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Mass optimized self-actuated panels for several truss core topologies

Bram de Jager

Abstract—Sandwich panels with truss cores offer possibilities for self-actuation. They are analyzed, with emphasis on mass optimal design for failure and stiffness. It is shown that introducing additional freedom in selecting the topology of tetrahedral or pyramidal cores, by 1) making the core units, or even all core members, disjoint, so varying the distance between core units becomes possible, 2) using multiple layers, brings significant advantages. For some conditions, the performance of the modified panel becomes better by a factor of two, causing actuated truss core panels to perform at least as well as, or even better than, panels with, *e.g.*, honeycomb cores, that do not easily allow core actuation. A special case of the analysis is the mass optimal design of panels with 3D-Kagomé truss units, which can be regarded as a special geometry for a tensegrity prism.

I. INTRODUCTION

When sandwich panels are self-actuated, they can bend, tilt, twist, *etc.*, which is particularly useful for applications where shape changes of light weight structures are desired, like in aerospace. To self-actuate a panel, one normally employs truss faces or truss cores, where the length of truss members is varied to change shape. The use of truss faces always comes at a performance penalty, compared to solid face sheets, when the load causes a planar stress or strain field in the faces. It was not really clear if the use of truss cores, compared to, *e.g.*, honeycomb or corrugated cores, also comes with a performance penalty. When this performance penalty can be avoided, the use of truss cores may be more advantageous for actuation than the use of truss faces.

In a series of papers, starting with [1], continuing with [2], and culminating in [3], [4], an extensive analysis of mass optimal, strength limited, panels with truss/prismatic cores has been performed. The continuing emphasis was on panels with a tetrahedral core topology, which were compared with panels employing several competing core topologies: pyramidal [4], hexagonal [1], [2] or square honeycomb [4], and corrugated/diamond [3], [4]. All core topologies considered were fully connected and, except for the diamond core (a corrugated core with corrugation order greater than one, [3]), all cores were structured as a single layer. The designs were aimed at minimal mass meeting failure conditions.

The initial conclusion [1, Fig. 10] was that the structural performance of panels with honeycomb cores is better than of those with truss cores, the latter being preferred only for their multifunctional ability, like the possibility to be used as storage or as transportation medium, or to effect changes in shape. In [2] which, compared to [1], included a more refined buckling analysis, due to restrictions on the relative panel height and careful selection of loading conditions by adding a crushing load besides bending/shear, the authors were able to find a case [2, Fig. 10] where a panel with tetrahedral truss core is more mass efficient with respect to failure than a hexagonal honeycomb panel. The honeycomb panel may still be stiffer, however. Contrary to this, [3, Fig. 12] showed a marked preference for a hexagonal honeycomb core, citing [2], by not including the effect of the crushing load. In [4, Fig. 10], however, the authors showed a better

performance of panels with tetrahedral cores, compared to square honeycomb and other cores, even without an additional crush load, but for another value of yield strain and only for lower levels of bending/shear load. For higher load levels the differences between the cores became insignificant. The offset in performance of a tetragonal core panel for low loads may be (partly) due to the use in [4] of a non-conservative face sheet wrinkling condition, taken from [1], that was later, in [2], replaced by a more realistic face sheet buckling condition. On the other hand, for the shear strength constraint of the tetragonal core the orientation of the core unit was chosen that was most limiting, but for the pyramidal core the orientation was chosen that was least limiting.

A special class of cores, named 3D-Kagomé, has been analyzed [5] and tested [6], [7]. Kagomé core units are similar to a tensegrity prism in a specific geometry. Despite the fact that these cores are exhibiting nice post-yield behavior, are reputed to be mass efficient, and are proved to be excellent for actuation purposes, strangely, no explicit mass optimal panel designs have been noted in the literature.

The main conclusion for applications of panels in controlled structures is that performance is not necessarily hindered by employing a truss core that enables the panel to change shape in a controlled way, but apparently truss cores also do not directly improve performance.

