( \mathbf{n} \) and could be written as

\[ \mathbf{p} = f_n \mathbf{n}. \quad (15) \]
The gap distance was finally related to the conjugate pressure by a (multivalued non-differentiable) unilateral contact law. At the interface, the frictionless contact law was locally characterized by three complementary Signorini conditions

\[ d_a \geq 0, \quad f_a \leq 0, \quad d_a f_a = 0, \]

respectively, expressing that the contacting bodies could not penetrate each other, could not pull on each other and were either separated or pressing on each other.

**Problem formulation.** The principle of virtual work (5), the constitutive laws (6), (7) or (8), the principle of action of reaction (13) and the contact law (16) together with proper boundary condition form a well posed boundary value contact problem; yet it is difficult to solve in this form. By assuming that all the applied forces were conservative, a first alternative form was found by considering the total energy \( n(u) \) of the system, given by the difference between the internal elastic strain energy \( \phi(u) \) stored in the patella and the external potential energies \( \eta(u) = \psi(u) \)

\[ \pi(u) = \phi(u) - \eta(u). \]

Adding the first Signorini condition to this last equation, the large slip contact problem reduced to finding a constrained local minimum of \( \pi(u) \):

\[ \begin{align*}
\min u \pi(u) &= \phi(u) - \eta(u), \\
\text{s.t.} \quad d_a(u) &\in \mathbb{R}^M
\end{align*} \]

where \( d_a(u) \) represented an \( M \)-dimensional vector, whose components contained the gap distances of the \( M \) contact elements discretizing the interface between the contacting bodies.

The resulting inequality constrained minimization problem was then transformed into an unconstrained saddle point problem using augmented Lagrangian multipliers, which at equilibrium hold the contact forces. A detailed description of the method has been discussed in (Heegaard and Curnier, 1993). The main features of this method are to lead to well conditioned problems and to provide exact solutions with respect to contact, in contrast to penalty methods.

**Model specifications**

The ground for the computer model input was provided by a preliminary experimental study (Heegaard *et al.*, 1994) on two cadaver knees. In that study, Roentgen stereophotogrammetric analysis (RSA) techniques (Selvik, 1989) were used to precisely measure patellar tracking during knee flexion in which the femur was rigidly fixed while the tibia was flexed from full extension (0°) to full flexion (150°) in 15° increments, with a constant 40 N force pulling on the quadriceps muscle. The corresponding numerical model was built by generating a finite element mesh of the extensor apparatus and by specifying proper boundary conditions.

**Finite element mesh characteristics.** From a structural point of view, the patella was assumed to consist of three constitutive regions: (1) a core of cancellous bone, (2) a shell of cortical bone and (3) a cartilage layer delimiting the articular surface (Fig. 1). The geometry of these parts was obtained by means of computerized tomography (Somatron DR3, SIEMENS); the shape and internal structure of each patella were digitized from a set of 4 mm spaced sagittal slices. Hereby, the three-dimensional mesh generation problem was reduced to a simpler two-dimensional problem. The three-dimensional mesh of the patella was obtained by assembling these two-dimensional meshes. The corresponding mechanical properties were taken from published values and are listed in Table 1.

The femur being considered as a rigid obstacle, only its geometry was taken into account: it was accurately measured using a stereophotogrammetric curve reconstructions (SCR) system (Meijer *et al.*, 1989) and discretized into a regularly spaced set of 250 points fitting the femoral groove surface.

The joint's interface was modeled by a set of 100 large slip contact elements. Typical values for cartilage-cartilage friction coefficients are below 0.01 (e.g. Mow and
Fig. 2. Patellar three-dimensional tracking, including (from top to bottom) flexion, tilt, rotation and shift, as a function of knee flexion for knee 1 (left column) and knee 2 (right column). Solid line curves (filled dots) represent numerical results and dashed curves (hollow dots) represent experimental results (from Heegaard et al., 1994).
Soslowsky, 1991) so that contact could be assumed frictionless. The striker nodes belonged to the external surface of the patellar cartilage layer. Adjacent nodes from the femoral groove were connected together into 16-nodes bi-cubic Hermite target patches. Associated Lagrangian nodes holding the unknown contact force were added and connected to their respective striker nodes.

