Technical Note

Optimization of the Bending Stiffness and Strength of Composite Sandwich Panels

ABSTRACT

The minimum weight criterion of a sandwich panel with respect to a given bending stiffness and strength is verified. For this purpose a number of sandwich constructions consisting of a foamed PVC core and glass fibre reinforced epoxy faces are tested in a four-point bending test. An optimum is found for both the bending stiffness and strength as design parameters.

INTRODUCTION

Sandwich panels are composed of two thin sheets (or faces) of a strong and stiff material separated by a thick core of low density material possessing less strength and stiffness. This arrangement combines high bending stiffness and strength with lightness.

Depending on the specific requirements of the various applications, sandwich panels are composed of a wide variety of face and core materials. Typical face materials include aluminium, fibre reinforced plastics, plywood, etc. For the core, both the material and geometric shape can vary. Besides foamed polymers and inorganic cements, corrugated and honeycomb cores of all kinds of materials are used extensively.

In this paper the optimization of a sandwich panel as reported by Kuenzi is experimentally verified using a four-point bending test. The weight of the panels is minimized for a given bending stiffness or strength. In these experiments sandwich panels consisting of a foamed PVC core and glass/epoxy faces are used.
THEORY

In order to derive the appropriate formulas for minimizing the sandwich weight, as described by Kuenzi, three assumptions are made:

1. The shear strength of the core is high enough to prevent failure in the core.
2. The Young's modulus $E$ of the face is many times higher than that of the core ($E_f \gg E_c$).
3. The face is very thin in relation to the thickness of the core ($t_f \ll t_c$).

**Bending stiffness**

For a sandwich panel the bending stiffness can be expressed by

$$D = E_c I_c + 2E_f I_f$$

(1)

Substitution of the moments of inertia $I_c$ and $I_f$ results in

$$D = E_c \left( \frac{1}{2} b t_c^3 + 2E_f \left[ \frac{1}{2} b t_f^2 + b t_f (\frac{1}{2} s)^2 \right] \right)$$

(2)

where $b$ is the panel width and $s$ is the distance between the centres of the two faces. Using the second and third assumption one can find

$$D = \frac{1}{2} E_f b t_f s^2$$

(3)

The sandwich weight per square unit is

$$W = \rho_c t_c + 2 \rho_f t_f$$

(4)

where $\rho_c$ and $\rho_f$ are the densities of the core and face respectively. The weight of any adhesive between the faces and core is assumed constant and therefore omitted from the calculations. Using eqn (3) and remembering that $t_c \approx s$ gives

$$W = \rho_c s + 2 \rho_f \left( \frac{2D}{E_f b s^2} \right)$$

(5)

Minimizing the sandwich weight with respect to $s$

$$\frac{dW}{ds} = \rho_c - 8 \rho_f D / E_f b s^3 = 0$$

(6)

or

$$s^3 = \frac{8 \rho_f D}{\rho_c E_f b}$$

(7)

Substituting eqn (3) into eqn (7) gives

$$s = \frac{4 t_f \rho_f}{\rho_c}$$

(8)
Since the core weight \( W_c \approx \rho_c s \) and the weight of the faces \( W_f \approx 2\rho_s t_f \) eqn (8) yields

\[
W_c = 2 W_f
\]  
(9)

Thus, for a minimum weight sandwich of specified stiffness the core weight should be twice that of the faces, independent of the materials used.

**Bending strength**

The maximum bending moment per metre panel width \( M^* \) (in Nm/m) which the sandwich panel can transfer is approximated by

\[
M^* = \sigma_s t_f s
\]  
(10)

Rewriting and substituting eqn (10) into eqn (4) and minimizing with respect to \( s \) gives analogue to the previous derivation

\[
W_c = W_f
\]  
(11)

Thus, for a minimum weight sandwich of specified bending strength, the core-to-face weight ratio equals one.

**EXPERIMENT**

The faces of the sandwich panels used in the experiments consisted of glass fibre woven fabric (plain weave) with a weight of 80 g/m² impregnated with a common epoxy system; Ciba Geigy's Araldite LY556/HY2962. In order to get a good bond between faces and core, the PVC-foam panels (Klegecel, 75 kg/m³) were pre-impregnated on both sides with the same epoxy system, mixed with a thickener (Aerosil) to prevent extensive penetration of the epoxy into the core.

The composed sandwiches were cured for 30 min under combined vacuum and pressure conditions in a hot press at 80°C and were post-cured in an oven for 4 h at 110°C. Test specimens with the dimensions of 30 mm x 300 mm were cut from these panels.

For experimental verification of the optimization theory as described in the previous section, a number of panels was prepared with various core-to-face weight ratios and designed with respect to constant stiffness and strength.

