Counterflow pulse-tube refrigerators

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Chapter 1

Introduction

Cryocoolers, which have no moving parts at low temperatures, are studied intensively since they have many advantages over existing cooling technologies: there is no need of a liquid-helium bath cryostat, the cost can be low, the reliability is high, the level of mechanical vibrations is low, and the magnetic interferences are small. In recent years there have been impressive developments in cryocoolers, especially in the field of Pulse-Tube Refrigerators (PTRs). A short review on the history of PTRs is given. Some applications of PTRs are briefly discussed. This research focuses on a special type of PTR, the so-called CounterFlow Pulse-Tube Refrigerator (CFPTR). The difference between CFPTRs and PTRs and the possible advantages of CFPTRs are described. Finally, a general description of the contents of this thesis is given.

1.1 Brief history of pulse-tube refrigerators

In 1963 Gifford and Longsworth introduced a new type of cryocooler, the pulse-tube refrigerator [1]. Compared to Stirling or Gifford-McMahon coolers [2] the construction of PTRs is simple and the reliability is high. The main distinction is the absence of moving parts in the low-temperature region resulting in less mechanical vibrations and electromagnetic interferences. The type of cooler invented in 1963 is nowadays known as the basic pulse-tube refrigerator. The operation is based on surface heat pumping [3], which is caused by heat exchange between the working fluid, usually helium, and the pulse-tube walls. The lowest temperature reached is 124 K with a single-stage and 79 K with a two-stage system [4].

The main breakthrough came in 1984 when Mikulin et al. added an orifice and a buffer volume to the basic PTR and in this way introduced the so-called orifice pulse-tube refrigerator [5]. A schematic diagram is given in figure 1.1. The system consists of a compressor, rotary valve, regenerator, heat exchanger, pulse tube, another heat
1. Introduction

Figure 1.1: Schematic diagram of the orifice-type pulse-tube refrigerator. The cold heat exchanger is indicated with CHX and the hot heat exchanger with HHX. The high- and low-pressure sides of the compressor and the high and low temperatures are also indicated.

exchanger, orifice, and buffer. These components will be described in more detail in chapter 2. Due to the presence of the orifice and the buffer the gas, flowing back through the CHX to the regenerator during the expansion phase, has a lower temperature than when it left the regenerator during the compression phase. This will be explained in more detail in chapter 2.

Already two years after the introduction of the buffer and the orifice, Radebaugh et al. [6] reached 60 K with this type of PTR. After that the development of the PTR went really fast. In 1990 Zhu et al. [7] introduced the double-inlet pulse-tube refrigerator, where the hot end of the pulse tube is connected to the hot end of the regenerator. This orifice is not indicated in figure 1.1. It is indicated in figure 2.1. Gas can flow directly to the hot end of the pulse tube, without passing the regenerator. The flow through the regenerator is smaller. This reduces the dissipation in the regenerator, which effects the cooling power in a positive way. Also a closed circuit is formed by the regenerator, the pulse tube, and the double-inlet orifice. In this circuit an unwanted net mass flow (the so-called DC-flow) can exist. To suppress this DC-flow Wang et al. [8] introduced the minor orifice in 1998. In the meantime, in 1994, Matsubara et al. [9] reached temperatures below the technologically important value of 4.2 K. In 1999 the Low Temperature Group of the Eindhoven University of Technology [10] reached a temperature below the lambda point of helium (2.17 K). The 1.78 K reached was the record lowest temperature until 2004, when the Giessen University developed a PTR that reached 1.27 K [11]. A lot of research is still going on to improve the performance of PTRs and explore the possibilities.
1. Introduction

1.2 Applications

With no moving parts in the low-temperature region of the cooler, PTRs are reliable, have a long lifetime, a low cost, and operate without interference with the object to be cooled. Therefore, the PTR is used in many applications where cryocoolers are needed. Generally speaking, PTRs can be used in situations where small liquid-helium baths keep the temperature around 4 K. For instance, MRI systems use a superconducting magnet, surrounded by liquid-helium. Pulse-tube refrigerators can cool the heat shields used to reduce helium boil-off. However, it is also possible to reach zero boil-off by means of a PTR. In that case the PTR recondenses all evaporated helium. The next step is to avoid the liquid-helium bath at all and cool the magnet directly by a PTR. This would make the MRI systems much smaller and cheaper, because no helium is consumed [12]. Also in applications with other superconductors (for example SQUIDs) PTRs are used to reach the desired temperatures [13].

Another major application of PTRs is the cooling of various detectors for space, military, and commercial purposes (like infrared detectors). Also for cryopumping as used in semiconductor industry for high and clean vacuum PTRs can be used. The PTR can also be used in storage of liquid gases, even helium [14].

1.3 Counterflow pulse-tube refrigerators

In all existing PTRs the regenerator is a vital component in which heat is alternatively stored and released. Therefore, the regenerator material should have a high heat capacity. In practice the regenerator is complicated, large, heavy, and expensive. It also contributes significantly to the losses in the cooler. In this research regenerators are avoided by using two identical pulse-tube refrigerators operating in opposite phase. The two regenerators are replaced by one counterflow heat exchanger. The complete system is called CounterFlow Pulse-Tube Refrigerator (CFPTR). The schematic diagrams of the three CFPTRs, used in this research, are given in figure 1.2. Figure 1.2a and b are single-stage coolers. Figure 1.2a has the rotary valve at room temperature and therefore an oscillating flow in the heat exchanger. This system is called CFPTR1. Figure 1.2b is CFPTR2, with the rotary valve at low temperature and steady flow in the heat exchanger. CFPTR3 (given in figure 1.2c) is a hybrid system that consists of a CFPTR precooled by a conventional PTR. At high temperatures it uses regenerators and at low temperatures heat exchangers. The three systems will be described in more detail in chapter 2.

Basically the concept of counterflow pulse-tube refrigerators was introduced by Matsubara [15], who proposed to replace the low-temperature part of the regenerator
1. Introduction

Figure 1.2: Schematic diagrams of the three counterflow pulse-tube refrigerators considered in this thesis. The rotary valve is indicated with RV, the counterflow heat exchanger with CFHX, the pulse tubes with t, the orifices with O, and the buffers with B. a: CFPTR1, a single stage CFPTR with oscillating flow in the heat exchanger. b: CFPTR2, a single stage CFPTR with steady flow in the heat exchanger. c: CFPTR3, a hybrid system with oscillating flow in the heat exchanger and a precooling heat exchanger PHX.
by a heat exchanger. In this research, we replaced the low-temperature regenerator, but we also built systems with no regenerators at all. The University of Giessen has done some preliminary research [16] on a CF PTR of the type as given in figure 1.2a. Due to the high temperatures, the oscillating flow in the heat exchanger, and the geometry of the heat exchanger, the performance of their system was almost completely based on the regenerative effect of the walls of the heat exchanger instead of the heat exchange effect between the two gas flows.

If a simple heat exchanger, e.g. made of tubes with diameters of a few millimeters and lengths of a few meters could be used in a CF PTR, the weight would be significantly lower than that of conventional regenerators. Furthermore, the thermal response would be faster and the valve losses, resulting from the void volume, would be smaller. Another advantage of the CF PTR is that the buffer volumes can be avoided as will be discussed in section 2.2. In this way, the total volume and weight of the system is reduced.

1.4 Introduction to this thesis

The main objective of this research has been to answer the question whether heat exchangers can be advantageous over regenerators. The main emphasis in this thesis is on the performance of counterflow pulse-tube refrigerators, both from fundamental and experimental points of view. The basic principles of PTRs in general and CF PTRs in particular are treated in chapter 2.

In the theoretical analysis of the pulse-tube refrigerator it is usually assumed that the working fluid is an ideal gas. However, the behavior of real systems depends on the real gas properties. The influence of real gases as the working fluid on the coefficient of performance of ideal PTRs is given in chapter 3.

The main component of a CF PTR is the heat exchanger. The performance of a heat exchanger is characterized by a combination of heat conduction, heat exchange, and flow losses. In chapter 4 flow measurements in steady and oscillating flow are compared to empirical relations describing the flow losses in different geometries at different velocities.

PTRs with an optimized regenerator and with an optimized heat exchanger are compared in a fundamental way. In order to optimize the regenerator or the heat exchanger we treat their performances by calculating the entropy production rates from the four relevant irreversible processes. This comparison is described in chapter 5.

Chapter 6 is dedicated to an analytical treatment of CF PTR2. The results of the model are compared with measurements and predictions are given for other geometries.
1. Introduction

and operating systems.

The two subsystems of a CFPTR are operated by opposite-phase pressure oscillations. In GM-type PTRs, as will be discussed in chapter 2, rotary valves are used to periodically connect the high- and low-pressures sides of the compressor. In this research, two different types of rotary valves have been used. Both are designed to have balanced forces on the rotor. Chapter 7 is dedicated to the description of the so-called 'no-contact' and the 'balanced contact' rotary valves.

Measurements are done with the three different CFPTRs, as given in figure 1.2. The descriptions and results of CFPTR1 and CFPTR2 are given in chapter 8. The measurements and the results of CFPTR3 are described in chapter 9.
Bibliography


Chapter 2

Basic concept of pulse-tube refrigeration

In this chapter the basic concept of pulse-tube refrigeration is treated. The operation principle is explained. The concept and the main components of the counterflow pulse-tube refrigerator are discussed. Finally, the energy balance is described.

2.1 Operation principle of pulse-tube refrigerators

Depending on the generation of the pressure oscillations two types of PTRs can be distinguished. In the Stirling-type PTR, pressure oscillations are created by the movement of a piston. In the Gifford-McMahon (GM)-type PTR, a compressor produces high and low pressures continuously. A rotary valve (or a set of switching valves) is used to generate pressure oscillations in the pulse tube. In this research GM-type PTRs are used. They operate at a low frequency (1-5 Hz), which makes it easier to reach lower temperatures than at higher frequencies [1]. Another reason for using GM-type PTRs in this investigation is that the performance of heat exchangers (pressure and heat exchange losses) is studied for steady flow most of the time. With the low frequencies of the GM-type PTR, steady-flow assumptions can be used in the analysis.

Now we will describe the main components of the GM-type PTR, as schematically given in figure 2.1. The system is filled with helium at an average pressure of typically 20 bar. The cold part of the system is placed in a vacuum chamber. The compressor has two buffer volumes: $V_H$ at high pressure and $V_L$ at low pressure. The rotary valve (RV) periodically connects the high- and low-pressure sides of the compressor ($p_H$ and $p_L$ respectively) to the system, generating pressure oscillations. Three heat exchangers are used: the compressor aftercooler (AC), the cold heat exchanger (CHX), and the hot heat exchanger (HHX). In the aftercooler the compression heat is released to room
2. Basic concept of pulse-tube refrigeration

Figure 2.1: Schematic representation of a double-inlet minor-orifice GM-type pulse-tube refrigerator. The compressor aftercooler is indicated with AC, the cold heat exchanger with CHX, and the hot heat exchanger with HHX. The orifices O, D, and M are flow resistances. Room temperature is indicated with $T_H$, the lowest temperature is $T_L$. The other symbols are explained in the text.

temperature. From the cold heat exchanger, heat is extracted from the application and is rejected at the hot heat exchanger. The cold heat exchanger is the coldest spot of the PTR. The pulse tube, which is a tube filled with nothing else than the working fluid, is positioned between the CHX and the HHX. To flatten the velocity profile in the pulse tube, we use flow straighteners at both ends of the pulse tube. Before the gas enters the pulse tube it is cooled by the regenerator. Subsequently, the regenerator should warm up the gas when it returns to the compressor. Therefore, the regenerator is filled with porous material with a high heat capacity. The flow in the pulse tube is influenced by three orifices (O, D, and M). These orifices are adjustable flow resistances. Valves $E_h$, $E_l$, and the minor orifice, are used to compensate for DC-flow (which will be explained later on). The buffer has a large volume which is typically ten times larger than the volume of the pulse tube. The pressure fluctuations in the buffer are one order of magnitude smaller than in the pulse tube.

2.1.1 Basic cooling concept

The cooling principle of PTRs is explained following small gas parcels as they move from the regenerator through heat exchanger CHX to the pulse tube and back. To
simplify the explanation several assumptions are made. The heat exchangers are ideal, resulting in a constant temperature and perfect heat exchange. There is a one-dimensional flow without mixing or turbulence. The processes in the pulse tube are adiabatic. In particular, there is no heat exchange from the gas to the wall of the pulse tube and no heat conduction.

The pressure in the pulse tube is considered to be cyclic with a shape according to figure 2.2. The pressure oscillation is divided in four steps. The orifice is assumed to be closed during steps I and III, and open during steps II and IV. Step I is the compression. The system is connected to the high-pressure side of the compressor. The pressure increases from the low pressure $p_l$ to the high pressure $p_h$. During step II the pressure remains constant at $p_h$. Step III is the expansion. The system is connected to the low-pressure side of the compressor. The pressure decreases from $p_h$ to $p_l$. In step IV the pressure remains constant at $p_l$. After step IV the cycle starts again.

The temperatures and positions of a particular gas parcel during the cycle are given in figure 2.3. The cycle starts with the gas parcel in point a, which is inside the regenerator.

Step I: from a via b to c. The orifice is closed. The gas parcel moves to the pulse tube due to compression. The temperature of the gas parcel remains constant as long as it is inside the regenerator and the cold heat exchanger (a to b). When the gas parcel is in the pulse tube the compression is adiabatic (b to c) and the temperature rises.

Step II: from c to d. The orifice is open. The pressure in the pulse tube is higher than in the buffer, so gas moves to the buffer. In order to keep the pressure constant

Figure 2.2: Idealized pressure in the pulse tube as a function of time, divided in four steps. The symbols a to f are explained in the text.
2. Basic concept of pulse-tube refrigeration

Figure 2.3: Temperatures of a gas parcel as a function of the positions. Step I is the compression and step III the expansion. The displacements during steps II and IV are caused by flow through the orifice. In a the cycle starts. In points b and f the gas parcel is at the interface of the pulse tube and the CHX.

at $p_h$ gas flows in the pulse tube from the regenerator. The pressure is constant, so the temperature of the gas element is constant as well.

