A multi-item periodic replenishment policy with full truckloads

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A multi-item periodic replenishment policy with full truckloads

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Abstract

In this paper we consider a stochastic multi item inventory problem. A retailer sells multiple products with stochastic demand and is replenished periodically from a supplier with ample stock. At each order instant it is decided which product to order and how much to order. For the delivery of the products trucks with a finite capacity are available. The dispatched trucks arrive at the retailer after a constant leadtime and with each truck fixed shipping costs are charged independent on the number of units shipped. Additionally, linear holding and backorder costs at the end of a review period are considered. Since fixed transportation costs are high coordination of orders and full truckload shipments can benefit from economies of scale.

We propose a dynamic order-up-to policy where initial ordersizes can be reduced as well as enlarged to create full truckloads. We show how to compute the policy parameters and in a detailed numerical study we compare our policy with a lower bound and an uncoordinated periodic replenishment policy. An excellent cost performance of the proposed policy can be observed when average time between two shipments is not too large and fixed shipping costs are high.

keywords: inventory, stochastic modelling, coordinated replenishments, joint setup costs

1 Introduction

In many practical situations a reduction of supply chain costs could be obtained by a close coordination of inventory and transportation management. However, transportation managers aim on a high truck utilization while inventory managers try to lower inventory costs resulting in many small replenishment orders. But with increasing transportation costs purchases in truckload quantities become more attractive. Moreover, transportation capacity reservation contracts (Serel et al. (2001)) reserve a fixed transportation capacity in each period and the shipper has to pay a fixed amount each period, independent on use and utilization of the reserved capacity. Therefore, in order to benefit from economies of scale its is important to coordinate replenishments for multiple products and to allocate the available transportation capacity in a smart way.

While there is a large body of literature on stochastic inventory models only a small part of it is devoted on multi-item stochastic inventory control, where opportunities exist for

coordination of replenishments (see for an overview on the stochastic joint replenishment problem Aksoy and Erenguc (1987) or Goyal and Satir (1989)). In these models it is in general assumed that a fixed charge (major setup costs) has to be paid whenever an order is placed, independent on the number of items ordered. For each item included in the replenishment order an additional fee (minor setup costs) is charged. Several coordinated replenishment policies for stochastic demands are suggested and analyzed in the literature.

A well-known continuous joint replenishment policy is the can-order strategy, first proposed by Balintfy (1964), which works as follows. Whenever the inventory position of an individual item is dropping below its reorder level, a replenishment order is placed and all items with their inventory positions below the can-order levels are also included in the replenishment order. This paper is followed by a series of papers (see for example Silver (1974), Silver (1981), Federgruen et al. (1984), van Eijs (1994), Melchior (2002)) discussing the challenging problem how to compute the optimal policy parameters.

A periodic coordinated replenishment policy is studied in Atkins and Iyogun (1988) and Fung and Lau (2001). The coordination is achieved by the choice of the review periods of the items, which are all integer multiples of a base period. This means that ordering of a specific item is not necessarily allowed at each review point but only at specified order moments. Another approach allows ordering of each item in principle at each review point, but controls the ordering decisions by a reorderlevel. An item is only included in the replenishment order when the inventory position of the item is smaller than its reorder level (see Viswanathan (1997)). The analysis of a can-order policy under periodic review can be found in Johanson and Melchior (2003).

All the replenishment policies mentioned above have in common that they only charge fixed costs for a replenishment order and that they ignore capacity restrictions on the order volume. As a consequence the total order volume can be 1% of the truck capacity as well as 300% of truckcapacity which results in reality in three full truck loads and therefore in different fixed joint order costs compared to order volumes smaller than a full truckload.

Joint replenishment policies taking into account capacity restrictions on the total order volume are limited. Under continuous review of the inventory position a QS-policy (Renberg and Planche (1967)) can deal with the capacity restriction, since an order is triggered whenever the total order volume is equal to a full truckload. For continuous as well as for periodic review Miltenburg (1985) is proposing an algorithm to allocate the total order volume to each item, assuming the total order volume to be known. His allocation rule is not based on costs but aims on maximizing the expected elapsed time before the next order takes place. His analysis is based on the assumption that the inventory position of an individual item can be modelled as a diffusion process. An application of this allocation rule is studied in Carlson and Miltenburg (1988) where replenishment orders are triggered based on the expected service level.

