An efficient train timetable scheduling approach with regenerative-energy supplementation strategy responding to potential power interruptions

Citation for published version (APA):
https://doi.org/10.1109/TITS.2021.3125781

Document license:
TAVERNE

DOI:
10.1109/TITS.2021.3125781

Document status and date:
Published: 01/09/2022

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
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Abstract—The timetable of a metro system is essential for trains running safely and efficiently. For the design of a timetable, the potential power interruptions of energy supplies from the substations to the trains should be borne in mind. In case potential power interruptions occur, the backup running plan should be completed responsively to accommodate the passenger and energy demands. In this study, we propose an energy supplementation strategy utilizing regenerative energy from decelerating trains in the event of power interruptions and develop a tri-objective optimization model incorporating passenger travel time, potential power interruption, and energy consumption. Particularly, for calculating regenerative energy utilization, we suggest a many-to-many energy allocation mechanism between decelerating and accelerating trains based on the real-time energy demands and supplies. A heuristic algorithm is developed to obtain a regular and cyclic timetable for minimizing passenger travel time, the potential power interruptions, and energy consumption. The suggested model and algorithm are tested based on the smart-card data collected from a bidirectional metro line in Beijing (China). The results show that the suggested approach significantly improves energy efficiency, reduces passenger travel time, and decreases power interruption risks, compared with the currently used scheduling method.

Index Terms—Potential power interruption, energy allocation, regenerative energy supplementation, multi-objective optimization.

I. INTRODUCTION

METRO systems play an increasingly important role in big cities for passenger mobility. Owing to the rapid expansion of the metro, the reliability of the traction power supply system (TPSS) becomes an essential issue for the stability and safety of daily operations [1]. In practice, power interruptions may happen on the trains, overhead contact lines, or substations. When a power interruption arises, the associated trains lose power and fail in traction, which may cause danger, disrupt passenger schedules, and even lead to traffic jams on the roads. Thus, ensuring reliable power supply has been a central goal of metro service providers. Existing studies have paid much attention to risk evaluation for the TPSS and recovery management of disruptions (see [2]–[10]).

The TPSS operations risks are usually assessed by the power interruption indicator of each component in the electric system. For example, Chen et al. [2] evaluated the critical components associated with reliability in a structured system. They used the fault tree analysis method to identify the critical components and integrated the reliability functions in a quantitative evaluation method. Depending on the evaluation results, they listed the impacts of each maintenance activity for the TPSS. Yang et al. [3] proposed serval reliability indices for the TPSS and overhead contact lines on condition that the substations can offer cross-sectional energy. A Monte Carlo simulation method was used to obtain random samples and speed up the evaluation process. Concerning wind influences, Li et al. [5] proposed a point estimate method to assess the overloading risks in the load power generation process. In their study, the correlation between overloading risks of the contact lines and the power generation was simulated by the spatial transformation of a probabilistic distribution.

To evaluate the TPSS operations associated with lightning, Hu et al. [11] pointed out several factors of lightning risks for the TPSS and introduced a lightning damage function to evaluate the probable consequences. Integrating the lightning information from a weather report system, the model obtained the risks of the contact lines and substations. The above studies do not cover the evaluation of the reliability of the TPSS when the trains are running on a whole metro line [12]. According to a fixed train timetable, Lin et al. [1] provided two probability risk sets for the components of the TPSS and the
variances of the electricity quality. Combining both functions yielded a weighted evaluation model. Since the model captured the internal risks of the TPSS in real-time, the potential power interruptions were treated explicitly. Nevertheless, all the above studies have captured the relationship between the instant power and the electrical components but ignored the mutual influence for multiple trains. By mitigating the power interruption risks of the TPSS, a well-design timetable can cut down the highest instant energy supply of the substations by distributing the accelerating time of trains. However, for saving energy while uncompromising passenger travel time, energy-efficient timetabling has gained increasing attention in the literature (see [13]–[24]). For example, considering constant speed limitations, a running section is divided into several speed-limited segments based on the gradient and curvature of the metro line. Then, the speed profile of a train is formulated by a mathematical formula. Finally, to produce a train speed profile embedded in the timetable, an energy-efficient optimization model is developed for reducing energy consumption in a bidirectional metro line [25]. Considering energy transmitting within independent electricity supply intervals, Yang et al. [26] attempted to coordinate train arrival and departure times in both directions and then developed a solution method based on a genetic algorithm (GA). In the same vein, Tian et al. [27] formulated a multi-train traction energy optimization method to generate a timetable and applied a probabilistic method to solve the problem. For a similar problem, Zhao et al. [28] solved the integrated optimization problem using a two-step method. In the first step, a brute-force algorithm was implemented for a single train trajectory optimization; in the second step, GA was developed for obtaining the train timetable. A few studies considered bi-objective timetable optimization models to fulfill passengers’ travel time and energy-saving requirements (see [29]–[34]). For example, Yang et al. [34] developed a bi-objective nonlinear programming model to determine train timetables and speed profiles for minimizing energy consumption and passenger waiting time in a bidirectional metro line. An exact algorithm was developed after the problem was transformed into a quadratic programming problem.

Alongside the timetable optimization model development, emerging electricity technologies, such as regenerative energy devices (RED), were developed to save energy and address environmental concerns. In a metro line installed with RED, the energy regenerated by the decelerating trains can be utilized by any accelerating trains within the maximum electricity transmission distance. In that sense, enlarging the overlapping between the accelerating and decelerating phases of trains is a direct way of saving energy. With the same objectives as [34], Sun et al. [35] considered the time-variant passenger demands and one-to-one energy transmission method between accelerating and decelerating trains. Then, a bi-objective model is developed and GA is applied for producing satisfactory solutions. Further, to reduce the total passenger travel time and increase the utilization of regenerative energy, Yang et al. [36] proposed a passenger assignment method and a one-to-many energy transmission mechanism (one decelerating train to multiple accelerating trains). Then, a solution method based on the non-dominated sorting genetic algorithm II (NSGA-II) is developed for obtaining near-optimal irregular timetable solutions. The results showed that the approach could significantly improve the performance of the above two objectives.

To reduce the effects of disruptions, a few studies addressed the recovery of train timetables. For example, Gao et al. [6] compared the proposed stop-skipping patterns with all-stopping patterns. For reducing passenger waiting time, they rescheduled all service trains during the peak hours after a disruption and then applied a heuristic decomposition method to determine the stop-skipping patterns. The results showed that the suggested approach could reduce total passenger waiting time by 17.0% and total train running time by 9.5%. Considering the disruptions in one metro line of a two-line metro system, Xu et al. [7] suggested that the trains could use the crossover line to recover the operations and decrease the delay. They developed a rigorous rescheduling strategy to obtain the least delay, in which a capacity check algorithm was used for preventing deadlock. The results showed that the optimality gap is 9.8% of the feasible solutions between the proposed method and the GAMS (General Algebraic Modeling System). For the same problem, Ghaemi et al. [37], [38] calculated the optimal stations, routes, and platforms using a short-turning operation method. The results showed that the recovery plan needs to cancel more than 60 trains and the delay time is 30 minutes. Considering the time-dependent passenger demands, Huang et al. [39] compared two recovery strategies (alternation operation and short-turning operation) with two objectives, i.e., the minimum number of remaining passengers and minimum average headway deviation. Using the method of weighted sum, a two-stage algorithm was developed to solve the bi-objective problem. The results showed that non-boarding passengers are decreased from 30% to 13.8% and the headway deviation is sufficiently small.