Step III: from d to e. The orifice is closed again. The gas is expanded adiabatically. It cools and moves back in the direction of the regenerator.

Step IV: from e via f to a. The orifice is open again. Now the pressure in the pulse tube is lower than in the buffer, so the gas continues to move towards the regenerator. From e to f the pressure is constant, so the temperature of the gas parcel is constant as well. The gas parcel enters the cold heat exchanger CHX with a low temperature $T_l < T_L$. It absorbs heat from the heat exchanger and cooling takes place. The heat taken from the heat exchanger constitutes the cooling power. Finally, the gas element moves back to its original position a.

2.2 Concept of counterflow pulse-tube refrigeration

In this section, the basic concept of counterflow pulse-tube refrigeration is described. The operation principle of the counterflow heat exchanger is discussed. The different phenomena that determine the performance of the heat exchanger are given.

2.2.1 Design principle

The counterflow pulse-tube refrigerator is a special type of PTR. Here, we will only discuss the type of CFPTR as given in figure 1.2a. CFPTR2 can be described in the
same way. The differences between CFPTR1 and CFPTR2 will be discussed in section 2.2.4. Figure 2.4 gives a schematic representation. The system consists of two identical subsystems, labeled 1 and 2, working in opposite phase. Both subsystems consist of a pulse tube with a cold and a hot heat exchanger, an orifice, and a buffer as usual. The heat, normally stored and released in the regenerator material, is now transported from one subsystem to the other in the CounterFlow Heat eXchanger (CFHX). The compressor (not shown in the figure) generates a high pressure \( p_h \) and a low pressure \( p_l \). During half of the period the rotary valve connects subsystem 1 to \( p_h \) while \( p_l \) is connected to subsystem 2. Half a period later it connects subsystem 1 to \( p_l \) and subsystem 2 to \( p_h \) etc. In figure 2.4 the flow directions are given by arrows. The flows in the two subsystems are opposite at all times.

The flows to the buffers are also always opposite, so the buffers can be omitted by making a direct connection between the hot ends of the pulse tubes via an orifice. This reduces the total volume of the system. For most measurements in this research the buffers are not omitted, because the pressures in the buffers give valuable information on system characteristics (e.g. flow, velocity).

### 2.2.2 Operation principle counterflow heat exchanger

Consider as the counterflow heat exchanger, a tube-in-tube heat exchanger placed in vacuum. The geometry and the radial temperature profile in the heat exchanger are given in figure 2.5. Two extreme temperature profiles are given qualitatively. The arrows indicate the corresponding gas flow directions.

In general the thermal penetration depth is defined as

\[
\delta_T = \sqrt{\frac{\kappa}{\pi \nu \rho c}},
\]

with \( \kappa \) the thermal conductivity, \( \nu \) the frequency, \( \rho \) the density, and \( c \) the specific heat
2. Basic concept of pulse-tube refrigeration

Figure 2.5: Schematic representation of the tube-in-tube heat exchanger (figure a). Figure b gives the temperature profiles. The continuous curve gives the temperature profile during one half of the cycle. The dashed curve gives the temperature profile during the other half of the cycle. The arrows indicate the corresponding gas flow directions.

Figure 2.6: Thermal penetration depths of copper, stainless steel, and helium at 20 bar as functions of the temperatures at a frequency of 1 Hz.

capacity of the material. For copper, stainless steel, and helium at 20 bar the thermal penetration depths are given as functions of temperatures in figure 2.6 for $\nu=1$ Hz. In figure 2.5 we assumed that the inner diameter of the tube $d_1$ is much smaller than the thermal penetration depth of the gas $\delta_{Tg}$

$$ d_1 \ll \delta_{Tg}. $$

(2.2)

The thermal penetration depth of the gas has the same order of magnitude as the viscous penetration depth of the gas $\delta_{vg}$, so also

$$ d_1 \ll \delta_{vg}. $$

(2.3)
This means we have a well-developed flow. In figure 2.5 the situation is drawn where the wall thickness is much smaller than the thermal penetration depth of the wall \( \delta_{T_w} \).

From figure 2.5 it can be seen that the direction of heat flow changes sign every half cycle, but that the temperature of the inner wall is practically constant. On the other hand the temperature of the outer wall changes. To show the effects of these phenomena on the performance of the heat exchanger, we will look at two extreme situations. If \( \delta_{T_w} \) is much smaller than the wall thickness, the two gases are thermally isolated from each other and the heat exchange is poor. Snap shots of the temperature profiles in this situation are given in figure 2.7a for the inner wall and figure 2.7b for the outer wall. If the wall thickness is much smaller than the thermal penetration depth

\[
d_2 - d_1 \ll \delta_{T_w},
\]

the temperature profiles look like figure 2.7c and 2.7d. So, the amount of heat exchange between the two gas flows depends on the ratio between the thickness of the wall and its thermal penetration depth.

### 2.2.3 Performance heat exchanger

Due to the complexity of the time-dependent flow in the system, it is very difficult to derive simple analytical relations to describe the heat-exchange phenomena. We will give here a rough estimation to get a feeling for the main parameters determining the heat exchange. As described in the previous section the wall allows heat exchange if condition (2.4) is satisfied. However, this is not sufficient to get good heat exchange.
2. Basic concept of pulse-tube refrigeration

Naturally, the amount of heat exchange also depends on the gas properties. We will look at an oscillating fully developed gas flow in a tube. There can only be heat exchange if, in principle, the wall is able to follow the temperature of the gas, so if the temperature profile of the wall varies like in figure 2.7d. We assume that the temperatures of the wall material and the gas have some effective value, which oscillates with amplitudes $T_{wa}$ and $T_{ga}$ respectively. If $d_2 - d_1 \ll \delta_{Tw}$, the temperature of the material is practically homogeneous and the effective temperature will be equal to the real temperature.

Due to energy conservation the heat-exchange rate between the gas and the wall over a length $dl$ is equal to the heat transported by the gas to the heat exchanger in the same length. So we can write

$$\frac{\pi}{4} (d_2^2 - d_1^2) \rho_w c_w dl \frac{dT_w}{dt} = \pi d_1 dl \frac{N_u \kappa_g}{d_1} (T_g - T_w), \quad (2.5)$$

with $\rho_w$ the density of the wall material, $N_u$ the Nusselt number, and $c_w$ the specific heat of the wall material. This can be simplified to

$$T_g = T_w + \frac{\rho_w c_w}{4N_u \kappa_g} (d_2^2 - d_1^2) \frac{dT_w}{dt}. \quad (2.6)$$

Introducing the harmonic approximation

$$T_w = T_{wa} \cos \omega t, \quad (2.7)$$

with

$$\omega = 2\pi \nu, \quad (2.8)$$

we can write for equation (2.6)

$$T_g = T_{wa} \cos \omega t - \frac{\pi \nu \rho_w c_w}{2N_u \kappa_g} (d_2^2 - d_1^2) T_{wa} \sin \omega t. \quad (2.9)$$

Introducing

$$k = \frac{\pi \nu \rho_w c_w}{2N_u \kappa_g} (d_2^2 - d_1^2), \quad (2.10)$$

we get

$$T_{ga} = T_{wa} \sqrt{1 + k^2}. \quad (2.11)$$

The ratio of the amplitudes $T_{wa}$ and $T_{ga}$ is written as

$$\tau = \frac{T_{wa}}{T_{ga}} = \frac{1}{\sqrt{1 + k^2}}. \quad (2.12)$$

If the values of $\tau$ are close to 1, which means $k \ll 1$, the temperature of the wall can oscillate together with the temperature of the gas. As a result heat exchange can take place.
2. Basic concept of pulse-tube refrigeration

As an example we give in figure 2.8 the calculated temperature dependences of \( \tau \) for the tube-in-tube heat exchanger, as described in section 8.2.3, for laminar and turbulent flow. The Nusselt numbers are calculated using the equations (5.42) and (5.54). The frequency is 1 Hz. Using typical values for this research, we can write the condition for \( k \) to have good heat exchange as

\[
(d_2^2 - d_1^2) \ll 0.01 \delta_{T_w}^2.
\]

If condition (2.13) is satisfied, \( \tau \) will be close to 1, and good heat exchange is possible. If \( \tau \ll 1 \), the heat exchange is poor.

![Figure 2.8: The parameter \( \tau \) for laminar and turbulent flow as functions of the temperature for a tube-in-tube heat exchanger for \( \nu=1 \) Hz.](image)

2.2.4 This research

To find out the best performance of the CFPTR and to find out how the heat-exchange and regenerative effect influence each other, we decided to study the true regenerative effect, the true heat-exchange effect, and a combination of both, experimentally. To study pure heat exchange, the system should be operated in a situation where the values of \( \tau \) are close to 1. From figure 2.8 it can be seen that this takes place with turbulent flow at low temperatures. Another possibility to impose pure heat exchange, is by using steady flow instead of AC flow in the heat exchanger.

In this research we used three different CFPTRs to study the influence of the heat exchange and the regenerative effect separately and combined. Figure 2.9a (CFPTR1) represents a system which operates with an oscillating flow in the heat exchanger. The performance of the heat exchanger is based on a combination of the regenerative and the heat-exchange effect. We do not use double inlet and minor orifices in this system. Figure 2.9b shows CFPTR2 with the rotary valve at low temperatures. The flow in
2. Basic concept of pulse-tube refrigeration

Figure 2.9: Schematic diagram of the three different CFPTRs used in this research. The rotary valve is indicated with RV, the regenerators with Reg, the counterflow heat exchanger with CFHX, the tubes with t, the orifices with O, the buffers with B, and the precooling heat exchanger with PHX. Figure a shows CFPTR1, a single-stage PTR with oscillating flow in the heat exchanger. The rotary valve is at room temperature. Figure b gives CFPTR2, a single-stage PTR with steady flow in the heat exchanger. The rotary valve is at low temperatures. Figure c represents CFPTR3, a hybrid system with oscillating flow in the heat exchanger. The upper part is the CFPTR; the low part is a single-stage orifice-type PTR.
the heat exchanger is steady and its performance is based on the counterflow heat exchange only. In this system we do not use double inlet and minor orifices as well. Figure 2.9c gives CFPTR3. This is a hybrid system. Although not shown, we do use double inlet and minor orifices here. The CFPTR is precooled by a PTR, so the heat exchanger works in a low temperature region. With turbulent flow its performance is mainly based on heat exchange if $T<70\text{ K}$ even with oscillating flow. For laminar flow this happens if $T<30\text{ K}$ (see figure 2.8).

2.3 Main components

In this section, the main components of the CFPTR and their characteristics are discussed.

2.3.1 Compressor and rotary valve

The compressor produces high and low pressures continuously. The rotary valve connects the cooling unit periodically to the high- and low-pressure sides of the compressor. Together they create the pressure oscillations in the subsystems. Two types of rotary valves (the 'no-contact' and the 'balanced contact' rotary valve) have been used in this research. They are described in chapter 7.

2.3.2 Heat exchanger

The performance of the counterflow heat exchanger is determined by the pressure and the heat exchange losses. The pressure losses are described in chapter 4. The rate of heat exchange is expressed in the efficiency.

Efficiency

In this section, we will introduce some concepts which are commonly used in the description of heat exchangers. Consider a heat exchanger that consists of two copper tubes, making perfect thermal and mechanical contact e.g. due to a soldering connection (see figure 2.10). The inner diameter of the tubes is indicated with $d_1$, the outer diameter with $d_2$. The length of the heat exchanger is $L_x$. Both tubes carry a molar flow $n$. The temperatures of the gas flows are defined in figure 2.10. The rate of heat exchange is hampered by heat flow resistances $R$. The heat transfer rate $\dot{Q}_e$ is defined as [3]

$$\dot{Q}_e = \frac{\Delta T}{R},$$

(2.14)
2. Basic concept of pulse-tube refrigeration

Figure 2.10: Schematic diagram of a counterflow heat exchanger. The arrows give the flow directions.

with

\[ \Delta T = T_{hi} - T_{co} = T_{ho} - T_{ci}. \]  \hspace{1cm} (2.15)

The heat flow resistance from the gas to the copper \( R_{gc} \) and vice versa \( R_{cg} \) are much larger than the thermal resistances in the copper. Using the dimensions of figure 2.10, they are described by

\[ R_{gc} = R_{cg} = \frac{1}{\alpha_h A_1}, \]  \hspace{1cm} (2.16)

with \( \alpha_h \) the heat exchange coefficient and

\[ A_1 = \pi d_1 L_x. \]  \hspace{1cm} (2.17)

The total resistance for the heat flow from one gas to the other is

\[ R_t = 2R_{gc}. \]  \hspace{1cm} (2.18)

The total resistance is needed to calculate the Number of Transfer Units (NTU), which is the ratio [3]

\[ NTU \equiv \frac{1}{R_t \hat{n} C_p}, \]  \hspace{1cm} (2.19)

with \( \hat{n} \) the flow in the heat exchanger, so

\[ NTU = \frac{T_{hi} - T_{ho}}{T_{hi} - T_{co}}. \]  \hspace{1cm} (2.20)

Conventionally, the efficiency \( \varepsilon \) of a heat exchanger is defined as the actual heat transfer rate compared to the maximum heat transfer rate, which is calculated with the maximum possible temperature difference for fixed \( T_{hi} \) and \( T_{ci} \). As we use the same gas in both heat exchanger channels we can write (see figure 2.10)
2. Basic concept of pulse-tube refrigeration

\[ \varepsilon = \frac{T_{hi} - T_{ho}}{T_{hi} - T_{co}}. \] (2.21)

For a counterflow heat exchanger the efficiency can be expressed in NTU like [4]

\[ \varepsilon = \frac{NTU}{1 + NTU}. \] (2.22)

Although equation (2.22) is called the efficiency of the heat exchanger, it does not describe the whole performance of the heat exchanger. First of all, a good heat exchanger has an \( \varepsilon \) value close to 1, but a difference of 0.01 in \( \varepsilon \) results in a big difference in the system performance. Secondly, the total performance of the heat exchanger is determined by a combination of heat exchange and pressure losses. If \( \varepsilon \) is close to 1 but the pressure losses over the heat exchanger are large, the overall performance of the heat exchanger is bad.