The goal of this research is therefore to improve the performance of sandwich panels with truss cores that are suitable for actuation. This will do away with the perceived performance disadvantages of panels with truss cores, leaving only lower cost as a reason for considering other cores. To achieve this goal, several topological modifications of the tetrahedral and pyramidal truss core are proposed and analyzed. The two main modifications are

- 1) not requiring the cores to be fully connected, the core units or even all core members being connected by the face sheets only, making it possible to vary the distance between units without varying the unit geometry,
- 2) using multiple layers in the core, like the corrugation order for diamond cores, making it possible to change the unit geometry without changing the panel height.

These two modifications suffice, as a special case, to analyze the properties of panels with 3D-Kagomé cores, which can be treated as a two-layer tetrahedral core with disjointed units.

We start with summarizing the problem formulation of [2]. This is followed by a description of the modifications made to the problem, optimal design results, and a discussion of these. Conclusions finish the paper.

II. PROBLEM FORMULATION

The panel investigated in [2] is built up from a “tetragonal” truss core and two identical face sheets, using the same material throughout. In [1] also face trusses were considered, but these are inefficient for carrying a load causing a state of plain stress or plain strain in the faces, so not advisable if strength or stiffness is emphasized.

The analysis in [2] is based on a constrained nonlinear optimization problem with a very small number of design variables, so it is easy to solve. Only four design variables

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characterize the panel, t_f , the face sheet thickness, R_c , the core member radius, H_c , the core height, and $d = \sqrt{L_c^2 - H_c^2}$, with $L_c = \sqrt{H_c^2 + d^2}$ the core member length, see Fig. 1.

Fig. 1. Variables characterizing a panel with tetrahedral core (front view)

The small number of design variables is partly due to the fixed core topology, and partly due to the regularity and repetitiveness of the design. The objective to minimize is the panel mass per unit area. Constraints are face sheet yield and buckling, and core member yield and buckling. Two load cases are considered, a moment, M , and shear force, V , both per unit length, *e.g.*, due to three-point bending (Fig. 2(a)), and compression, with a crush load, σ_c , per unit area (Fig. 2(b)). The panel should be able to carry either bending/shear load or crush load, but not simultaneously.

Fig. 2. Load cases: (a) three-point bending, (b) crush; (front views)

A. Problem formulation

In dimensionless form, with

$$x = [t_f \quad R_c \quad H_c \quad d] / l$$

as dimensionless design variables, and with

$$\begin{aligned} l &= \frac{M}{V}, \\ \lambda &= \frac{V^2}{EM} = \frac{V}{El} = \frac{M}{El^2}, \\ \gamma &= \frac{\sigma_c}{E} = \frac{\sigma_c}{E} \varepsilon_Y, \end{aligned}$$

the characteristic length and dimensionless load parameters, respectively, the nonlinear optimization problem in dimensionless form is stated as follows

$$\min_x 2 \left(\underbrace{x_1}_{\text{face}} + \underbrace{\frac{\pi}{\sqrt{3}} \frac{x_2^2 \sqrt{x_3^2 + x_4^2}}{x_4^2}}_{\text{core}} \right) \quad (1)$$

$$\text{sub } \frac{\lambda}{x_1 x_3} - \varepsilon_Y \leq 0 \quad (2)$$

$$\frac{\lambda}{x_1 x_3} - \frac{\pi^2}{27(1 - \nu^2)} \frac{x_1^2}{x_4^2} \leq 0 \quad (3)$$

$$\frac{\sqrt{3} \lambda x_4}{\pi} \frac{\sqrt{x_3^2 + x_4^2}}{x_2^2 x_3} - \varepsilon_Y \leq 0 \quad (4)$$

$$\frac{\sqrt{3} \lambda x_4}{\pi} \frac{\sqrt{x_3^2 + x_4^2}}{x_2^2 x_3} - \pi^2 \frac{x_2^2}{x_3^2 + x_4^2} \leq 0 \quad (5)$$

