Observer based model predictive control

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OBSERVER BASED MODEL PREDICTIVE CONTROL

Bas Roset* Henk Nijmeijer**

*Technische Universiteit Eindhoven, Department of
Mechanical Engineering, Dynamics and Control Group,
P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

**e-mail: B.J.P.Roset@tue.nl*

***e-mail: h.nijmeijer@tue.nl*

Abstract: Model Predictive Control in combination with discrete time nonlinear observer theory is studied in this paper. Model Predictive Control, generally based on state space models, needs the complete state for feedback. In this paper the complete state is assumed not to be known and only outputs and inputs of the system are measured. To obtain knowledge of the full state an observer is used to obtain an estimate of the state. An extended nonlinear observer is used for this purpose and potentially allows for successful output based model predictive controllers.

Keywords: Output feedback, Nonlinear observers, Linear error dynamics, Nonlinear model predictive control, Discrete time

1. INTRODUCTION

General model predictive control is based on the knowledge of the complete state of the system. In a control environment it often happens that measurement of the complete state of the system is often not practical, very costly or even impossible. The solution to this problem is then the use of an observer. Recent work on the general output feedback stabilization problem for nonlinear systems is presented in [1] and [2]. Also significant progress has been achieved in [5] and [6]. The papers propose different versions of the nonlinear separation principle which originates from [4] and others.

In this paper we propose to use a general applicable nonlinear observer theory in discrete time which uses, comparable to the continuous time observer theory used in [1], a coordinate transformation of the original coordinates of the nonlinear system equations. The model resulting from this state transformation is the so called *Extended*

Nonlinear Observer Canonical Form, which was studied in [8] and [9]. Designing an observer in this *Extended Nonlinear Observer Canonical Form* results in *linear error dynamics*, which means that the observer design can be performed using standard linear observer design techniques. A major disadvantage using this observer type is that there are, in general, future input values required in the observer structure. This is a problem if the input sequence is not known in advance, which is the case in a general control environment.

In this paper, and also in [11], it is proposed to *predict* future input values, needed by the extended observer, by using the Model Predictive Control scheme and subsequently feeding them to the observer. We consider single input single output systems defined as

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{f}(\mathbf{x}(k), u(k)) \\ y(k) &= g(\mathbf{x}(k)) \end{aligned}, \quad (1)$$

$$\mathbf{x}(k=0) = \mathbf{x}_0, \text{ and } k \in \mathbb{N}_0,$$

with state variable $\mathbf{x}(k) \in \mathbb{R}^n$, input $u(k) \in \mathbb{R}^1$, output $y(k) \in \mathbb{R}^1$. The functions $\mathbf{f}$ and g are considered to be smooth.

The paper is organized as follows. First the Model Predictive Control scheme is briefly explained. The observer theory is explained in section 3. In section 4 the proposal to combine the used Model Predictive Control scheme and the observer theory is set forth. An example is given in section 5. Finally some conclusions are drawn in section 6.

2. THE MODEL PREDICTIVE CONTROL SCHEME

A nice overview of different issues about Model Predictive control theory is given in [7]. One of the open questions and research areas of the Model Predictive Control strategy is stability. Some results on the stability of Model Predictive Control using state space models and stabilizing properties of Model Predictive Control based on I/O models is obtained in [3] and [12] respectively. Model Predictive Control is a receding horizon principle. At time step k the future control sequence (control horizon), over a horizon of m time steps ahead is determined using the predicted system behavior over a prediction horizon of p time steps ahead, such that a cost criterion J is minimized subject to constraints. At time step k the first element of the optimal sequence ($u(k)$) is applied to the system. At the next time instant the horizon is shifted. The model, used to make the prediction of the system behavior, is re-initialized with new information by means of a state measurement. Subsequently a new optimization at time step $k+1$ is executed. Formally the problem is defined as

$$\begin{aligned} \min_{\bar{u}(k|k), \dots, \bar{u}(k+p|k)} \{J(\mathbf{x}(k), \bar{u}(l|k), m, p)\}, \\ l \in [k, \dots, k+p], \end{aligned} \quad (2)$$