Apart from evaluating the electric components, it is necessary to examine whether the used timetable involves power interruption risks of the TPSS given the high frequency of traction operations. TPSS usually uses the double-sided power supply mode, of which one substation at a metro station offers energy to trains running on the neighboring section(s). If the overloading time of a substation is longer than the regulated threshold, the power interruption would happen with a high probability. Recalling that the accelerating trains can directly use the regenerative energy, it is worth investigating whether the regenerative energy can be supplementary for the TPSS whenever a power interruption occurs. To that end, we develop a responsive train timetable scheduling method to determine an energy supplementation strategy. The method concerns a tri-objective timetable optimization model incorporating passenger travel time, power interruption cumulation (PIC), and total energy consumption. To calculate the regenerative energy utilization, we propose a many-to-many energy allocation approach to take advantage of the RED. To calculate the energy supply from the TPSS, the double-sided power supply model is considered and utilized energy is subtracted correspondingly. An efficient heuristic algorithm is developed to obtain a regular and cyclic timetable for minimizing passenger travel time, the maximum PIC, and
energy consumption. Particularly, using a space-time path representation (see [40], [41]), we generate timetables to satisfy the constraints of a regular timetable. Finally, the proposed model is verified in a bidirectional metro line with real passenger demands.

Considering regenerative energy and an energy supplementation strategy in the TPSS, the main contributions of this paper are listed below.

1. For regenerative energy, we formulate a many-to-many energy allocation mechanism among the accelerating and decelerating trains. With the maximum energy transmission distance, we develop an approach for allocating energy according to train energy demands and supplies.

2. For energy supply from the substations, we introduce a microscopic evaluation method according to the double-sided power supply mode. Besides the utilized regenerative energy, we develop a modified method to calculate the real-time traction and remaining regenerative energy.

3. To evaluate the power interruption risk of a substation, we propose a piece-wise function based on the linear relationship between the power interruption indicator and the overloading time.

4. To cope with potential power interruptions at a substation, we propose an energy supplementation strategy for replenishing the energy supply at the neighbor substations.

5. To meet the timetable constraints, a method is developed to obtain a feasible headway, and a running-dwelling modification method is suggested to adjust running and dwelling time for holding the total running time unchanged. These methods are integrated into a heuristic algorithm to obtain a regular and cyclic timetable.

The remainder of the paper is organized as follows. The problem descriptions and basic assumptions are introduced in Section 2. In section 3, a tri-objective optimization model is proposed. In section 4, we discuss a fast heuristic algorithm to solve the timetable optimization problem. Section 5 provides the numerical examples and then discusses the results. Finally, Section 6 gives the conclusions and discusses future researches.

II. PROBLEM DESCRIPTION

During the electricity supply interval, the energy for towing the trains is mainly provided by the electric substations in the power supply system. The switch installed at a substation is disconnected (at status “off”) with other substations for ensuring the independence of each power supply section. One substation offers energy to accelerating trains running on its neighboring sections according to the double-sided power supply mode. One section affords one decelerating train and one accelerating train simultaneously running in one direction. After simplifying the energy-flow diagram, there are three different working mechanisms, as shown in Fig. 1.

For regenerative energy flow (green arrows), the decelerating trains (green) transmit energy through the overhead contact lines to the accelerating trains (red) running in both directions only in sections (Fig. 1a). For the energy supply (red arrows), one substation (except at the two termini) offers energy to trains running on the two neighboring sections (Fig. 1b), while the terminal substations offer energy to trains on only one section. The two types of energy may co-exist. If multiple decelerating trains generate regenerative energy at the same time, multiple accelerating trains can use directly, which compose a parallel circuit with multiple power sources. In this circuit, the power sources are in series connection and the resistors are in parallel connection. Meanwhile, substations can save energy when regenerative energy is utilized. Once there is no overlapping between the decelerating and accelerating trains, the regenerative energy is wasted.

The power ratings of a substation are usually predetermined. However, when multiple trains on the neighboring sections are accelerating at the same time, they need substantial energy supply instantly. If the power supply is higher than a power rating for a certain period, the substation would fail with a high probability. If the power at one substation is cut off, the switch of the substation is at status “on” and the circuit is connected to the neighboring sections. Consequently, a substation may supply energy to accelerating trains on a combined section. The maximum distances for transmitting regenerative energy are changed and then the overlapping between accelerating and decelerating trains is considered on a combined section. The transmission of regenerative energy is unavoidable attenuated owing to the prolonged distances between the running trains. Once a power interruption occurs, the trains should be reorganized immediately, and thus a back-up plan is needed. The regenerative energy, if utilized efficiently, can supplement the energy supply from the neighboring substations. Note that power interruptions do not happen at the two terminal substations as there is only one neighboring section. In Fig. 1c, the neighboring substations (left and right) supply energy (red arrows) to the accelerating train near the power-off station (middle) in the up-direction. The regenerative energy (green arrow) from the decelerating train in the down-direction is used by the accelerating train simultaneously.

A. Assumptions

The following assumptions are made to facilitate the presentation of the modeling framework.

(A1) Passenger arrival rates vary with time at stations, but the O-D (origin-destination) matrix is deterministic (see [13], [25]).

(A2) Non-served passengers must wait for the next train due to the train loading capacity limitation. (see [6], [9], [34]–[36]).

(A3) A train has three working phases: acceleration-coasting-deceleration. The conversion factors of energy are constants (see [13], [14], [26], [35], [36], [42]).

(A4) Two energy flows of one accelerating train from two substations are the same.

(A5) There is a linear relationship between the PIC and the overloading time at a substation.

(A6) At one time, the power interruption may occur at multiple substations except for the termini.

(A7) Substations are placed at stations. Usually, the number of substations is no more than that of stations.
**B. Notations**

The following notations are used in this study.

- **Parameters**
  - \( i, j \) station index, \( i = 1, 2, \ldots, N; j = 1, 2, \ldots, N \)
  - \( s \) section index, \( s = 1, 2, \ldots, N - 1 \)
  - \( \mathbf{X}(p) \) set of potential power-off substations for timetable \( p \)
  - \( k \) train index \( k = 1, 2, \ldots, 2K \)
  - \( t, \tau \) time instances
  - \( tr \) turnaround index, \( tr = 0, 1, \ldots, TR \)
  - \( z, z' \) decelerating and accelerating phase index respectively, \( z = 1, 2, \ldots, (TR + 1) \cdot (N - 1) \);
  - \( z' = 1, 2, \ldots, (TR + 1) \cdot (N - 1) \)
  - \( n \) element index, \( n = 1, 2, \ldots, (TR + 1) \cdot (N - 1) \)
  - \( a, b \) acceleration and deceleration rates
  - \( r \) resistance rate
  - \( M \) train mass
  - \( \theta_1, \theta_2 \) conversion factors from electricity to kinetic energy and from kinetic energy to electricity
  - \( d_s \) length (distance) in section \( s \)
  - \( \bar{T}_{\text{dwe}}, T_{\text{dwe}} \) upper and lower bounds of dwelling times at stations
  - \( \bar{T}, T \) upper and lower bounds of running times on sections

- **Notations**
  - \( \bar{H}, H \) upper and lower bounds of headway between two consecutive trains
  - \( T_{\text{turn}} \) turnaround time
  - \( t_{B,\text{acc}}^{i,k,i}, t_{E,\text{acc}}^{i,k,i} \) beginning and ending times of the accelerating phase of \( k \) departing from \( i \)
  - \( t_{B,\text{dec}}^{i,k,i}, t_{E,\text{dec}}^{i,k,i} \) beginning and ending times of the decelerating phase of \( k \) running toward \( i \)
  - \( C \) train loading capacity
  - \( T_{\text{walk}} \) walking-out time per traveler at a station
  - \( \lambda_{i}(t) \) arrival rate of \( i \) during time \( t \)
  - \( w_{i,j} \) proportion of passengers from \( i \) to \( j \)
  - \( \gamma_i \) PIC of \( i \)
  - \( E_{\text{sub}}^{\text{ut}} \) upper bound of the energy supply
  - \( E_{\text{acc}}^{i,k,i}, E_{\text{dec}}^{i,k,i} \) traction and regenerative energy of \( k \) running from and departing toward \( i \) respectively.

