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How to Realize Uniform Three-Dimensional Ellipsoidal Electron Bunches

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Uniform three-dimensional ellipsoidal distributions of charge are the ultimate goal in charged particle accelerator physics because of their linear internal force fields. Such bunches remain ellipsoidal with perfectly linear position-momentum phase space correlations in any linear transport system. We present a method, based on photoemission by radially shaped femtosecond laser pulses, to actually produce such bunches.

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Uniform three-dimensional (3D) ellipsoidal distributions continue to be the prime theoretical objects in systems governed by Poisson's equation:

$$\vec{\nabla}^2\Phi(\vec{r}) = -k\rho(\vec{r}), \tag{1}$$

where $\Phi(\vec{r})$ is, for example, the electrostatic potential due to a charge density distribution $\rho(\vec{r})$, in which case $k = 1/\epsilon_0$, or the gravitational potential due to a mass density distribution $\rho(\vec{r})$, in which case $k = 4\pi G$.

The importance of uniform 3D ellipsoidal distributions stems from the fact that they represent the *only* class of physically relevant and finite 3D distributions, whose self-field $\Phi(\vec{r})$ can be calculated analytically. Despite contemporary computational power and the deceivingly simple appearance of Eq. (1), it remains highly challenging to accurately calculate the fields $\Phi(\vec{r})$ in arbitrary distributions $\rho(\vec{r})$. In addition, uniform 3D ellipsoids are the *only* distributions whose internal force fields are linear functions of position. This gives rise to particularly simple dynamical behavior: a uniform ellipsoid under the influence of its self-field (either electrostatic or gravitational) will change its size and its shape, but it will remain a uniform ellipsoid with linear internal fields.

Uniform 3D ellipsoids are part of the paradigm in disciplines as diverse as astrophysics [1], hydrodynamics [2], and accelerator physics [3]. Mostly they represent highly idealized distributions, important for basic physical understanding, but remote from actual conditions. In accelerator physics uniform 3D ellipsoids are used nowadays to benchmark Poisson solvers [4] and to quickly estimate the effect of space-charge forces in beam lines [5].

In fact, because of their well-behaved linear internal fields, uniform 3D ellipsoids are the *ideal* particle distributions for controlled, high-brightness charged particle acceleration. (For a recent overview of high-brightness applications, see Ref. [6].) This was recognized already many years ago, but up to now no one seriously considered the possibility of actually realizing a hard-edged, uniform 3D ellipsoid of charge in free space. Several methods have been proposed to improve the linearity of the space-charge forces (e.g., Ref. [7]), but they all fail to set the correct 3D distribution as a whole. In this Letter we show that the ideal electron bunch with a true uniform 3D ellipsoidal distribution can be realized with established technology, by ultrashort pulsed-laser photoemission with an appropriate radial intensity profile.

Consider a uniform ellipsoidal charge distribution containing N electrons, whose confining surface is given by

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 + \left(\frac{z}{C}\right)^2 = 1. \tag{2}$$

Inside the ellipsoid, the electric field $\vec{E} = -\vec{\nabla}\Phi$ is a linear function of position $\vec{r} = (x, y, z)$:

$$\vec{E} = (E_x, E_y, E_z) = \frac{\rho_0}{\varepsilon_0}(M_x x, M_y y, M_z z), \tag{3}$$

where $\rho_0 = 3Ne/4\pi ABC$, and the form factors M_x, M_y, and M_z, with $M_x + M_y + M_z = 1$, depend only on the ratios A/B and B/C of the lengths of the primary axes of the ellipsoid [8]. For a spheroid ($A = B$)

$$M_z = \frac{1+\Gamma}{\Gamma^3}(\Gamma - \arctan\Gamma), \qquad M_x = M_y = \tfrac{1}{2}(1 - M_z) \tag{4}$$

with the eccentricity $\Gamma \equiv \sqrt{A^2/C^2 - 1}$ [8].