Another approach taking into account transportation capacity can be found in van Eijs (1994b). He is studying a periodic replenishment policy where items can be ordered at every review instant. The initial ordersizes are determined based on an uncoordinated periodic replenishment policy and then in a second step he allows for enlarging the initial order quantities in order to benefit from economies of scale. He is using a Markovian

decision model to approximate the expected extra ordering and holding costs for enlarging the initial order quantities and is comparing these extra costs with the approximated extra shipping costs. In case of positive expected cost savings the initial ordersize is enlarged to a full container load. While the numerical examples reveal large cost improvements compared to the uncoordinated periodic replenishment policy, this policy can still lead to small total replenishment volumes resulting in low container utilization.

A periodic policy which avoids low truck utilization is studied in Cachon (2001), where orders are not shipped when a specified truck utilization cannot be reached. Then the orders have to wait until the next replenishment instant where the specified truck utilization is met. In contrast to the policy proposed by van Eijs (1994b), where initial orders are only allowed to be enlarged, the total order volume to be shipped at a specific review instant can only be reduced under the policy studied in Cachon (2001). He shows how to compute policy parameters when demand can be modelled as a Poisson process and when the decision about which products to be shipped is done using a first-come-first-serve rule.

Since both approaches mentioned above can result in considerable cost savings we combine in this paper both ideas and come up with a periodic joint replenishment policy where at each review instant all items can be ordered and the initial order quantities can be enlarged as well as reduced. At each review instant based on the aggregate inventory position it is first decided about how many full truckloads to be shipped and then in a second step the available transportation capacity is allocated among all products. In contrast to Miltenburg (1985) and Cachon (2001) we propose an allocation which is based on the stochastic characteristics of the demands and on cost considerations. While the overall objective is the minimization of long-term average transportation, inventory and backorder costs, the policy parameters are computed using a myopic approach. In our cost model we assume linear holding and backorder costs and a fixed delivery charge for each truckload. In our numerical study we compare our suggested policy with a lower bound and an uncoordinated periodic replenishment policy and we describe the characteristics of the situations where our proposed policy has an excellent cost performance. As long as the average time between two shipments is not too large the total cost of the proposed policy are closed to the lower bound and large cost savings compared to the uncoordinated replenishment policy can be achieved.

The rest of the paper is organized as follows. In section 2 a description of the studied problem is given. In section 3 we come up with a dynamic replenishment policy and we show how to compute the policy parameters at each review instant. In section 4 we compare our policy with a lower bound and an uncoordinated periodic replenishment policy. Finally, a summary and conclusions are given in section 5.

2 Problem description

In this paper we consider a single stock location as a node in a decomposed logistical distribution network. At this warehouse a family of N items is kept on stock in order to satisfy customer demand, which is assumed to be stochastic. In opposite to a lot of inventory models we do not consider time as a continuum. Rather in our model time is divided into periods of fixed length (e.g. days). We assume that inventory is reviewed every

period and that the length of such a review period is given and therefore an exogenous variable in our model. Demand for item i in period t is modelled as a random variable $D_{i,t}$ with a continuous cumulative distribution function $F_{D_{i,t}}$ and we assume that unsatisfied demand is backordered. Further, demand for item i in subsequent periods are assumed to be independent identically distributed and demand processes for the various items are independent of each other.

The following replenishment policy is used at the warehouse. At the end of each review period the inventory position, which is defined as stock on-hand minus backorders plus outstanding orders, of each item is reviewed and based on the inventory status of the item an order is placed. At every review point the warehouse may place an order for one or more items at an external supplier with ample supply. This means that there is always enough inventory at the supplier to fulfill the replenishment request.

The warehouse is replenished from the supplier via trucks. It is assumed that each truck has a fixed capacity V and there is no limit on the number of trucks available. Regardless of which items are included and how many items are shipped a fixed shipping cost A is charged for each dispatched truck. A replenishment order of a warehouse results in an arrival of a shipment exactly L time periods later. Besides the shipping costs also holding and backorder costs are charged. We assume linear holding and backorder costs where product i is charged h_i per unit inventory and b_i per unit backordered at the end of a period. We do not consider pipeline inventory costs since they can not be influenced in this setting.