Length

With the energy balance an expression for the length of the heat exchanger can be derived. The heat-exchange rate between the gas and the heat exchanger \( \dot{Q}_e \) and the heat transported by the gas to the heat exchanger \( \dot{Q}_f \) [1] are used. The heat-exchange rate between the gas and the heat exchanger is given by a combination of the equations (2.14) and (2.16)

\[ \dot{Q}_e = \frac{\alpha h A_1}{2} \Delta T = \frac{\alpha h A_1}{2} (T_{hi} - T_{co}) . \] (2.23)

The heat transported by the gas into the heat exchanger is given by

\[ \dot{Q}_f = n C_p (T_{hi} - T_{ho}) . \] (2.24)

With the energy balance the length of the heat exchanger can be calculated

\[ L_x = \frac{2nC_p}{\alpha h \pi d_1} \frac{T_{hi} - T_{ho}}{T_{hi} - T_{co}} = \frac{2nC_p}{\alpha h \pi d_1} NTU . \] (2.25)

2.3.3 Flow straighteners

The pulse tube is basically just an empty tube except for the working fluid. However, the pulse tube is connected to the rest of the system with tubes with small cross sections. Therefore, turbulent eddies are formed on the sides of the main jet (see e.g. figure 2.11) [5]. The effects of turbulence can be reduced by flow straighteners. The position of the flow straighteners in the pulse tube can be seen in figure 2.1.
For flow straighteners we used plates of sintered phosphor bronze particles. The thickness of these plates is calculated by requiring that the difference in dynamic pressures in the connection tube (with cross section $A_c$) and the pulse tube (with cross section $A_t$) has to be much smaller than the pressure drop over the flow straightener. Both are much smaller than the pressure of the gas. The dynamic pressure difference is given by

$$\Delta p_B = \frac{1}{2} \rho^* V^2 \left[ \left( \frac{1}{A_c} \right)^2 - \left( \frac{1}{A_t} \right)^2 \right],$$

(2.26)

with $^*V$ the volume flow of the gas and $\rho$ the density. The pressure drop over the flow straightener is given by [6]

$$\Delta p_{fs} = \frac{V s}{A_t} \left[ \frac{\eta}{\delta^2} + \frac{\rho^* V}{\delta A_t} \right].$$

(2.27)

In this equation

$$\delta = 0.13 d_p,$$

(2.28)

with $d_p$ the pore size, $\eta$ the viscosity of the gas, and $s$ the thickness of the sintered plate. Requiring, $\Delta p_B \ll \Delta p_{fs}$ and assuming that $A_c \ll A_t$, we can rewrite equations (2.26) and (2.27) into

$$s \gg \frac{1}{2} \frac{\rho^* V \delta^2}{\eta A_t + \rho^* V \delta} \left( \frac{A_t}{A_c} \right)^2.$$

(2.29)

Since

$$\frac{\rho^* V \delta^2}{\rho^* V \delta} > \frac{\rho^* V \delta^2}{\eta A_t + \rho^* V \delta},$$

(2.30)

condition (2.29) will certainly be satisfied if

$$s \gg \frac{\delta}{2} \left( \frac{A_t}{A_c} \right)^2.$$

(2.31)
2. Basic concept of pulse-tube refrigeration

2.3.4 First orifice

The first orifice is indicated in figure 2.1 with 'O'. The first orifice and the buffer contribute to the gas flow at the hot end of the pulse tube. Due to this gas flow, steps II and IV of the cooling cycle (see figure 2.3) can happen. Therefore, all the gas parcels, that have contact with the cold heat exchanger and enter the pulse tube during the cycle, contribute to the cooling power.

2.3.5 Double-inlet orifice

The double-inlet orifice connects the hot end of the pulse tube to the hot end of the regenerator. It is indicated with 'D' in figure 2.1. Gas can enter the pulse tube also via the double-inlet orifice. The reason why this extra orifice improves the performance is not trivial [7]. Due to the double inlet the flow rate through the regenerator is smaller. This is effecting the cooling power negatively as it reduces the amount of gas passing the cold heat exchanger. The dissipation in the extra orifice also has a negative influence on the total system performance. On the other hand, the dissipation in the regenerator is reduced. Whether the net cooling power increases or decreases by opening the bypass orifice, depends on the difference in reduction of cooling power and loss terms in the regenerator.

2.3.6 Minor orifice

In a PTR with a double-inlet orifice a closed circuit is formed by the regenerator, the pulse tube and the orifice D. Any asymmetrical flow resistance in this circuit leads to an internal circulation of the working fluid. This net flow is called DC-flow. The DC-flow decreases the performance of the PTR most of the time, since it partly shorts-circuits the cold and the hot end of the system. However, sometimes a net DC-flow seems to improve the performance.

The DC-flow is defined positive if the gas flows from the regenerator via the cold heat exchanger to the pulse tube. The way to reduce DC-flow, is to inject an opposite flow using so-called minor orifices as shown in figure 2.1. A positive DC-flow can (partly) be compensated by connecting the hot end of the pulse tube to the high-pressure side of the compressor via E_h. Negative DC-flow can be compensated using E_l. Orifice M is used as flow resistance that can be regulated to optimize the system performance.
2. Basic concept of pulse-tube refrigeration

2.4 Losses

In this section we discuss the losses in CFPRTR2, using the energy balance over a system as indicated in figure 2.12. The energy balance is used to quantify the influences of all processes taking place. A schematic diagram of CFPRTR2 with the different parasitic energy flows is given in figure 2.12. The rotary valve is drawn in more detail. The motor is at room temperature. It is connected with the rotor of the rotary valve by an axis. These three parts are placed in one housing, filled with helium gas.

![Schematic diagram of CFPRTR2 with the rotary valve at low temperature.](image)

Figure 2.12: Schematic diagram of CFPRTR2 with the rotary valve at low temperature. The different parasitic energy flows are indicated. The flow direction of the gas in the rotary valve housing is given by the dotted arrow. The dotted rectangle gives the thermodynamic system under investigation.

The energy balance is given for the system in the dotted box in figure 2.12 as

\[
\dot{H}_t = \dot{Q}_{Ln} + \dot{H}_{hx} + \dot{H}_{non} + P + \dot{Q}_{hsv} + \dot{Q}_{hct} + \dot{Q}_{hs}. \tag{2.32}
\]

The ideal cooling power \(\dot{H}_t\) is equal to the net cooling power of the system \(\dot{Q}_{Ln}\) plus all the losses. The different energy flows of equation (2.32) are defined as:

- the ideal cooling power of the system \(\dot{Q}_{Ln}\), which is equal to \(\dot{H}_t\) the average enthalpy flow in the pulse tube,
- the average enthalpy flow in the heat exchanger for an ideal gas \(\dot{H}_{hx}\),
- the contribution to the enthalpy flow due to the nonideality of the \(^4\)He \(\dot{H}_{non}\),
- two losses in the rotary valve: dissipated motor input power \(P\) and shuttle heat leak \(\dot{Q}_{hsv}\),
- the heat conduction in the pulse tubes \(\dot{Q}_{hct}\),
- the heat-shuttle effect in the pulse tubes \(\dot{Q}_{hs}\).
2. Basic concept of pulse-tube refrigeration

We will now describe the different energy flows in more detail.

2.4.1 Cooling power

To calculate the cooling power we first look at the pulse tube with its two heat exchangers. Applying the first law of thermodynamics to this system [7] we find that the cooling power $\dot{Q}_L$ is equal to the enthalpy flow in the pulse tube. Assuming small pressure amplitudes compared to the absolute pressures, the total cooling power of a pulse-tube refrigerator is defined as [7]

$$\dot{Q}_L = 2\nu \int V_o^* dp_t. \tag{2.33}$$

The factor two in equation (2.33) is due to the fact that we are dealing with a double-tube system. In this equation $p_t$ is the pressure in the pulse tube and $V_o^*$ the volume flow of the gas at the hot end of the pulse tube

$$V_o^* = C_1 p_1, \tag{2.34}$$

with $p_1$ the pressure amplitude in the pulse tube. The flow at the hot end of the pulse tube is used because of the first law of thermodynamics as described above. In this equation $C_1$ is the conductance of the orifice, which is related to a characteristic parameter $\alpha$ according to the relation [7]

$$C_1 = \frac{1}{\alpha} \frac{C_V V_t \omega}{p_0}. \tag{2.35}$$

In this equation $\omega$ is the angular frequency, $V_t$ is the volume of a pulse tube, $p_0$ the average pressure in the pulse tube, $C_V$ the specific heat at constant volume, and $C_p$ the specific heat at constant pressure. For a square-wave form $\alpha$ is defined as

$$\alpha = \omega p_1 \frac{V_t}{V_B} \left( \frac{dp_B}{dt} \right)^{-1}, \tag{2.36}$$

with $p_B$ the pressure in the buffer, $V_B$ the volume of the buffer, and $t$ the time. For a sinusoidal-wave form $\alpha$ is given by

$$\alpha = \frac{p_1 V_t}{p_{BA} V_B}, \tag{2.37}$$

with $p_{BA}$ the amplitude of the pressure in the buffer.

Equation (2.33) can be simplified to

$$\dot{Q}_L = 2c C_1 p_1^2. \tag{2.38}$$

The factor $c$ describes the shape of the pressure wave. If the pressure wave is sinusoidal $c=0.5$; for a block shape $c=1$. The factor 2 comes from the double-tube system.
2. Basic concept of pulse-tube refrigeration

2.4.2 Enthalpy flow heat exchanger

The average enthalpy flow in the heat exchanger for an ideal gas can be calculated from

\[ \dot{H}_{\text{hx}} = C_p \dot{n} \Delta T, \]  

(2.39)

with \( \dot{n} \) the molar flow and \( \Delta T \) the temperature difference between the two gas flows in the heat exchanger. To obtain the total enthalpy flow in the heat exchanger the effect of the nonideality of the working fluid must be taken into account. The enthalpy flow due to the nonideality of \( ^4 \text{He} \) can be calculated from

\[ \dot{H}_{\text{non}} = 2 \left( 1 - T \alpha_v \right) V_m \dot{n} p_1, \]  

(2.40)

with \( \alpha_v \) the volumetric thermal expansion coefficient and \( V_m \) the molar volume. Helium shows nonideal gas effects already at room temperature [8].

2.4.3 Rotary valve

There are two different heat loads due to the rotary valve. First the dissipated motor input power \( P \)

\[ P = VI, \]  

(2.41)

with \( V \) the voltage and \( I \) the current of the motor. In experiments with the rotary valve at room temperature the motor housing feels hot. However, if the valve is in the cold, the motor housing is cool. So, almost all the motor input power is dissipated in the low-temperature region of the system, presumably due to friction of the moving surfaces.

The second energy flow in the rotary valve is due to the pressure oscillations in the valve. They are caused by a leak in the rotary valve. Therefore, the amount of gas in the valve housing at room temperature is changing

\[ \dot{n} = \frac{\Delta p_v V_v}{RT_H}. \]  

(2.42)

This causes a heat-shuttle effect, which can be estimated from

\[ \dot{Q}_{\text{hsv}} = C_p V_v \frac{\Delta p_v V_v}{RT_H} \left( T_H - T_v \right), \]  

(2.43)

with \( V_v \) the volume of the housing of the rotary valve, \( \Delta p_v \) the peak-to-peak value of the pressure oscillation in the rotary valve, and \( T_v \) the temperature of the rotary valve.

The heat conduction of the shaft of the rotary valve can be neglected.
2. Basic concept of pulse-tube refrigeration

2.4.4 Heat conduction in the pulse tube

The average heat leak due to conduction through the pulse tube walls can be calculated from

\[ \dot{Q}_{\text{hct}} = 2A_{tw} \left( \kappa_w \frac{dT}{dx} \right)_{L}, \]  

where the factor 2 comes from the double-tube system. The wall cross-section area is given with \( A_{tw} \) and the length coordinate is \( x \).

2.4.5 Heat-shuttle effect in the pulse tube

There are no exact formulae for the heat-shuttle effect in the pulse tube, because of the complexity of the phenomenon. To estimate the order of magnitude of the effect we use the relation derived by Swift [9]

\[ \dot{Q}_{\text{hs}} = 2c^2 \pi d_l \delta_{Tg} \frac{p_1 \langle u_1 \rangle}{(1 + P_r)} \Lambda \left[ \frac{\nabla T_{\text{in}}}{\nabla T_{\text{crit}}} \frac{1 + \sqrt{T_r + P_r}}{1 + \sqrt{T_r}} \right] \left( 1 + \sqrt{P_r} - \frac{2 \delta_{vg}}{d_l} \right). \]  

The factor 2 comes from the double-tube system. In this equation \( c \) describes the shape of the pressure wave. We use \( d_l \) for the diameter of the pulse tube, \( P_r \) is the Prandtl number, which is for helium equal to 0.67, and \( \delta_{vg} \) the viscous penetration depth of the gas, which is calculated from [9]

\[ \delta_{vg} = \sqrt{\frac{\eta RT}{M p_0 \pi \nu}}. \]  

In this equation \( M \) is the molar mass of \(^4\text{He}\). For the pulse tube we can write

\[ \frac{\delta_{vg}}{d_l} \ll 1 \]  

and

\[ \frac{\delta_{Tg}}{d_l} \ll 1. \]

Furthermore, \( \langle u_1 \rangle \) is the maximum velocity of the gas at the cold end

\[ \langle u_1 \rangle = 2\pi x_L \nu L_t, \]

with \( L_t \) the length of the pulse tube and \( x_L \) the reduced inlet length given by [1, p.37]

\[ x_L = 2 \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \frac{C_V p_1}{C_p p_0}. \]  

In equation (2.45) \( \Lambda \) is defined as
\[ \Lambda = 1 - \frac{2\delta v_g}{d_t} + \frac{2\delta^2 v_g}{d_t^2} \approx 1. \]  

(2.51)

The two temperature gradients given in equation (2.45) are the average temperature gradient \( \nabla T_m \)

\[ \nabla T_m = \frac{T_H - T_L}{L_t} \approx \frac{T_H}{L_t} \]  

(2.52)

and the critical temperature gradient \( \nabla T_{\text{crit}} \). The critical temperature gradient can be derived from adiabatic compression and expansion in the pulse tube as follows:

Consider a gas parcel that is about to enter the pulse tube from the CHX when the pressure is \( p \). It is at position \( x=0 \) and has a temperature \( T_L \). Some time later, when the pressure is \( p_h \) the gas parcel has entered the tube as far as possible. It is then in position \( x \), with temperature \( T \). The temperature at that point can be calculated from the relation describing the adiabatic process

\[ T = T_L \left( \frac{p_h}{p} \right)^{R/C_p}. \]  

(2.53)

For the position we can write

\[ L_t - x = L_t \left( \frac{p}{p_h} \right)^{C_V/C_p}. \]  

(2.54)

Combining equations (2.53) and (2.54) we find a relation for the temperature profile at the cold end

\[ T = T_L \left( \frac{L_t}{L_t - x} \right)^{R/C_V}. \]  

(2.55)

The temperature gradient at the cold end of the pulse tube is

\[ \left( \frac{dT}{dx} \right)_L = \frac{R}{C_V L_t} \frac{T_l}{T} = \nabla T_{\text{crit}}. \]  

(2.56)

All the parameters of the equation for the heat-shuttle effect are defined now and we can simplify equation (2.45) to

\[ \dot{Q}_{hs} = 13.4cd p_1 x_L L_t \sqrt{\kappa \nu c} \left[ 2.1 \frac{T_H}{T_L} - 1.8 \right]. \]  

(2.57)

In chapter 8 we will show that the heat-shuttle effect in the pulse tubes is significant.
Bibliography


Chapter 3

Ideal pulse-tube refrigerators with real gases

1In most analyses of pulse-tube refrigerators the working fluid is assumed to be an ideal gas. In this chapter the influence of real gases on the coefficient of performance of ideal PTRs is described.