$$\frac{\sqrt{3} \gamma x_4^2}{2\pi} \frac{\sqrt{x_3^2 + x_4^2}}{x_2^2 x_3} - \varepsilon_Y \leq 0 \quad (6)$$

$$\frac{\sqrt{3} \gamma x_4^2}{2\pi} \frac{\sqrt{x_3^2 + x_4^2}}{x_2^2 x_3} - \pi^2 \frac{x_2^2}{x_3^2 + x_4^2} \leq 0 \quad (7)$$

The objective represents the mass per unit area, m , made dimensionless, or normalized, as $m/\rho/l$, with ρ the specific density of the material used. Constraints (2)–(3) relate to face yield and buckling, while the remaining constraints relate to core yield and buckling. There are only two material parameters remaining explicitly in the problem formulation, due to the normalization and due to using the same material for sheet and core, namely yield strain, $\varepsilon_Y = \frac{\sigma_Y}{E}$, the ratio of yield stress and Young's modulus, and Poisson's ratio, ν . The values used in [2] are, respectively, 0.007 and 1/3, representative for a high yield aluminum alloy. These will be used here also.

III. PROBLEM MODIFICATIONS

As mentioned, the core in [2] is labeled by the authors as “tetragonal,” with tetrahedrons as basic building blocks, the core units. See Fig. 3(a) for the layout. This core is not very efficient for panels, mainly because the number of nodes on the face sheets is limited, so the sheets are relatively prone to buckling. This is due to the nodes collecting three core members each. By splitting up all the nodes on one face, so we get free standing tetrahedrons, the number of nodes is increased three-fold, see Fig. 3(b) for the effect of this increase in nodes on the lower face sheet. On average, the nodal distance will then be reduced by a factor $\sqrt{3}$ after moving the nodes away from each other, but the reduction in distance between straight lines connecting the nodes may be more significant, see, *e.g.*, Fig. 3(b), where the inter-line distance is reduced by approximately a factor 2. In the following, however, we consider only the general case, where the distances between nodal points and straight lines connecting them are assumed to be inversely proportional to the square root of the number of nodes per unit area. It is good to keep in mind that specific arrangements of the units may give better results, the present approach being generic.

Fig. 3. Core layout examples: (a) standard tetrahedral core, (b) core disjointed on lower sheet; core member: —, tetrahedral unit: - - -, repeating unit: ---, a • indicates a collecting node on the top sheet (top views)

To get an equal number of nodes on both faces, half of the tetrahedrons have to be flipped upside down, so then there is a twofold increase of nodes on both face sheets. This will lead to a nodal spacing that is $\sqrt{2}$ times as dense in terms of their average distance. If the basic pattern of face sheet buckling does not change, nor is a higher order buckling mode more limiting, this will lead to a two-fold increase in face sheet buckling load limit, being inversely proportional to the nodal distance squared, $1/x_4^2$, see (3).

The same procedure can be applied for a pyramidal core, see Figs. 4(a)–4(b).

Fig. 4. Core layout examples: (a) standard pyramidal core, (b) core disjoined on lower sheet; core member: —, pyramidal unit: - -, repeating unit: ---, a • indicates a collecting node on the top sheet (top views)

For a pyramidal core, it is possible to split up the nodes on both top and bottom sheets, getting a core consisting of single members, grouped together by the face sheets, while keeping the distribution and mutual distance of the nodes regular. Compare Fig. 4(a) with Fig. 5(b) for the transition from a connected pyramidal core unit to a core with units containing single members only. This transition conserves the main mechanical properties of a standard pyramidal core, because the same members with the same orientation are used, they are just positioned differently in the core unit volume by simple translations. The number of nodes on each face is increased by a factor of four, leading to a decrease of average nodal distance by a factor of two, and so giving a four-fold increase in the face sheet buckling load limit. The same procedure can be used for a tetrahedral core, but here it is not possible to make the nodal pattern on top and bottom face sheets congruent, see Fig. 5(a).