The objective of the inventory manager is twofold. On the one hand he aims to minimize transportation costs. Therefore, if fixed shipping costs are high, a large utilization of the truck is economic beneficially and the manager aims to have full truckloads. On the other hand holding and backorder costs should be minimized. But these two aims can be contradictory. With flexible transportation capacity and many small shipments low inventory and backorders costs can be obtained but transportation costs may be large. On the other hand, a few big shipments reduce transportation costs but may lead to larger inventory and backorder costs due to less supply flexibility. So we are interested in a replenishment policy which minimizes the long-run average total costs per period, consisting of transportation, inventory and backorder costs, which is given as follows:

$$\lim_{t \rightarrow \infty} \frac{1}{t} \sum_{s=1}^t \left(AE[M_s] + \sum_{i=1}^N (h_i E[I_{i,s}^+] + b_i E[I_{i,s}^-]) \right) \quad (1)$$

where $I_{i,s}^+$ denotes the stock on hand of item i in period s , $I_{i,s}^-$ denotes the backorders of item i in period s and M_s denotes the number of trucks dispatched in period s .

In this paper we do not aim to find the optimal policy structure but we propose a class of dynamic order-up-to policies and show how to compute the policy parameters. Moreover, we investigate the performance of this heuristic. Before we describe our heuristic to compute replenishment orders, we summarize the used notation:

N :	Number of different items
V :	Capacity of the truck
A :	Fixed costs for dispatching one truck
M_t :	Number of trucks dispatched in period t
$D_i(T)$:	Demand of item i during T periods
D_0 :	Total demand volume during a review period, $D_0 := \sum_{i=1}^N D_i(1)w_i$
$Y_{i,t}$:	Inventory position of item i at review point t , before the decision is made
$I_{i,t}$:	Net-stock (Stock on-hand minus backorders) of item i at the end of review period t
w_i :	The volume of item i
h_i :	Holding cost parameter for item i
b_i :	Backorder cost parameter for item i
L :	Replenishment leadtime
$E[X]$:	Expectation of a random variable X
$VAR[X]$:	Variance of a random variable X
F_X :	Cumulative distribution function of a random variable X
c_X^2 :	Squared coefficient of variation of a random variable X ($c_X^2 = \frac{VAR[X]}{E[X]^2}$) .
X^+ :	$\max(0, X)$
X^- :	$\max(0, -X)$

3 The replenishment policy

We have observed situations where fixed costs for a shipment are quite high compared to inventory costs. Then it is reasonable to aim at a high truck utilization. Therefore, instead of minimizing transportation costs, we adapt our mathematical model and require full truck-loads to benefit from economies of scale. In the following we propose a periodic order-up-to policy which leads to full truck loads. In contrast to a general base stock policy, where constant order-up-to levels are used, dynamic order-up-to levels are required to create full truck loads. Moreover, the heuristic is composed of two steps. First the number of trucks to be used is determined and in a second step the available space is allocated to the different products. We also decompose the problem for the computation of the policy parameters.

3.1 The decisions

In order to determine the necessary number of trucks the aggregate inventory position, measured in volume, is considered. This means, we have to multiply the inventory position $Y_{i,t}$ of item i , which is defined as the stock-on hand minus backorders plus outstanding orders and measured in number of items, with the volume w_i . An adapted order-up-to policy is used where S_0 denotes the order-up-to level for the aggregate inventory position.

Then, at each review instant t the number of trucks to be dispatched is computed using

$$M_t := \left\lceil \frac{S_0 - \sum_{i=1}^N Y_{i,t} w_i}{V} \right\rceil \quad (2)$$

where $\lceil x \rceil$ denotes the closest integer to x . Note that this rule does not only allow the delay of shipments in case of a low truck utilization (see for example Cachon (2001)), which is similar to reducing total order volume at this review instant, but it also allows for enlarging ordersizes to get full truck loads in case the initial order already leads to a truck utilization larger than 50 %. An ordering strategy only allowing enlarging orders is studied in van Eijs (1994).