3.1 Introduction

In recent years there have been impressive developments in pulse-tube refrigerators. New ingenious concepts led to improvements in cooling performances and in temperature range, which are reviewed, e.g., by Radebaugh [2], De Waele [3], and Ravex [4]. However, until now many of its fundamental properties are not well understood. In particular, the very large discrepancy between the Coefficient Of Performance (COP) (ratio between the cooling power $\dot{Q}_L$ and the input power $\dot{W}$), derived for ideal PTRs with ideal gases, and the COP realized in actual practice has not been explained. The ideal COP is equal to the ratio of the low temperature $T_L$ and room temperature $T_H$ [5]

$$\xi_0 = \frac{\dot{Q}_L}{\dot{W}} = \frac{T_L}{T_H}.$$  \hspace{1cm} (3.1)

Equation (3.1) implies that 0.5 W of cooling power at $T_L=4.2$ K with $T_H=300$ K requires 36 W compressor power. In reality, however, 5 kW are needed, about two orders of magnitude more. A factor of four is due to the dissipation in the room-temperature components (compressor, valves) [6], leaving still a gap of 1.3 kW and 36 W, which is a factor of 29 discrepancy. It is tempting to attribute this large

1This chapter is based on an article [1], published in Journal of Applied Physics.
discrepancy to the many imperfections of the cooler such as flow resistances, bad heat transfer, turbulence, heat leaks, etc. However, we will explain that the largest part of the gap between the highly idealized case and the more realistic COP is due to the nonideality of the working gas. We calculate the COP of ideal pulse-tube refrigerators with nonideal gases. We will show that the pressure dependence of the enthalpy of the gas, which is zero in the case of an ideal gas, has a large influence on the COP. This results in a much lower COP, even if the imperfections of the system are minimized. It limits the technological innovations and has a similar scope of validity as the Carnot efficiency for cryocoolers, but on a more realistic basis. This result has tremendous conceptual and technological implications.

3.2 Ideal pulse-tube refrigerator

![Schematic diagram of a Stirling-type single-orifice PTR](image)

Figure 3.1: Schematic diagram of a Stirling-type single-orifice PTR. From left to right the system consists of a compressor with moving piston (piston), a heat exchanger at room temperature (AC), a regenerator, a low-temperature heat exchanger (CHX), a pulse tube (tube), a third heat exchanger (HHX) which is at room temperature, an orifice (O), and a reservoir (res). The space between the piston and AC is called the compressor space. The cooling power is generated at low temperature $T_L$. The room temperature is $T_H$. The part in between AC and HHX is isolated thermally from the surroundings, e.g., by a vacuum chamber.

In this section we discuss a Stirling-type single-orifice PTR (see figure 3.1). We assume that the system is ideal. This means that: the flow resistances of all components, except the orifice, are zero; all heat exchangers are ideal, i.e., they have a constant temperature and the gas flowing out always has the temperature of the heat-exchanger body; the processes in the compressor space and in the pulse tube are reversible and adiabatic; the heat capacity of the matrix material in the regenerator is very high; the thermal contact between the gas and the regenerator material is very good; and the pressure variations are small. The heat conduction and the thermal conductivity play a special role as will be discussed in detail later on. All these assumptions are the usual assumptions which hold for an ideal system. If, in addition, we assume that the
working fluid is an ideal gas equation (3.1) can be derived.

The first law of thermodynamics is used in the form

$$\dot{U} = \sum_k \dot{Q}_k + \sum_k \dot{n}_k H_{mk} - \sum_k p_k \dot{V}_k + \dot{W},$$

(3.2)

with $\dot{U}$ the rate of change of the internal energy of the system under consideration, $\dot{Q}_k$ the heat flows into the system at the various boundaries which are labeled with the index $k$, $\dot{n}_k$ the molar flow of matter into the system, $H_{mk}$ the molar enthalpy of this matter, $\dot{V}_k$ the rates of change of the volume of the system at various moving boundaries such as a piston, and $p_k$ the corresponding pressure behind it. Finally, $\dot{W}$ takes into account all other forms of power applied to the system. The potential and kinetic energies are neglected. The heat flows and molar flows are positive when they flow into the system. We use the notation $\dot{Y}$ for the flow of a thermodynamic state function $Y$ and $\dot{\dot{Y}}$ for the rate of change of $Y$ of a system. Even though the dimensions of $\dot{\dot{Y}}$ and $\dot{Y}$ are the same their physical meaning is distinctly different. In the case of properties which are not properties of state, this distinction is meaningless and we will use the asterisk notation to indicate flow rates.

The second law will be used in the form

$$\dot{\dot{S}} = \sum_k \frac{\dot{Q}_k}{T_k} + \sum_k \dot{n}_k S_{mk} + \sum_k \dot{S}_{ik} \text{ with all } \dot{S}_{ik} \geq 0.$$

(3.3)

Here $\dot{S}$ is the rate of change of the entropy of the system and $T_k$ the temperatures at which the heat flows $\dot{Q}_k$ enter the system. With $S_{mk}$ we represent the molar entropy of the matter flowing into the system. Finally, $\dot{S}_{ik}$ represent the entropy production rates due to the various irreversible processes inside the system. We discuss a state in which all system parameters are perfectly periodic. As a result the time-averaged values $\overline{U}$ and $\overline{S}$ are zero.

In our analysis we assume that the motion of the piston is controlled in such a way that the pressure varies stepwise between $p$ and $p + \Delta p$ so that there are two time intervals each with constant pressure and constant flow. In this way we avoid complicated averaging formulae. As $\Delta p$ is small the pressure difference, driving the gas through the orifice is small, and the amount of gas $n_r$ flowing through the regenerator during a half cycle tends to become small as well. However, $n_r$ can still be macroscopic if the orifice $O$ is opened far enough or if the cycle time $t_c$ is long enough. Therefore, we will treat $n_r$ as a macroscopic quantity. In this situation the enthalpy flows, cooling power, etc., are first order in $\Delta p$. During the change in pressure a small amount of gas flows through the regenerator to compensate for the compressibility of the gas in the tube, but this extra flow only leads to second-order effects and will be neglected.
3.3 Influence of a real gas

In order to derive a new expression for the COP we concentrate on the energy balance near the regenerator. The total energy flow $\dot{E}$ in the regenerator, i.e. in the material and in the gas, is the sum of the heat flow

$$\dot{Q}_c = -\kappa A_r \frac{dT}{dl},$$  \hbox{(3.4)}

and the enthalpy flow $\dot{H}$, with $\kappa$ an effective thermal conductivity, $A_r$ the area of the cross section of the regenerator, and $l$ the length coordinate. Energy conservation requires that $\overrightarrow{E}$ = constant, so independent of $l$.

The temperature and pressure dependence of the molar enthalpy $H_m(p,T)$ can be derived from the general relation

$$dH_m = C_p dT + H_p dp,$$  \hbox{(3.5)}

with $C_p$ the molar heat capacity at constant pressure and

$$H_p = V_m (1 - \alpha_V T),$$  \hbox{(3.6)}

where $\alpha_V$ is the volumetric thermal expansion coefficient at constant pressure and $V_m$ the molar volume. Isothermal enthalpy changes, as in the regenerator, are equal to $\Delta H_m = H_p \Delta p$, so the average energy flow in the regenerator is

$$\overrightarrow{E} = \dot{Q}_c + \dot{H}_r H_p \Delta p,$$  \hbox{(3.7)}

(with $\dot{n}_r = n_r / (T_c / 2)$). Using equation (3.4) and integrating over the regenerator length $L_r$, we get the relation

$$1 = \int_{T_h}^{T_h} \frac{\kappa A_r / L_r}{\overrightarrow{E} - \dot{n}_r H_p \Delta p} dT,$$  \hbox{(3.8)}

which determines the value of $\overrightarrow{E}$.

For the molar enthalpy also holds the general relation

$$dH_m = T dS_m + V_m dp,$$  \hbox{(3.9)}

so, for isentropic changes (as in the compressor space and in the pulse tube), $\Delta H_m = V_m \Delta p$. The average power $\overrightarrow{W}$ applied to the compressor is equal to the average energy flow in the compressor space, so

$$\overrightarrow{W} = \dot{n}_r V_H \Delta p,$$  \hbox{(3.10)}
3. Ideal pulse-tube refrigerators with real gases

with $V_H = V_m(p, T_H)$. (The quantities $V_L$, $H_{pL}$, and $H_{pH}$, which will be used later on, are defined in a similar way.) The time-averaged enthalpy flow in the pulse tube can be written as

$$
\overline{H_t} = \overline{E_r V_L \Delta p} .
$$

(3.11)

With these relations the first law, applied to CHX, gives for the cooling power

$$
\dot{Q}_L = \overline{E_r V_L \Delta p} - \overline{E} .
$$

(3.12)

So the COP is

$$
\xi = \frac{\dot{Q}_L}{\dot{E}} = \frac{\overline{E_r V_L \Delta p} - \overline{E}}{\overline{E_r V_H \Delta p}} .
$$

(3.13)

Equation (3.13) shows that the maximum value for COP is found for the minimum value for $\dot{E}$. This is the case if the heat flow is zero. With equation (3.4) it can be seen that this happens when

$$
\kappa A_r/L_r = 0 .
$$

(3.14)

In this limit the integral of equation (3.8) can only give finite values if the denominator is equal to zero somewhere in the temperature interval $[T_L, T_H]$.

The only realistic working fluid for applications in low temperature cryocoolers on a large scale is $^4$He. Figure 3.2 is a plot of $H_p$ for $^4$He at 15 bar as a function of temperature [7]. The horizontal lines in figure 3.2 give the $H_p$ value at the indicated temperatures. There are two possibilities. When $T_L < 6.6$ K or $T_L > 12.3$ K the

![Figure 3.2: The parameter $H_p$ as a function of $T$ for $^4$He at 15 bar. The horizontal lines give the $H_p$ values at the indicated temperatures. For 8 and 10 K the intersections with the high-temperature branch determine the values of $T_\times$ which are used to determine the validity of equation (3.22).](image-url)
horizontal line does not cross the $H_p - T$ function again at a higher temperature. For $12.3 \, K < T_L < 6.6 \, K$ there is an intersection of the horizontal line with the $H_p - T$ function. The temperature at this intersection is marked as $T_x$ for later use. Due to equation (3.7) and the fact that $\dot{Q}_c \geq 0$, the minimum energy flow is found at the maximum of $H_p$. Inspection of the $H_p - T$ plot for helium shows that the minimum energy flow is found if

$$E = \dot{n}_t \Delta p H_{p0} \text{ with } H_{p0} = \max[H_{pL}, H_{pH}].$$

(3.15)

Equation (3.15) expresses that the total energy flow is equal to the enthalpy flow in one of the two ends of the regenerator. At this particular end the heat flow and the temperature gradient are both equal to zero. At the other end the heat flow is nonzero and, due to equation (3.14), the temperature gradient is infinitely large [8]. With equations (3.13) and (3.15) we arrive at the basic relation for the COP with a real gas

$$\xi = \frac{V_L - H_{p0}}{V_H}.$$  

(3.16)

In order to compare equation (3.16) with the COP of an ideal gas (equation (3.1)) we introduce the nonideality factor $\psi$

$$\psi = \frac{\xi}{\xi_0} = \frac{V_L - H_{p0} T_H}{V_H T_L}.$$  

(3.17)

From equation (3.15) it can be seen that $\psi$ always assumes the lowest value possible in the $[T_L, T_H]$ interval. For an ideal gas $H_{p0} = 0$ and $V_L/V_H = T_L/T_H$ so $\psi = 1$, as it should be.

