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FABRIC AND ELASTIC PRINCIPAL DIRECTIONS OF CANCELLOUS BONE ARE CLOSELY RELATED

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Abstract—Cancellous bone architecture and mechanics are intimately related. The trabecular architecture of cancellous bone is considered determined by its mechanical environment (Wolff's law), and the mechanical properties of cancellous bone are inversely determined by the trabecular architecture and material properties. Much effort has been spent in expressing these relations, but the techniques and variables necessary for this have not been fully identified. It is obvious, however, that some measure of architectural anisotropy (fabric) is needed. Within the last few years, volume-based measures of fabric have been introduced as alternatives to the mean intercept length method, which has some theoretical problems. This paper seeks to answer which of four different fabric measures best predicts finite element calculated mechanical anisotropy directions.

Twenty-nine cancellous bone specimens were three-dimensionally reconstructed using the automated serial sectioning technique. A series of large-scale finite-element analyses were performed on each of the three-dimensional reconstructions to calculate the compliance matrix for each specimen, from which the mechanical principal directions were derived. The architectural anisotropy was determined in three-dimensional space for each specimen using mean intercept length (MIL), volume orientation (VO), star volume distribution (SVD) and star length distribution (SLD). Each of the architectural anisotropy results were expressed by a fabric tensor. Architectural main directions were determined from the fabric tensors and compared with the FE-calculated mechanical anisotropy directions.

All architectural measures predicted the mechanical main directions rather well, which supports the assumption that mechanical anisotropy directions are aligned with fabric directions. MIL showed a significant, though very small (1.4°), deviation from the primary mechanical direction. VO had difficulty in determining secondary and tertiary mechanical directions; its mean deviation was 8.9°. SVD and SLD provided marginally better predictors of mechanical anisotropy directions than MIL and VO. © 1997 Elsevier Science Ltd. All rights reserved.

Keywords: Cancellous bone; Trabecular architecture; Anisotropy; Fabric; FEM.

INTRODUCTION

Bone continually adapts to its mechanical environment (Wolff's law), and for cancellous bone this adaptation results in varying trabecular architectures. The adaptation has the consequence that the elastic properties of the fully adapted bone fit forces in the mechanical environment (Cowin, 1986). Adaptation thus links usage, trabecular architecture and elastic properties, and it has the implication that it may be possible to 'read' the elastic properties and the mechanical usage from the trabecular architecture. Much effort has been invested in specifying the suggested relation between architecture and elastic properties, but it has been a problem to identify the architectural parameters of relevance. Cancellous bone is mechanically and architecturally anisotropic, i.e. mechanical and architectural properties depend on test direction. A requirement for formulating a relation between architecture and mechanics should be an ability to predict mechanical anisotropy from architectural anisotropy.

Cowin (1985, 1986) introduced the term *fabric* in bone mechanics as a description of the local anisotropy of a material's microstructure, and a *fabric tensor* was de-

fined as any positive definite second rank tensor, which quantitatively describes fabric. Based on an assumption of orthotropy, the elastic mechanical properties may be formulated as a function of fabric and density (Cowin, 1985, 1986). In these relations, it is implicitly assumed that fabric and mechanical main directions are aligned.

Different fabric measures exist. The *mean intercept length* (MIL) method (Harrigan and Mann, 1984; Whitehouse, 1974) quantifies the anisotropy of trabecular surfaces. Obviously anisotropic structures with isotropy of the surface exist (Fyhrie *et al.*, 1992; Odgaard, 1994), and to overcome this potential problem with MIL, a new set of methods for describing architectural anisotropy was introduced with the *volume orientation* (VO) method (Odgaard *et al.*, 1990b). This method does not depend on trabecular surfaces but on the typical distribution of trabecular volume around a typical point in bone. Recently, a variation of the VO method known as the *star volume distribution* (SVD) method has been introduced (Cruz-Orive *et al.*, 1992; Karlsson and Cruz-Orive, 1993). The MIL method and the VO and SVD methods are fundamentally different. No known theoretical relation exists between them, and all are candidates for being used in a formulation of a fabric-mechanics relation for cancellous bone using Cowin's (1985, 1986) relations.

A method has recently been introduced, which allows for numerical simulation of elastic mechanical tests of cancellous bone specimens using microstructural large-

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scale finite element models (LS-FEM) generated from three-dimensional reconstructions (Hollister, 1995; van Rietbergen *et al.*, 1995a,b). By simulating different loading situations, it is possible with LS-FEM to calculate elastic anisotropy properties and principal directions (van Rietbergen *et al.*, 1996a).