Fig. 5. Core layout examples: (a) tetrahedral core disjoined on top and bottom sheet, (b) pyramidal core disjoined on top and bottom sheet, core member: —, pyramidal unit: - -, repeating unit: ---, an arrow indicates where a single member connects to the top sheet (top views)

The assumption that the members are fully clamped at the nodes is not really justifiable now, so the core member buckling constraint can better be based on a conservative simple supported beam assumption, leading to a four-fold decrease in critical core member buckling load. This change is also more commensurate with cores that are fully ball-jointed, like tensegrity ones.

All constraints mentioned up till now are strength related, and stiffness does not play a role. As already mentioned in [10], the equations normally employed in panel analysis and design are only valid for small deflections. Specifically, [10, pp. 212–213], if the deflection of the panel is not small with respect to its height, H_c , or, in the case of a weak core, even its face sheet thickness, t_f , the analysis is deemed to be inappropriate. We therefore add a constraint to limit the panel deflection relative to the panel height, but even stricter constraints could be enforced.

In the literature, experiments with “near optimal” panels with tetrahedral and other cores have been reported, e.g., [7], [11]–[14]. Mostly, in these experiments, the face sheet thickness was larger than optimal, dictated either by the properties of the manufacturing process, or by the desire to highlight a specific core failure mode, or by the need to avoid penetration of the face sheet by core members. The last

reason leads us to consider also local face sheet indentation as one of the failure mechanisms.

The problem setup resulting after these four modifications,

- 1) increase the number of contact points between core and face sheets,
- 2) assume all joints to be friction-less ball-joints,
- 3) incorporate stiffness constraints,
- 4) include local indentation,

will be used as the baseline to compare with. Other core topology modifications considered here are the following.

1) *Core replication*: The core truss ensemble is replicated, shifted a distance d perpendicular to the load line direction, which implies that the cores are nested, and that the core unit density, n_c , is doubled. When core member properties are not changed, also the core relative density, ρ_c , the volume fraction occupied by core material, is doubled. The motivation for this modification is mainly the better face sheet buckling condition, a result of the increased number of nodes supporting the face sheets. Without this beneficial effect, the optimal panel would just be heavier due to less efficient buckling properties of the core, which for the same mass and twice as many members, would employ beams that are more slender.

This nesting idea can be repeated a number of times, if so desired, or it can be used to make the truss more sparse, by introducing holes in the pattern of basic core units, choosing $n_c < 1$. In those cases n_c can be changed freely, as long as $n_c > 0$, although normally a regular pattern for n_c will be chosen, to facilitate the manufacturing process. See Figs. 6(a) and 6(b) for examples where $n_c = 1/4$.

Fig. 6. Core layout examples: (a) tetrahedral core with $n_c = 1/4$, (b) pyramidal core with $n_c = 1/4$; core member: —, pyramidal unit: - -, repeating unit: ---, a • indicates a collecting node on the top sheet, an arrow indicates where a single member connects to the top sheet (top views)

2) *Core layering*: The core truss ensemble is used again, in a second layer on top of the existing one, using a mirrored/rotated version so the nodal points of the two layers are all at the same location in the intermediate plane where they contact each other. The motivation for this modification is the effective reduction of the length of the core members, so core member buckling will be less critical, when the core height is not changed.

For the tetrahedral core it is now better to start with a layer where half of the tetrahedrons are not flipped upside down after splitting up the nodal points on one face, leading to a three-fold increase of the number of nodes on one face, so on average a change by a factor $\sqrt{3}$ in nodal distance, and no increase on the other. Now, when a second layer is used, the number of nodes on the plane separating the layers is chosen as the low number of nodes, while the outsides then form the high number of nodes planes. The number of nodes in the intermediate plane has no material influence on the problem formulation, while a high number of nodes on the face sheets is beneficial for increasing the buckling load of the face sheets. In this case the core essentially becomes a collection of free-standing 3D-Kagomé truss units, see [5], [6], but with a different model for the central joint: not completely rigid but ball-jointed when buckling is considered.