- **Intermediate variables**
  - \( t_{E,\text{acc}}^{i,k,i}, t_{B,\text{dec}}^{i,k,i} \) switching time points from acceleration to coasting and from coasting to decelerating of \( k \) near \( i \), respectively.
  - \( E_{\text{ut}}^{i,k,i} \) utilization of regenerative energy of \( k \) running from \( i \).
  - \( u \) utilization rate of regenerative energy.
  - \( p_{W,\text{tr},i,j}^{k}, p_{B,\text{tr},i,j}^{k} \) waiting and boarded passengers for \( k \) at \( i \) to \( j \), respectively.
  - \( p_{W,\text{tr},i,j}^{k}, p_{B,\text{tr},i,j}^{k} \) waiting and boarded passengers for \( k \) at \( i \), respectively.
  - \( p_{L,\text{tr},i}^{k}, p_{R,\text{tr},i}^{k} \) alighted and stranded passengers for \( k \) at \( i \), respectively.

- **Decision variables**
  - \( t_{\text{arr},i,j}, t_{\text{dep},i,j} \) arrival and departure times of train \( k \) at station \( i \).

**III. MODELING FORMULATION**

This section presents a tri-objective optimization model to integrate passenger demand, train running, and energy supply for minimizing passenger travel time, train energy consumption, and the maximum potential power interruption. To calculate the potential of power interruption, indicated by PIC, the model considers the accelerating trains, the power ratings, and the overloading status of the substations. To calculate the passenger travel time, the trip is divided into walking-in, waiting, on-train, and walking-out stage. The following part presents the constraints and detailed formulation of the objectives associated with a regular timetable.

**A. Energy Consumption Analysis**

1) **Power Interruption Analysis**: First, the PIC of every substation is measured. During the energy supply period, if the instant power supply from a substation is too large, the substation is overloaded. In general, two neighboring substations are linked by one section. In case of no power-off substations, substation \( i \) offers energy for trains accelerating on the two neighboring sections \( s \) and \( s + 1 \). For calculating...
the instant energy $E_{i,\text{sub}}^\text{acc}(\tau)$ from $i$, the utilized regenerative energy should be deducted from the energy supply to the trains. $E_{i,\text{sub}}(\tau)$ is formulated as

$$E_{i,\text{sub}}(\tau) = \begin{cases} \sum_{k \in K_{i,\text{acc}}} \left( \frac{E_{k,\text{acc}}^\text{acc}(\tau)}{2} - \frac{E_{k,i}^{\text{uti}}(\tau)}{2} \right), \\ 0, \end{cases}$$

$$(\tau \in \Gamma_s \cup \Gamma_{s+1} \wedge i \neq 1 \wedge i \neq N) \cup (\tau \in \Gamma_s \wedge i = 1 \vee i = N)$$

$$(\tau \not\in \Gamma \wedge \tau \not\in \bigcup_{k \in [1,2K]} \left( \bigcup_{i \in Q(E)} \left[ i_{k,i}^{\text{B,dec}}, t_{k,i}^{\text{E,dec}} \right] \right)$$

$$\gamma_i = \sum_{k=1}^{K} \sum_{t \in [1,2K, \infty]} (t - i_{k,i}^{\text{B,acc}}) \cdot \pi_i(\tau),$$

where $\gamma_i$ is the instant energy from station $i$ at time $\tau$ and $E_{i,\text{sub}}^\text{acc}(\tau)$ is the use of regenerative energy from $k$ decelerating to $i$ at $\tau$. According to the double-sided power supply mode, the energy of one accelerating train is offered by two neighboring substations. Based on Assumption 4, half of the traction energy of one train is supplied from one substation. During the overlapping time intervals of accelerating-decelerating train pairs near the substation, similarly, the utilized regenerative energy should be deducted from two substations. Note that the terminal substations offer energy to trains only on one section (i.e., the second condition of Eq. (1)). $\Gamma_s$ is an overlapping time set of decelerating-accelerating train pairs and records the intersection between the union of the decelerating time intervals and the union of the accelerating time intervals on $s$, given as

$$\Gamma_s = \left\{ \left( \bigcup_{i \in Q(s), k \in [1,2K]} \left[ i_{k,i}^{\text{B,dec}}, t_{k,i}^{\text{E,dec}} \right] \right) \cap \left( \bigcup_{i \in Q(s), k \in [1,2K]} \left[ i_{k,i}^{\text{B,acc}}, t_{k,i}^{\text{E,acc}} \right] \right) \right\}$$

where $i_{k,i}^{\text{B,dec}}$ and $t_{k,i}^{\text{E,dec}}$ represent the beginning and ending times of decelerating phase of $k$ running toward $i$, respectively; $i_{k,i}^{\text{B,acc}}$ and $t_{k,i}^{\text{E,acc}}$ represent the beginning and ending times of accelerating phase of $k$ departing from $i$, respectively. $Q(s)$ denotes a set of stations at both ends of $s$. If trains are accelerating at the same time on section $s$, the indices are recorded by $K_{s}^{\text{acc}}$; if trains are decelerating at the same time, they are recorded by $K_{s}^{\text{dec}}$. $K_{s}^{\text{acc}}$ and $K_{s}^{\text{dec}}$ are recorded by Eq. (3.1) and Eq. (3.2) respectively.

$$K_{s}^{\text{acc}} = \left\{ k \mid \bigcup_{i \in Q(s), k \in [1,2K]} \left[ i_{k,i}^{\text{B,acc}}, t_{k,i}^{\text{E,acc}} \right] \right\}$$

$$K_{s}^{\text{dec}} = \left\{ k \mid \bigcup_{i \in Q(s), k \in [1,2K]} \left[ i_{k,i}^{\text{B,dec}}, t_{k,i}^{\text{E,dec}} \right] \right\}$$

In a bidirectional metro line, there are at most one decelerating train and one accelerating train in one direction on a section, making at most four trains in each above set at one time [26]. Thus, it may cause the instant power volume higher at substations. Commonly, the instant energy supply from a substation can tolerate certain supply levels exceeding the power rating $E_{i,\text{sub}}^\text{acc}$, which is pre-determined according to the characteristics of the electrical components. We introduce a piece-wise function between the exceeding energy and the overloading period (Eq. 4). Given three levels ($\sigma, 1.5\sigma$, and $3\sigma$) of exceeding power rating, the PIC of $i$ ($\gamma_i$) during the entire energy supply period considered is formulated as

$$\gamma_i = \sum_{k=1}^{K} \sum_{t \in [1,2K, \infty]} (t - i_{k,i}^{\text{B,acc}}) \cdot \pi_i(\tau),$$

where $\sigma$ and $\zeta$ are parameters of the energy levels and tolerance levels [43]. To calculate $\gamma_i$, the volumes of the instant power supply and the tolerated times for overloaded operations are decision variables. For the first condition on the right-hand side of Eq. (4), if the energy volumes from a substation are higher than $3\sigma \cdot E_{i,\text{sub}}^\text{acc}$ for duration $\zeta$ or above, the power interruption of the substation occurs; likewise, if the energy volumes are higher than $1.5\sigma \cdot E_{i,\text{sub}}^\text{acc}$ and lower than $3\sigma \cdot E_{i,\text{sub}}^\text{acc}$ for duration $10\zeta$ or above, the power interruption occurs; if the energy volumes are higher than $1.5\sigma \cdot E_{i,\text{sub}}^\text{acc}$ and lower than $\sigma \cdot E_{i,\text{sub}}^\text{acc}$, the power interruption would never occur.