Bone adaptation is mediated through cell-based remodeling of trabecular surfaces, but the mechanical properties seem more related to the arrangement of trabecular volume. This suggests a functional association between surface and volume, but no theoretical or empirical knowledge exists, which relates trabecular surface and volume anisotropy. The hypothesis behind the present study is that the volume- and surface-based fabric measures differ in real specimens. We furthermore wish to provide experimental data on the implicit assumption of alignment of fabric and elastic anisotropy directions.

MATERIALS AND METHODS

For the present study we wanted the cancellous bone material to be as homogeneous as possible, and it was chosen to use cancellous bone from a vertebra of a large whale. A vertebra from an 11 m long sperm whale (*Physeter catodon*) was obtained from a museum of natural history. The whale had been stranded on the west coast of Denmark. The vertebral body had a diameter of 22 cm. CT scanning of the vertebra was performed to identify regions with different densities and textures, and based on this, 29 specimens were harvested. Of these, 8 came from the anterior part of the vertebral body, 13 from the center, and 8 from the spinous process. All specimens were cubic with a side length of 10 mm.

Each specimen was three-dimensionally reconstructed using the automated serial sectioning technique (Odgaard *et al.*, 1990a, 1994) with an x -, y -, and z -resolution of 20 μm . Each three-dimensional data set consisted of about 500 images (1024×1024) with a total size of 524 Mb unsegmented and 60 Mb in segmented form. Previously described segmentation (Odgaard *et al.*, 1994) and purification (Odgaard and Gundersen, 1993) methods were used.

The microstructural geometry of each individual specimen encoded in the three-dimensional reconstructions were used for LS-FEM. The three-dimensional reconstructions were remeshed into a coarser data set by grouping $4 \times 4 \times 4$ voxels into a new one. The FE-models were then generated by directly converting the new bone voxels to equally sized 8-node brick elements (van Rietbergen *et al.*, 1995b). A special purpose FE-code was used to solve these large FE-problems. The solving algorithm makes use of an iterative solver in combination with a 'row-by-row' matrix-vector multiplication algorithm (van Rietbergen *et al.*, 1996c). In the present study, the number of elements in the FE-models varied between 352,679 and 653,533 depending on the volume fraction. The computations were performed on a Cray C92. Six mechanical tests (three normal tests and three shear tests) were simulated for each specimen in the specimen coordinate system, and this allows all components of the apparent compliance matrix to be determined for each specimen using a standard mechanics approach (Hollis-

ter and Kikuchi, 1992). The average CPU-time per specimen was about 2 h.

Assuming orthotropy, a method was developed to find the mechanical principal directions directly from the components of the compliance matrix, as measured in the specimen coordinate system. This method determines the coordinate transformation, which yields the best orthotropic representation of the compliance matrix by minimizing the sum of squares of the matrix entries, which are zero for pure orthotropic materials (van Rietbergen *et al.*, 1996a). The matrix is then forced to orthotropic symmetry by setting these entries to zero. For each specimen, the principal mechanical axes and Young's moduli (E_{1-3}) in these directions were calculated. Axes and moduli were sorted such that $E_3 \leq E_2 \leq E_1$, and moduli were normalized, so that $E_1 + E_2 + E_3 = 1$.

The basic principle of the MIL method is to count the number of intersections between a linear grid and the bone/marrow interface as a function of the grid's orientation ω (Whitehouse, 1974); see Fig. 1(A). The mean intercept length (an intercept is the line between two intersections) is the total length L of the line grid divided by the number of intersections: $\text{MIL}(\omega) = L/I(\omega)$. Three-dimensional MIL measurements may be fitted to an ellipsoid (Whitehouse, 1981), which can be expressed as the quadratic form of a second rank tensor $\mathbf{M}$, i.e. a fabric tensor (Harrigan and Mann, 1984). Cowin (1986) defined a MIL fabric tensor $\mathbf{H}$ as the inverse square root of $\mathbf{M}$. The main advantage of this modification is that larger values of $\mathbf{H}$ will be associated with larger values of Young's modulus, and that the eigenvalues of $\mathbf{H}$ are the MIL values in the main directions. In this paper the fabric tensor $\mathbf{H}$ will be used to express MIL anisotropy.

The volume orientation (VO) method was introduced to shift the interest of anisotropy measurements from the bone/marrow interface to the distribution of trabecular volume (Odgaard *et al.*, 1990b). Instead of measuring a length for a number of directions, the result of a VO measurement is a sample of orientations. The local volume orientation is defined for any point within bone as the orientation of the longest intercept through the point [Fig. 1(B)], and it is assumed that this orientation is uniquely defined for any point. The local volume orientation does not have any positive or negative direction, so the orientation can be identified by a hemispherical orientation. A convenient statistic of a sample of local volume orientations is given by the orientation matrix (see the Appendix), which we define as the VO fabric tensor $\mathbf{V}$.