Also when using a multi-layer pyramidal core, it is better to start with a core where only the nodes on one of the faces are disjointed, see Fig. 4(b), two of these layers can be connected directly, without the need for additional members. On the sides of the core facing the face sheets the number of nodes is then still four times that of the core with shared nodes. Note that the inter-line distance in Fig. 4(b) is reduced by a factor of 3, which is better than the factor 2 in Fig. 5(b), but by shifting the even rows of units a quarter unit length with respect to the odd rows in Fig. 5(b), a factor 4 is obtained, for a single direction.

The layering idea can be repeated a number of times, if so desired, allowing for the number of layers, n_l , to be a natural, positive number, although normally powers of 2 will be used, to make the contact points in the intermediate contact plane compatible.

When the number of layers exceeds two, additional members are needed to avoid mechanisms in the core structure. Failure conditions of these member can be added to the constraints, and the mass contribution can be added to the objective. It is also possible to explicitly solve for the sizing parameter of the additional members, meeting failure conditions, to avoid increasing the number of design variables and constraints.

These two topological modifications can be combined. Details of the analysis needed to deal with all these modifications are provided next. We present the analysis for a tetrahedral core in some detail. The modifications for a pyramidal core are only tabulated.

A. Dealing with variable core density and layers

The modifications necessary for the two possible core topology modifications are as follows.

1) *Core duplication*: With the addition of a second truss ensemble the core load is shared by both, so in the core constraints (4)–(7) the line load, λ , becomes $\frac{\lambda}{\sqrt{2}}$ and the area load, γ , is halved. The area density of the nodes on the face sheet is doubled, so, assuming the same critical buckling mode, the critical distance is reduced by a factor $\sqrt{2}$, being the average change in distance between attachment points. This change in the face shield buckling constraint effectively allows a buckling load increase by a factor 2. The core mass contribution is doubled.

2) *Core layering*: The addition of a second layer implies that the effective panel height, x_3 , is halved in the core constraints (4)–(7), because they are related to a single layer, so the effective layer height, $x_3/2$, should be used. Both the shear and crush load are invariant for the number of layers, if no intermediate layer or other mechanism is used that transfers the load to the face sheets or support, other than the unit immediately below. This is different from the situation with corrugated cores, with order of corrugation larger than one, where the shear load per layer may be inversely proportional to the number of layers [3]. The constraints for the face sheet need not be modified, except when the number of nodal points is chosen to be larger on the face sheet then in the intermediate plane between the layers. In this case, the face sheet buckling load is increased proportional to the increase in nodal points on the face sheet, being 3/2. The core mass contribution is twice that of a single layer, but the effective layer height, $x_3/2$, has to be used to compute the mass.

3) *Core duplication and layering*: When using n_c truss ensembles and n_l layers we combine the measures outlined above, so the layer height is the panel height, x_3 , divided by n_l , the core loads, λ and γ , are divided by $\sqrt{n_c}$ and n_c respectively, the face sheet buckling condition is adapted as

detailed below in (17), and the core mass contribution is $n_c n_l$ times that of a single truss, single layer version core with identical member geometry, the baseline core, with additional contributions of the members in the connecting plane if $n_l > 2$, as detailed below.

4) *Interlayer members*: The additional mass for the members in the connecting planes, parallel to the face sheets, the core interlayer members, needed to satisfy equilibrium for $n_l > 2$ without bending in the core members, or, if the core members are ball-jointed, without mechanisms, is computed directly, based on static equilibrium for each unit separately and with a yield constraint for these members. Member buckling is assumed to be less likely to occur, these members mostly being loaded in tension. Furthermore, additional instability mechanisms are neglected, it being assumed that these can be suppressed with negligible mass. This is true as long as $\varepsilon_Y \ll 1$ and the geometry is not too extreme, i.e., if $\alpha > 45^\circ$.