### 3.4 Discussion

Let us consider the case that $H_{pL} > H_{pH}$. Now equations (3.15) and (3.17) give

$$\psi_{LT} = \frac{V_L - H_{pL} T_H}{V_H T_L}.$$  

(3.18)

With the equations (3.5) and (3.6) this relation can be simplified to

$$\psi_{LT} = \left(\frac{\partial V}{\partial T}\right)_{pL} \frac{T_H}{V_H}.$$  

(3.19)

This is an elegant relation expressing the effect of the nonideal nature of the working fluid in terms of the properties of the fluid at the low-temperature and at the high-temperature ends of the system. Equation (3.19) consistently explains for the first time why $^3$He works better than $^4$He as a working fluid in cryocoolers, since $(\partial V/\partial T)_{pL}$ is
3. Ideal pulse-tube refrigerators with real gases

higher for $^3$He than for $^4$He. For $2.5 \text{ K} < T < 10 \text{ K}$ the molar volume of $^4$He can be approximated by

$$V_m = V_0 \left[1 + \left(\frac{T}{T_0}\right)^3\right],$$

(3.20)

with $V_0=23.8 \text{ cm}^3/\text{mol}$ and $T_0=10.8 \text{ K}$. With this relation equation (3.19) gives

$$\psi_{LT} = 3\frac{V_0 T_H T_L^2}{V_H T_0^3},$$

(3.21)

which shows that $\psi$ goes down quadratic with $T_L$.

For combinations of $T_L$ and $T_H$ where $H_{pL} < H_{pH}$ the nonideality factor is

$$\psi_{HT} = \frac{V_L - H_{pH} T_H}{V_H T_L}.$$  

(3.22)

We have found no way to simplify this expression further.

Now we will have a closer look at figure 3.2. The highest values of $H_p$ are found in the low-temperature region. In fact, if $T_L < 6.6 \text{ K}$, $H_{pL} > H_{pH}$ for all values of $T_H$, so equation (3.18) holds over the whole temperature range. On the other hand, for $T_L > 12.3 \text{ K}$ (which is the temperature where $H_p$ has its minimum) $H_{pL} < H_{pH}$ for all values of $T_H$. Now equation (3.22) holds over the whole range. In the intermediate temperature range there is a crossover value $T_x$ for $T_H$. For $T_H < T_x$ equation (3.18) holds, while for $T_H > T_x$ equation (3.22) should be used.

Figure 3.3 gives plots of $\psi - T_H$ relationships for six values of $T_L$. In the majority of cases $\psi < 1$. Figure 3.3 shows that, for low $T_L$ values, $\psi$ is significantly lower than 1, indicating a strong reduction of the COP due to the nonideality of the gas. If

![Figure 3.3: $\psi - T_H$ relationships for $^4$He at 15 bar at six values of $T_L$. The numbers give the respective values of $T_L$ in kelvin. The dots in the 8 K curve and the 10 K curve indicate the crossover points $T_x$.](image-url)
3. Ideal pulse-tube refrigerators with real gases

$T_L = 4.2$ K and $T_H = 300$ K we get for helium at 15 bar $\psi = 0.163$. With this value 0.5 W of cooling power at 4.2 K requires at least 0.22 kW compressor power. With a real power of 1.3 kW this leaves only a factor of six discrepancy instead of the factor of 29 mentioned earlier. The remaining discrepancy of a factor of six can be attributed to imperfections in the system such as flow resistance, turbulence, bad heat transfer, etc., which are not unusual.

There are also some cases where $\psi > 1$, if $H_{pL}$ and $H_{pH}$ are both negative (hence $H_{p0} < 0$). This means that the COP can be enhanced due to the nonideality of the gas. This is a surprising result since it is generally believed that equation (3.1) gives an upper limit for the COP of pulse-tube refrigerators. One may wonder whether our result is in conflict with the second law. The entropy production rate in the regenerator is given by

$$\dot{S}_{ir} = \int_{T_L}^{T_H} \frac{Q_c}{T^2} dT = \dot{n}_r \Delta p \int_{T_L}^{T_H} \frac{H_{p0} - H_p}{T^2} dT. \quad (3.23)$$

Since $H_{p0} - H_p > 0$ (even if $H_{p0} < 0$) and all other quantities in equation (3.23) are positive as well, we may conclude that $\dot{S}_{ir} > 0$ and that there is no conflict with the second law. The enhanced cooling power can be considered as coming from Joule-Thomson-like effects in the PTR.

The enhancement of the COP for helium at 15 bar takes place only in the temperature range between 8.2 and 36.4 K. So it can only be used in the second stage of the pulse-tube refrigerator if it operates in that temperature range. However, one might consider using other substances (such as nitrogen or methane) which show cooling-power enhancement in different temperature ranges. Devices which cool from room temperature to 130 K have economic prospects, e.g., cooling machines for the process industry [9]. A PTR working with two stages, one with nitrogen and one with methane, has a COP which is 24% higher than that of a single-stage machine with helium.

3.5 Conclusions

In conclusion, in this section we have shown that the limits of technological innovation in this field are not set by equation (3.1), but by equation (3.16) which has a similar scope of validity as the Carnot efficiency of cryocoolers but on a much more realistic basis. This is due to the thermodynamic properties of the working fluid which determine the performance of pulse-tube refrigerators. At very low temperatures the COP, calculated with real gas parameters, is very much lower than for an ideal gas. This explains, to a large extent, the gap between the ideal and the real performance. We also have shown that there is an intermediate temperature range where the COP is enhanced.
Bibliography


Chapter 4

Flow losses

An optimized heat exchanger (good heat exchange, low flow resistance, and simple construction) is of key importance for the CFPTR. In literature pressure losses in fully developed steady flow have been studied widely. However, the effect of an oscillating flow on the flow resistance is only studied in a limited number of cases. These cases are conflicting and confusing. Therefore, we have built a set-up in which we can measure the pressure drop over a sample both for steady and oscillating flow with frequencies from 0 to 40 Hz.

4.1 Empirical relations

In the analysis of flow losses we found that losses due to changing flow area, bends, or coils can have a significant contribution. Therefore, we will start by giving a short overview of the relevant loss terms found in the literature. Only for laminar flow in straight pipes and ducts these loss terms are based on theory. For all other situations the loss terms are empirical.

The change in static pressure for incompressible steady flow through a series of pipeline components can be expressed by [1, p.38]

\[ p_1 - p_2 = \frac{1}{2} \rho \left( v_2^2 - v_1^2 \right) + \frac{1}{2} \rho v_c^2 K_c + \rho g (z_2 - z_1), \]  

with \( p \) the static pressure, \( \rho \) the fluid density, \( v \) the velocity, \( K_c \) the irreversible loss coefficient of the component, \( g \) the acceleration due to gravity, and \( z \) the elevation. The irreversible loss coefficient \( K_c \) depends on the velocity (or at least the sign of it), the density, the viscosity, and the dimensions of the flow channel. This chapter focuses on these irreversible losses. The indices 1 and 2 refer to the start and end situations of the flow. Furthermore, \( v_c \) is the flow velocity averaged over the cross section in the component with the smallest diameter. So, the change in static pressure consists of
4. Flow losses

a reversible change associated with the velocity, an irreversible loss associated with friction and viscosity, and a hydrostatic contribution. In this chapter we will neglect the hydrostatic pressure changes.

4.1.1 Straight pipes and ducts

The pressure gradient along a homogeneous straight pipe, in which the flow is incompressible and fully developed, can be written as [1, p.42]

\[
\frac{dp}{dl} = -\frac{1}{2} \rho v^2 f_r \frac{d_h}{d_h},
\]

(4.2)

with \(f_r\) the Darcy friction factor\(^1\), \(l\) the length, and \(d_h\) the hydraulic diameter which is defined as [1, p.42]

\[
d_h = \frac{4A}{\Omega},
\]

(4.3)

with \(A\) the flow area of the cross section and \(\Omega\) the wetted perimeter of the cross section.

The friction factor for fully developed flow is a function of the geometry of the cross section, the Reynolds number, and the surface roughness of the tube. For laminar flow in a circular tube, the friction factor can be derived from the Poiseuille equation [2, p.171]

\[
f_r = \frac{64}{R_e},
\]

(4.4)

with \(R_e\) the Reynolds number based on the hydraulic diameter. Somewhere between Reynolds numbers of 2000 and 10,000 the flow will not remain laminar. The critical Reynolds number \(R_{ec}\) where this happens, depends among other things on the geometry of the entrance for the flow. In the transition region between laminar and turbulent flow the friction factor increases but there is a large uncertainty in it. This uncertainty consists of two contributions. First, the increase from laminar to turbulent flow can follow different paths. Secondly, there is a fluctuation in the flow losses. For one \(R_e\) different flow losses are possible.

For turbulent flow \(f_r\) is a function of both \(R_e\) and the ratio of the surface roughness \(\varepsilon\) to the hydraulic diameter. All correlations for \(f_r\) in turbulent flow are empirical. For smooth pipes \(f_r\) can be written as [1, p.55]

\[
f_r = \frac{0.316}{R_e^{0.25}} \text{ if } 4 \times 10^3 < R_e < 2 \times 10^5.
\]

(4.5)

In figure 4.1 the friction factors described by the equations (4.4) and (4.5), are compared to measurements. The uncertainties in the transition area can be seen here.

\(^1\)In this thesis we will use \(f_r\) for the friction factor instead of the generally used \(f\). This is done to make a clear distinction between the friction factor and the frequency.
4. Flow losses

Figure 4.1: Comparison of the experimental and the theoretical results for the friction factor for a smooth pipe [1, p.51]. In this figure it can be seen that more paths are possible in the transition area, indicated with circles, dots, and triangles.

The friction factor is given as function of $R_e$ in a Moody diagram for different relative roughnesses in figure 4.2.

The distance between the inlet, where gas enters the channel and the point at which fully developed flow is established, is called the inlet length $L$. For laminar flow through a circular pipe it can be written as [1, p.39]

$$\frac{L}{d_h} = 0.061R_e + \frac{0.72}{0.04R_e + 1} \quad \text{if } 1 < R_e < 2000,$$

(4.6)

based on experiments. In Ward-Smith [2, p.203] it is assumed that the velocity profile can be represented by a boundary layer in which the velocity distribution is parabolic together with an inviscid core in which the velocity is uniform. This results in an analytical solution for the entrance flow through a circular pipe

$$\frac{L}{d_h} = 0.056R_e.$$

(4.7)

Another analytical solution is reached when the differential momentum equation is approximated by a linear equation through a circular pipe [2, p.221]

$$\frac{L}{d_h} = 0.026R_e.$$

(4.8)
4. Flow losses

Figure 4.2: Friction factor as function of the Reynolds number for various relative roughnesses [1, p.52].

For turbulent flow through a circular pipe the inlet length is found empirically [1, p.40]

$$\frac{L}{d_h} = 14.2 \log R_e - 46. \quad (4.9)$$

These relations for the inlet length are plotted in figure 4.3. From the figure it can be seen that the entrance length for laminar flow is much longer than for turbulent flow. The equations describing the laminar flow are not consistent. In the rest of this chapter we will use equation (4.6). Using this inlet length makes sure that the flow will be fully developed according to all three equations. In the region where the flow is not developed, there is an extra inlet resistance coefficient $K_{in}$. For laminar flow [1, p.43] this can be written as

$$K_{in} = 1.20 + \frac{38}{R_e}. \quad (4.10)$$

4.1.2 Curved pipes and bends

The pressure drop to impel a steady flow through a curved pipe or bend is higher than that for an equivalent flow in a straight pipe. It is described in the dimensionless loss coefficient $K$ as fraction of the dynamic pressure [1].

"For relatively long continuous curved pipes such as coils, the pressure drop can be computed from the dimensionless curved pipe friction factor. As fluid flows through
4. Flow losses

Figure 4.3: Relative inlet lengths for laminar flow according to the equations (4.6), (4.7), and (4.8), and for turbulent flow according to the equation (4.9).

A continuously curved pipe, the momentum of the flow tends to drive the fluid in a straight line while the curvature of the pipe forces the fluid to accelerate centripetally around the bend. As a result the point of maximum axial flow is displaced outward from the pipe centerline. Fluid is driven outward across the core of the pipe flow, and a secondary flow in the radial plane is established as this fluid returns back along the pipe walls. This secondary (radial) flow can be considered to consist of two vortices of opposite sign, each occupying one-half the cross section (see figure 4.4a and b). The vortices rotate the flow, causing the fluid elements to follow helical paths. Due to the

Figure 4.4: Flow in a tube coil showing the helical flow path induced by the double vortex system and displacement of maximum axial flow toward outside of the tube [1, p.66]. c: Schematic representation of a coiled heat exchanger with the diameter of the coil and the hydraulic diameter of the tube.
interference of the secondary flow with the main flow, a velocity profile originates with a maximum displaced from the middle to the outside of the coil [1].

In a tube the secondary (radial flow) is a function of the Dean number $D_e$ [1, p.68] [3, p.5,6]

$$D_e = Re \left( \frac{d_h}{D} \right)^{0.5}. \quad (4.11)$$

The dimensions of the helical tube as used in equation (4.11) are given in figure 4.4c. For Dean numbers smaller than around 30, there is no influence of the curving on the pressure drop. If $D_e$ is larger, the pressure drop over the pipe increases compared to a straight tube. The critical Reynolds number of the flow now depends on the hydraulic diameter of the pipe and the diameter of the coil. The disturbances (originating when $Re$ increases) are first absorbed by the secondary flow (the vortices). Only at higher flow velocity, dependent on the bending of the pipe, the flow will change from laminar to turbulent. In a curved-duct flow, it is difficult to identify $Re_c$ by a change in the slope of the $Re$-$f_r$-curve because of the gradual change (see for example figure 4.17a) instead of the discontinuity seen for straight-duct flows. In the literature different empirical formulae are given for $Re_c$. We will give one of them [1, p.58] [4]

$$Re_c = 2100 \left[ 1 + 12 \left( \frac{d_h}{D} \right)^{0.5} \right]. \quad (4.12)$$

To simplify the expressions we will introduce a parameter $r_d$ for the ratio of the hydraulic diameter and the coil diameter

$$r_d = \frac{d_h}{D}, \quad (4.13)$$

so for equation (4.12) we can write

$$Re_c = 2100 \left[ 1 + 12r_d^{0.5} \right]. \quad (4.14)$$

The influence of the coiling on the pressure drop and heat transfer coefficient is largest in the laminar region and gets smaller in the turbulent region. All correlations for $f_r$ for coiled-tube flow are empirical, resulting in many different published formulae. For laminar flow the friction factor is given by the following formulae, in which $f_r$ is the friction factor in a straight-pipe flow and $f_{rc}$ the friction factor in a coil. All these formulae are valid for components (coiled pipes) attached to relatively long, straight inlet and outlet pipe sections [1, p.58]
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\[ f_{rc} = f_r \text{ if } D_e < 11.6, \]  \hspace{1cm} (4.15a)

\[ f_{rc} = \frac{f_r}{1 - \left[ 1 - \left( \frac{11.6}{D_e} \right)^{0.45} \right]^{2.2}} \text{ if } 11.6 < D_e < 2000, \]  \hspace{1cm} (4.15b)

\[ f_{rc} = 0.11D_e^{0.5}f_r \text{ if } D_e > 2000. \]  \hspace{1cm} (4.15c)

The following relations hold for \( r_d < \frac{1}{3} \) \[3, \text{p.5-7}\]

\[ f_{rc} = f_r \text{ if } D_e < 30, \]  \hspace{1cm} (4.16a)

\[ f_{rc} = 0.419D_e^{0.275}f_r \text{ if } 30 < D_e < 300, \]  \hspace{1cm} (4.16b)

\[ f_{rc} = 0.1125D_e^{0.5}f_r \text{ if } D_e > 300. \]  \hspace{1cm} (4.16c)

The following relation holds for \( 1 < D_e < D_{ec} \), with \( D_{ec} \) the critical Dean number, calculated at \( R_{ec} \) \[4\]

\[ f_{rc} = f_r \left[ 1 + 0.033(\log D_e)^4 \right]. \]  \hspace{1cm} (4.17)

The ratios of these relations to \( f_r \) are plotted in figure 4.5. It can be seen that the values agree nicely. For square cross section formulae for the friction factor are given in \[1, \text{p.58}\].