The star volume distribution (SVD) is conceptually closely related to both the VO measure and the star volume (Gundersen and Jensen, 1985), which is defined as the average volume, that can be seen unobscured from a random point within a structure. The star volume may be expressed as $\bar{v}_V^* = \frac{1}{3}\pi L^3$ where L is the average length of an intercept with random orientation through a random point (Gundersen and Jensen, 1985; Vesterby, 1993). Instead of determining the average of intercepts with random orientations, one may estimate the volume from a single orientation ω of the test line through a random point [Fig. 1(C)]:

$$\bar{v}_V^*(\omega) = \frac{\pi}{3} L^3(\omega). \quad (1)$$

Fig. 1. Two-dimensional illustrations of the architectural anisotropy measures. For these examples ω is a semicircular orientation, and extended to three-dimensional space ω will be a hemispherical orientation. (A) The mean intercept length (MIL) method. A parallel line grid is placed on the structure, and the number of intersections between test lines and structure interfaces are counted as a function of orientation ω . The count is normalized by the test line length and expressed as mean test line length between intersections (mean intercept length). (B) The volume orientation (VO) method. The local volume orientation is defined for any point within bone as the orientation of the longest intercept through the point. By randomly selecting a number of points, a sample of local volume orientations is obtained, which may be treated by a number of statistical methods. The figure shows four local volume orientations sampled by a randomly translated point grid. (C) The star volume distribution (SVD) and star length distribution (SLD) methods. For each of a number of orientations through a point in bone the intercept lengths are determined. This process is repeated over a number of points, which yields the mean volume or length as a function of orientation ω . The figure shows intercepts through six random points sampled by a point grid. The orientation of the lines are 30° apart. The intercept length is used directly for SLD and cubed for SVD.

which defines a *star volume component* (Cruz-Orive *et al.*, 1992; Karlsson and Cruz-Orive, 1993). The variation of the star volume component with orientation defines the star volume distribution. As suggested by Karlsson and Cruz-Orive (1993) the SVD fabric tensor $\mathbf{S}$ was calculated using the orientation matrix method (ellipsoid of inertia) with the modification that each direction is weighed by its star volume component (see the Appendix).

An obvious modification to SVD is simply to weigh the orientation by the observed length, and we have chosen to call this the *star length distribution* (SLD):

$$s(\omega) = \frac{1}{n} \sum_{i=1}^n L_i(\omega), \quad (2)$$

where s is the *star length component*. This definition was used by Goulet *et al.* (1992). The sample is analyzed by the orientation matrix method, and the fabric tensor is called $\mathbf{L}$.

For all architectural measures, a central cubic sampling volume was defined in each specimen. The planar faces of this volume were parallel to and 1.5 mm away from the faces of the specimen. The argument for this guard region is that the results of volume-based measurements will be affected by artificial edges, if these edges can be seen from the sampling point. The specific choice of 1.5 mm was made after considering the architecture of the specimens. Local volume orientations were sampled in the three-dimensional sampling volume for 500 random points falling within the trabeculae. An exhaustive search algorithm was used, which searches all possible directions. The star volume and star length distributions were determined in three-dimensional space for 1000

random orientations for 1000 random points within the trabeculae. Separate measurements of SVD and SLD were made. Mean intercept lengths were determined in three-dimensional space for 1000 random orientations, and the total line length was about 1700 mm for each orientation. Random points were determined by a randomly translated and rotated cubic grid. Random directions were equally distributed in the probability field $[0 \leq \phi < 2\pi; 0 \leq \sin \theta \leq 1]$, where (θ, ϕ) are polar coordinates (see the Appendix). If the random directions were equally distributed in the field $[0 \leq \phi < 2\pi; 0 \leq \theta \leq \pi/2]$, i.e. fixed steps in (θ, ϕ) , the orientations would be highly concentrated around $\theta = 0$.