Finally, for one timetable, if the PIC of a substation is higher than a threshold $\bar{\gamma}$, the substation is added to the set of potential power-off substations (POSs) $\chi$. $\chi$ is given as

$$\chi = \{ i \mid \gamma_i \geq \bar{\gamma} \}$$

2) Overlapping Rearrangement: If there are POSs ($\chi \neq \emptyset$), the switches of substations $\forall i \in \chi$ are turned on. The electricity circuit is connected and thus the sections near the POSs are combined as one. In the combined section, the maximum energy transmission distance is changed and thus the decelerating-accelerating train pairs may change correspondingly. Considering the POSs from $i$ to $i'$ ($i' \leq N - 1$), the maximum energy transmission distance is updated as $d_{i'} = \sum_{i=i'-1}^{i} d_i$, where $d_i$ is the length of the original section $s$. $s'$ is an index corresponding to the sections after combination. On the combined sections, the accelerating and decelerating train indices ($K_{s'}^{\text{acc}}$ and $K_{s'}^{\text{dec}}$) are needed to be updated for Eq. (1). $K_{s'}^{\text{acc}}$ is updated as

$$K_{s'}^{\text{acc}} = \bigcup_{s \in Q'(i)} K_{s}^{\text{acc}}, i \in \chi$$

A similar way can be applied to update $K_{s'}^{\text{dec}}$. $Q'(i)$ denotes a set of neighboring sections of each station in $\chi$. To record the overlapping time intervals of decelerating-accelerating train
pairs, \( \Gamma' \) in Eq. (1) is updated as

\[
\Gamma' = \bigcup_{s \in Q(i)} \Gamma_s, \quad i \in \chi
\]  

(7)

In the above situation, the traction energy is offered by the neighboring substations of the POSs and the regenerative energy is utilized by accelerating trains on the combined section.

3) Energy Allocation: In one (combined) section, in case of multiple trains accelerating and decelerating at the same time, an energy allocation mechanism is needed for distinguishing energy sources and amounts. The overlapping time intervals are recorded for each accelerating-decelerating train pair to determine how much regenerative energy is utilized (Eq. 2). Two sets record the train indices when there are accelerating or decelerating at the same time (Eq. 3). If the regenerative energy from multiple decelerating trains can be used by multiple accelerating trains, a utilization coefficient is involved for each decelerating-accelerating train pair. At \( \tau \), if the regenerative energy generated by train \( k \) can be used by \( k' \) in \( s \), the utilization coefficient \( u_{k,k',s}(\tau) \) is formulated as

\[
u_{k,k',s}(\tau) = \frac{a_{k,k',s}(\tau) \cdot \eta_{k,k',s}(\tau) \cdot \mu_{k,k',s}(\tau)}{\sum_{k' \in K_{acc}^s} a_{k,k',s}(\tau) \cdot \eta_{k,k',s}(\tau) \cdot \mu_{k,k',s}(\tau)}, \quad \tau \in \Gamma_s
\]  

(8)

where \( a_{k,k',s}(\tau) \) is the electricity decay factor between \( k \) and \( k' \) at \( \tau \) on \( s \). \( \eta_{k,k',s}(\tau) \) is a proportion of energy allocation for the situation of one-to-many transmission and \( \mu_{k,k',s}(\tau) \) is an allocation rate for the situation of many-to-many energy transmission. Since the series connection and parallel connection in one circuit, the power sources share the energy supplies simultaneously and the supply volumes are determined by the resistances. The larger the resistance, the less power is offered to them. When the regenerative energy transmitting to multiple accelerating trains, each decelerating train shares a part of the supplies. In this circuit, the resistances changes with the relative distances between the decelerating-accelerating train pairs. The decay factor determines \( \eta_{k,k',s}(\tau) \) as

\[
\eta_{k,k',s}(\tau) = \left( \frac{a_{k,k',s}}{\sum_{k' \in K_{acc}^s} a_{k,k',s}} \right)
\]  

(9)

To determine how much regenerative energy from decelerating trains is allocated to the accelerating trains, an allocation rate \( \mu_{k,k',s}(\tau) \) is determined by the decay factors and the numbers of accelerating and decelerating trains at \( \tau \), formulate as

\[
\mu_{k,k',s}(\tau) = \left[ (a_{k,k',s} \cdot \eta_{k,k',s} \cdot E_{k,i}^{dec}) / \left( \sum_{k \in K_{acc}^s} (a_{k,k',s} \cdot \eta_{k,k',s} \cdot E_{k,i}^{dec}) \right) \right] 
\]  

(10)

\[
\sum_{k \in K_{acc}^s} \sum_{k' \in K_{acc}^s} (a_{k,k',s} \cdot \eta_{k,k',s} \cdot E_{k,i}^{dec}) \geq \sum_{k' \in K_{acc}^s} E_{k,i}^{dec}(\tau)
\]  

\[
\sum_{k \in K_{acc}^s} \sum_{k' \in K_{acc}^s} (a_{k,k',s} \cdot \eta_{k,k',s} \cdot E_{k,i}^{dec}) < \sum_{k' \in K_{acc}^s} E_{k,i}^{dec}(\tau)
\]

Fig. 2. A diagram for many-to-many energy transmission.

where \( E_{k,i}^{dec}(\tau) \) is the regenerative energy generated by \( k \) decelerating toward \( i \). If the regenerative energy supply is higher than the traction energy demand, a part of the regenerative energy from each decelerating train is utilized by the accelerating trains; if the supply is less than the demand, all the regenerative energy from each is utilized.

The resistance is a key factor to determine \( a_{k,k',s}(\tau) \), which decreases with the relative distance increasing.

\[
a_{k,k',s}(\tau) = \begin{cases} 
\beta_1 \cdot \exp \left( -\beta_2 \cdot \frac{d_{k,k',s}(\tau)}{d_s} \right), & d_{k,k',s}(\tau) \leq d_s \land \tau \in \Gamma_s \\
0, & d_{k,k',s}(\tau) > d_s \land \tau \in \Gamma_s 
\end{cases}
\]  

(11)

where \( \beta_1 \) and \( \beta_2 \) are two resistance parameters [36].

The relative distance \( (d_{k,k',s}(\tau)) \) between the accelerating train \( k' \) and decelerating train \( k \) is given as

\[
d_{k,k',s}(\tau) = \left| p_k(\tau) - p_{k'}(\tau) \right|, \quad \tau \in \Gamma_s
\]  

(12)

where \( p_k \) and \( p_{k'} \) are positional information of \( k \) and \( k' \) at \( \tau \), respectively; operator \( \left| \cdot \right| \) gives the distance between the two trains along the track.

The allocation mechanism proposed by Eq. (9) takes effect if multiple accelerating trains are using the regenerative energy at the same time. Since the electricity attenuation is inevitable during transmission, the energy decay function is defined as a monotonically decreasing function according to Eq. (11). The decay factor ranges from 0 to 1, which is an indicator of the loss of energy exchanges between accelerating and decelerating trains. Fig. 2 is an example to show the energy allocation process. In Fig. 2, there is a section including two decelerating trains (trains 1 and 11 denoted by the green dotted and solid lines respectively) and two accelerating trains (trains 2 and 12 denoted by red dashed and solid lines respectively). The black points denote binary variables, which indicate whether energy exchanges occur or not. The green numbers show the decay factors between decelerating and accelerating trains. The red numbers show the proportions of the decelerating-accelerating pairs. A similar rule is adopted for many-to-many energy transmission. If the regenerative energy from one
decelerating train can be used by two accelerating trains at the same time, the proportion is calculated based on the second part of Eq. 10 (Fig. 2(a)); if the regenerative energy from two decelerating trains can be used by two accelerating trains, the allocation rate is calculated based on the first part of Eq. 10 (Fig. 2(b)).