with

$$J(\mathbf{x}(k), \bar{u}(l|k), m, p) = \sum_{l=k}^{k+p} \mathbf{F}(\bar{y}(l+1|k), \bar{u}(l|k)), \quad (3)$$

subject to

$$\begin{aligned} \bar{\mathbf{x}}(l+1|k) &= \mathbf{f}(\bar{\mathbf{x}}(l|k), \bar{u}(l|k)), \\ &\quad l \in [k, \dots, k+p], \\ \bar{y}(l+1|k) &= g(\bar{\mathbf{x}}(l+1|k)), \\ \bar{\mathbf{x}}(k|k) &= \mathbf{x}(k), \bar{u}(l|k) \in U, \\ &\quad l \in [k, \dots, k+m-1], \\ \bar{u}(l|k) &= \bar{u}(k+m-1|k), \\ &\quad l \in [k+m, \dots, k+p-1], \\ \bar{\mathbf{x}}(l+1|k) &\in X, \quad l \in [k, \dots, k+p-1], \\ u(k) &= \bar{u}(k|k), \quad \forall k. \end{aligned} \quad (4)$$

Note that a distinction is made between the true inputs and outputs and the predicted *future* input and output signals predicted in the Model Predictive Controller. They are denoted by y , u and $\bar{y}$, $\bar{u}$ respectively. The function $\mathbf{F}$ specifies the desired cost criteria. A standard quadratic form is the simplest and most used one

$$\begin{aligned} \mathbf{F}(\bar{y}(l+1|k), \bar{u}(l|k)) &= \\ Q(\bar{y}(l+1|k) - y_r(k+1))^2 &+ R(\bar{u}(l|k) - u_r(k))^2 \end{aligned} \quad (5)$$

Further, $\bar{\mathbf{x}}(l|k)$ is the l -th predicted future state, predicted at time step k . The future inputs predicted at time step k ($\bar{u}(l|k)$, $l \in [k, \dots, k+p-1]$) are chosen such that the cost functional J in (3) is minimized with respect to (4) and also to possible constraints on input, state or output. These are defined in their simplest form as

$$\begin{aligned} u(k) &\in U, \quad U := \{u \in \mathbb{R}^1 | u_{min} \leq u \leq u_{max}\}, \\ \mathbf{x}(k) &\in X, \quad X := \{\mathbf{x} \in \mathbb{R}^n | \mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}\}, \\ y(k) &\in Y, \quad Y := \{y \in \mathbb{R}^1 | y_{min} \leq y \leq y_{max}\} \end{aligned} \quad (6)$$

The eventually implemented input to the system at time step k is the first input value $\bar{u}(k|k)$ of the predicted input sequence $\bar{u}(k|k), \dots, \bar{u}(k+p-1|k)$. The parameter Q in (5) is a parameter, weighting the output in the cost function to be minimized. R is the parameter weighting the input in the cost function to be minimized. (y_r, u_r) are the reference trajectory which should be tracked. For a certain trajectory (y_r, u_r) there should also exist a certain state trajectory $\mathbf{x}_r$ such that the system equation (1) is fulfilled.

3. OBSERVER DESIGN IN THE EXTENDED NONLINEAR OBSERVER FORM (ENOCF)

The nonlinear observer strategy proposed in [8] and [9] is based on nonlinear state equations that have a model structure which is called the *Extended Nonlinear Canonical Form*

$$\begin{aligned} \mathbf{z}(k+1) &= \mathbf{A}_z \mathbf{z}(k) + \mathbf{f}_z([y(k-n+1), \dots, \\ &\quad y(k)], [u(k-n+1), \dots, u(k+n)]]) \\ y(k) &= h_z(z_n(k), [u(k-n+1), \dots, u(k)]), \\ z_n(k) &= \mathbf{C}_z \mathbf{z}(k), \quad \mathbf{z}(k=0) = \mathbf{z}_0, \end{aligned} \quad (7)$$

where $\mathbf{z}(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^1$, $y(k) \in \mathbb{R}^1$,

$$\mathbf{A}_z = \begin{bmatrix} 0 & \dots & 0 & 0 \\ 1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 \end{bmatrix}, \quad \mathbf{C}_z = [0 \dots 0 \ 1].$$