The patellar tendon (PT), connecting the patella to the tibial tuberosity, was discretized into three string-like elements (Fig. 1) representing lateral, central and medial fibers whose insertion sites were measured using the RSA system.

The characteristics of the final mesh for both knees are summarized in Table 2. The foregoing continuum mechanics problem with unilateral contact was implemented and solved with the program TACT (Curnier, 1985).

Boundary conditions. The three tibial insertion points of the PT were subjected to a prescribed motion which controlled the flexion process. The successive positions of these points were recorded during the preliminary experiment using the RSA system. In the model, knee flexion angles were only considered in the range between 30 and 135° for stability reasons (Heegaard et al., 1994).

A constant 40 N pulling force, representing the rectus femoris (RF) actions, was equally distributed on three nodes of the patellar base central part. The direction of these forces varied during flexion (due for example to muscle interaction with the femoral groove). An estimate of these directions was obtained during the experimental measurements of patellar kinematics.

Model sensitivity. When the articular surface topography or the patellar structure geometry were measured, the reconstruction errors were small (≈1/10 mm) when compared to the uncertainty on localization of PT and RF insertion points on the patella (≈1–2 mm). Therefore the influence on computed patellar kinematics of a 2 mm posterior shift of the PT insertions and of a 2 mm posterior or anterior shift of the RF insertions, was also analyzed.

Results

The output from the computer model includes patellar kinematics, PT tension, joint contact pressure and patellar stresses, as a function of knee flexion.

Table 1. Material constants assumed for the extensor apparatus elements

<table>
<thead>
<tr>
<th>Element</th>
<th>λ (MPa)</th>
<th>μ (MPa)</th>
<th>E (MPa)</th>
<th>ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cortical bone</td>
<td>8.65 × 10³</td>
<td>5.76 × 10³</td>
<td>1.5 × 10⁴</td>
<td>0.30</td>
</tr>
<tr>
<td>Cancellous bone</td>
<td>1.73 × 10²</td>
<td>1.15 × 10⁶</td>
<td>3.0 × 10²</td>
<td>0.30</td>
</tr>
<tr>
<td>Cartilage</td>
<td>1.065 × 10⁻³</td>
<td>6.8 × 10⁻³</td>
<td>2.0</td>
<td>0.47</td>
</tr>
<tr>
<td>Patellar tendon</td>
<td>0</td>
<td>5.0 × 10¹</td>
<td>10⁵</td>
<td>0.00</td>
</tr>
</tbody>
</table>

*Taken from Reilly et al. (1972), Townsend et al. (1976) and Mow et al. (1991). λ and μ represent the Lamé coefficients, E the Young modulus and ν the Poisson ratio.

Table 2. Finite element mesh specifications of the two knees used in the model

<table>
<thead>
<tr>
<th>Knee</th>
<th>Problem dimension</th>
<th>DOF per mode</th>
<th>Total number of nodes</th>
<th>Total DOE</th>
<th>Displacement BC</th>
<th>Force BC</th>
<th>Number of sets</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1445</td>
<td>1371</td>
<td>903</td>
<td>9</td>
<td>6</td>
<td>Cortex</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3452</td>
<td>3120</td>
<td>993</td>
<td>9</td>
<td>9</td>
<td>Cancellous bone</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Contact (two-node line)</td>
<td>3</td>
<td>3</td>
<td>Cartilage</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Contact (two-node large slip)</td>
<td>99</td>
<td>90</td>
<td>Tendon</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Femur (16-nodes Hermite)</td>
<td>148</td>
<td>169</td>
<td>Flexion increment</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Number of increments</td>
<td>14</td>
<td>14</td>
<td>7.5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Convergence criteria</td>
<td>10⁻⁴</td>
<td>10⁻⁴</td>
<td>7.5°</td>
</tr>
</tbody>
</table>

Kinematics

The motion of the patella is expressed by three Eulerian rotations and three translations relative to a fixed coordinate frame εf attached to the femur (Fig. 1). Adopting the Tait-Bryan convention, the rotations are carried first around the x-axis (patellar flexion), then around the y-axis (patellar tilt) and finally the z-axis (patellar rotation). Patellar shift denotes translation along the x-axis.