Let the eigenvalues of any of the fabric tensors be designated by $\hat{\tau}_1$, $\hat{\tau}_2$ and $\hat{\tau}_3$, where $\hat{\tau}_3 \leq \hat{\tau}_2 \leq \hat{\tau}_1$, then the fabric tensor is normalized by dividing by $\hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3$. All fabric tensors were normalized by this method. The eigenvectors corresponding to $\hat{\tau}_1$, $\hat{\tau}_2$ and $\hat{\tau}_3$ were designated $\hat{\mathbf{u}}_1$, $\hat{\mathbf{u}}_2$ and $\hat{\mathbf{u}}_3$, respectively. The direction given by $\hat{\mathbf{u}}_1$ is called the primary direction, $\hat{\mathbf{u}}_2$ defines the secondary direction, and $\hat{\mathbf{u}}_3$ the tertiary direction. Collectively, these directions are the *main* directions, which are orthogonal. In some situations the sign of a direction may change to preserve a right-hand coordinate system.

For each of the 29 specimens a LS-FEM calculated compliance matrix and four fabric tensors ($\mathbf{H}$, $\mathbf{V}$, $\mathbf{S}$ and $\mathbf{L}$) were calculated. The task at hand is to compare the eigenvectors of the fabric tensors with the mechanical main directions. A general test for comparison of fabric main directions (rotated to mechanical symmetry coordinate system) with the mechanical main directions was performed using a non-parametric method (Fisher *et al.*,

1987).^{*} For comparison of the spread of samples of unit vectors, a non-parametric method based on the *mean resultant length* (mrl) was used. The mean resultant vector of a sample of n unit vectors is defined as

$$\hat{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \hat{y} = \frac{1}{n} \sum_{i=1}^n y_i, \quad \hat{z} = \frac{1}{n} \sum_{i=1}^n z_i, \quad (3)$$

where $(\hat{x}, \hat{y}, \hat{z})$ are its coordinates. The mean resultant length is defined as the length of $(\hat{x}, \hat{y}, \hat{z})$. If all unit vectors of a sample are identical, then the mean resultant length will be 1. The larger the spread of the sample, the smaller the mean resultant length. The mean resultant length can readily be calculated for each sample, and its standard error may be estimated using the jackknife method (Fisher *et al.*, 1987).

RESULTS

Two sample specimens (Fig. 2 and Table 1) will demonstrate the type of results obtained from individual specimens. For specimen B the primary mechanical direction has coordinates $(-0.776, 0.625, 0.080)$ and the primary directions of the architectural measures are in close agreement with this. For the secondary and tertiary directions, there is also relatively good agreement between mechanical and architectural main directions. For specimen A, the primary mechanical direction has coordinates $(0.092, 0.031, 0.995)$, which is well predicted by all architectural measures. In order to better compare the architectural main directions with the mechanical main directions, all architectural directions were rotated to the mechanical symmetry coordinate system (Table 2). After this rotation, the z -axis corresponds to the primary mechanical direction, the y -axis to the secondary, and the x -axis to the tertiary mechanical direction. Table 2 also shows deviations between architectural and mechanical main directions.

The analysis of all 29 specimens showed that all fabric measures come very close to the mechanical main directions (Fig. 3), but some of the deviations are nevertheless significant (Table 3). At a 5% level, MIL deviates significantly from the primary mechanical direction, although this deviation is only 1.4° on average. VO has difficulty in determining the secondary and tertiary mechanical directions (both 8.9° off). MIL also deviates for the tertiary mechanical direction, although this deviation is only 3.8° . In conclusion, SLD and SVD provide the most accurate estimates of the mechanical main directions.

All architectural measures show similar spread around the mechanical main directions (Fig. 3 and Table 4). For the primary direction data, MIL has the smallest mrl, but for secondary and tertiary directions, MIL has the largest mrl. Testing for differences using a t -test fails, however, to reveal any significant difference between mrl for the different groups.

As an indication of the degree of anisotropy, the ratio between normalized stiffness or fabric eigenvalues is shown in Table 5. The secondary to tertiary ratios are all close to unity indicating that some of the specimens have been close to transversely isotropic.

Fig. 2. Graphical three-dimensional reconstructions of two characteristic specimens. The specimens are shown in three different rotations given by the coordinate system axes: the upper row is seen from the z -axis, the middle row from the x -axis and the lower row from the y -axis. Specimen A is visually close to isotropic, while specimen B is highly anisotropic. Specimen B has visually a primary direction close to $(-\sqrt{2}/2, \sqrt{2}/2, 0)$. The specimens are shown with gradient-coded shading.