The new state $\mathbf{z}$ appears in the equations linearly and all the nonlinearity is present in a function $\mathbf{f}_z$, which is only a function of the inputs and outputs. Notice however, that the input and output variables present in $\mathbf{f}_z$ consist of past, current

and future input variables. The possible future dependency means that the *Extended Nonlinear Observer Canonical Form* is *noncausal* in general. Without loss of generality, one can make the following simple choices for the function $\mathbf{f}_z$.

$$\mathbf{f}_z = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ f_{z,n-1}([y(k-n+1), \dots, y(k)], [u(k-n+1), \dots, u(k+1)]) \end{bmatrix}, \quad (8)$$

where the function $f_{z,n-1}$ is an arbitrary function of its arguments. An observer in the *Extended Nonlinear Observer Canonical Form*, with the chosen function structure, will typically have the following structure.

$$\begin{aligned} \hat{\mathbf{z}}(k+1) &= \mathbf{A}_z \hat{\mathbf{z}}(k) + \mathbf{f}_z([y(k-n+1), \dots, \\ &\quad y(k), [u(k-n+1), \dots, u(k+1)]] + \\ &\quad \underbrace{\mathbf{L}(h_{z,u_{fixed}}^{-1}(y(k), [u(k-n+1), \dots, u(k)])) - \hat{z}_n(k)}_{z_n(k}), \\ \hat{y}(k) &= h_z(\hat{z}_n(k), [u(k-n+1), \dots, u(k)]), \\ \hat{\mathbf{z}}(k=0) &= \hat{\mathbf{z}}_0, \end{aligned} \quad (9)$$

where $\mathbf{L}^T = [l_0 \ l_1 \ \dots \ l_{n-1}]$ is the observer innovation gain and $h_{z,u_{fixed}}^{-1}$ is the inverse of h_z for fixed inputs $[u(k-n+1), \dots, u(k)]$ that maps y onto z_n . Note that the estimated state $\hat{\mathbf{z}}(k)$ appears linear in the observer equations. This fact leads to linear error dynamics

$$\mathbf{e}_z(k+1) = \begin{bmatrix} 0 & \dots & 0 & -l_0 \\ 1 & \dots & 0 & -l_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & -l_{n-1} \end{bmatrix} \mathbf{e}_z(k), \quad (10)$$

where the error is defined as

$$\mathbf{e}_z(k) = \hat{\mathbf{z}}(k) - \mathbf{z}(k)$$

In [8] and [9] it has been proven that if the system equation given in (1) is strongly locally observable, then there exists a locally invertible coordinate transformation map Ξ between the z -coordinates in which the proposed observer is defined and the x -coordinates on which the model of the Model Predictive Controller in section 2 is based (1). System (1) is strongly locally observable if the following condition is fulfilled

$$\begin{aligned} \text{rank}\left\{\frac{\partial \psi(\mathbf{x}(k), [u(k) \dots u(k+n-1)])}{\partial \mathbf{x}}\right\} &= n, \\ \forall \quad [u(k) \dots u(k+n-1)] &\in \mathcal{U}, \end{aligned} \quad (11)$$

where $\mathcal{U}$ is some region in $\mathbb{R}^n$ and ψ is defined as

$$\psi(\mathbf{x}(k), [u(k) \dots u(k+n-1)]) := \begin{bmatrix} g(\mathbf{f}^0(\mathbf{x}(k)), u(k)) \\ g(\mathbf{f}^1(\mathbf{x}(k), u(k)), u(k+1)) \\ \vdots \\ g(\mathbf{f}^{n-1}(\mathbf{x}(k), [u(k) \dots u(k+n-2)]), u(k+n-1)) \end{bmatrix}, \quad (12)$$

where

$$\begin{aligned} \mathbf{f}^0(\mathbf{x}(k)) &= \mathbf{x}(k) \\ \mathbf{f}^i(\mathbf{x}(k), [u(k) \dots u(k+i-1)]) &= \\ \mathbf{f}(\mathbf{f}(\dots \mathbf{f}(\mathbf{f}(\mathbf{x}(k), u(k)), u(k+1)), \dots), u(k+i-1)) & \end{aligned} \quad (13)$$