Model validation. The computed kinematics of the patella (Fig. 2; solid line, black dots) match almost perfectly the corresponding experimental curves (dashed line, hollow circles). The average root mean square (RMS) errors between numerical and experimental curves are less than 0.5° for both specimens. Maximum deviations and RMS errors are listed in Table 3 for both knees. The maximum errors occur at high flexion angles in both specimens (except for the second knee patellar flexion). Patellar shift RMS error is below 0.5 mm for both specimens.

Sensitivity analysis. Patellar kinematics were not significantly altered after slightly shifting the tendon insertions, as shown on Table 4 which summarizes the maximum and RMS tracking deviations between altered and original tendon insertions. In all cases the RMS deviations are below 0.2° for the rotations and 0.03 mm for the translations.

Patellar tendon tension

The computed tension ||Fpr|| in the PT (representing the magnitude of the three fibers resultant) is reported in Fig. 3 as a function of knee flexion for both knees. This tension remains below 40 N for both specimens. The quadriceps pulling force magnitude ||Fq|| being constant (40 N), it is readily seen that the ratio ||Fpr||/||Fq|| is below unity throughout the considered flexion range (30–135°).
The biomechanics of the human patella during passive knee flexion

Table 3. Maximum absolute error ($e_{max}$) and RMS error ($\bar{e}$) between experimental and computed patellar tracking parameters during knee flexion

<table>
<thead>
<tr>
<th>Flexion</th>
<th>Tilt</th>
<th>Rotation</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee 1</td>
<td>2.51 (139.20°)</td>
<td>0.78</td>
<td>1.12 (105.00°)</td>
</tr>
<tr>
<td>Knee 2</td>
<td>1.91 (42.08°)</td>
<td>0.60</td>
<td>1.21 (119.80°)</td>
</tr>
</tbody>
</table>

Note: Errors for the rotations are in degrees and in mm for the translations. The tibial flexion angle corresponding to the maximal error is indicated between parentheses.

Table 4. Maximum absolute error ($e_{max}$) and RMS error ($\bar{e}$) between the standard simulation and perturbed simulations: (a) PT 2 mm posterior shift, (b) RF 2 mm posterior shift and (c) RF 2 mm anterior shift.

<table>
<thead>
<tr>
<th>Flexion</th>
<th>Tilt</th>
<th>Rotation</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee 1</td>
<td>0.47 (89.06°)</td>
<td>0.13</td>
<td>0.12 (72.23°)</td>
</tr>
<tr>
<td>Knee 2</td>
<td>0.68 (9.48°)</td>
<td>0.19</td>
<td>0.17 (72.23°)</td>
</tr>
<tr>
<td>(c)</td>
<td>0.20 (89.06°)</td>
<td>0.00</td>
<td>0.27 (9.48°)</td>
</tr>
</tbody>
</table>

Note: Errors for the rotations are in degrees and in mm for the translations. The tibial flexion angle corresponding to the maximal error is indicated between parentheses.

Fig. 3. Patellar tendon tension as a function of knee flexion, for knee 1 (triangles) and for knee 2 (squares). The dotted line corresponds to the applied 40 N quadriceps force.

The tension $||F_{PT}||$ decreases until 90° of flexion where it reaches a minima (33.3 N for specimen 1 and 22.5 N for specimen 2), and then increases again during the last 40° of knee flexion.

Contact pressures

Both knees present similar contact pressure distributions up to 90° of knee flexion. Contact areas are divided into a smaller medial zone displaying higher pressure gradients than over the lateral facet (Fig. 4). These areas shift proximally during flexion. However, between 90° and full flexion, inter-specimen differences appear: for the first knee, the contact area is still split over the medial and lateral facet and is located in the upper third of the articular surface, whereas in the second knee, the medial and lateral zones merge together through a narrow contact zone across the central ridge. In the latter specimen, a second medial contact zone appears on the medio-distal edge. Except in the mid-flexion range, the lateral facet bears more pressure (mean peak pressure: $p_{max} = 0.51$ MPa for knee 1, and 0.52 MPa for knee 2) than the medial one ($p_{max} = 0.4$ MPa for knee 1, and 0.48 MPa for knee 2) (Fig. 5).