A uniform ellipsoidal bunch initially at rest will expand and, unless it has a perfectly spherical shape, be deformed due to space-charge forces. However, as one may easily show (see Ref. [9] for the gravitational case), its uniform ellipsoidal character will be preserved during the expansion. The internal space-charge forces will therefore remain linear, although the ratios of the lengths of the major axes, and therefore the form factors M_x, M_y, and M_z, will change. As a result, the velocities which the particles acquire due to space-charge forces will also remain linear functions of position. The 6D phase space distribution $f(\vec{r}, \vec{p})$, normalized to the total charge, is therefore given by [10]

$$\frac{f(\vec{r},\vec{p})}{\rho_0} = \Theta\left[1 - \left(\frac{x}{A}\right)^2 - \left(\frac{y}{B}\right)^2 - \left(\frac{z}{C}\right)^2\right]\delta(\vec{p} - \mathbf{D}\vec{r}), \tag{5}$$

where $\vec{p}$ is the particle momentum, $\Theta(x)$ the Heaviside step function, and $\delta(\vec{x})$ the 3D Dirac delta function. The 3×3 matrix $\mathbf{D}$ is diagonal for a purely space-charge-force driven expansion. An arbitrary linear 6D phase space transformation, i.e., any form of linear accelerator transport, will rotate and deform the ellipsoid, and add nondiagonal elements to $\mathbf{D}$, but the linear character of $f(\vec{r},\vec{p})$ will be maintained [10]. This makes a uniform ellipsoid the ideal bunch.

Our idea to create a uniform 3D ellipsoid by means of pulsed-laser photoemission was inspired by the observation from theoretical astrophysics that a uniform oblate spheroid [Eq. (2) with $A = B > C$] will collapse under its own weight into a flat disk, i.e., an oblate spheroid with the short axis C reduced to zero [9], whose density distribution is given by

$$\rho(r,z) = \sigma_0\sqrt{1 - (r/A)^2}\,\delta(z), \tag{6}$$

where σ_0 is the surface density at the center and $r = \sqrt{x^2 + y^2}$. This implies that if one is able to create a surface *charge* density with a "half-circle" radial profile, as given by Eq. (6), then the electrostatic repulsion will cause this flat distribution to evolve into a full-fledged, hard-edged, uniform 3D ellipsoidal bunch.

A possible way to create ultrathin sheets of charge is the combination of radio-frequency (rf) photoguns and femtosecond laser technology. The rf photoguns are compact rf accelerators, in which electron bunches with charges of up to 1 nC are created by pulsed-laser photoemission, and subsequently accelerated in fields of, typically, 100 MV/m [11]. These devices deliver the brightest relativistic electron beams currently available. The length of the shortest electron bunches that can be created is fundamentally limited by the time scale of the photoemission process itself, which is of the order of 10 fs for pure metals like copper. Recently we have implemented 30 fs photoemission laser pulses in an rf photogun [12], resulting in $\sim$10 nm thick sheets of charge at the cathode surface, 5 orders of magnitude smaller than the typical bunch radius of 1 mm. By shaping the transverse laser intensity profile, the desired initial condition (6) may thus be approached very closely.

It is not *a priori* clear that an ultrathin sheet of electrons with a radially shaped surface charge distribution produced in an rf photogun, will actually evolve into a true uniform 3D ellipsoid. First, the initial longitudinal density distribution is determined by the temporal intensity profile of the laser and the photoemission process and is therefore different from the hard-edged longitudinal distribution of the uniform ellipsoid into which it should evolve. Second, since the front part of the bunch is created earlier than the back part, the external acceleration field will induce potentially nonlinear velocity-position correlations, which may interfere with the desired evolution towards a uniform ellipsoid. Third, space-charge and image-charge forces during initiation may introduce additional undesired velocity-position correlations. These three issues are addressed below, using a simple but effective approximate description of bunch evolution in the so-called "pancake" regime [13], which accurately describes the first stage of acceleration when the rest-frame bunch length is much smaller than the bunch radius.