The coordinate transformation map Ξ , which appears later also to be a function of inputs and outputs of the system, represents locally a diffeomorphism between $\hat{\mathbf{z}}$ and $\hat{\mathbf{x}}$ iff the inputs and outputs are fixed. (denoted by $\Xi_{u_{fixed}}$). In fact Ξ can be decomposed in two coordinate transformations ψ and $\mathcal{Z}$ respectively. This decomposition of the coordinate transformation, results in a new state variable $\mathbf{s}$ which is obtained as follows

$$\mathbf{s}(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+n-1) \end{bmatrix} = \psi(\mathbf{x}(k), [u(k), \dots, u(k+n-1)]) \quad (14)$$

The difference equation describing the evolution of the $\mathbf{s}$ state variable is of the form

$$\mathbf{s}(k+1) = \begin{bmatrix} s_2(k) \\ \vdots \\ s_n(k) \\ f_s(\mathbf{s}(k), [u(k) \dots u(k+n)]) \end{bmatrix} \quad (15)$$

$$y(k) = s_1(k), \quad \mathbf{s}(k=0) = \mathbf{s}_0,$$

where

$$f_s(\mathbf{s}(k), [u(k) \dots u(k+n)]) = g(f^n(\mathbf{x}(k), [u(k), \dots, u(k+n-1)]), u(k+n))|_{\mathbf{x}(k)=\psi_{u_{fixed}}^{-1}}$$

If the inputs $[u(k) \dots u(k+n-1)]$ are fixed then ψ , or to be more precise $\psi_{u_{fixed}}$, represents a local diffeomorphism between the states $\mathbf{x}$ in (1) and the states $\mathbf{s}$ in (15).

The diffeomorphism between $\mathbf{s}$ and $\mathbf{z}$ for fixed input and output values is represented by $\mathcal{Z}_{u_{fixed}}$. $\mathcal{Z}$ has the following form

$$\mathbf{z}(k) = \mathcal{Z}(\mathbf{s}(k), [y(k-n+1) \dots y(k)], [u(k-n+1) \dots u(k+n-1)]) \quad (16)$$

The motivation for the coordinate transformation $\mathcal{Z}$, can be explained by considering the structure of the *Extended Nonlinear Observer Canonical Form* (7) and taking into account that $\mathbf{s}(k) = [y(k) \dots y(k+n-1)]^T$. Now, note that $z_n(k) = h_{z,u_{fixed}}^{-1}(y(k), [u(k-n+1), \dots, u(k)])$. Replacing $y(k)$ by $s_1(k)$ one obtains $z_n(k) = h_{z,u_{fixed}}^{-1}(s_1(k), [u(k-n+1), \dots, u(k)])$. From the last component of the state equation in (7) it

follows that

$$z_{n-1}(k) = z_n(k+1) - f_{z,n-1}([y(k-n+1), \dots, y(k)], [u(k-n+1), \dots, u(k+1)]),$$

where $z_n(k+1)$ can be replaced by $h_{z,u_{fixed}}^{-1}(s_1(k+1), [u(k-n+2), \dots, u(k+1)])$. Continuing in this way, the state variables $z_i(k)$, $i = 1, 2, \dots, n$ result from $s_i(k)$, $y(k-n+2), \dots, y(k-1)$ and $u(k-n+1), \dots, u(k+n-1)$ in the following form

$$\begin{aligned} z_n(k) &= h_{z,u_{fixed}}^{-1}(s_1(k), [u(k-n+1), \dots, u(k)]) \\ z_{n-1}(k) &= h_{z,u_{fixed}}^{-1}(s_2(k), [u(k-n+2), \dots, u(k+1)]) - f_{z,n-1}([y(k-n+1), \dots, y(k-1)], [u(k-n+1), \dots, u(k+1)]) \\ z_{n-2}(k) &= z_{n-1}(k+1) \\ &= h_{z,u_{fixed}}^{-1}(s_3(k), [u(k-n+3), \dots, u(k+2)]) - f_{z,n-1}([y(k-n+2), \dots, y(k-1)], [u(k-n+2), \dots, u(k+2)]) \\ &\vdots \\ z_1(k) &= z_2(k+1) \\ &= h_{z,u_{fixed}}^{-1}(s_n(k), [u(k), \dots, u(k+n-1)]) - f_{z,n-1}([y(k-1), s_1(k), \dots, s_{n-1}(k)], [u(k-1), \dots, u(k+n-1)]) \end{aligned} \quad (17)$$