Cancellous bone stresses

The stresses occurring in the patellar cancellous bone are expressed in terms of hydrostatic and deviatoric (von Mises) invariants of the stress tensor. Throughout knee flexion, stresses are concentrated mainly at the periphery of the cancellous bone, near the cortical shell, whereas in the bulk the stresses remain low. In the sagittal plane, the compressive hydrostatic pressures are primarily located beneath the patello-femoral contact region (Fig. 6). In the frontal plane, the highest stresses are located beneath the lateral contact area (Fig. 7). Tensile hydrostatic stresses are mainly concentrated at the distal subchondral bone, beneath the patellar tendon insertion region (Fig. 8). In the frontal plane, tensile stresses are found beneath the lateral facet (knee 1) or the medial one (knee 2) (Fig. 9). Von Mises are primarily located at the distal pole (patellar apex) beneath the patellar tendon insertion region and along the anterior face of the cancellous bulk (Fig. 10).

DISCUSSION

The purpose of this research was to adopt a global approach to the analysis of patello-femoral biomechanics.
by simultaneously assessing its kinematics and dynamics. Such an approach became essential when realizing how the loads acting on the patella vary during its motion.

The computed three-dimensional motion of the patella during knee flexion could be characterized by an increasing flexion, a wavy tilt, a small lateral rotation and a medio-lateral shift. Such trends were already found in previous experimental studies (Heegaard et al., 1994; van Kampen and Huiskes, 1990). Patellar tracking could thus be used to validate the model by comparing computed results with the corresponding experimental ones.

Validation of the model rested on a specimen-related comparison: predicted numerical results were analysed and compared to equivalent experimental ones. Here, 'equivalence' means that the model used the same articu-
lar surface geometries and boundary conditions as those used in preliminary experimental study (Heegard et al., 1994). During this experimental study, it was shown that a simplified description of the extensor apparatus, in which all soft tissue structures (except for the PT and RF) were removed, is still able to predict meaningful results in the knee flexion range from 30° to full flexion. The present numerical results have in turn shown that the proposed mathematical model representing this simplified extensor apparatus accurately reproduced the experimental tracking within the specified flexion range.

Small variations of the tendon insertion locations produced only small changes in patellar tracking (Table 4), reflecting the good stability of the model with respect to positioning of the extensor elements (patellar tendon and rectus femoris). Thus, some tolerance in the positioning of the bony insertion sites of these structures was allowed. This was of particular importance in the case of the patella, where it is difficult (indeed even impossible) to locate RF and PT insertion sites precisely, as both structures seem to blend together over the anterior margin of the patella. Furthermore, the effects of quadriceps tension characteristics on the patellar trajectory have been shown to be small (Ahmed et al., 1989; van Kampen, 1987) especially in the considered flexion range.

The predicted patellar tendon force magnitudes were consistent with published experimental results (e.g. Ahmed et al., 1987; Ellis et al., 1980; Huberti et al., 1984) or numerical results (e.g. Hirokawa, 1991; van Eijden et al., 1986; Yamaguchi and Zajac, 1989), all of which found a decreasing ratio up to about 90° of knee flexion. According to Ellis et al., (1980) the difference of tensions in the PT and quadriceps is not due to frictional forces but due to the geometry of the patello-femoral joint. This could be further deduced in the present frictionless model by considering the equilibrium of moments of the pulling forces $f_T$ and $f_Q$ acting on the patella: the ratio $\frac{\|f_T\|}{\|f_Q\|}$ reflected, in first approximation, the ratio between the
area proximal and distal to the contact region, which decreased until 90° of flexion as the contact region moved from distal to proximal.