B. Modified problem

The modifications outlined above lead to the following modified optimal design problem

$$\min_x \quad 2 \left(\underbrace{x_1}_{\text{face}} + \underbrace{n_c n_l \frac{\pi}{\sqrt{3}} \frac{x_2^2 x_l}{x_4^2}}_{\text{core}} \right) + \underbrace{n_c \frac{x_4}{x_h \varepsilon_Y} \max(0, n_l - 2) \max(2 \frac{\lambda}{\sqrt{n_c}}, \frac{\gamma}{n_c} x_4)}_{\text{core interlayer}} \quad (8)$$

$$\text{sub} \quad \frac{\lambda}{x_1 x_3} - \varepsilon_Y \leq 0 \quad (9)$$

$$\frac{\lambda}{x_1 x_3} - \frac{\pi^2}{27(1 - \nu^2)} \left(\frac{x_1}{a x_4} \right)^2 \leq 0 \quad (10)$$

$$\lambda \left(\frac{3}{2} \frac{1 - \nu^2}{x_1 x_3^2} + \frac{\sqrt{3}}{\pi} \frac{x_l^3}{n_c n_l x_2^2 x_h^2} \right) - x_3 \leq 0 \quad (11)$$

$$2\lambda \frac{a x_4}{x_1^2} - \varepsilon_Y \leq 0 \quad (12)$$

$$\frac{\sqrt{3}}{\pi} \frac{\lambda}{\sqrt{n_c}} \frac{x_4 x_l}{x_2^2 x_h} - \varepsilon_Y \leq 0 \quad (13)$$

$$\frac{\sqrt{3}}{\pi} \frac{\lambda}{\sqrt{n_c}} \frac{x_4 x_l}{x_2^2 x_h} - \frac{\pi^2}{4} \frac{x_2^2}{x_l^2} \leq 0 \quad (14)$$

$$\frac{\sqrt{3}}{2\pi} \frac{\gamma}{n_c} \frac{x_4^2 x_l}{x_2^2 x_h} - \varepsilon_Y \leq 0 \quad (15)$$

$$\frac{\sqrt{3}}{2\pi} \frac{\gamma}{n_c} \frac{x_4^2 x_l}{x_2^2 x_h} - \frac{\pi^2}{4} \frac{x_2^2}{x_l^2} \leq 0 \quad (16)$$

where $x_h = x_3/n_l$, the dimensionless height of a single layer, $x_l = \sqrt{x_h^2 + x_4^2}$, the dimensionless length of core members in a single layer, and

$$a = \begin{cases} \max \left(\frac{1}{\sqrt{n_c}} - 1, \min \left(\frac{1}{\sqrt{2}\sqrt{n_c}}, \frac{1}{\sqrt{2}\sqrt{\sqrt{n_c}}} \right) \right) & \text{if } n_l = 1, \\ \max \left(\frac{1}{\sqrt{n_c}} - 1, \min \left(\frac{1}{\sqrt{3}\sqrt{n_c}}, \frac{1}{\sqrt{3n_c} \frac{\log(\sqrt{3})}{2\log(2)}} \right) \right) & \text{if } n_l \geq 2. \end{cases} \quad (17)$$

Equation (11) represents deflection and (12) represents indentation constraints.