After determined how many trucks to be used, the available space is allocated to the different products. This decision is again done using an order up to policy where the replenishmentsize $O_{i,t}$ for item i at time t is computed as follows:

$$O_{i,t} := \max\{0, \hat{S}_{i,t} - Y_{i,t}\} \quad (3)$$

This means that at each review point the replenishment policy for N items is described by $N + 1$ parameters $(S_0, \hat{S}_{1,t}, \hat{S}_{2,t}, \dots, \hat{S}_{N,t})$ where the parameters have to be chosen such that full truck loads are obtained. Note that the parameter S_0 is constant over all review instants while the other parameters will change. How the parameters can be computed is described below.

3.2 Computation of the policy parameters

In order to determine a numerical value for the order-up-to level S_0 we ignore the full truck-load restriction and rely on the unconstrained problem where only holding and backorder costs are considered. This means we minimize the following function

$$\begin{aligned} G(S_1, S_2, \dots, S_N) \\ = \sum_{i=1}^N h_i E[(S_i - D_i(L+1))^+] + b_i E[(D_i(L+1) - S_i)^+] \end{aligned} \quad (4)$$

It is well known that the optimal values S_i^* are given as a solution of the following newsboy equation

$$F_{D_i(L+1)}(S_i^*) = \frac{b_i}{b_i + h_i} \quad (5)$$

In order to compute a numerical value of the order-up-to level S_0 we suggest to use the following formula:

$$S_0 := \sum_{i=1}^N S_i^* w_i \quad (6)$$

This means that the total order volume should be close to the optimal order volume in case of no transportation costs and capacity restrictions.

For the computation of the numerical values of the order-up-to levels $(\hat{S}_{1,t}, \hat{S}_{2,t}, \dots, \hat{S}_{N,t})$ we use a myopic approach. This means that for the replenishment decision in period t we try to minimize the average holding and backorder costs in period $t + L + 1$ and we ignore the effect of the replenishment decision on the subsequent periods. The mathematical formulation of the problem, which has to be solved at every review instant t , is given as follows:

$$\min_{(\hat{S}_{1,t}, \hat{S}_{2,t}, \dots, \hat{S}_{N,t})} \sum_{i=1}^N h_i E[(\hat{S}_{i,t} - D_i(L+1))^+] + b_i E[(D_i(L+1) - \hat{S}_{i,t})^+] \quad (7)$$

s.t.

$$\sum_{i=1}^N (\hat{S}_{i,t} - Y_{i,t}) w_i = M_t V \quad (8)$$

$$\hat{S}_{i,t} - I_{i,t} \geq 0 \quad (9)$$

where restriction (8) guarantees full truck loads and condition (9) reflects the requirement of positive order sizes.

We first make the assumption that negative orders are allowed. However, there is no meaningful interpretation of negative orders from a practical point of view. Therefore, we will later adapt the solution such that only positive order sizes are obtained. But for the moment we ignore the non-negativity restriction (9) of the orders which allows us to use Lagrange multipliers to formulate an unconstrained optimization problem and get necessary conditions for an optimal solution. The Lagrange function is then defined as

$$\begin{aligned} \mathcal{L}(\hat{S}_{1,t}, \hat{S}_{2,t}, \dots, \hat{S}_{N,t}) \\ := \sum_{i=1}^N \left((h_i + p_i) \int_0^{\hat{S}_{i,t}} (\hat{S}_{i,t} - x) f_{D(L+1)}(x) dx + p_i (E[D_i(L+1)] - \hat{S}_{i,t}) \right) \\ - \lambda \left(\sum_{i=1}^N (\hat{S}_{i,t} - Y_{i,t}) w_i - M_t V \right) \end{aligned} \quad (10)$$

Using some algebra we obtain the following necessary conditions for optimal values for $(\hat{S}_{1,t}^*, \hat{S}_{2,t}^*, \dots, \hat{S}_{N,t}^*)$

$$F_{D(L+1)}(\hat{S}_{i,t}^*) = \frac{p_i + \lambda w_i}{h_i + p_i}, \quad i = 1, 2, \dots, N \quad (11)$$

Based on these newsboy equations together with the condition (8) the allocation of available space can be reduced to the determination of the optimal value of the lagrange multiplier λ . A simple iterative algorithm using a bisection method, the newsboy equations (11) and condition (8), can be implemented and the optimal order-up-to levels $(\hat{S}_{1,t}^*, \hat{S}_{2,t}^*, \dots, \hat{S}_{N,t}^*)$ can easily be computed at every review point.