![Figure 4.5](image)

**Figure 4.5:** Empirical relations for laminar flow in coiled tubes with \( r_d = 0.02 \).

For turbulent flow through a smooth tube for \( r_d < 0.01 \) we can write \[1\]

\[ f_{rc} = \frac{0.336}{R_e^{0.2}r_d^{0.1}}. \]  \hspace{1cm} (4.18)
Other empirical relations are found in Kakaç [3]

\[ f_{rc} = 0.029r_d^{0.5} + \frac{0.30}{Re^{0.25}} \text{ if } 0.034 < Re_d < 300, \]  
\[ f_{rc} = 0.336 R_e^{0.1} \text{ if } Re_d^2 < 700 \text{ and } 7 < \frac{1}{r_d} < 104. \]  

The last relation is found in the Wärmeatlas [4]

\[ f_{rc} = \frac{0.316}{Re^{0.25}} + 0.03r_d^{0.5} \text{ if } Re < R_e < 10^5. \]  

The ratio of these relations for turbulent flow to \( f_r \) are plotted in figure 4.6 as functions of the Reynolds number. It can be seen that the calculated values vary slightly. This results in a minimum and maximum friction factor.

\[ \text{Figure 4.6: Empirical relations for turbulent flow in coiled tubes with } r_d = 0.02. \]

The tube roughness may cause higher friction factors than for smooth tubes. Presently, no explicit correlations are available to account for tube-roughness effects in coiled tubes [1]. In the absence of definitive data, it is reasonable to assume that the surface roughness influences the pressure loss in curved pipes and ducts in the same way as in straight pipes. The correction for surface roughness is [1]

\[ \frac{\text{loss in rough coil}}{\text{loss in smooth coil}} = \frac{f_r \text{ (rough straight)}}{f_r \text{ (smooth straight)}} \]  

with \( f_r \) the friction factor for an equivalent turbulent flow in a straight pipe with the same cross section and relative roughness.
4. Flow losses

4.1.3 Abrupt changes in flow area

The geometry of the entrance to a pipeline has a large influence on the pressure drop over the total system. If there are abrupt expansions or compressions turbulent eddies are formed around the edges (see figure 4.7). The velocity used to calculate the pressure drop, is the velocity in the part with the smallest diameter. The irreversible loss coefficient for abrupt contraction is for laminar flow \[1, \text{p.71}\]

\[ K = 1.2 + \frac{38}{R_e} = K_{in}. \] (4.22)

For turbulent flow

\[ K = \frac{1}{2} \left( 1 - \frac{A_2}{A_1} \right), \] (4.23)

with \(A_2\) the area with the smallest diameter and \(A_1\) the area with the largest diameter. For abrupt expansion from area \(A_2\) to \(A_1\) the irreversible loss coefficient is \([1, \text {p.79}]\)

\[ K = -2.66 \frac{A_2}{A_1} \left( 1 - \frac{A_2}{A_1} \right), \] (4.24)

for laminar flow. For turbulent flow

\[ K = \left( 1 - \frac{A_1}{A_2} \right)^2. \] (4.25)

If the entrance has the shape of a gradual contraction or expansion with an angle smaller than 7°, there is no extra loss for turbulent flow \([1]\).

If the flow is split into a multichannel core (see figure 4.8) the irreversible loss coefficient in turbulent flow is \([1, \text{p.77}]\)

\[ K = \frac{1}{2} \left( 1 - \frac{A_{i2}}{A_1} \right), \] (4.26)

with \(A_{i2}\) the sum of all the channel areas

\[ A_{i2} = \sum_i A_{i2}. \] (4.27)
4. Flow losses

\[ A_1 \quad A_1 \quad A_2 \]

Figure 4.8: Multichannel structure with the different cross sections.

For expansion from a multichannel core we can write \[1, \text{p.78}\]

\[ K = \left( 1 - \frac{A_{i2}}{A_1} \right)^2, \tag{4.28} \]

again with equation (4.27) to describe \( A_{i2} \).

4.1.4 Concentric and eccentric annuli

According to the literature \[3\][5], the friction factor in concentric annuli \( f_{ro} \) for laminar flow (outer channel of tube-in-tube configuration) cannot be calculated just by using the hydraulic diameter in the formulae. The different diameters of the channels are explained in figure 4.9. Using equation (4.3) for the hydraulic diameter of the outer channel gives

\[ d_h = \frac{\pi \left( d_3^2 - d_2^2 \right)}{\pi \left( d_3 + d_2 \right)} = d_3 - d_2. \tag{4.29} \]

The friction factor for concentric annuli can be written as ratio to \( f_r \), which is the friction factor calculated with the hydraulic diameter. According to Miller \[5\]

\[ \frac{f_{ro}}{f_r} = 1.05. \tag{4.30} \]

Kakaç et al. \[3\] give the following relation
4. Flow losses

\[ \frac{f_{\infty}}{f_r} = 1.5 \left( \frac{d_2}{d_3} \right)^{0.035} \quad \text{if} \quad \frac{d_2}{d_3} > 0.005. \]  

(4.31)

According to Blevins [1] the following relation can be used

\[ \frac{f_{\infty}}{f_r} = \frac{(d_2 - d_3)^2}{d_2^2 + d_3^2 - \frac{(d_2^2 - d_3^2)}{\ln(d_2/d_3)}}. \]  

(4.32)

For the geometry as used in the experiments as given in table 4.1 equations (4.31) and (4.32) are in agreement, but equation (4.30) gives a lower result. For turbulent flow the friction factor for concentric annuli can be given as [4, p.4-117]

\[ \frac{f_{\infty}}{f_r} = 1 + 0.0925 \frac{d_2}{d_3}. \]  

(4.33)

For eccentric annuli the friction factor differs from concentric annuli. For laminar flow the ratio is [3, p.3-114]

\[ \frac{f_{\infty}}{f_r} = \frac{1.5}{1 + 1.5 \left( \frac{e}{2d_3 - e} \right)^2}, \]  

(4.34)

with \( e \) the eccentricity. For turbulent flow no such equation has been found. Figures with empirical relations are given in Kakaç et al. [3, p.4-138].

4.1.5 Oscillating flow

The influence of an oscillating flow on the flow resistance is only studied in a limited number of cases. The results of these studies are confusing, because they are conflicting. Some studies show a large influence of oscillating flow (up till a factor of four) and others show a small influence (only 10%).

According to Zhao et al. [6] there will be an increase in friction factor only if the oscillating frequency is larger than a characteristic value related to the viscous penetration depth \( \delta_{vg} \). If the typical size of the flow channel is larger than the penetration depth, turbulent eddies will develop and the friction factor will rise. The characteristic frequency \( \nu_c \) can be calculated from the expression for the viscous penetration depth [6]

\[ \delta_{vg} = \sqrt{\frac{\eta}{\pi \nu \rho}}, \]  

(4.35)

so, if \( r = \delta_{vg} \),

\[ \nu_c = \frac{\eta}{\pi r^2 \rho}. \]  

(4.36)
4. Flow losses

In these formulae \( r \) is the typical size of the flow channel, \( r = \frac{1}{2} d_h \). The dimensionless parameter describing the effect of oscillating flow is the Womersley number \( \alpha_W \)

\[
\alpha_W = r \sqrt{\frac{2 \nu \rho}{\eta}} = \sqrt{2} \frac{r}{\delta_{vg}} . \tag{4.37}
\]

The problem of oscillating flows has been studied for more than 100 years, but a thorough understanding of the phenomena has not yet been achieved, because of its complexity and the lack of experimental data. The typical characteristic frequency of the measurements reported in literature is \( \nu_c = 300 \) Hz for typical flow size channels of 100 \( \mu \)m. However, most measurements are done at a frequency of only a few Hz. Therefore, most researches on PTRs have adopted correlations for the pressure drop based on steady flow experiments, although the flow is oscillating.

Some researchers [7][8][9] performed pressure drop experiments in oscillating flow in the regenerator and presented the maximum or the cycle-averaged friction factors for low frequencies (under 10 Hz). This maximum friction factor \( f_{rm} \) is calculated with the maximum pressure drop at maximum gas velocity \( v_{max} \). The cycle-averaged friction factor \( f_{rav} \) is calculated from Zhao et al. [7]

\[
f_{rav} = \frac{\sqrt{\sum \Delta p dh}}{\frac{1}{2} \rho v_{max}^2 L} , \tag{4.38}
\]

with the rms of the pressure drop and the velocity. They use \( R_{em} \) for the maximum \( R_e \) calculated with [7, p.334]

\[
R_{em} = \frac{\rho v_{max} d_h}{\eta} , \tag{4.39}
\]

with \( d_h \) the hydraulic diameter of the screens based on the porosity. The measurements are done up to \( R_{em} = 100 \). The maximum pressure drop is 30 mbar. The maximum friction factor as found in their experiments is [8]

\[
f_{rm} = \frac{198}{R_{em}} + 1.737 . \tag{4.40}
\]

Compared to the \( f_{rs} \) for steady flow through a stack of woven screens [7]

\[
f_{rs} = \frac{33.6}{R_e} + 0.337 , \tag{4.41}
\]

this is an increase by a factor of five. Especially at these low \( R_e \) this seems to be too large.

Zhao also found that the cycle-averaged pressure drop of oscillating flow in a packed column is two times higher than of steady flow at the same \( R_e \) based on cross-sectional
mean velocity. These results only apply to cryocooler regenerators and heat exchangers with small pressure drops (up to 50 mbar).

In case of oscillating flow with larger pressure drops (up to 1.5 bar), Helvensteijn et al. [10] presented the friction factor of a regenerator for oscillations of 30 to 70 Hz. They measured up to $Re=100$ with an average pressure of 10 bar. The mass flow rate in the regenerator was not measured directly but calculated from the pressure in the reservoir. They calculated the instantaneous $Re$ and the corresponding $f_r$, and found that the oscillating flow friction factors deviate from the steady flow values, indicated with the line (see figure 4.10). The deviation slightly increases with frequency. They also found asymmetries in $f_r$, being larger for increasing flow than for decreasing flow.

Figure 4.10: Oscillating flow friction factor as function of the Reynolds number for various frequencies. A compressor stroke (expressed in LVDT) of 0.25 corresponds with a frequency of 36 Hz, 1.5 LVDT corresponds to 70 Hz [10].

Ju et al. [10] measured the mass flow rate and presented a correlation of the maximum friction factor only at 50 Hz for $Re$ up to 40. Their average working pressure is 8 bar, with pressure amplitudes up to 1.2 bar. They found that the cycle-averaged pressure drop in oscillating flow is two to three times higher than in steady flow at the same Reynolds numbers based on the cross-sectional mean velocity. Choi et al. [11] also measured the pressure drop of oscillating flow with frequencies from 50 to 70 Hz with $Re$ up to 100. The pressure amplitude is up to 1.5 bar. They show that the friction factor for oscillating flow is a factor of four higher than for steady flow (see figure 4.11).
In view of the similarity in the $f_r$ of a steady flow in a bundle of tubes and in a packed column, it can be conjectured that the similarity parameters for an oscillating flow in a bundle of tubes and in a packed column are also the same, using in both cases the hydraulic diameter [6]. Overall there seems to be an increase of the friction factor with increasing frequency even at low Reynolds numbers.

### 4.2 Experimental set-up

We have built a set-up in which we can measure the pressure drop over a sample both with steady and oscillating flow. Schematically the set-up is given in figure 4.12. The working fluid is helium. A compressor is used to drive the flow. The rotary valve (RV) in the system makes it possible to have an oscillating flow through the sample. It is described in more detail in chapter 7. To stabilize the pressure in front of the rotary valve during oscillating flow, buffers are put in front of the valve. Buffer volumes are indicated with B. The gas at the entrance of the sample is water cooled by two heat exchangers indicated with w. They prevent heating of the gas, due to the cyclic compression and expansion of the gas, causing a heat pumping effect. The flow $n_f$ is measured by a DC-flow meter (F). The flow meter uses the heat capacity of the gas to determine the mass flow. It has a time constant of a few seconds and cannot be used to measure oscillating flows directly. The pressure is measured by absolute or differential pressure sensors (p) with a sampling rate of 35 kHz. Needle and shut off valves (V) are used to set the flow through the system.
4. Flow losses

Figure 4.12: Schematic diagram of the set-up. The gas flow is provided by the compressor. The rotary valve (RV) in the system makes it possible to have an oscillating flow through the sample. The buffers in the system are indicated with B. The sample is water-cooled by two heat exchangers indicated with w. The needle and shut off valves are indicated with V, the pressure sensors with p, and the flow meter with F. The arrows indicate the flow directions. The flow through the sample (\( \hat{n}_s \)) consists of two components: the buffer flow (\( \hat{n}_B \)) and the flow measured by the flow meter (\( \hat{n}_f \)).