The dimensionless critical distance factor a appearing in (10) and (12) is computed based on the fact that the critical distance between nodes for buckling and indentation is not proportional to $\frac{1}{\sqrt{n_c}}$ for $n_c < 1$, as it is assumed to be for $n_c > 1$, which would be too pessimistic, but to $\frac{1}{\sqrt{n_c}} - 1$ for $n_c \leq 1/4$. Note, *e.g.*, from Fig. 6(a), that it is possible for a core at $n_c = 1/4$ to have the same inter-line distance as for the standard tetrahedral core. So the factor a should be at least smaller than 1 for $n_c > 1/4$. The terms $\sqrt{2}$ and $\sqrt{3}$ in (17) are the result of decoupling the tetrahedral units. The terms at the right of (17), with powers of n_c unequal to $1/2$, are used to get a continuous function connecting the values for $n_c = 1/4$ and $n_c = 1$. See Fig. 7 for a graphical representation of a .

Fig. 7. Critical distance factor a as function of unit density for several topologies

Equations (8)–(17) are for a tetrahedral core in a specific orientation. For the changes needed for a pyramidal core, see Table I, that details how factors in (8)–(17) are changed for a pyramidal core. Additionally, the last term in (17) is not needed for the pyramidal core. See Fig. 7 for the factor a as used with a pyramidal core.

Table I also lists the modifications needed to use another orientation (labeled II), see [9]. For the tetrahedral core that orientation is obtained by rotating over 30° , and it will worsen face sheet buckling and indentation, but will improve core shear. For the pyramidal core it is obtained by rotating over 45° , and the results are just the other way around. See Fig. 8 for these orientations.

Fig. 8. Overview of core unit orientations: (left) tetrahedral, (right) pyramidal, (top) orientation I, (bottom) orientation II (top views)

The change in face sheet buckling and indentation depends on what one considers the appropriate critical distance for these failure conditions, and it could be defended that for the tetrahedral core in orientation II this distance will be smaller by a factor 2 as compared to the one used for Table I, for which $\sqrt{3}d$ was chosen, although this would lead to a discrepancy between characteristic distances in width and length direction by almost a factor 2, which might trigger another buckling/wrinkling mode than the one assumed.

The shear strength properties of the pyramidal core differ by a factor $\sqrt{2}$ between the orientations. If it is required that the panel properties are more isotropic than corresponding

to this factor, mixing different core unit orientations in the panel core will achieve that.

TABLE I

DIFFERENCES BETWEEN TETRAHEDRAL AND PYRAMIDAL CORE AND ORIENTATION I (ON WHICH THE DESIGN IS BASED) AND ORIENTATION II

Equation	tetrahedral I	II	pyramidal I	II	comment
(8)	$1/\sqrt{3}$	same	1	same	core
(8)	2	same	$\sqrt{2}$	same	core interlayer
(9)	-	-	-	-	identical
(10)	$1/27$	$1/36$	$1/24$	$1/12$	
(11)	$\sqrt{3}$	same	1	same	isotropic
(12)	2	$4/\sqrt{3}$	$4\sqrt{2}/3$	$4/3$	
(13)	$\sqrt{3}$	$3/2$	$1/\sqrt{2}$	1	
(14)	$\sqrt{3}$	$3/2$	$1/\sqrt{2}$	1	
(15)	$\sqrt{3}$	same	1	same	
(16)	$\sqrt{3}$	same	1	same	
(17)	$1/\sqrt{2}$	same	$1/2$	same	for $n_l = 1$
(17)	$1/\sqrt{3}$	same	$1/2$	same	for $n_l \geq 2$

IV. OPTIMAL DESIGN RESULTS

Studied are 4 main cases for the tetrahedral and pyramidal core each: crush load, γ , present or not, and core height, x_3 , restricted, so $x_3 \leq 1/10$, or free. It appears that these constraints are never active simultaneously. For small loads the crush constraints may be active, while for high loads the panel height limit may be active. This implies that from the results of only two cases, the results for the other two cases can be easily derived.

The mass results of some optimization runs are given in Figs. 9(a)–10(d) for different values of the load, λ , together with a comparison of the best panel at each load against the baseline panel. Designs are performed for a series of gridded values for n_l and n_c , and for a grid of values for n_l and the optimal value of n_c , making for 20 curves to be compared. It is possible to solve the constraint equations explicitly in some cases, but these analytical results are not presented here due to space limitations.