Let us replace the Dirac delta function in Eq. (6) by a realistic longitudinal density distribution function $\lambda(r,z)$ (in principle also dependent on r), which is symmetrical around $z = 0$ and normalized to unity, $\int \lambda(r,z)dz = 1$, but otherwise arbitrary. From Gauss's theorem it follows that a particle initially at position (r,z) will experience a longitudinal space-charge field given by

$$E_z = \frac{\sigma_0}{2\varepsilon_0}\sqrt{1 - (r/A)^2}\int_0^z \lambda(r,z')dz'. \tag{7}$$

While the bunch is in the pancake regime, E_z is approximately constant. Note that E_z is proportional to the *integrated* distribution, and therefore independent of the detailed shape of $\lambda(r,z)$. As a result, *the particle experiences the same acceleration as if it were part of an ideal uniform ellipsoidal distribution.* Consequently, the only effect of using a nonideal initial distribution $\lambda(r,z)$ is that the particles start out with an offset in position with respect to the ideal case. While still in the pancake regime, the expanding disk will therefore automatically assume a uniform ellipsoidal shape once its longitudinal size is sufficiently larger than the initial thickness (but still much smaller than the bunch radius). The deviation from the ideal uniform ellipsoid will be expressed only by "soft" edges: the density will not fall off to zero abruptly, but over a distance comparable to the initial thickness. As the bunch expands further, it will leave the pancake regime and evolve from there on as a full-fledged uniform 3D ellipsoid, independent of the precise shape of the initial longitudinal density distribution.

Let us now turn to the second issue: under what conditions can the acceleration-field-induced velocity-position correlations associated with the finite duration τ_l of the photoemission process be neglected? Let us assume that an N-electron bunch with radius A is created truly instantaneously ($\tau_l = 0$) at the cathode surface ($z = 0$), and is subsequently accelerated in the z direction by a uniform field E_0. Using the pancake regime description one may then derive straightforwardly that at position z the duration Δt of the bunch is given by [13]

$$\Delta t(\gamma) = \Delta t(\infty)\sqrt{\frac{\gamma - 1}{\gamma + 1}}, \tag{8}$$

where $\Delta t(\infty) = mc\sigma_0/e\varepsilon_0 E_0^2$ is the asymptotic bunch duration, $\gamma = 1 + eE_0 z/mc^2$ is the Lorentz factor, and $\sigma_0 = 3Ne/(2\pi A^2)$. Clearly, the photoemission process may be considered instantaneous if $\tau_l \ll \Delta t(\infty)$.

Finally, the characteristic space-charge (and thus image-charge) field strength at the cathode surface is given by σ_0/ε_0. This value should be much smaller than E_0 to ensure that any space-charge or image-charge related velocity-position correlations which might develop during initiation, remain negligible. The requirements may be summarized in the following condition:

$$\frac{eE_0\tau_l}{mc} \ll \frac{\sigma_0}{\varepsilon_0 E_0} \ll 1, \tag{9}$$

which states that (i) the maximum velocity acquired by the electrons during the photoemission process, expressed in units of c, should be much smaller than the ratio of the surface charge field and the acceleration field, and (ii) this ratio should be much smaller than unity. By substituting the realistic values $\tau_l = 30$ fs, $E_0 = 100$ MV/m, $Ne = 100$ pC, and $A = 1$ mm, we find $eE_0\tau_l/mc \approx 0.002$ and $\sigma_0/\varepsilon_0 E_0 \approx 0.05$, which seems to fulfill condition (9) quite reasonably.