DISCUSSION

All mechanical tests of cancellous bone are based on the continuum assumption. For human cancellous bone this means that a cubic specimen should have a minimal size of the order of 3–6 mm (Goulet and Hollister, 1996; Harrigan *et al.*, 1988), but because of large architectural gradients in most epi- and metaphyseal regions, this means that there may be considerable architectural variation within a test specimen (Goulet *et al.*, 1988; Zhu *et al.*, 1994). Because of scaling constraints (Mullender *et al.*, 1997; Schmidt-Nielsen, 1984), the trabeculae of a whale are not much wider or more widely separated than in a human, but the size of a large whale vertebral body (20–40 cm) seems to promise a much more homogeneous mechanical environment—and architecture—for a test specimen of size 10 mm. This motivates the choice of the material used in this study. We do not imply that the mechanical properties of cetaceous and human cancellous bone are identical, or that the relations between mechanical and architectural properties are the

^{*} For the 1987, 1st edition, the list of errata must be consulted.

Table 1. Results of mechanical and architectural anisotropy analyses for the two specimens shown in Fig. 3. The coordinates of eigenvectors are given in rows

	Specimen A				Specimen B			
Normalized stiffness and directional cosines of mechanical main directions	0.435	(0.092, 0.031, 0.995)	0.306	(0.383, 0.922, -0.064)	0.273	(0.116, 0.266, -0.957)	0.182	(0.620, 0.734, 0.279)
MIL fabric tensor H	$\begin{bmatrix} 0.317 & -0.012 & -0.006 \\ -0.012 & 0.333 & -0.003 \\ -0.006 & -0.003 & 0.350 \end{bmatrix}$				$\begin{bmatrix} 0.373 & -0.091 & -0.022 \\ -0.091 & 0.337 & 0.008 \\ -0.022 & 0.008 & 0.290 \end{bmatrix}$			
Eigenvalues and eigenvectors	0.353	(0.260, 0.294, 0.920)	0.337	(0.384, 0.843, -0.378)	0.451	(0.770, -0.623, -0.136)	0.289	(-0.080, -0.307, 0.948)
	0.310	(0.886, -0.451, -0.106)	0.260	(0.632, 0.720, 0.286)				
SLD fabric tensor L	$\begin{bmatrix} 0.295 & 0.004 & 0.002 \\ 0.004 & 0.320 & -0.003 \\ 0.002 & -0.003 & 0.385 \end{bmatrix}$				$\begin{bmatrix} 0.384 & -0.140 & -0.023 \\ -0.140 & 0.327 & -0.002 \\ -0.023 & -0.002 & 0.289 \end{bmatrix}$			
Eigenvalues and eigenvectors	0.385	(0.019, -0.051, 0.999)	0.321	(0.172, 0.984, 0.047)	0.500	(0.774, -0.629, -0.076)	0.291	(-0.068, -0.202, 0.977)
	0.294	(0.985, -0.171, -0.028)	0.209	(0.630, 0.751, 0.199)				
SVD fabric tensor S	$\begin{bmatrix} 0.229 & 0.013 & 0.021 \\ 0.013 & 0.263 & -0.011 \\ 0.021 & -0.011 & 0.508 \end{bmatrix}$				$\begin{bmatrix} 0.517 & -0.386 & -0.039 \\ -0.386 & 0.372 & 0.017 \\ -0.039 & 0.017 & 0.111 \end{bmatrix}$			
Eigenvalues and eigenvectors	0.510	(0.077, 0.048, 0.996)	0.226	(0.298, 0.952, -0.069)	0.839	(0.769, -0.637, -0.056)	0.111	(-0.083, -0.187, 0.979)
	0.224	(0.951, -0.302, -0.059)	0.049	(0.634, 0.748, 0.197)				
VO fabric tensor V	$\begin{bmatrix} 0.223 & -0.004 & 0.039 \\ -0.004 & 0.287 & 0.013 \\ 0.039 & 0.013 & 0.491 \end{bmatrix}$				$\begin{bmatrix} 0.512 & -0.357 & -0.047 \\ -0.357 & 0.371 & 0.021 \\ -0.047 & 0.021 & 0.118 \end{bmatrix}$			
Eigenvalues and eigenvectors	0.497	(0.140, 0.060, 0.988)	0.286	(-0.093, 0.995, -0.047)	0.808	(0.772, -0.632, -0.071)	0.119	(-0.148, -0.287, 0.946)
	0.217	(0.986, 0.085, -0.145)	0.073	(0.619, 0.720, 0.315)				

Table 2. Architectural main directions rotated into mechanical symmetry coordinate system. For each set of rotations, two of the three deviations should be identical. This may not be the case because of round-off errors