4) Energy Utilization: For each accelerating-decelerating train pair, the utilized regenerative energy takes the smaller values between the needs of the accelerating train and the supply from the decelerating train as Eq. (13), of which the definitions and formulations of the notations are presented below.

\[
E_{\text{uni}}^{\text{acc}}(\tau) = \begin{cases} 
\sum_{k \in K_{\text{dec}}} u_{k,k'}(\tau) \cdot E_{\text{dec}}^{\text{acc}}(\tau), \\
\sum_{k \in K_{\text{acc}}} u_{k,k'}(\tau) \cdot E_{\text{acc}}^{\text{dec}}(\tau), 
\end{cases}
\]

\[
u_{k,k'}(\tau) = 1 \land \tau \in \Gamma_{k} \land i \neq N
\]

\[
u_{k,k'}(\tau) < 1 \land \tau \in \Gamma_{k} \land i \neq N
\]

(13)

The formulations of traction and regenerative energy of \( k \) from and towards \( i \) are given respectively as

\[
E_{k,i}^{\text{acc}}(\tau) = M \cdot \frac{(v(\tau + 1)^2 - (v(\tau))^2)}{2\theta_1},
\]

\[
E_{k,i}^{\text{dec}}(\tau) = M \cdot \theta_2 \cdot \frac{(v(\tau + 1)^2 - (v(\tau))^2)}{2},
\]

\[
t_{k,i}^{\text{acc}} \leq \tau < \tau + 1 \leq \frac{E_{k,i}^{\text{acc}}}{\theta_1} \land i \neq N
\]

\[
t_{k,i}^{\text{dec}} \leq \tau < \tau + 1 \leq \frac{E_{k,i}^{\text{dec}}}{\theta_2} \land i \neq 1
\]

(14)

where \( M \) is the train mass, \( \theta_1 \) and \( \theta_2 \) are the conversion factors, and \( v(\tau) \) is the train velocity at \( \tau \). Particularly, according to the formula \( (P = F \cdot S) \), the traction force \( F \) is calculated by the formula \( M \cdot a \) and the displacement \( S \) is calculated by the formula \( \int_0^t (v^2 - v_0^2)/2a \), where \( a \) is the accelerate rate, and \( v_1 \) and \( v_0 \) are final velocity and initial velocity.

B. Passenger Demand Analysis

Given an O-D timetable, passenger arrival rates, and a timetable, we can calculate the numbers of passengers at different stages in a metro trip according to a passenger assignment mechanism [36]. During peak hours, a proportion of passengers may not be able to board the incoming trains due to the train capacity constraint. Since the stranded passengers wait for the next trains, the number of stranded passengers of each O-D pair should be known to calculate the number of the boarding passengers of each O-D pair for the next train. It means that the waiting passengers may include the incoming passengers and stranded passengers from previous trains for the next train. The boarding passengers come from the waiting group. Yang et al. [36] have introduced formulas to calculate the numbers of tapping-in and boarding passengers of each O-D pair, but the passengers left on the platforms and waiting for the next trains were ignored. As the calculation of the boarding passengers, the proportions of the stranded passengers are equal to the proportions of the waiting passengers of each O-D pair. To calculate the stranded passengers of each O-D pair, we supplement new equations according to the principle of proportional fitting as

\[
P_{\text{tr},k,i,j}^{\text{L,up}} = \frac{P_{\text{tr},k,i,j}^{\text{W,up}}}{P_{\text{tr},k,i,j}^{\text{L,up}}}, \quad P_{\text{tr},k,j,i}^{\text{L,down}} = \frac{P_{\text{tr},k,j,i}^{\text{W,down}}}{P_{\text{tr},k,j,i}^{\text{L,down}}}, \quad P_{\text{tr},k,j,i}^{\text{L,down}} = \frac{P_{\text{tr},k,j,i}^{\text{W,down}}}{P_{\text{tr},k,j,i}^{\text{L,down}}}, \quad P_{\text{tr},k,j,i}^{\text{L,down}} = \frac{P_{\text{tr},k,j,i}^{\text{W,down}}}{P_{\text{tr},k,j,i}^{\text{L,down}}}
\]

(15)

where \( P_{\text{tr},k,i,j}^{\text{L,up}} \) and \( P_{\text{tr},k,j,i}^{\text{L,down}} \) denote the numbers of stranded passengers from \( i \) to \( j \) for \( k \) in the up-direction and from \( j \) to \( i \) in the down-direction, respectively. \( P_{\text{tr},k,i,j}^{\text{W,up}} \) and \( P_{\text{tr},k,j,i}^{\text{W,down}} \) denote the numbers of stranded passengers at \( i \) in the up-direction and at \( j \) in the down-direction, respectively; \( P_{\text{tr},k,i,j}^{\text{W,up}} \) and \( P_{\text{tr},k,j,i}^{\text{W,down}} \) are the waiting numbers of passengers from \( i \) to \( j \) for \( k \) and from \( j \) to \( i \) for \( k \), respectively; \( P_{\text{tr},k,i,j}^{\text{W,up}} \) and \( P_{\text{tr},k,j,i}^{\text{W,down}} \) are the waiting numbers at \( i \) and at \( j \), respectively.

In [36], the waiting time, on-train time, and walking-out time have been calculated without the consideration of walking-in time. Focusing on the complete metro trips, it is necessary to consider walking-in time as well. Nevertheless, the total walking-in time is constant due to the fixed passenger demands. The walking-in time in the up-direction is given as Eq. (15), while the walking-in time in the down-direction has a similar form.

\[
WIT_{\text{up}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{T} (P_{\text{tr},k,i,j}^{\text{W,up}} \times T_{\text{walk}})
\]

(16)

where \( T_{\text{walk}} \) is the walking time.

The times of waiting, boarding, alighting, on-train, and stranded passengers at each stage were formulated by [36].

C. Completed Model

The first objective is to minimize the total passenger travel time, which relates to the service quality of a metro system. A passenger trip is recorded from the tapping-in gate of an origin station to the tapping-out gate of the destination station: walking-in time (WIT), waiting time (WT), on-train time (OT), and walking-out time (WOT). The objective is expressed as

\[
\min f_1 (W) = \text{[WIT + WT + OT + WOT]}
\]

(17)

The second objective is to minimize the total energy consumption as

\[
\min f_2 (U) = \sum_{k=1}^{K} \sum_{i=1}^{N} \left( E_{k,i}^{\text{acc}} - E_{k,i}^{\text{uni}} \right)
\]

(18)

The third objective is to minimize the maximum PIC across all substations. The objective is shown as

\[
\min f_3 (\gamma) = \max_{1 \leq i \leq N} \left[ \gamma_i \right]
\]

(19)

In view of the above, we rearrange a two-step optimization model. The first step is to minimize the number of the potential power-off substations, shown as follows.

\[
\min f_3 (\gamma)
\]

s.t.

Eqs. (1) – (16) and \( (A1) - (A7) \) (20.1)
The main process is listed as follows and the pseudo-code is covering power-off stations to maintain the overlap unchanged. For decreasing the traction energy consumption, we increase energy supply. When generating the timetable, we first enlargeative energy could be used more efficiently for supplementingvals between decelerating and accelerating trains, the regener-

At a power-off station, by enlarging the overlapping time inter-
relieving the supply pressure of the neighboring substations.

metro line and thus the timetable needs to be produced for
sections lengths and passenger demands. Using the notations
defined in section 2.2, the constraints related to a regular timetable are given in Appendix A.