In order to check the validity of our analysis, we have performed simulations with the above parameter set ($E_0 = 100$ MV/m, $Ne = 100$ pC, and $A = 1$ mm), using the General Particle Tracer (GPT) code [14]. In the simulations the electrons are ejected isotropically from the cathode surface with an energy of 0.4 eV. The photoemission quantum efficiency is assumed homogeneous across the cathode, and the laser pulse has a half-circle radial intensity profile [Eq. (6)]. The finite duration of the photoemission process is taken into account by initiating the particles sequentially in time, using a Gaussian temporal profile $\sim\exp(-t^2/2\sigma_t^2)$, with a full-width-at-half-maximum duration of $2.355\sigma_t = 30$ fs. Figure 1 shows the projection in the $x - z$ plane of the resulting particle distribution 50 ps after initiation (centroid position $z_c \approx$ 10 mm), corresponding to an electron energy of approximately 1 MeV. The bunch duration is 600 fs, corresponding to a peak current of 250 A. Clearly the distribution has evolved into an ellipsoidal shape. Also shown is a solid-line ellipsoidal contour, corresponding to the confining surface of the uniform ellipsoid which would result from the ideal initial condition (6). The contour has been calculated by direct numerical integration of the relativistic equations of motion of electrons at the extremal points of the ellipsoid, making use of Eq. (4). The GPT simulation predicts an ellipsoidal shape slightly more elongated than the solid-line contour, which we attribute to the finite value of τ_l. This is confirmed by the fact that the difference in expansion is of the order of $c\tau_l \approx 10$ μm.

From the particle cloud shown in Fig. 1 it is hard to judge to what extent it is truly a *uniform* ellipsoid. A much more stringent test of the supposedly linear character of its 6D phase space distribution is obtained by investigating its projection on $x - p_x$ transverse phase space. Integrating Eq. (5) over y, z, p_y, and p_z reveals that ideally the $x - p_x$ projection should be a straight line of finite length, through the origin. From Eq. (4) one may derive that inside an infinitely flat oblate spheroid $E_x = 3Nex/16\varepsilon_0 A^3$. In the pancake regime the following relation between x and p_x can then be obtained [13]:

$$\frac{p_x}{mc} = \frac{3Nex}{16\varepsilon_0 A^3 E_0}\log(\gamma + \sqrt{\gamma^2 - 1}). \tag{10}$$

Figure 2 shows the $x - p_x$ transverse phase space projection of the simulated bunch 50 ps after initiation, at an energy of approximately 1 MeV (red-dotted distribution). The distribution is characterized by a linear relationship, which agrees well with Eq. (10) (black solid line), predicted by pancake regime theory. The additional broadening of the simulated distribution is due to the finite velocities and the random photoemission angle at

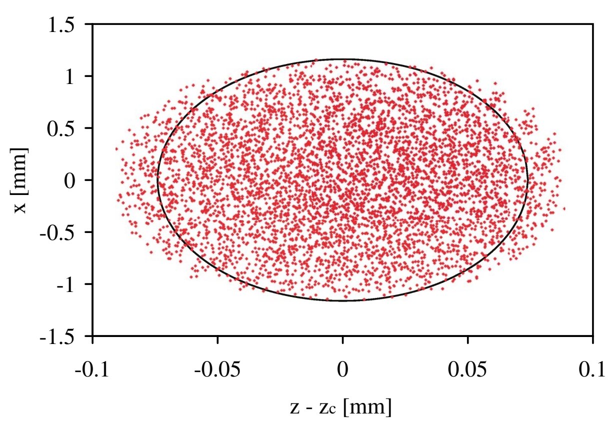

FIG. 1 (color). Bunch distribution in $x - z$ space 50 ps after initiation ($z_c \approx 10$ mm). Red dots: GPT simulation; solid line contour: ideal initiation.

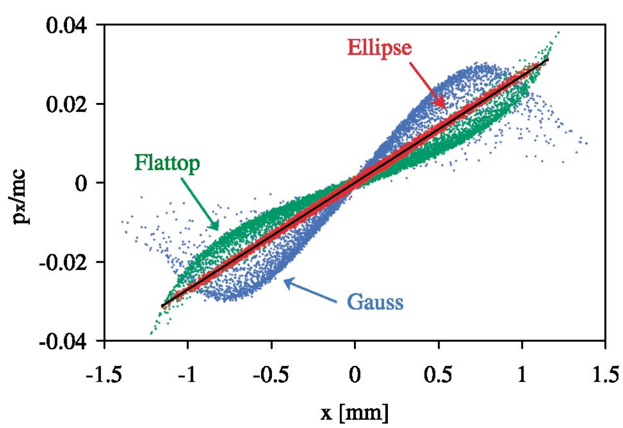

FIG. 2 (color). GPT simulations of the distribution in $x - p_x$ phase space 50 ps after initiation with an ellipsoidal (red dots), a flattop (green dots), and a Gaussian (blue dots) initial radial profile. Black solid line: pancake theory.