V. OBSERVATIONS AND DISCUSSION

Based on the results the following remarks can be made.

- 1) At least one of the deflection and indentation constraints was always active for some range of the load λ , so the resulting designs for the baseline case are slightly heavier than those reported in [2]. It is believed that the present baseline design is more practically useful, however.
- 2) Using multiple layer cores with optimal unit density, n_c , gives an improvement with respect to the baseline core up to a factor of two for low loads, see, *e.g.*, Fig. 10(a).
- 3) For a single or dual layer tetrahedral core, varying n_c is hardly advantageous, because the optimal value for n_c appears to be approximately equal to $1/4$. For multiple layer tetrahedral cores, the optimal value for n_c varies quite a lot. For $n_l = 4$ and $n_l = 8$ it is in a range of values from, until significantly below, $n_c = 1/4$.
- 4) For pyramidal cores the same pattern arises, except that for certain loads with a 1-layer core n_c may be larger than $1/4$, and with a 2-layer core it may be smaller than $1/4$.
- 5) For low loads a high number of layers is advantageous, for high loads $n_l = 2$ is best, due to the fact that the interlayer members for $n_l > 2$ make the panel less mass efficient. For intermediate loads, $n_l = 4$ seems an appropriate choice, but the differences with $n_l = 2$

are small, so probably a larger number of layers is not worth the effort.

- 6) In all cases with $n_l > 2$, the optimal n_c needs to be picked as function of design load to get the best results, the optimizing value of n_c varying quickly in these cases.
- 7) For a certain range of higher loads and for $n_l = 2$, $n_c = 1/4$ is best. This design point corresponds more or less to a design with 3D-Kagomé trusses as tested experimentally in [6], although there a slightly larger value for n_c seems to have been used (no exact data was provided in [6]).
- 8) The main difference in design variables between crush load or not is in the aspect ratio of core members, L_c/R_c , which increases more with decreasing load when there is no crush load. Also, with crush load, α tends to go down for lower loads, while without crush, α tends to go up for lower load.
- 9) The main difference in design variables between relative panel height limitation present or not, is in the growth rate of face sheet thickness, t_f , which increases faster with load if the panel height is limited, leading to a less efficient design.
- 10) Comparing the performance of tetrahedral and pyramidal cores, Figs. 10(a)–10(d), it can be observed that for low loads the tetrahedral core performs better, while for medium and high loads the pyramidal core performs better. As discussed, the properties of the cores can be varied by changing the orientation of the core units. The differences created in this way overwhelm the differences between the tetrahedral and pyramidal core versions, as presented here.

VI. CONCLUSIONS

The properties of sandwich panels with various core topologies employing tetrahedral and pyramidal units have been analyzed. The results obtained indicate that it is possible, by using multiple layers and varying core unit density, to improve the performance of panels, even significantly for lower design loads.

However, the improvement in performance, for more than 2 layers, is offset by a more complicated manufacturing process, requiring additional members in the planes separating the layers. This indicates that a general advisory for a two layer core with $n_c = 1/4$, using 3D-Kagomé trusses, or tensegrity prisms, as units, either three or four-legged, may be appropriate, especially for higher design loads. This allows to conclude that cores that are amenable for actuation do not impose any performance penalty, on the contrary, they perform better.

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Fig. 9. Mass of sandwich panel as function of load, for different core topologies (see legend): (a) tetrahedral core with crushing load but without panel height limit, (b) tetrahedral core without crushing load but with panel height limit, (c) pyramidal core with crushing load but without panel height limit, (d) pyramidal core without crushing load but with panel height limit

Fig. 10. Mass of best sandwich panel compared to baseline panel: (a) tetrahedral core with crushing load but without panel height limit, (b) tetrahedral core without crushing load but with panel height limit, (c) pyramidal core with crushing load but without panel height limit, (d) pyramidal core without crushing load but with panel height limit