The total coordinate transformation map from $\hat{\mathbf{x}}$ to $\hat{\mathbf{z}}$ is then given by

$$\hat{\mathbf{z}}(k) = \Xi(\hat{\mathbf{x}}(k), [y(k-n+1) \dots y(k)], [u(k-n+1) \dots u(k+n-1)]) \quad (18)$$

which is obtained by substituting (14) into (17) and forms a diffeomorphism between the state $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$. Note that the coordinate transformation Ξ is totally determined by the function $f_{z,n-1}$ and h_z , which must fulfill the following condition

$$f_{z,n-1}(s_1(k), s_2(k), \dots, s_n(k), [u(k), \dots, u(k+n)]) = h_{z,u_{fixed}}^{-1}(f_s(\mathbf{s}(k), [u(k), \dots, u(k+n)]), [u(k+1), \dots, u(k+n)]) \quad (19)$$

For the derivation of (19) the reader is referred to [8] and [9].

4. MODEL PREDICTIVE CONTROL VIA EXTENDED OBSERVER

Often a Model Predictive Control strategy, like the one discussed in this paper, is based on the knowledge of the full state. In this paper the state is estimated by the observer in the *Extended Nonlinear Observer Canonical Form* as presented. In the observer design one can in certain situations select the output function h_z in (7) such that a causal observer structure is obtained. However in the general case this is nontrivial. In some cases there even might not at all exist a function h_z that realizes a causal observer structure. In order

to generalize the observer design in the *Extended Nonlinear Observer Canonical Form*, one chooses for the function h_z a trivial function, for example

$$y(k) = h_z(z_n(k), [u(k-n+1), \dots, u(k)]) = z_n(k) \quad (20)$$

Now the proposal is, if the obtained observer appears to be noncausal, to make a *prediction* of the *future* input values, that are responsible for the *noncausality*. The *prediction* is made by a model predictive controller, which simultaneously controls the process. Subsequently the model predictive controller is provided with the required state variables by the *noncausal* observer in the *Extended Nonlinear Observer Canonical Form*. For the state estimation the observer in *Extended Nonlinear Observer Canonical Form* also requires past input and output data of the controlled system. This data is buffered in two buffer systems B1 and B2, one for the systems input and one for the output values respectively.

The choice for h_z in (20) leads to the following observer

$$\begin{aligned} \hat{\mathbf{z}}(k+1) &= \mathbf{A}_z \hat{\mathbf{z}}(k) + \mathbf{f}_z([y(k-n+1), \dots, y(k)], \\ &\quad [u(k-n+1), \dots, u(k)], \bar{u}(k+1|k)) + \\ &\quad \mathbf{L}(y(k) - \hat{z}_n(k)) \end{aligned} \quad (21)$$

with coordinate transformation map

$$\begin{aligned} \hat{\mathbf{x}}(k) &= \Xi_{u_{fixed}}^{-1}(\hat{\mathbf{z}}(k), [y(k-n+1), \dots, y(k-1)], \\ &\quad [u(k-n+1), \dots, u(k-1)], \\ &\quad [\bar{u}(k|k-1), \dots, \bar{u}(k+n-1|k-1)]) \end{aligned} \quad (22)$$

where $\bar{u}(k+1|k)$, $\bar{u}(k|k-1)$, $\dots$, $\bar{u}(k+n-1|k-1)$ are the *predicted* future input variables predicted by the model predictive controller.