To minimize the total passenger travel time only, a timetable with the minimized headway, running and dwelling times could simply achieve the objective. However, smaller running times generally lead to higher traction energy consumption, which contradicts the second objective for saving energy. Min-

imizing the overloading risks at the substations is important for the safety of the TPSS operations (see [2], [5]). Once a substation is power-off, the traditional solutions are either to halt all running trains in the metro line or to sharply magnify the headways, referred to as the magnifying headway method. If the regenerative energy from the decelerating trains is prop-
erly utilized, energy supplies are substantially supplemented at the power-off stations and thus the normal TPSS operations can recover immediately.

As indicated in Section 3.2.2, if one substation is power-off, the calculations of the regenerative energy and the traction energy for the neighboring substations are changed. Partic-

ularly, the neighboring substations undertake the task of supplying energy to trains accelerating near the power-off stations. If no adjustment is made, domino effects may occur at other substations. Therefore, a fast-responsive and efficient solution algorithm is needed for the above tri-objective model.

IV. HEURISTIC SOLUTION ALGORITHMS

Given a feasible timetable, the PIC of one or more substa-
tions may be higher than the threshold. When such a substation exists, the substation is considered a potential power-off station in the metro line and thus the timetable needs to be produced for relieving the supply pressure of the neighboring substations. At a power-off station, by enlarging the overlapping time intervals between decelerating and accelerating trains, the regenerative energy could be used more efficiently for supplementing energy supply. When generating the timetable, we first enlarge the overlap of the decelerating and accelerating phases. Then, for decreasing the traction energy consumption, we increase the running times of trains on some sections except those covering power-off stations to maintain the overlap unchanged. These two adjustments are at the core of the suggested fast heuristic algorithm to the tri-objective optimization problem. The main process is listed as follows and the pseudo-code is given in Algorithms I and II.

First, based on Eqs. (A1-A7), the space-time matrix $X$ is generated for each solution (timetable). To guarantee the diversity of the power-off substation set, a population ($pop$) of solutions is introduced based on the lower and upper bounds of the headway time ($H$, $\tilde{H}$). In one solution, the running and dwelling times vary with the sections and stations. The running time constraints also vary with longer-distance and shorter-distance sections.

Second, to evaluate $X$, the beginning ($t_{B,dec}^{k,i}$ and $t_{B,acc}^{k,i}$) and ending ($t_{E,dec}^{k,i}$ and $t_{E,acc}^{k,i}$) time points of decelerating and accelerating phases are rearranged corresponding to station indices ($t_{k,i,i+1}^{B,dec}$, $t_{k,i,i+1}^{B,acc}$, $t_{k,i,i+1}^{E,dec}$, and $t_{k,i,i+1}^{E,acc}$). The traction and regenerative energy of each train are calculated. The PIC for each substation is calculated based on Eqs. (1-16). The power-off substation set $\chi$ is recorded. The passenger numbers and time expenses on each trip stage are calculated by [36].

Third, at the first iteration, two-stage modifications are applied. The first stage focuses on the regenerative energy supplementation strategy to adjust the headway times for enlarging the overlapping times between the accelerating and decelerating trains at the power-off stations. A difference set \( \left\{ t_{k,i,i+1}^{B,acc} - t_{k,i}^{B,dec} \right\} \), \( i \in \chi \), for each accelerating-decelerating train pair, is introduced. The beginning times of the decelerating phases that are earlier than the accelerating phase will be considered. At potential power-off station $i$, if the minimum adjusted value ($\sigma^{\alpha}$) between the beginning time of accelerating phase of $k$ and the beginning time of $k'$ is $H$ times larger than the headway time $H^{dep}$, the arrival time of $k'$ is postponed by $\sigma^{H} - 1 \cdot H^{dep}$, where $\sigma^{H}$ is a given amplification coefficient; if the adjusted value is less than $H$ times of $H^{dep}$, the arrival time of $k'$ is postponed by the value (Eq. 21.2). Eq. 21.2 calculates the adjusted value. The first difference set records the values between the beginning time of accelerating phase of $k$ at $i$ and the beginning time of decelerating phase of $k'$ at $i - 1$; the other set concerns the differences between $k$ at $i$ and $k'$ at $i$. The following adjustments are made.

$$
\sigma^{\alpha} = \min \left( \min_{i \in \chi, \ k \in [1,2K]} \left\{ t_{k,i}^{B,acc} - t_{k',i-1}^{B,dec} \right\}, \min_{i \in \chi, \ k \in [1,2K]} \left\{ t_{k,i}^{B,acc} - t_{k',i}^{B,dec} \right\} \right)
$$

(21.2)

The second stage adjusts the running times on the sections except for the neighboring sections near POSs. With the fixed total travel times, decreasing the traction energy by increasing the running time in a section has been verified by the previous studies [21]. In this stage, by decreasing the dwelling times in the next stations correspondingly, we increase the running times of trains on sections except for the neighboring sections near the power-off substation. $t_{i+1}^{temp}$ is the modified value of
the arrival time at $i + 1$, formulated as

$$t_{i+1}^{temp} = \begin{cases} \min[(\bar{T} - T_{i,i+1})(T_{i,i+1}^{dwe} - T_{i,i+1}^{dwe})], & \bar{T} > T_{i,i+1} \wedge i + 1 \notin \chi \\ 0, & \bar{T} = T_{i,i+1} \wedge i + 1 \notin \chi \vee T_{i,i+1}^{dwe} = T_{i,i+1}^{dwe} \end{cases}$$

(22.1)

where $T_{i,i+1}$ is the running time between stations $i$ and $i + 1$, $\bar{T}$ is the upper bound of the running time, $T_{i,i+1}^{dwe}$ is the dwelling time at $i + 1$, and $T_{i,i+1}^{dwe}$ is the lower bound of the dwelling time.

Fourth, the space-time matrix $X$ is updated correspondingly.

Finally, when one of the termination conditions is satisfied, the algorithm stops and outputs a satisfactory solution.

As shown above, these two modifications supplement the energy supply at the power-off stations and decrease the total energy consumption. At the first iteration, the headway modification aims to obtain the perfect overlaps of decelerating-accelerating train pairs at the power-off stations. In the following iterations, the modification applies to the running and dwelling times.

Remark 1: After power interruption occurs, the two-stage modification method contributes to finding large overlaps between accelerating-decelerating train pairs and energy-efficient satisfactory solutions.

Recall that if the beginning times of the decelerating and accelerating phases are the same on a section, the energy supply may decrease. In this case, we have $t_{k,i}^{acc} = t_{k,i}^{dec}$ and $t_{k,i}^{D,acc} = t_{k,i}^{D,dec}$. According to Eq. (13), the energy supply for a train on section $i$ is formulated as

$$\sum_{k' \in \chi} u_{k',i}^D \cdot E_{k',i}^{acc}(\tau) - \sum_{k \in \chi} t_{k,i}^{D,acc} \cdot E_{k,i}^{dec}(\tau).$$

If $t_{k,i}^{acc} - t_{k,i}^{dec} < t_{k,i}^{D,acc}$ or the regenerative energy is wasted from $t_{k,i}^{dec}$ to $t_{k,i}^{acc}$ since $u_{k,k'}(\tau)$ is equal to zero. The energy supply is increasing. If $t_{k,i}^{acc} + T_{i,i+1}^{dwe} > t_{k,i}^{D,acc}$ or $t_{k,i}^{D,dec} + T_{i,i+1}^{dwe} > t_{k,i}^{D,dec}$, the regenerative energy is wasted from $t_{k,i}^{D,dec}$ to $t_{k,i}^{D,dec}$ + $t_{k,i}^{D,dec}$. The energy supply is also increasing. Therefore, if $T_{i,i+1}^{dwe} = 0$, the results are better than the above two situations. When the acceleration rate, deceleration rate, and section length are fixed, the traction energy is decreasing by increasing the running time (see [20], [21], [26], [36]). Thus, the second-stage modification reduces traction energy consumption.