For steady flow measurements the rotary valve is put in a steady position where it is open. The flow through the sample is measured directly by the flow meter. For oscillating flow measurements the valve is turning. Then, there is a steady flow in the left part of the system and an oscillating flow in the right part, as indicated by the arrows in figure 4.12. The oscillation frequency can be varied between 1 and 40 Hz.

As the flow meter cannot measure oscillating flow, the flow is determined differently. The flow through the sample \( \hat{n}_s \) consists of two components (see figure 4.12)

\[
\hat{n}_s = \hat{n}_f + \hat{n}_B. \tag{4.42}
\]

The time dependent buffer flow \( \hat{n}_B \) results from the fact that the valve is closed during a part of the cycle. When the valve is closed the pressure at the high-pressure side of the compressor increases and at the low-pressure side it decreases. When the valve opens again, there will be a flow from the high-pressure side buffer to the low-pressure side buffer. Considering the expansion and the compression in the buffer volumes to be adiabatic, the buffer flow can be written as [13]

\[
\hat{n}_B = \frac{C_V}{C_p} \frac{V_B}{RT_H} \frac{dp_B}{dt}, \tag{4.43}
\]

with \( V_B \) the volume of the buffer and \( p_B \) the pressure in the buffer measured by \( p_3 \) and \( p_4 \) (see figure 4.12). The other component \( \hat{n}_f \) is steady and measured by the flow meter.
Measurements have been done for different sample geometries. The results will be discussed in the next section.

4.3 Experimental results

First we will discuss the results of some steady flow measurements and compare the results with the empirical values as reported in the literature. Three different samples are used. Sample number 1 has an entrance and exit with abrupt changes in cross section. Sample number 2 has a smooth (70°) entrance and exit. Sample number 3 is a coiled tube-in-tube configuration. Afterwards we will look at oscillating flow measurements in sample number 2.

4.3.1 Steady flow

The first geometry we looked at is sample number 1 with abrupt changes in the flow area (see figure 4.13). We choose this geometry because in our systems some of the changes in cross section are abrupt. It is multi-stage because that geometry was in stock. In order to compare the measurements to the empirical relations, we have plotted the raw measured data, as well as the results corrected with equations (4.22), (4.23), and (4.24) for laminar and turbulent flow in figure 4.14. The results in the transition region are corrected both for laminar and turbulent flow. The empirical relations plotted are the equations (4.4) for laminar and (4.5) for turbulent flow. From figure 4.14 it can be seen that the influence of the entrance and exit is around 20% of the total ft. Both for the laminar and the turbulent region the measurements and the empirical relations agree within the experimental error, which is 5%. For this sample the transition region is from Re=2000 to 5000. In this region it can be seen that the measured ft increases, but does not agree with the empirical relations for laminar or

Figure 4.13: Schematic diagram of sample number 1 with the dimensions in mm.
4. Flow losses

Figure 4.14: Measurement results and empirical relations for sample number 1 with abrupt contractions and expansions. The points give the raw measured data, as well as the measurements corrected for laminar or turbulent entrance flow with equations (4.22), (4.23), (4.24) and (4.25). The empirical relations are equations (4.4) for laminar and (4.5) for turbulent flow.

Sample number 2 has a smooth entrance and exit from $d = 4$ mm to $d = 1.58$ mm with an angle of $70^\circ$. The dimensions are given in figure 4.15. The results are given in figure 4.16. Again the raw measured data are plotted as well as the laminar corrected results (equation (4.22)) and the empirically found laminar (equation (4.4)) and turbulent (equation (4.5)) results. The transition takes place between $Re = 5000$ and $9000$. In this region $f_r$ increases and at one $Re$ different values of $f_r$ are possible. The

Figure 4.15: Schematic diagram of sample number 2 with the dimensions in mm.
transition takes place at higher Reynolds numbers because the sources of disturbance within the flow are minimized by the smooth entrance. According to Ward-Smith [2, p.19] laminar flow can be maintained up to $R_e=40,000$ in a straight pipe of constant circular section when experiments are designed to minimize sources of disturbance within and external to the flow. After correction for the inlet in the laminar region the measurements agree with the laminar relation. For the transition and turbulent flow the data are not corrected. For turbulent flow the results agree with the empirical relation.

Next, the results of measurements on a coiled tube-in-tube system will be described. The influence of the coiling and the fact that there is a tube-in-tube configuration, have quite some influence on the pressure drop over the heat exchanger. We will call this sample number 3. Its dimensions are given in table 4.1. For the inner tube of sample number 3, both the raw measured data and the empirical relations (equation (4.17) for laminar and equations (4.18) and (4.19a) for turbulent flow) are given in figure 4.17a.

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$D$</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 mm</td>
<td>1.2 mm</td>
<td>1.6 mm</td>
<td>2.0 mm</td>
<td>40 mm</td>
<td>1.5 m</td>
</tr>
</tbody>
</table>

Table 4.1: Dimensions of sample number 3.
4. Flow losses

The measurements are done at three different pressures. Although the pressures vary a factor of three, the friction factor versus Reynolds number nicely coincide (see figures 4.17a). The critical Reynolds number is calculated with equation (4.12) and is 5500. From the figure it can be seen that the slope of the curve changes around $R_e = 5000$. The laminar and turbulent slopes both agree with the empirical relations. In the laminar region the measured $f_r$ is larger than the empirical values. This means that we measure a larger pressure drop than expected, perhaps due to entrance effects. The raw measured data for turbulent flow are in between the empirical values, which show an upper and lower limit as given in figure 4.6, although our results confirm the lower values.

The sample with the smooth entrance (sample number 2) is used to compare steady flow to oscillating flow. Typical $p$-$t$-diagrams for oscillating flow with $\nu = 1$ Hz are given

4.3.2 Oscillating flow
Figure 4.18: \( p \)-\( t \)-diagrams for 1 Hz. Between the time moments a and b the valve is closed. The valve is completely open between the time moments c and d. In figure a \( p_3 \) and \( p_4 \) give the buffer pressures. The pressure over the sample is measured by \( p_1 \) and \( p_2 \). In figure b the difference of the buffer pressures is given as \( \Delta p_B = p_3 - p_4 \). The pressure over the sample is \( \Delta p_s = p_1 - p_2 \). The diagrams are measured with different flows in the sample.

in figure 4.18 for measurements with absolute (figure 4.18a) and differential pressure sensors (figure 4.18b). The positions of the sensors are given in figure 4.12. The pressure sensors \( p_3 \) and \( p_4 \) give the buffer pressures before and after the rotary valve. These pressures are needed to calculate the buffer flow with equation (4.43). The sensors \( p_1 \) and \( p_2 \) give the pressures at both ends of the sample. In figure 4.18b the differential sensors are positioned so that \( p_1 - p_2 = \Delta p_s \) and \( p_3 - p_4 = \Delta p_B \). In between the time moments a and b the pressure difference over the sample is zero because the valve is closed and there is no flow. When the valve is closed, \( p_4 \) increases and \( p_3 \) decreases, causing extra flow through the sample when the valve is open. As a result the flow through the sample (\( \dot{n}_a \)) is larger than the value \( \dot{n}_f \) given by the flowmeter. Between the time moments c and d the valve is completely open and there is flow.

The Womersley number for this sample, using helium gas, is calculated with equation (4.37). For the lowest used frequency (\( \nu = 1 \) Hz) \( \alpha_W = 0.19 \), for the highest frequency (\( \nu = 40 \) Hz) \( \alpha_W = 1.18 \). So the Womersley number is smaller than 1 or slightly higher than one. Therefore, the influence of phase shifting between the pressure and the velocity can be neglected. The characteristic frequency for this sample, using helium gas, is calculated with equation (4.36) \( \nu_c = 29 \) Hz. Oscillating flow measurements are done up to 40 Hz. Higher frequencies are not possible because the buffer flow becomes impossible to measure. The bufferflow is almost 30% of \( \dot{n}_a \), so at higher frequencies \( \dot{n}_a \) cannot be determined anymore. This can be seen from figure 4.19 where
the $p-t$-diagrams are given for 8, 18, 28, 40, and 50 Hz. With increasing frequency
the $p-t$-diagrams show higher harmonics in the pressure oscillation. Up to 30 Hz the
pressure over the sample stays constant for a small period of time when the valve is
completely open (between c and d). For frequencies of 40 Hz and higher $\Delta p_s$ does not
become constant anymore. The time derivative of the buffer pressures is impossible
to obtain, especially above 50 Hz. Already from frequencies of 20 Hz, we can see that
there is still a pressure drop over the sample, even if the valve is closed (between the
time moments a and b). This is due to the void volume of the connections from the
valve to the sample, which is quite large (around 100 cm$^3$). This will act as a buffer
volume. For all measurements $f_t$ is obtained at the moment the valve is completely
open. We calculate the maximum friction factor at the maximum Reynolds number
during the cycle, so for quasi-steady flow.

In order to make a good comparison between the steady and the oscillating $f_t$ we
will plot the raw, uncorrected data. First we give the steady flow $f_t$ for sample number
2 in figure 4.20 as well as the error bars. The errors are caused by the noise in the
pressure sensors and the flow meter. As can be seen from figure 4.20 the noise in the
transition region is large. This is due to intrinsic noise in the pressure over the sample.
The noise on the measured $f_t$ is plotted as function of $R_e$ in figure 4.21. It has been
measured with two differential pressure sensors independently.

In figure 4.22a the steady flow friction factor is given, as well as the friction factors
for frequencies of 1 and 3 Hz. The error in the oscillating flow measurements is caused
by the pressure sensors and the flow meter, and by the error in the calculation of the
buffer flow. The overall errors in the laminar and turbulent regions are 10%. It can
be seen that, within the error, the results are the same. In the transition region (at
$R_e=5000$ to 9000) the uncertainty in $f_t$ is larger, because the noise is almost 20%.

In figure 4.22b the steady flow friction factor is compared to the oscillating flow
results with frequencies between 8 and 40 Hz. To get a closer look at the influence of
the frequency, we plotted in figure 4.23a for two $R_e$ in the laminar region ($R_e=3000$
and $R_e=4000$) and two $R_e$ in the turbulent region ($R_e=15,000$ and $R_e=30,000$) $f_t$
as functions of the frequency. For the transition area $R_e=7000$ is given in figure 4.23b. In
the transition region the inaccuracy in the measurements is so large that we cannot see
whether $f_t$ increases with increasing $\nu$. For the laminar and the turbulent region we see
an upward tendency of $f_t$ when the frequency is larger than 28 Hz. The characteristic
frequency, calculated from equation (4.36), is 29 Hz. So an increase in friction factor
from 30 Hz can be expected.

Although the flow through the sample is determined indirectly in these measure-
ments, the results do not have a large error. This can be seen especially for larger
frequencies, where even with higher harmonics added to the signal, we still get rea-
Figure 4.19: $p$-$t$-diagrams for frequencies of 8, 18, 28, 40, and 50 Hz.
4. Flow losses

Figure 4.20: Measured friction factor as function of $R_e$ for steady flow of sample number 2 with error bars.

Figure 4.21: Relative noise in the pressure drop.
Figure 4.22: Measured friction factors as function of $R_e$ for steady flow measurements and for low frequencies (fig a) or for high frequencies (fig b).
4. Flow losses

Figure 4.23: Friction factor as function of the frequency. Figure a gives the laminar ($R_e=3000$ and $R_e=4000$) and the turbulent region ($R_e=15,000$ and $R_e=30,000$). Figure b gives the transition region with $R_e=7000$.

sensible results.

4.4 Conclusions

For steady flow measurements the measured friction factors agree with the empirical relations found in the literature both for the laminar and the turbulent region. For the oscillating flow with frequencies below 20 Hz we found no difference with the steady flow. For frequencies over 28 Hz the friction factor has an upward tendency both in the laminar and the turbulent region. This agrees with the calculated characteristic frequency. Comparing the oscillating-flow results with published data, it can be seen that we do not have an increase by a factor of two to five as is found for regenerators \cite{7,8,9,11,12}. Looking at the characteristic frequency $\nu_c$ the large increase they found, is questionable. Our results correspond more to the small increase found by Helvensteijn \cite{10}. The fact that in this research a tube with $d=1.58$ mm is used instead of regenerators with typical size of 0.2 mm, makes sure that if there is an influence of the frequency on the friction factor, it should be seen at lower frequencies, because in equation (4.36) we use a larger flow channel $r$. 
Bibliography


Chapter 5

Heat exchanger versus regenerator: a fundamental comparison

1 Irreversible processes in regenerators and heat exchangers limit the performance of cryocoolers [2]. In this chapter we compare the performance of CFPTRs, operating with regenerators and heat exchangers from a fundamental point of view. The losses in the two systems are calculated from the entropy productions due to the various processes.

5.1 Introduction

The purpose of our investigation is to compare a pair of PTRs operating with regenerators (see figure 5.1a) to a pair of PTRs operating with a counterflow heat exchanger (see figure 5.1b). Both systems consist of a compressor, rotary valve (RV), two identical pulse tubes (t), two orifices (O), and two buffers (B). Due to the position of the rotary valve we have oscillating flow in the regenerator (figure 5.1a). In the heat exchanger we consider steady flow (figure 5.1b). The main system parameters of CFPTR2 are given in table 5.1. These parameters are typical for medium size PTRs. In order to optimize the regenerator or the heat exchanger we treat their performances from a fundamental point of view by calculating the entropy production rates from the four relevant irreversible processes:

- axial thermal conduction in the gas ($\dot{S}_{cg}$),
- axial thermal conduction in the material ($\dot{S}_{cm}$),

1This chapter is based on an article [1], published in Cryogenics.
5. Heat exchanger versus regenerator: a fundamental comparison

![Diagram](image)

Figure 5.1: Schematic diagrams of the two systems in comparison. The components are: a compressor, a rotary valve (RV), a regenerator (Reg) or a heat exchanger (CFHX), two identical pulse tubes (t), two orifices (O), and two buffers (B). Figure a is a regular PTR with the rotary valve at room temperature and a regenerator. The pulse tube is split in two in order to facilitate comparison with the counterflow system of figure b. Figure b is a CFPTR with a heat exchanger and the rotary valve at low temperatures.