	Specimen A			Specimen B		
	Coordinates	Deviation (deg)		Coordinates	Deviation (deg)	
<i>MIL</i>						
Primary direction	0.058, 0.311, 0.949	18.4		0.022, -0.053, 0.998	3.6	
Secondary direction	-0.054, -0.948, 0.314	18.6		0.010, 0.999, 0.052	2.6	
Tertiary direction	-0.997, 0.070, 0.038	4.4		-1.000, 0.009, 0.022	0.0	
<i>SLD</i>						
Primary direction	-0.035, -0.104, 0.994	6.3		0.007, 0.006, 1.000	0.0	
Secondary direction	-0.226, 0.970, 0.093	14.1		0.083, -0.997, 0.005	4.4	
Tertiary direction	0.974, 0.221, 0.058	13.1		-0.997, -0.082, 0.008	4.4	
<i>SVD</i>						
Primary direction	-0.020, 0.010, 1.000	0.0		0.011, 0.028, 1.000	0.0	
Secondary direction	0.089, -0.996, 0.011	5.1		0.084, -0.996, 0.027	5.1	
Tertiary direction	0.996, 0.089, 0.020	5.1		-0.996, -0.084, 0.013	5.1	
<i>VO</i>						
Primary direction	0.033, 0.045, 0.998	3.6		0.010, 0.012, 1.000	0.0	
Secondary direction	0.467, -0.884, 0.025	27.9		-0.038, -0.999, 0.012	2.6	
Tertiary direction	-0.884, -0.465, 0.051	27.9		-0.999, 0.038, 0.009	2.6	

same for human and cetaceous cancellous bone. But it seems plausible that conclusions concerning fabric measures should also be valid in human cancellous bone.

In the present study, the mechanical principal directions as obtained from the microstructural finite element

analyses were used as a golden standard. The accuracy of this golden standard is dependent on the accuracy of the FE-analyses and that of the procedure for finding the orthotropic principal directions. The accuracy of FE-results can be affected by modeling errors due to inaccuracies in the reconstruction, and by numerical errors due

Fig. 3. Equal area projections (Fisher *et al.*, 1987) for all 29 specimens of architectural main directions rotated to corresponding mechanical main direction coordinate system. The upper row represents the primary direction, the middle row the secondary direction, and the lower row the tertiary direction. The first column represents MIL data, the second SLD data, the third SVD, and the fourth VO data. In each plot the orientations are viewed from one of the coordinate system axes, and the other two axes are marked. Each minor tic along the periphery of a plot marks 10° longitude, and each tic from the center to the periphery marks 10° colatitude. It should be noted that all architectural measures provide very good estimates of the primary mechanical direction. The reason for the spread around secondary and tertiary directions is probably relatively transverse isotropy.

Table 3. Result of tests for coincidence of mechanical and architectural main directions

	Main direction			Deviation	p
<i>Primary direction (mechanical direction = (0, 0, 1))</i>					
MIL	0.015,	0.018,	1.000	1.4°	0.0374*
SLD	0.009,	-0.011,	1.000	0.8°	0.0836
SVD	0.001,	-0.001,	1.000	0.1°	0.9784
VO	0.006,	-0.001,	1.000	0.3°	0.5375
<i>Secondary direction (mechanical direction = (0, 1, 0))</i>					
MIL	-0.064,	0.998,	-0.017	3.8°	0.3118
SLD	0.042,	0.999,	0.009	2.4°	0.0759
SVD	0.009,	1.000,	-0.006	0.6°	0.5602
VO	-0.155,	0.988,	0.008	8.9°	0.0370*
<i>Tertiary direction (mechanical direction = (1, 0, 0))</i>					
MIL	0.998,	0.063,	-0.019	3.8°	0.0170*
SLD	0.999,	-0.041,	-0.003	2.4°	0.6870
SVD	1.000,	-0.009,	-0.000	0.5°	0.9942
VO	0.988,	0.155,	-0.004	8.9°	0.0551

*Significance at the 5% level.

Table 4. Mean resultant lengths (mrl) and jackknife-estimated standard errors (SEM) for samples of main directions of architectural measures

	mrl	SEM
<i>Primary direction</i>		
MIL	0.996840	0.001608
SLD	0.998963	0.000375
SVD	0.999064	0.000253
VO	0.998924	0.000292
<i>Secondary direction</i>		
MIL	0.441851	0.150085
SLD	0.262516	0.147029
SVD	0.289184	0.153658
VO	0.103464	0.153137
<i>Tertiary direction</i>		
MIL	0.222290	0.166084
SLD	0.060928	0.096700
SVD	0.175825	0.158781
VO	0.156587	0.106466

to a relatively large element size, jagged boundaries, and incorrect boundary conditions. Since the same three-dimensional reconstructions were used as a basis for the fabric measurements and for the FE-analyses, possible errors in the reconstruction would not affect the results. The numerical errors are somewhat harder to address. In

a mesh convergence study (van Rietbergen *et al.*, 1995b), it was found that the largest difference in the calculated modulus for models with an element size in the range $20\text{--}80\text{ }\mu\text{m}$ was of the order of 10%. Additionally, since the elements are the same size in all directions, the errors related to element size are not likely to show significant