5. EXAMPLE

A bioreactor is a reactor in which micro-organisms grow by eating a substrate (the feed of the reactor). These two components, the micro-organisms and the substrate, are assumed to be present at low concentrations in the reactor, so that a "constant volume assumption" is realistic. The concentrations of micro-organisms and substrate in the reactor are denoted by x_1 and x_2 respectively. Note that from a physical point of view those variables cannot become negative, so $x_1 \geq 0$ and $x_2 \geq 0$. According to [10], the growth rate of micro-organisms is of the form $\frac{a_1 x_2(k)}{a_2 x_1 + x_2}$, where a_1 and a_2 are positive constants, a_1 being the maximum specific growth rate and a_2 the so-called Contois's constant. Since the dynamics of the bioreactor is slow enough, a Euler discretization of the continuous time model is sufficient to give a representative model. Denote by T the sampling time interval. So, the Euler discretization gives the following mapping $\mathbf{f}$ and output function g :

$$\mathbf{f}(x_1(k), x_2(k), u(k)) =$$

$$\begin{bmatrix} x_1(k) + T \frac{a_1 x_1(k) x_2(k)}{a_2 x_1(k) + x_2(k)} - T x_1(k) u(k) \\ x_2(k) - T \frac{a_3 a_1 x_1(k) x_2(k)}{a_2 x_1(k) + x_2(k)} - T x_2(k) u(k) + T a_4 u(k) \end{bmatrix},$$

$$g(x_1(k), x_2(k)) = x_1(k) = y(k) \quad (23)$$

where $u(k)$ is the control which represents the output flow rate of the reactor. The concentrations of micro-organisms $y(k)$ is the variable which has to be controlled and is also measurable. Note that from the model it follows that in addition to the constraints on x_1 and x_2 also the constraint $a_2 x_1 + x_2 > 0$ should be satisfied. To cope with all the given constraints in the control synthesis, Model Predictive control is used to control the system. The cost function in the model predictive controller is given by (5), with reference output $y_r(k) = 0.1 - 0.05e^{-0.1k}$ and reference input $u_r(k)$

$$u_r(k) = \frac{1}{T} + \frac{a_1 x_{2,r}(k)}{a_2 y_r(k) + x_{2,r}(k)} - \frac{y_r(k+1)}{T y_r(k)} \quad (24)$$

where

$$x_{2,r}(k+1) = x_{2,r}(k) - T \frac{a_3 a_1 y_r(k) x_{2,r}(k)}{a_2 y_r(k) + x_{2,r}(k)} - T x_{2,r} u_r(k) + T a_4 u_r(k), \text{ with } x_{2,r}(k=0) = 0.06$$

Because negative input values are physically not possible, the following additional constraint on the input is used $u \geq 0$. The state variables needed in the Model Predictive Controller are reconstructed by the observer explained earlier. For this example (12) will result in

$$\psi = \begin{bmatrix} x_1(k) + \frac{T a_1 x_1(k) x_2(k)}{(a_2 x_1(k) + x_2(k))} - T x_1(k) u(k) \end{bmatrix} \quad (25)$$

Verifying the observability condition

$$\det \left(\frac{\partial \psi}{\partial \mathbf{x}} \right) = \frac{T a_1 x_1(k)^2 a_2}{(a_2 x_1(k) + x_2(k))^2} \quad (26)$$

According to (26) condition (11) is met as long as $x_1(k) \neq 0$ and $a_2 x_1(k) + x_2(k) \neq 0$, meaning that the system defined by (23) is strongly locally observable. In this case even global observability is obtained for $x_1(k) \neq 0$ and $a_2 x_1(k) + x_2(k) \neq 0$ because (26) is independent of the input. Note that the earlier determined constraint on x_1 will thus become $x_1(k) > 0$, in addition with the earlier determined constrained $a_2 x_1(k) + x_2(k) > 0$, one has sufficient conditions for the observability map (25) to be a smooth diffeomorphism for all possible input sequences, and so also the total coordinate transformation map given by (18) is well defined on that domain. In the theory presented in section 4 the output function h_z in (7) is chosen in the simplest form given in (20). However, if in this example h_z is put in this simplest structure, the resulting observer will not be noncausal. In order,

to still be able to give a low dimensional illustrative and transparent example of the idea how to deal with the noncausality, that is generally present in the presented observer design in section 4, we introduce the following function for h_z