V. NUMERICAL EXAMPLE

In this section, the performance of the proposed model and the heuristic algorithm is tested by a numerical example.

A. Example Description

The total length of the Yizhuang line (Beijing, China) is 23.23 km. The line is a bidirectional metro line including 13 stations (12 running sections). A feasible solution (timetable) has the following characteristics. The dwelling time is ranged from 20 s to 60 s ($T^{DW} \in [20, 60]$). In long sections ($d_i > 1500$ m), the running time is ranged from 135 s to 200 s ($T \in [135, 200]$); in short sections ($d_i < 1500$ m), $T \in [90, 120]$. The headway time is ranged from 90 s to 180 s ($H \in [90, 180]$). Train turnaround takes $T_{\text{turn}} = 150$ s. The train fleet size is $K = 10$. Table I shows the lengths of the sections and station indices. Table II shows the values and units of train operating parameters. The passenger arrival rate and OD are applied from the previous study [36]. Each train turns around 3 times. The train loading capacity is 1200 passengers/train.

The first trips of each fleet starting at 7:00 am appear in the timetable as 0. Trains running in both directions from the origin to the termini comprise a sub-timetable (block). Thus, a solution is including 4 blocks. For the solution population, the headway times have a common difference of 2 s, making the population size pop as $46 = (H - H) / 2 + 1$.

B. Results Analysis

According to the above setup, the heuristic algorithm obtains a solution after five iterations. Each iteration takes 90 s on average, of which around 0.17% (0.15 s) of the time is used for evolution and 12% (10.8 s) of the time is used for crossover and mutation.
to adjust the solution population. Thus, the short computation time satisfies the scheduling requirement.

For comparisons, we first display the PIC of each substation associated with the initial solutions in Fig. 3. Except for the termini, the stations have varied PICs in different solutions. At stations 4, 8, 9, and 10, the average PIC is higher than those of other substations. The preliminary results confirm that the power-off interruptions need to be considered in the timetabling problem. To show the energy changes before and after the power-off interruptions, we display the energy supply fluctuations of all substations of the initialsolutions in Fig. 4. Since timetables are regular, the fluctuations of energy supplies from all substations are periodic, while they differ in the two running directions. In Fig. 4a, the colored lines show the energy supplied by different substations. The x-axis refers to the accelerating phase index. When 20 trains running through each station 4 times, each substation offers energy 80 train times in total. The numbers from 1 to 20 correspond to the 20 trains in order. The odd indices represent the trains first starting from station SJ and the even indices represent the trains starting from station CQ. The train indices are updated from 21 to 40 corresponding to the trains running after turning around. If substation 4 is power-off, for example, it has effects on all substations, especially on the neighboring substations (Fig. 4b). The energy supplies of substations 3 (yellow line) and 5 (green line) increase significantly, compared with the results before the power-off interruption.

Fig. 5 shows the POSs of the initial solutions. In Fig. 5a, the blue stars (with reference to the left y-axis) show the maximum PICs. As seen, some substations are power-off in the different solutions (for example, the black rectangles in Fig. 5a). This implies that larger headways may not avoid power-off interruptions. In Fig. 5b, the squares show the maximum PICs generated by the optimized solutions. The circles show the PIC of the initial solutions, while the stars show the PIC of the magnifying headway solutions. The average maximum PIC is decreased from 0.85% in the initial solutions to 0.48% in the optimized solutions. Comparing the results of the magnifying headway solutions and our optimized solutions, the average value of the maximum PIC is decreased from 0.62% to 0.48% (Fig. 5b). The POSs are obtained at the first iteration and fixed in the subsequent iterations; then, the PICs of these substations are zero. Thus, the substation indices of the maximum PIC calculated by the optimized solutions are different. The magnifying headway solutions may not decline the power interruption risk. For example, in Fig. 5b, the maximum PIC rises after applying the magnifying headway method (solution indices 2, 3, 5, 8, 9, 10, 11, 12, 19, 20, 21, 23, 30, 35).

To compare the outcomes of the different methods, we calculate the energy consumption and travel times of the produced solutions. In Fig. 6(a, b), the stars refer to the total energy consumption and total travel time associated with the magnifying headway method. The squares and circles represent the results of the initial solutions and the optimized solutions, respectively. The energy consumption of the optimized solutions is in general decreased compared with those found by the magnifying headway method. The maximum decrease is 7.40% by solution 42 (from $5.459 \times 10^{10}$ J to $5.0829 \times 10^{10}$ J). The average energy consumption of the optimized solutions is decreased by 13.04% from $5.8453 \times 10^{10}$ J to $5.0829 \times 10^{10}$ J, compared with the results of the initial solutions. The average passenger travel time of the optimized solutions is decreased by 51.12% from $1.8952 \times 10^{8}$ s to $0.9263 \times 10^{8}$ s, compared with the results of magnifying headway solution.

We use solution 21 to illustrate the many-to-many energy allocation results in detail. In Fig. 7, the generated regenerative energy curves of train 1 (up-direction) and train 10 (up-direction) are represented by black solid and dashed lines respectively. ‘X 1220’ corresponds to the time 1220 s on the x-axis and ‘Y -7.916e+04’ corresponds to the regenerative energy value $7.9160 \times 10^4$ on the y-axis. During the time period

### TABLE I

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<thead>
<tr>
<th>Index &amp; sections</th>
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<tbody>
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<tr>
<td>2 XC-XH</td>
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</tr>
<tr>
<td>3 XH-JG</td>
<td>2366</td>
</tr>
<tr>
<td>4 JG-YZQ</td>
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<td>5 YZQ-WH</td>
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<table>
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### TABLE II

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Fig. 3. The PIC of each substation of each initial solution.
Fig. 4. The supplied energy from each substation of each timetable.

Fig. 5. The PICs and POSs for each solution.

Fig. 6. Total energy consumption and total travel time comparison.

from 1219.3 s to 1226 s, trains 1 and 10 transmit regenerative energy to train 9 (down-direction) at the same time.

The utilized regenerative energy is obtained as 7.9160 \times 10^4 (3.2040 \times 10^4 \times 0.1727 + 1.3670 \times 10^5 \times 0.1743) since the sum of
the utilized regenerative energy from the two decelerating trains is less than the traction energy demand of the accelerating train.

In case of double calculation of the utilized regenerative energy for the same accelerating train, the utilized regenerative energy is subtracted during the decelerating phase of train 10 correspondingly (from 1219.3 s to 1226 s). During the period from 1227 s to 1227.3 s, train 10 transits the regenerative energy to train 3.

The calculation of the utilized regenerative energy is shown as $6.5330 \times 10^4 \times (1.5960 \times 10^5 \times 0.4039)$. After the power interruption, to compare the energy supply from each substation, we calculate the average energy consumption of solution 21, for example, associated with the initial, optimized, and magnifying headway solutions. When substation 8 power-off, the energy previously supplied from substation 8 is supplied by substations 7 and 9; hence, the supplied energy from these two substations will be dramatically increased (Fig. 8). In Fig. 8, the blue, black, and red stars represent the average energy supply of each substation from the initial solution, optimized solution, and magnifying headway solution, respectively. The results of the optimized solution are superior to the results of the other two solutions.