Using the order-up-to levels obtained with this algorithm may lead to negative ordersizes which makes it infeasible to apply the obtained solution. In such a situation the items with negative ordersizes are not ordered at all (ordersizes are set equal to zero) and for the remaining items the optimization problem given by (7) and (8) is solved again. This can be repeated until all ordersizes are positive.

3.3 Benchmarks

Since the optimal policy of the multi item replenishment problem with truck capacity restrictions is not known we have proposed in the previous paragraph a policy and a method to compute the policy parameters which seems to be reasonable. In order to get an idea about the quality of the proposed policy and heuristic we compare the average cost with a lower bound. The lower bound consist of a lower bound for the inventory cost and the lower bound for the transportation costs.

It is obvious that on average the number of trucks to be used in a period must be equal to the average total demandvolume in a period divided by the volume of the truck. Further, the average holding and backorder costs can never be lower than the minimal costs obtained without taking into account transportation costs and capacity restrictions. Therefore, a lower bound for the total average costs in a period is given as

$$LOW = \sum_{i=1}^N h_i E[(S_i^* - D_i(L+1))^+] + b_i E[(D_i(L+1) - S_i^*)^+] + A \frac{E[D_0]}{V} \quad (12)$$

where the values S_i^* are defined by the newsboy equation (5).

Besides comparing our policy with a lower bound we additionally compare our policy with a situation where only inventory and backorder costs are minimized without taking into account transportation issues. This corresponds with many organizational structures in companies where two different persons are responsible for transportation and inventory management. Moreover, it is a common policy in literature and in reality. For example, in Cachon (2001) this policy is called "full service periodic review policy" because each product's order is shipped and he mentioned a large grocery retailer in the Netherlands using such a policy. The average costs under such a policy are given as

$$\begin{aligned} & \sum_{i=1}^N \left(h_i E[(S_i^* - D_i(L+1))^+] + b_i E[(D_i(L+1) - S_i^*)^+] \right) \\ & + A \sum_{j=1}^{\infty} j \cdot P\left((j-1)V < D_0 \leq jV\right) \end{aligned} \quad (13)$$

4 Numerical Study

In this section we investigate the performance of the proposed replenishment policy using discrete event simulation. In order to get accurate point estimates for the relevant performance measures we did 10 simulation runs with different seeds, and depending on the variability of the demand between 5000 and 10000 periods are simulated. In the simulation demand per review period is mixed Erlang distributed and we have taken into account that usually only for some items large demand sizes can be observed (large accounts). The ratio between small and large accounts is approximately 3:1. The average demand sizes for the large accounts are uniformly drawn on the interval (50, 80) while for the small accounts average demand sizes are uniformly drawn on the interval (10, 40). All experiments with the same number of items are done with the same average demand sizes. We further have set the factor w_i equal to one for all items.

We simulated the replenishment policy as proposed in section 3 and as a performance measure we computed the total average costs (consisting of inventory, backorder and transportation costs) per period C_{FT} . Additionally, the average costs C_P (13) for the uncoordinated periodic replenishment policy (full service policy) are computed and the lower bound LOW (12). We are interested in the relative deviation of the costs of our proposed policy to the lower bound defined as

$$\Delta_{LOW} := \frac{C_{FT} - LOW}{LOW} \cdot 100\% \quad (14)$$

and in the relative savings compared to the full service policy

$$\Delta_P := \frac{C_P - C_{FT}}{C_P} \cdot 100\%. \quad (15)$$

For the numerical study we have chosen the following parameter values:

$$\begin{aligned} N &\in \{4, 8, 16, 32\} \\ c_D &\in \{0.4, 1.0, 1.6\} \\ L &\in \{1, 4, 8\} \\ p &\in \{9, 19, 99\} \\ V &\in \{0.5 \cdot E[D_0]E[D_0], 2E[D_0], 4E[D_0]\} \\ A &\in \{250, 500, 1000, 2000\} \end{aligned}$$

resulting in a total number of experiments equal to 1728. In the following we will summarize the most important results.