The motion of the contact area evolved during knee flexion along the same trends as those reported by previous investigations (e.g. Ahmed 1983; Fujikawa et al., 1983) and could be characterized by a proximal shift of the contact region during the first part of flexion (up to 90° of flexion) and by a backward distal shift during the last phase of knee flexion. Noticeable differences ap-
peared between both specimens, stressing the influence of individual anatomy on these patterns. This description of the contact patterns evolution yielded to a global view of the pressure distribution across the articular surface, but did not provide precise quantitative information about the pressure magnitudes. Hence, to complete the description, peak pressures on the medial and lateral facets were also analyzed as a function of knee flexion (Fig. 5) and showed that the pressure distribution between the medial and lateral facets is characterized by higher lateral peak pressures near extension and full flexion, and by an almost even distribution in the mid-flexion range. This could further confirm the tendency to find more worn fibrils on the superficial layer of the lateral facet in patellar chondropathy (Mori et al., 1993).

Patellar trabecular bone architecture has been described as a non-homogeneous stacking-up of sheets and struts (Townsend et al., 1976), resulting from an optimized remodeling process. Moreover, the loads sustained by the patella depend on the location of the contact areas which vary during flexion. It follows that the mechanical properties of cancellous bone will be spatio-dependent. It can thus be hypothesized that the highest hydrostatic compressive stresses found primarily beneath the lateral
facet are related to the higher subchondral bone densities found in the proximal part of the lateral facet (Eckstein et al., 1992). In the present model, however, cancellous bone was considered as an isotropic and homogeneous material, suggesting some care in interpreting the computed stresses.

It is, nevertheless, instructive to relate these patterns to available results in the literature. Using a simple three-dimensional planar beam model, Minns et al. (1979) computed surface stress patterns along transverse sections, which were characterized by compression at the posterior facet (articular surface) and tension at the opposite anterior surface. They further observed that high tensile stress on the medial aspect may exist during knee flexion. Current results for the hydrostatic stress patterns computed in both specimens made it unclear, however, whether such tensile stress patterns occur: in the first specimen they appeared over the lateral side as opposed to the second specimen where some tensile stress was found in the posterior medial aspect. Most of the tensile stress was concentrated near the PT insertion area, whereas there was no such concentration at the quadriceps insertion, underscoring the importance of the bone shape in stress distribution (the PT inserts in a region where the sagittal radius of curvature of the cortical shell is smaller than the one near the quadriceps insertion, thus favoring stress concentrations).

The highest shear stress values were observed along the anterior surface of the cancellous bone, which in case of homogeneous and isotropic material agrees with previous results (Hayes et al., 1982). High shear stresses were also found beneath the patellar tendon insertion site. In terms of tissue differentiation, this could further explain how the tendon inserts in a reinforced region, by assuming that ossification of cartilage is accelerated in those regions exposed to higher shear (deviatoric) stresses (Wong and Carter, 1990).

At this point it is appropriate to mention the essential features of the computer model presently used to assess the global patello-femoral biomechanics. Currently available mathematical models in joint biomechanics usually deal either with rigid body systems, in which forces and moments are related to rigid body motions through the laws of classical mechanics (i.e. Newton Laws) or with deformable bodies, in which stresses are related to strains through constitutive laws in addition to the former relationships. On the one hand, rigid body systems provide only a coarse approximation of the joint's interface behavior while deformable body systems are fixed and are further submitted to fully prescribed external loads representing only an approximation of the unknown joint forces.

Most models dealing with joint kinematics are two-dimensional (resp. three-dimensional) rigid body models, where each component of the system has three (resp. six) degrees of freedom. Sometimes, these models also include a few additional degrees of freedom taking into account the joint components flexibility and are governed by a constitutive law (e.g. ligaments, articular contact). All these models end up with a set of non-linear equations expressing equilibrium in terms of rigid body kinematics parameters. Such models have been widely used for analyzing knee joint kinematics (e.g. Andriacchi et al., 1983; Blankevoort and Huiskes, 1991; Essinger et al., 1989; Wisman et al. 1980), with emphasis brought to the tibio-femoral joint (a comprehensive review can be found in (Huiskes, 1992)). However, less attention has been paid in modeling the patello-femoral joint (Hefty and Grood, 1988). Van Eijden et al. (1986) published the first mathematical model to compute patellar kinematics by describing the joint as a two-dimensional mechanism acting in the sagittal plane. Since then only a few more two-dimensional patellar models have been published up to this date (e.g. Reithmeier and Plitz, 1990; Yamaguchi and Zajac, 1989). Finally, Hirokawa (1991) presented a three-dimensional generalization of van Eijden's two-dimensional model.