Table 5. Expressions of degree of anisotropy

	E	MIL	SLD	VO	SVD
Primary-to-secondary	1.99 (1.42–2.72)	1.36 (1.05–1.70)	1.70 (1.20–1.98)	6.17 (1.74–8.89)	7.12 (1.92–10.69)
Secondary-to-tertiary	1.15 (1.03–1.54)	1.07 (1.01–1.22)	1.09 (1.01–1.40)	1.22 (1.01–1.91)	1.30 (1.03–2.25)
Primary-to-tertiary	2.30 (1.55–3.24)	1.55 (1.13–1.84)	1.89 (1.31–2.39)	7.38 (2.10–12.60)	10.25 (2.20–16.97)

Note: The first column (E) gives median of ratios between normalized stiffnesses, and the four right columns (MIL–SVD) give medians of ratios between fabric tensor eigenvalues. Numbers in parentheses are ranges.

anisotropy effects. Numerical errors due to the jagged surfaces have been shown to cause local inaccuracies but do not affect the apparent properties (Jacobs *et al.*, 1993). The boundary conditions chosen with the standard mechanics approach (uniaxial strain) can result in mechanical properties which are somewhat higher than the *in vivo* properties, but this effect also would affect all directions. Finally, an upper limit for errors in the calculated stresses and strains due to forcing the material properties to orthotropy was calculated and was found to be in the order of a few percent (van Rietbergen *et al.*, 1996a). In summary, although it is possible that the absolute values found for the mechanical properties are affected by numerical errors, it seems unlikely that the determination of the orthotropy axes is much affected by these errors.

It should be emphasized that the results of the FE-analyses give the mechanical properties of the trabecular architecture, and not that of the bone specimen. The latter is also dependent on the tissue material properties. For the comparisons made in the present study, this is an advantage, since the dependency of mechanical properties on tissue material properties clearly is not caught by fabric measures, and thus would yield unexplained variance. If, however, one is interested in the mechanical properties of the specimen from the FE-analyses, information on the tissue modulus is needed.

All specimens in this study showed anisotropy in the LS-FEM analysis (Table 5). If truly isotropic specimens had been included, the interpretation of the primary mechanical direction would have been impossible. The secondary-to-tertiary ratios (Table 5) indicate that some of the specimens are close to transversely isotropic. This may explain, why the fabric measures had difficulty in identifying the secondary and tertiary direction (Fig. 3 and Table 3).

Goulet *et al.* (1994) defined the degree of anisotropy (DA) as the ratio between MIL in the primary and tertiary direction, and a median value of 1.62 (range 1.11–2.54) was found. Turner *et al.* (1990) found a mean of about 1.73 ($\hat{\tau}_3(\mathbf{H}) = 0.246$ and $\hat{\tau}_1(\mathbf{H}) = 0.426$) for bovine, distal femoral samples and 1.54 ($\hat{\tau}_3(\mathbf{H}) = 0.278$ and $\hat{\tau}_1(\mathbf{H}) = 0.429$) for human, proximal tibial samples. In the present study, MIL DA had a median of 1.55 (range 1.13–1.84). Even though the latter value is within the range found by the authors mentioned, direct comparison is difficult because of different origins.

The data presented in this study suggest that both MIL and volume-based measures give good estimates of

the mechanical main directions (Fig. 3), but significant deviations do exist for some of the measures. The architectural measures which best predict the mechanical anisotropy directions are the SLD and SVD measures (Table 3). The VO measure has difficulty in identifying secondary and tertiary directions, and the MIL measure has a significant deviation from the primary direction. For each of the measures, there is some spread around the main directions, but this spread does not differ significantly between the architectural measures.

As for the hypothesis of this study, it may be concluded that there is only marginal differences between results of applying surface- and volume-based anisotropy measures, and the volume-based measures provide a fabric measure which is at least as good as MIL. Although it was found that the primary MIL direction deviated significantly from the primary elastic direction, the magnitude of this deviation (1.4°) probably excludes any practical consequence of this. The marginal differences found, however, indicate that the SVD and SLD measures may provide superior architectural measures of mechanical anisotropy in cancellous bone. The guard distance of 1.5 mm restricts the architectural examination volume to a subvolume of the specimen. Including the entire specimen volume in the MIL analysis did not significantly change the conclusions from the study.