In order to determine the weight ratios of these panels, the weight of the core was measured before preparation at constant temperature and
humidity. Therefore these factors were kept constant. Note that to calculate the core-to-face weight ratios, the weight of any adhesive is excluded because it can be assumed to be constant.

Verification of the prepared test panels with respect to a constant stiffness and strength was carried out using a four-point bending test with a total span length of 230 mm and 115 mm between the two centre pins at a rate of 1 mm/min. To prevent local buckling of the inner face during bending, load pads were used to distribute the load. The load pads were made of glass fibre reinforced epoxy about twice as thick as the face of the test panel.

In a four-point bending test the deflection of a sandwich panel can be regarded as the sum of a bending deflection $\delta_b$ and a shear deflection $\delta_s$. Consequently, the total deflection at the load point is given by:  

$$\delta = \left(\frac{PL^3}{96EI}\right) + \left(\frac{PL}{8S}\right)$$

(12)

where $S = G_c bs$ and $G_c$ represents the shear modulus of elasticity of the core.

The shear stiffness $S$ can be obtained by rewriting eqn (12) as:

$$\frac{\delta}{PL} = \left(\frac{L^2}{96D}\right) + \left(\frac{1}{8S}\right)$$

(13)

The deflection of the sandwich panel was measured for three span lengths and plotted in Fig. 1. Having calculated the shear stiffness $S$, the shear modulus $G_c$ appeared to be $25 \text{ N/mm}^2$. Now one can calculate the bending stiffness $EI$ using the result of the four-point bending test and eqn (12).

![Fig. 1. Determination of the core shear stiffness.](Image)
TABLE 1
Dimensions, Weight and Stiffness of Sandwich Panels

<table>
<thead>
<tr>
<th>Test panel</th>
<th>s (mm)</th>
<th>t (mm)</th>
<th>Weight ratio Wc/ Wf</th>
<th>Weight per cm (g/cm)</th>
<th>Stiffness per cm (Nm²/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>27.2</td>
<td>0.10</td>
<td>9.4</td>
<td>6.6</td>
<td>5.1</td>
</tr>
<tr>
<td>D4</td>
<td>19.0</td>
<td>0.15</td>
<td>3.4</td>
<td>5.5</td>
<td>4.6</td>
</tr>
<tr>
<td>D6</td>
<td>15.4</td>
<td>0.25</td>
<td>1.8</td>
<td>5.3</td>
<td>4.7</td>
</tr>
<tr>
<td>D8</td>
<td>13.5</td>
<td>0.30</td>
<td>1.2</td>
<td>5.8</td>
<td>4.1</td>
</tr>
</tbody>
</table>

TABLE 2
Dimensions, Weight and Maximum Four-Point Bending Load of Sandwich Panels

<table>
<thead>
<tr>
<th>Test panel</th>
<th>s (mm)</th>
<th>t (mm)</th>
<th>Weight ratio Wc/ Wf</th>
<th>Weight per cm (g/cm)</th>
<th>Maximum 4-p b load per cm (N/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3</td>
<td>14.2</td>
<td>0.12</td>
<td>4.5</td>
<td>5.1</td>
<td>89</td>
</tr>
<tr>
<td>M4</td>
<td>10.0</td>
<td>0.17</td>
<td>1.6</td>
<td>4.7</td>
<td>90</td>
</tr>
<tr>
<td>M6</td>
<td>7.2</td>
<td>0.25</td>
<td>0.65</td>
<td>4.6</td>
<td>107</td>
</tr>
<tr>
<td>M8</td>
<td>5.0</td>
<td>0.32</td>
<td>0.35</td>
<td>5.0</td>
<td>95</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSION

The weight ratios and the stiffness, strength and weight per cm panel width of the two panel series D and M are given in Tables 1 and 2. All data is an average value of four individual tests. The stiffness and strength of the panel appeared to be fairly constant for panels having various core-to-face weight ratios.

All panels failed by compressive failure of the inner face. This implies that the strength of the panel is controlled by the compressive strength of the glass fibre reinforced epoxy skin.

Figure 2 illustrates that the optimal core-to-face weight ratio for bending strength is about one and the optimum ratio for bending stiffness is about two. The theoretical curve which fits the experimental data fairly well shows that at core-to-face ratios greater than the optimum, the increase in panel weight is relatively small. For example, a 100% increase in weight ratio results in a 6% increase in panel weight.
CONCLUSIONS

A composite sandwich panel can be optimized with regard to some specific characteristics. The optimum core-to-face weight ratio for bending stiffness is about two and for bending strength it is about one. The increase in panel weight for panels with a core-to-face weight ratio near the optimum is relatively low.

REFERENCES


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