In contrast to rigid body models, continuous deformable models are characterized by an infinite number of degrees of freedom. Hence, equilibrium conditions take the form of a system of non-linear partial differential equation (NPDE) instead of a finite set of equations characteristic of rigid body systems. However, when confronted with these NPDE over such complex shapes as those found in human bones, analytical solutions do not exist. These NPDE are thus approximated (to a fixed degree of accuracy) by a set of non linear equations by discretizing the geometrical domain under study using the finite element method (FEM) (e.g. Curnier, 1994; Hughes, 1987; Zienkiewicz and Taylor, 1991). A general description of this method, its principles, its possibilities and limitations in orthopedic biomechanics has been published by Huiskes and Chao (1983). As for rigid body models, only a few mathematical studies have been devoted to patellar stress analysis (e.g. Hayes et al., 1982; Minns et al., 1979; Minns and Braiden, 1981).

Besides the rigid or deformable approach in joint biomechanics, a still challenging problem arises from the contact between the components constituting the joint (i.e. they are free to separate but cannot penetrate each other). The finite element method has provided the ground for a number of efficient solutions to this problem by the implementation of contact elements (e.g. Alart and Curnier, 1991; Chan and Tuba, 1971; Hughes et al., 1976). The effectiveness of such elements in joint biomechanics was illustrated by, for example, Chan and Rim (1976), Rapperport et al., (1987), Huber-Betzer et al., (1990), Weinas et al., (1990) and Rubin et al., (1993). However, these contact elements were characterized by a node-on-node geometry and were therefore restricted to small slips between the contacting bodies. Thus, moving joints, where relative motion becomes large, were precluded with these elements.

In the present mathematical model of the patello-femoral joint all the foregoing limitations were overcome. First, the geometrical non-linear continuum formulation adopted to describe deformable bodies kinematics allowed one to consider large displacements of the patella, including finite rotations, from which the pseudo-rigid body kinematics could be extracted. Secondly, by assum-
ing the patella as a moving deformable body sliding on the femur, stresses occurring in the patella could be evaluated during its motion and not only at a prescribed fixed position.

Finally an essential feature of this model was to consider the forces acting on the joint as part of the problem unknowns. For each successive patellar position occurring during knee flexion, the corresponding system of forces, including quadriceps forces (imposed), patellar tendon forces and contact forces, represented a loading system consistent with the computed patellar tracking, and therefore provided more realistic loading conditions acting on the patella than applying fully imposed forces. This was especially relevant when considering the wide range of contact pressure variations occurring during knee flexion, and which could directly affect subchondral bone stresses. It follows that the hydrostatic pressure and von Mises stress could be further related to these loading conditions, unlike the aforementioned stress studies, in which all the forces acting on the patella were prescribed. This important feature followed from the adopted method to include unilateral large slip contact in a non-linear continuum model of the joint: the problem could be stated as a constrained optimization one, where the total energy of the extensor system, including the patella and the patellar tendon, had to be minimized under the constraint that the gap distances between the patella and the femur be non-negative. This minimum was then characterized by means of an augmented Lagrangian functional. The resulting model highlighted the accuracy that could be obtained by combining a rigorous contact law and a precise geometric model.

A natural extension to the present model would be to consider the trabecular bone as inhomogeneous, with spatial dependent density (or equivalently spatial dependent stiffness) which according to Hayes et al. (1982) could lead to differences, from the homogeneous case, as large as 200% in the peak von Mises stresses. Such features are already available in the present model (Rakotomanana et al., 1992), but were neglected here for simplicity (identification of the associated material constants remains difficult). The articular cartilage could be further considered as a bi-phasic material, which would require further development. It can be noticed that the augmented Lagrangian formalism introduced to enforce exactly contact conditions could also be used to handle the cartilage fluid phase incompressibility condition. Finally, including bone remodeling in such a model could help to understand the relationship between joint motion and underlying bone morphology and structure better.

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