4.1 Comparison with the lower bound

For all our examples we could observe that the relative deviation related to the lower bound is decreasing with increasing leadtime. Therefore, we only illustrate the worst-case scenario $L = 1$. Further, we number our examples in the following way

example	c_D	p	example	c_D	p	example	c_D	p
1	0.4	9	4	1.0	9	7	1.6	9
2	0.4	19	5	1.0	19	8	1.6	19
3	0.4	99	6	1.0	99	9	1.6	99

We first illustrate the impact of the truck capacity V on the relative deviation to the lower bound. If on average a truck is only dispatched every 4 periods then the relative deviation between the lower bound and the average costs can be extremely large (400%). Thereby the largest values are obtained for small demand variability and large backorder costs. We therefore only depicted the results of the other examples in Figure 1. It can be seen that the relative deviation is increasing with increasing truck capacity, since the average time between shipments is increasing, resulting in larger backorder and holding costs. But as long as the average time between two shipments is not too large the relative deviation is very small and the average costs are close to the lower bound which means that the proposed policy is performing close to the optimal one.

Figure 1: Impact of the truck capacity

It can further be seen that for small demand variability the difference between the lower bound and the average costs is larger. Moreover, in that case the deviation is increasing with increasing backorder costs. For large demand variability the impact of the backorder costs on the deviation is less and we can observe decreasing deviations with increasing backorder costs. This can also be seen in Figure 2 where the impact of the number of items is illustrated, under the condition that the ratio between average demand during a review period and the truck capacity is fixed.

The relative deviation of the lower bound is increasing with increasing number of items, but when the truck capacity itself is fixed then the relative deviation of the lower bound is decreasing with increasing number of items, since the average time between shipments is decreasing (see Figure 3).

The relative deviation is decreasing with increasing shipment costs A , as illustrated in Figure 4, because then the influence of the first term in (13), which is responsible for the cost difference, is decreasing. Again it can be observed that for large demand variability our proposed policy is close to the optimal one.

Figure 2: Impact of the number of items

Figure 3: Impact of the number of items

4.2 Comparison with the full service policy

In the following we compare the proposed replenishment policy with an uncoordinated periodic replenishment policy, also called full service policy. We first investigate the impact of the truck capacity on the relative savings. Again we only illustrate the case $L = 1$ and for Figure 5 we have additionally chosen $N = 8$ and $A = 1000$.

It can be seen that for large average time between two truck departures our proposed policy is outperformed by the full service policy. This is in line with the findings in Cachon (2001). In such a situation our proposed policy is not flexible enough and much more backorders can be observed. But if on average at least one truck is dispatched at each review instant then large cost improvements can be obtained using the proposed policy. The transportation cost savings are much higher than the additional inventory and backorder costs compared to the full service policy.

In Figure 6 we illustrate the impact of the number of items on the relative cost savings while the ratio between period demand and truck capacity is fixed and in Figure 7 we fix

Figure 4: Impact of the shipment costs

Figure 5: Impact of the truck capacity

the truck capacity itself. With increasing number of items inventory and backorder costs are increasing and therefore, cost improvements are getting smaller.

Besides the effect of increasing inventory and backorder costs for larger number of items we also can observe in Figure 7 the effect that for a small number of items our proposed policy is not flexible enough, since average time between shipments is too large. However, the cost parameters have the largest influence on the costsavings. It is obvious that with increasing shipping costs the cost savings are increasing (see Figure 8).

5 Summary and Conclusions

In this paper we proposed a dynamic order-up-to policy resulting in full truck loads. In contrast to the existing literature, our policy allows for reducing total order volume as well as enlarging total order volume. Moreover, allocation of the truck capacity is based on item and cost characteristics and not on a first-come-first serve rule.

Figure 6: Impact of the number of items

Figure 7: Impact of the number of items

We have shown how to compute policy parameters and in a numerical study we have compared the total average costs of our- proposed replenishment policy with a lower bound and additional with the total costs of an uncoordinated periodic replenishment policy without taking into account truck capacity restriction. Our numerical study reveals that large cost savings compared to the periodic replenishment policy can be obtained if the average time between shipments is not too large and the shipping costs are large. Moreover, in these situations the proposed policy is closed to the optimal one.

Since under the just mentioned conditions the performance of this policy is excellent a direction of future research will be including line-item costs and other restrictions related to handling of replenishment orders.

Figure 8: Impact of the number of items

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