Cowin's fabric-mechanics relations (Cowin, 1985, 1986) implicitly assume mechanical and fabric main directions to be aligned. This may seem an intuitively acceptable assumption, but experimental support has not previously been given. Several multiple regression analyses based on Cowin's relations have been published (Goulet *et al.*, 1994; Snyder and Hayes, 1990; Turner *et al.*, 1990; van Rietbergen *et al.*, 1996b), but none of these have allowed an isolated study of alignment. One important consequence of the close alignment, which has been demonstrated between mechanical and fabric main directions, relates to experimental studies on Wolff's law. Demonstration of changes in fabric main directions secondary to experimentally induced changes in mechanical environment will provide experimental support of the trajectorial theory interpretation of Wolff's law.

In a study of the relation between fabric measures and elastic properties (van Rietbergen *et al.*, 1996b), it was found that the fabric measures explain a very high proportion of the variation in elastic properties using Cowin's relations (1985). One may conclude from the present study and the study by van Rietbergen *et al.* that not only are fabric and mechanical main directions

closely related, but the degrees of anisotropy found by the different fabric measures do also relate closely to mechanical anisotropy.

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APPENDIX: THE ORIENTATION MATRIX

A hemispherical direction ω may in a (e_1, e_2, e_3) coordinate system be expressed either in polar coordinates (θ, ϕ) or by directional cosines (x, y, z) . Using polar coordinates, the colatitude is the angle between ω and the e_3 -axis and the longitude ϕ is the angle measured anticlockwise between the e_1 -axis and the projection of ω onto the e_1e_2 -plane (Fig. A1). As only hemispherical directions are considered, $0 \leq \theta \leq \pi/2$ and $0 \leq \phi < 2\pi$. The orientation may be identified by a unit vector $\mathbf{a}$:

$$\mathbf{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}. \quad (\text{A1})$$

Fig. A1. Definition of polar coordinates for a hemispherical orientation. The angle θ between the z -axis and the orientation is known as the colatitude. The angle ϕ between the x -axis and the projection of the orientation onto the xy -plane (measured anti-clockwise) is known as the longitude.

If each orientation $\mathbf{a}_i$ is considered to represent the location of a unit mass, the second moment of inertia I_Λ around a line Λ with directional cosines (λ, μ, ν) is given by (Meriam, 1971)

$$I_\Lambda = \sum_{i=1}^n |\Lambda(x_i, y_i, z_i)|^2 = n - (\lambda, \mu, \nu) \mathbf{T} \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix}, \quad (\text{A2})$$

where $|\Lambda(x_i, y_i, z_i)|$ is the distance from Λ to (x_i, y_i, z_i) , and $\mathbf{T}$ is the orientation matrix defined by

$$\mathbf{T} = \sum_{i=1}^n \mathbf{a}_i \mathbf{a}_i^T = \sum_{i=1}^n \begin{pmatrix} x_i & y_i & z_i \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i & \sum x_i z_i \\ \sum x_i y_i & \sum y_i^2 & \sum y_i z_i \\ \sum x_i z_i & \sum y_i z_i & \sum z_i^2 \end{bmatrix}. \quad (\text{A3})$$

Calculating the eigenvalues $\hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3$ and eigenvectors $(\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \hat{\mathbf{u}}_3)$ of $\mathbf{T}$ may be thought of as minimizing or maximizing the moment of inertia for the set of scattered masses.

In the case where $\mathbf{a}$ has directional cosines (x, y, z) but a length $L \neq 1$, equation (A3) should be modified to

$$\mathbf{T} = \begin{bmatrix} \sum L_i^2 x_i^2 & \sum L_i^2 x_i y_i & \sum L_i^2 x_i z_i \\ \sum L_i^2 x_i y_i & \sum L_i^2 y_i^2 & \sum L_i^2 y_i z_i \\ \sum L_i^2 x_i z_i & \sum L_i^2 y_i z_i & \sum L_i^2 z_i^2 \end{bmatrix}. \quad (\text{A4})$$

This modification is used for calculation of the SVD fabric tensor $\mathbf{S}$ and the SLD fabric tensor $\mathbf{L}$.

The orientation matrix method has been described by several authors, including Scheidegger (1964) and Watson (1966). In parametric distribution models, the orientation matrix also plays an important role in statistical inference (Fisher *et al.*, 1987; Mardia, 1972; Woodcock, 1977), and the orientation matrix is the first element in the fabric tensor expansion described by Kanatani (1984).