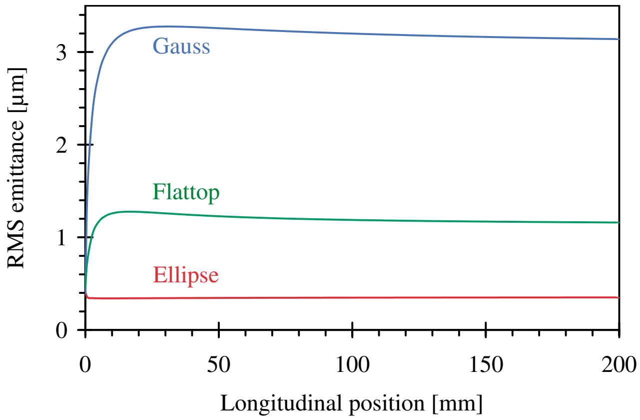

FIG. 3 (color). The rms emittance as a function of z for an ellipsoidal, a flattop, and a Gaussian initial radial profile.

initiation. The linear behavior of the ellipsoidal bunch is in sharp contrast with the results of similar simulations, also shown, which were initiated with the commonly used flattop (green dots) and Gaussian (blue dots) radial profile.

The quality of the $x - p_x$ transverse phase space distribution is usually expressed in terms of the normalized root-mean-square (rms) emittance $\epsilon_{n,x}$,

$$\epsilon_{n,x} = \frac{1}{mc}\sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle xp_x\rangle^2}, \tag{11}$$

where $\langle\,\rangle$ indicates averaging over the distribution. In Fig. 3 $\epsilon_{n,x}$ is plotted as a function of z for all three radial profiles. The rms emittances of the bunches initiated with either the flattop or the Gaussian radial profile rise sharply immediately after initiation, whereas for the ellipsoidal bunch $\epsilon_{n,x}$ remains constant at its initial value of approximately 0.4 μm. The zero emittance growth of the ellipsoidal bunch is the best possible result. This bunch remains ellipsoidal with an asymptotic bunch duration of 750 fs, corresponding to a peak current of 200 A. Equally impressive results are obtained when a realistic (time dependent and nonuniform) rf acceleration field is used with magnetic focusing. In this Letter we restrict ourselves to a constant and uniform acceleration field, because it illustrates the basic physics most clearly and allows comparison with a simple analytical model.

Proper radial shaping is clearly of vital importance for obtaining the highest possible beam quality. Although the desired radial shaping of the 266 nm, 30 fs laser pulses, required for ultrafast photoemission from copper, is not trivial, and the photoemission quantum efficiency may be nonuniform across the cathode surface, there are no fundamental obstacles. Moreover, the GPT simulations indicate that the evolution into a uniform 3D ellipsoid is quite robust and not extremely sensitive to the exact initial radial profile. We therefore claim that practical realization is feasible.

In summary, we conclude that the ideal uniform 3D ellipsoidal electron bunch can be realized in practice, using state-of-the-art accelerators and commercially available femtosecond laser technology. GPT simulations show that the $x - p_x$ phase space projection is characterized by perfectly linear behavior, resulting in zero emittance growth. Interestingly, this suggests perfectly linear behavior in $z - p_z$ phase space as well, and therefore highly efficient α-magnet bunch compression. Multi-kA electron beams with sub-μm rms emittance may thus be created, suited for the most demanding high-brightness applications [6].

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