The metro operator needs to trade-off the energy consumption, travel time, and PIC for choosing one fileable timetable. As travel time is monotonically increasing with increased headways, solution 1 has the least total travel times. As far as the power-off interruption is concerned, timetable 21 is the best choice. Taking solution 21 for example, we show the space-time diagram of a partial timetable (3rd block) in Fig. 9a, in which the different colors denote different trains running in both directions. The solid lines refer to the initial solution and the dashed lines refer to the optimized solution. Station RJ (O) (8-th station) is the power-off substation. All the accelerating phases are perfectly overlapped with the decelerating phase at the power-off station. To show more detail, we zoom in two accelerating train trajectories in Fig. 9b. In Fig. 9b, ‘X 6070’ corresponds to the time 6070 s on the x-axis and ‘Y 1.217e+04’ corresponds to the distance 12170 m on the y-axis.

If we extend the above partial timetable from solution 21 to cover the complete peak hours, the PICs are shown in Fig. 10. The blue and red dashed lines show the values of the magnifying headway solution and the extended solution, respectively. According to the results of the extended solution, the two highest PIC values of substations 5 and 9 are decreased, although values at the other three substations (7, 10, 11) are increased compared with the results of the magnifying headway solution. Taken together, the average PIC value calculated by the extended solution is decreased by 0.0054% compared with the result of the magnifying headway solution; moreover, the energy consumption is reduced by 2.10%, from $4.8409 \times 10^{10}$ J (13447 kWh) to $4.7391 \times 10^{10}$ J (13164 kWh); nevertheless, the total travel time is increased by 0.87%, from 9.0219 $\times 10^7$ s (26061 h) to 9.1008 $\times 10^7$ s (25280 h). The results show that the extended solution outperforms the one produced by the magnifying headway method.

For the optimality, we have compared the results calculated by a traditional algorithm (Non-Dominated Sorting Genetic Algorithm II) and our method (Fig. 11). The termination condition is that the frontier solutions are steady after five iterations (NSGA-II). For instance, with the same population solutions (purple stars), NSGA-II takes 3211.85 s to obtain the Pareto frontier with red lines after 40 iterations; our algorithm takes 2058.3 s to run 40 iterations and obtain the Pareto
frontier with black lines. We conclude that the two groups of solutions are all located at the same frontier since they cannot dominate each other. Comparing the running times of the two algorithms, our algorithm can save time by 1153.55 s (35.91%).

All in all, based on the above numerical analysis, the proposed heuristic algorithm can produce a high-quality solution for the tri-objective optimization problem. The above results demonstrate that our developed model and algorithm can capture many-to-many energy allocation among trains, decrease energy consumption, reduce the total travel time, and reduce the power interruption risks sufficiently.

VI. Conclusion

In this paper, we present a timetable scheduling model for reducing train energy consumption, passenger travel time, and potential power interruption at the substations considering many-to-many energy allocation in a metro system. If power interruption occurring at substations, the train operates is recovered based on an energy-regenerative supplementary strategy. To achieve an optimal tradeoff between the passenger travel time, energy consumption, and the potential power interruption, we suggest a tri-objective optimization model. With this model, we attempt to optimize the arrival, departure, and headway times simultaneously to generate a regular and cyclic timetable. To solve the problem, a heuristic algorithm is developed in combination with the headway and running-dwelling modifications.

The proposed model is tested by a numerical example to show the efficiency and effectiveness of the fast-scheduling method with consideration of the power interruption risks in a metro system. The total energy consumption is reduced by 7.40%. The average passenger travel time is decreased by 51.12%. The average maximum PIC is decreased from 0.85% to 0.48% compared with the currently used timetable. In general, the longer the running time on section, the less energy consumption. the smaller headway time, the shorter passenger travel time. However, to decrease or avoid the substation overloading risks, there is no obvious timetable to support the longer headway is positive or not. For safety concerns, we suggest that it is needed for enhancing the timetable evaluation combining electricity characteristics.

In this research domain, our future research will focus on the following aspects. First, we will use the real-time voltage values instead of the energy consumption representation. It may easily assess the power overloading risks since the voltage
fluctuation can be observed in the operations according to the installed electrical components. Second, with the energy-regenerative supplement strategy, it should be further discussed in more detailed situations such as the effects by train mass, accelerating rate fluctuations considering train running delays.

APPENDIX

Regular Timetable Constraints

Based on the regularity of timetables, the headway time in both directions is set as a fixed constant value. Whereas, as for cyclic property, headway times may change due to passenger fluctuations. The running times on different sections in the same direction are different. The dwelling times at different stations are different. After turning around, the train fleet symbols stay the same, but the turnaround index tr is updated as tr + 1. The timetable constraints are given as follows.

\[
H \leq H \left( t_{tr,k+1,i} - t_{tr,k,i} = \frac{\text{dep}}{tr,k+1,i} - \frac{\text{dep}}{tr,k,i} \right) \leq \hat{H},
\]

\[
1 \leq k \leq K - 1 \land 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A1.1) \]

\[
H \leq H \left( t_{tr,k+1,i} - t_{tr,k,i} = \frac{\text{arr}}{tr,k+1,i} - \frac{\text{arr}}{tr,k,i} \right) \leq \hat{H},
\]

\[
1 \leq k' \leq K' - 1 \land 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A1.2) \]

\[
\frac{\text{arr}}{tr+1,i} - \frac{\text{arr}}{tr,K,1} \geq H, \quad 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A2.1) \]

\[
\frac{\text{dep}}{tr+1,1,i} - \frac{\text{dep}}{tr,K,1} \geq H, \quad 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A2.2) \]

\[
t_{dwe} \leq \frac{\text{dep}}{tr,k,i} - \frac{\text{arr}}{tr,k,i} \leq \hat{t}_{dwe}, \quad 1 \leq k \leq K \land 0 \leq t_r \leq T
\]  
\[
(A3) \]

\[
T \leq t_{tr,k,i+1} - t_{tr,k,i} \leq \hat{T}, \quad i = N - 1 \land 1 \leq k \leq K \land 0 \leq t_r \leq T
\]  
\[
(A4) \]

\[
t_{tr+1,k,N} - t_{tr,k,N} = T_{\text{turn}}, \quad 1 \leq k \leq K \land 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A5) \]

\[
t_{\text{coa-dec}}(i) - t_{\text{arr}} - t_{\text{dep}}(i) = \sqrt{\frac{2R \cdot d(i+1)}{C} \cdot \left( \frac{t_{\text{arr}} - t_{\text{dep}}}{A} \right)^2},
\]

\[
i \leq N - 1 \land 1 \leq k \leq K \land 1 \leq t_r < t_r + 1 \leq T
\]  
\[
(A7) \]

The detailed processes can be found in (Yang et al. [36]).

Timetable Initialization and Rearrangement

Matrix Rearrangement

We adopt the method developed in Yang et al. [36] to generate a timetable population. A timetable matrix includes time and positional information for all operating trains in both directions.

\[
\mathbf{X}_{\text{acc}} = \begin{bmatrix}
B\text{acc} & E\text{acc} & \ldots & 0 & 0 \\
B_{tr,1,1} & E_{tr,1,1} & \ldots & B_{tr,1,N} & E_{tr,1,N} \\
0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
B_{tr,K,1} & E_{tr,K,1} & \ldots & B_{tr,K,N} & E_{tr,K,N} \\
0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & 0 \\
\end{bmatrix}
\]

where \( x_{tr,k,i} \) is the starting time and positional information of \( k \) accelerating from \( i \). \( x_{tr,k,i}^{E,\text{acc}} \) is the ending time and positional information. To relate with \( x_{tr,k,i}^{E,\text{acc}} \), the trains are not accelerating.
from the termini in the up-direction but needs to accelerate after train turned around. If \( tr = 0 \), \( k = 1 \) and \( i = 1 \), \( B_{1,1}^{B,acc} \) corresponds to \( x_{1,1}^{B,acc} \); if \( tr = 1 \), \( B_{1,1}^{B,acc} \) corresponds element \( x_{1,1}^{B,acc} \). The rest components can be obtained in the same way.

REFERENCES


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