$$y(k) = h_z(z_2(k), [u(k-1), u(k)]) = z_2(k) + u(k)$$

For the function $\mathbf{f}_z$ the structure given in (8) is used. This gives, by using condition (19) and using the $a_1 = a_2 = a_3 = 1$, $a_4 = 0.1$ and $T = 0.1$ as the model parameters, the following expression

$$\begin{aligned} f_{z,1}(y(k-1), y(k), [u(k-1), u(k), \bar{u}(k+1)]) = & 100y(k)u(k)y(k-1)^2 - 10y(k)u(k)y(k-1)^2 \\ & u(k-1) + 11y(k)u(k)y(k-1)u(k-1) - y(k) \\ & u(k)y(k-1)u(k-1)^2 - 121y(k-1)u(k-1) \\ & y(k) - 100\bar{u}(k+1)y(k-1)^2u(k-1) + \\ & 110\bar{u}(k+1)y(k-1)u(k-1) - 10\bar{u}(k+1)y(k-1) \\ & u(k-1)^2 - 100\bar{u}(k+1)u(k-1)y(k) + 11y(k-1) \\ & u(k-1)^2y(k) + 110y(k)y(k-1)^2u(k-1) - \\ & 100y(k-1)u(k-1)y(k)^2 - 10y(k)^2u(k) \\ & u(k-1) + 1000\bar{u}(k+1)y(k-1)^2 - \\ & 1100y(k)y(k-1)^2 + 110u(k-1)y(k)^2 + \\ & 1100y(k-1)y(k)^2 - 1000y(k)^3)/(100y(k-1)^2 \\ & u(k-1) - 110y(k-1)u(k-1) + 10y(k-1) \\ & u(k-1)^2 + 100u(k-1)y(k) - 1000y(k-1)^2) \end{aligned}$$

The function $\mathbf{f}_z$ and h_z are now known, which means that the observer in the *Extended Non-linear Observer Canonical Form* given in (9) is defined. Also the coordinate transformation map (22) is now defined.

$$\begin{bmatrix} \hat{x}_1(k) & \hat{x}_2(k) \end{bmatrix}^T =$$

$$\begin{bmatrix} \bar{u}(k) + \hat{z}_2(k) \\ -(\bar{u}(k) + \hat{z}_2(k))((1/1000\bar{u}(k)u(k-1) - 1/10\hat{z}_1(k) - 1/100\bar{u}(k) - 1/100\hat{z}_2(k) + 1/1000u(k-1)\hat{z}_2(k) + 1/100\hat{z}_1(k)u(k-1)) \\ y(k-1)^2 + (-11/10000\bar{u}(k)u(k-1) + 1/10000\bar{u}(k)u(k-1)^2 - 1/100u(k-1) \\ \bar{u}(k)^2 - 11/1000\hat{z}_1(k)u(k-1) + 11/50\bar{u}(k) \\ \hat{z}_2(k) - 1/100u(k-1)\hat{z}_2(k)^2 - 11/10000u(k-1)\hat{z}_2(k) + 1/10000u(k-1)^2 \\ \hat{z}_2(k) - 1/50\bar{u}(k)u(k-1)\hat{z}_2(k) + 11/100 \\ \hat{z}_2(k)^2 + 11/100\bar{u}(k)^2 + 1/1000\hat{z}_1(k)u(k-1) \\ y(k-1) + 1/100\hat{z}_1(k)u(k-1)\bar{u}(k) - 1/10\hat{z}_2(k)^3 - 1/10\bar{u}(k)^3 + 1/500\bar{u}(k)u(k-1) \\ \hat{z}_2(k) + 1/1000u(k-1)\hat{z}_2(k)^2 - 3/10\bar{u}(k)\hat{z}_2^2 + 1/100\hat{z}_1(k)u(k-1)\hat{z}_2(k) - 3/10\bar{u}(k)^2\hat{z}_2(k) + 1/1000u(k-1)\bar{u}(k)^2)/((1/100\hat{z}_1(k)u(k-1) \\ - 1/10\hat{z}_1(k))y(k-1)^2 + (1/1000\hat{z}_1(k) \\ u(k-1)^2 - 1/50\bar{u}(k)u(k-1)\hat{z}_2(k) - 1/100u(k-1)\hat{z}_2(k)^2 + 11/100\bar{u}(k)^2 + 11/100\hat{z}_2(k)^2 - 11/1000\hat{z}_1(k)u(k-1) + 11/50\bar{u}(k)\hat{z}_2(k) - 1/100u(k-1)\bar{u}(k)^2) \\ y(k-1) - 1/10\bar{u}(k)^3 + 1/100\hat{z}_1(k)u(k-1) \\ \bar{u}(k) - 1/10\hat{z}_2(k)^3 + 1/100\hat{z}_1(k)u(k-1) \\ \hat{z}_2(k) - 3/10\bar{u}(k)^2\hat{z}_2(k) - 3/10\bar{u}(k)\hat{z}_2(k)^2) \end{bmatrix}$$

Simulation result of the proposed control strategy are given in figure 1 and 2. The control parameters used in simulation are shown in table A.

Control parameters			
Q	R	l_0, l_1	m,p
1	0.01	0.01,0	3,12
Initial conditions at k=0			
$x_1(0), x_2(0)$	$\hat{z}_1(0), \hat{z}_2(0)$	$y(-1)$	$u(-1), \bar{u}(0), \bar{u}(1)$
0.05,0.06	0.1,0.1	0.1	0.1,0.1,0.1

Table A: Initial conditions and used control parameters

Fig. 1. Response of concentration of microorganisms y and the applied control u .

Fig. 2. Response of the estimated and the actual state.

6. CONCLUSION

In general it can be concluded that combining a *noncausal* nonlinear observer in the extended nonlinear observer canonical form, combined with a nonlinear model predictive control scheme may form a successful approach for tackling output based model predictive control. This has been shown in an illustrative nonlinear example. Further research on the closed-loop stability of the considered extended observer and Model Predictive Controller combination is required. This will be the subject of a future publication.

REFERENCES

- [1] R. Findeisen, L. Imsland, F. Allgöwer, and B.A. Foss. Output feedback stabilization of constrained systems with nonlinear predictive control, *Int. J. of Robust and Nonlinear Contr.* 2003; 13:211-227.
- [2] R. Findeisen, L. Imsland, F. Allgöwer, and B.A. Foss, Output feedback nonlinear Predictive control -a separation principle approach. *In Proc. of 15th IFAC World Congress, 2002.*
- [3] H. Chen and F. Allgöwer, A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability, *Automatica*, 34(10):1205-1218, 1998.
- [4] F. Esfandiari, H.K. Khalil, Output feedback stabilization of fully linearizable systems, *Int. J. Contr.* 1992; 56(5):1007-1037.
- [5] A.N. Atassi, H.K. Khalil, A separation principle for the stabilization of a class of nonlinear systems. *IEEE Trans. Aut. Control* 1999; 44(9):1672-1687.
- [6] M. Maggiore, Output feedback control: a state variable approach, PhD Thesis, Ohio State University, 2000.
- [7] D.Q. Mayne, J.B. Rawlings, C.V. Rao and P.O.M. Scokaert. Constrained model predictive control: stability and optimality, *Automatica*, 789-814, 2000.
- [8] T. Lilge, On Observer Design for Nonlinear Discrete-Time Systems, *European J. of Contr.*, (1998)4, pp. 306-319.
- [9] T. Lilge, Zum Entwurf nichtlinearer zeitdiskreter Beobachter mittels Normalformen, *Fortschr.-Ber. VDI Reihe 8 Nr. 752.* Düsseldorf 1999.
- [10] D. Contois, Kinetics of bacterial growth relationship between population density and specific growth rate of continuous culture, *J. Genetic Microbiol.*, 21, 40-50, (1959).
- [11] B. Roset, H. Nijmeijer, Observer based model predictive control, Submitted to: *Int. J. of Contr.*, 2003.
- [12] A. Bemporad, L. Chisci, E. Mosca, On the stabilizing property of SIORHC, *Automatica*, 30, 2013-2015, 1995.