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>pulse tube</td>
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<td>24 mm</td>
</tr>
<tr>
<td></td>
<td>length</td>
<td>275 mm</td>
</tr>
<tr>
<td></td>
<td>volume</td>
<td>124.4 cm³</td>
</tr>
<tr>
<td>regenerator / heat exchanger</td>
<td>flow area</td>
<td>10 cm²</td>
</tr>
<tr>
<td></td>
<td>filling factor</td>
<td>0.5</td>
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<td>system parameters</td>
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<td>0.1 mol/s</td>
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<td></td>
<td>average pressure</td>
<td>1.75 MPa</td>
</tr>
<tr>
<td></td>
<td>frequency</td>
<td>1 Hz</td>
</tr>
</tbody>
</table>

Table 5.1: Main geometry of CFPTR2 and typical system parameters.
5. Heat exchanger versus regenerator: a fundamental comparison

- flow resistance (\( \dot{S}_f \)),
- heat exchange between the gas and the material (\( \dot{S}_{hx} \)).

The entropy production in the valves (\( \dot{S}_v \)) is the same in both cases, because

\[
\dot{S}_v = \frac{\dot{V} \Delta p}{T} = \frac{n R \Delta p}{p},
\]

and the pressure difference is not temperature dependent. Therefore, it will not be included in the discussion.

The first part of this chapter describes this fundamental comparison, which is valid for all kinds of geometries and materials. In the second part we compare a certain regenerator and heat exchanger as an example of the application of the theoretical expressions in a practical case and to show some typical results. Other geometries or materials for the heat exchanger or the regenerator can easily be used by substituting the hydraulic diameter, friction factor, and material characteristics in the equations. The regenerator has a cross sectional area \( A_r \), filling factor \( f_f \), and is supposed to be filled with spherical particles with diameter \( d_h \). The heat exchanger is supposed to consist of \( N \) parallel tubes for the high- and \( N \) tubes for the low-pressure side each with inner diameter \( d_1 \). The tubes are assumed to have a very good thermal contact. This can be achieved by pairwise thermally connecting two tubes (one hot with one cold) and arranging them in bundles as schematically depicted in figure 5.2b. Each tube has a free flow cross section of

\[
A_{hx} = \frac{1}{4} \pi d_1^2.
\]

The area of cross section of the tube wall is \( A_{hw} \). Figure 5.2 is a schematic diagram showing the temperature profile near the wall of a tube and the cross section of the heat exchanger. The temperature difference between the high-pressure side \( T_h \) and the low-pressure side \( T_l \) is given by

\[
\Delta T = T_h - T_l.
\]

Assuming equal heat transfer in all tubes and \( T = T_w \), the gas temperature at the high-pressure side is

\[
T_h = T + \frac{1}{2} \Delta T,
\]

and the temperature of the gas at the low-pressure side is

\[
T_l = T - \frac{1}{2} \Delta T.
\]
5. Heat exchanger versus regenerator: a fundamental comparison

Figure 5.2: Schematic representation of a heat exchanger part. The arrows indicate the flow direction. Fig. a gives the temperature distribution assuming constant temperature in the wall. Fig. b shows the way the tubes are connected to each other. We assume that the tubes make contact to each other over the complete area and have a uniform temperature in the radial plain.

In order to avoid unnecessary complications we will treat the various contributions to lowest order in $\Delta T$. We will further assume ideal-gas conditions.

5.2 Losses

In this section we will describe the four relevant processes producing entropy. The equations are written in a uniform format as much as possible, to make an easy comparison possible.

5.2.1 Axial heat conduction in the gas

For a regenerator the entropy production per unit length due to heat conduction in the axial direction in the gas can be written as [3]

$$\frac{d\dot{S}_{cg}}{dl} = (1 - f_t) A_t \frac{C_t \kappa_{gr}}{T^2} \left( \frac{dT}{dl} \right)^2 \text{[reg]},$$

(5.6)

with $\kappa_{gr}$ the effective coefficient of thermal conductivity of the gas. Typically it differs a factor five from the undisturbed thermal conductivity of the gas ($\kappa_g$) due to the complex geometry of the matrix. The influence of the flow of the gas is described separately in the constant $C_t$ for the regenerator. Furthermore, $T$ is the gas temperature and $l$ the length coordinate. We take $l = 0$ at the cold end of the regenerator or the heat exchanger.

For the heat exchanger the entropy production due to axial heat conduction in the gas can be obtained from equation (5.6) with $f_t = 0$. The entropy production can be
written as

\[
\frac{d\dot{S}^c_{cg}}{dl} = 2N A_h \kappa_g \left( \frac{dT}{dl} \right)^2 \ [\text{hx}],
\]  
(5.7)

with \( C_{\text{hx}} \) the constant representing the flow effects in the heat exchanger.

### 5.2.2 Axial heat conduction through the material

For the regenerator the entropy production due to axial heat conduction through the matrix material can be given as [3]

\[
\frac{d\dot{S}^c_{cm}}{dl} = f_t A_r C_{\kappa m} \left( \frac{dT}{dl} \right)^2 \ [\text{reg}],
\]  
(5.8)

with \( \kappa_m \) the thermal conductivity of the material. The effective coefficient of thermal conductivity of the matrix is smaller than the bulk coefficient of the matrix material due to the reduced contact surface of the different layers of matrix material. This is included in the coefficient \( C_{\kappa} \) [4].

The entropy production due to axial heat conduction through the material for the heat exchanger can be obtained from equation (5.8) with \( f_t = 1 \)

\[
\frac{d\dot{S}^c_{cm}}{dl} = 2N A_{\text{hw}} \kappa_m \left( \frac{dT}{dl} \right)^2 \ [\text{hx}].
\]  
(5.9)

### 5.2.3 Flow resistance

For the regenerator the entropy production due to flow resistance can be written as [3]

\[
\frac{d\dot{S}^c_f}{dl} = C_{z} \eta \frac{s^2}{d^2} \left( \frac{V_m}{A_r T} \right)^2 \ [\text{reg}],
\]  
(5.10)

with \( C_{z}/d^2 \) the flow impedance factor, \( \eta \) the viscosity, \( V_m \) the molar specific volume, and \( s^2 \) the mean square molar flow rate.

For the high-pressure side of the heat exchanger the formulae for the entropy production due to flow resistance are

\[
\frac{d\dot{S}^c_{fh}}{dl} = n V_m d_{ph} \left( \frac{dp_h}{T_h} \right),
\]  
(5.11)

with

\[
\frac{dp_h}{dl} = \frac{1}{2} \rho v^2 \frac{f_r}{d_1}.
\]  
(5.12)
5. Heat exchanger versus regenerator: a fundamental comparison

with \( f_r \) the friction factor, \( \rho \) the density, \( p \) the pressure, and \( v \) the average velocity. Combining equations (5.11) and (5.12) we get

\[
\frac{d\dot{S}_{\text{fh}}}{dl} = \frac{1}{2} f_r M \frac{n^3}{N^2 d_1 A_{\text{hx}}^2} \frac{V_m}{T_h},
\]

(5.13)

with \( M \) the molar mass of helium. When we sum the contributions of the high- and the low-pressure sides and assume \( p_h \approx p_l = p \), we get

\[
\frac{d\dot{S}_l}{dl} = f_r M \frac{n^3}{N^2 d_1 A_{\text{hx}}^2} \frac{V_m}{T} [\text{hx}],
\]

(5.14)

where \( A_{\text{hx}} \) is described by equation (5.2).

5.2.4 Heat exchange between gas and material

For the regenerator this contribution to the entropy production can be written as [3]

\[
\frac{d\dot{S}_{\text{hx}}}{dl} = \beta A_r \frac{(T_r - T_g)^2}{T_r T_g},
\]

(5.15)

with \( \beta \) the volumetric heat exchange parameter, which can be written as

\[
\beta = \alpha_h F = 12 f r \kappa_{gr} N_{ur} d_h^2,
\]

(5.16)

with \( \alpha_h \) the heat exchange coefficient, \( N_{ur} \) the Nusselt number for the regenerator, and \( F \) the heat exchange surface area per unit volume.

We now make some general assumptions in order to simplify the treatment and to be able to transfer the formulae into the preferred format. We assume that the regenerator material has a high heat capacity, so the temperature of the matrix is not a function of time. The gas temperature will be a function of the flow through the regenerator. Furthermore, when we assume zero void volume in the regenerator, so \( f_t = 1 \), the temperature difference between the gas and the matrix is proportional to the molar flow. Finally we assume that \( T_r \) is a linear function of the axial position and we have an ideal gas without thermal conduction in the axial direction [4]. Using these assumptions we can write [4]

\[
T_r - T_g = \frac{C_p \dot{n}}{\beta A_r} \frac{dT}{dl} = \frac{C_p \dot{n}}{A_r} \frac{d_h^2}{12 f r \kappa_{gr} N_{ur}} \frac{dT}{dl},
\]

(5.17)

so

\[
\frac{d\dot{S}_{\text{hx}}}{dl} = \frac{1}{12 f r \kappa_{gr} N_{ur}} \frac{C_p \dot{n}^2}{T^2} \left( \frac{dT}{dl} \right)^2 \left[ \text{reg} \right],
\]

(5.18)

with \( C_p \) the molar heat capacity of the gas at constant pressure.
To calculate the entropy production rate due to the heat exchange between the gas and the material for the heat exchanger, the assumption is made that the tubes make contact to each other over the complete wall area. The $\Delta T$ is constant over the entire length of the heat exchanger. Using the temperature profiles as given in figure 5.2a [5]

$$\frac{dS_{\text{hx}}}{dl} = d\dot{Q} \left( \frac{1}{T_1} - \frac{1}{T_h} \right) = d\dot{Q} \frac{\Delta T}{T_1 T_h}, \quad (5.19)$$

with $\dot{Q}$ the heat flow in the radial direction,

$$d\dot{Q} = \frac{1}{2} \pi \kappa_g N_{\text{uh}} N \Delta T d_l, \quad (5.20)$$

with $N_{\text{uh}}$ the Nusselt number for the heat exchanger, so

$$\frac{dS_{\text{hx}}}{dl} = \frac{1}{2} \pi N \kappa_g N_{\text{uh}} (\Delta T)^2 \frac{T_1}{T_1 T_h}. \quad (5.21)$$

When we have a steady state with small differences between the low- and high-pressure sides

$$\kappa_{gl} N_u = \kappa_{gh} N_{\text{uh}} = \kappa_g N_u, \quad (5.22)$$

the energy balance gives

$$\frac{n^*}{N} C_p dT = \frac{1}{2} N_u \kappa_g \frac{\pi d_1 dl}{d_1} \Delta T, \quad (5.23)$$

so

$$\Delta T = 2 \frac{C_p n^*}{\pi \kappa_g N} \frac{1}{N_{\text{uh}}} \frac{dT}{dl}, \quad (5.24)$$

and

$$\frac{dS_{\text{hx}}}{dl} = 2 \frac{C_p n^*}{\pi N \kappa_g N_{\text{uh}}} \frac{1}{T^2} \left( \frac{dT}{dl} \right)^2 [\text{hx}], \quad (5.25)$$

### 5.2.5 Summary

We now collect the entropy production rates in the regenerator and in the heat exchanger (see table 5.2).

### 5.3 Optimization

In this section, we apply the theoretical expressions derived above in the optimization of regenerators and heat exchangers and compare an optimized regenerator with an
5. Heat exchanger versus regenerator: a fundamental comparison

Table 5.2: Entropy production rates in the heat exchanger and the regenerator. The last column gives the ratios.

<table>
<thead>
<tr>
<th></th>
<th>heat exchanger</th>
<th>regenerator</th>
<th>ratio (hx/reg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>axial cond. gas</td>
<td>$2N A_{\text{hx}} \frac{C_{\text{h}} \kappa_{\text{g}}}{T^2} \left( \frac{dT}{dt} \right)^2$</td>
<td>$(1 - f_l) A_r \frac{C_{\text{r}} \kappa_{\text{gr}}}{T^2} \left( \frac{dT}{dt} \right)^2$</td>
<td>$\frac{2N A_{\text{hx}} C_{\text{h}} \kappa_{\text{g}}}{(1 - f_l) A_r C_{\text{r}} \kappa_{\text{gr}}}$</td>
</tr>
<tr>
<td>axial cond. wall</td>
<td>$2N A_{\text{hw}} \frac{C_{\text{h}}}{T^2} \left( \frac{dT}{dt} \right)^2$</td>
<td>$f_l A_r \frac{C_{\text{r}} \kappa_{\text{gr}}}{T^2} \left( \frac{dT}{dt} \right)^2$</td>
<td>$\frac{2N A_{\text{hw}}}{f_l A_r C_{\text{r}}}$</td>
</tr>
<tr>
<td>flow resistance</td>
<td>$f_r M \left( \frac{V^2}{T^2} \right)$</td>
<td>$\frac{C_{\text{r}} \eta^2 V^2}{T^2} \frac{d^2 A_r}{d^2}$</td>
<td>$f_r M \left( \frac{\eta}{\pi} \right) \frac{d^2 A_r}{d^2}$</td>
</tr>
<tr>
<td>heat exchange</td>
<td>$\frac{2 N \pi}{\eta N_{\text{ub}} N_{\text{ah}}} \delta_w^2$</td>
<td>$\frac{1}{2f_l A_r \kappa_{\text{gr}} N_{\text{ur}} \eta^2} \left( \frac{dT}{dt} \right)^2$</td>
<td>$\frac{24 f_r A_r \kappa_{\text{gr}} N_{\text{ub}} \kappa_{\text{gr}}}{\eta N_{\text{ub}} N_{\text{ah}} \kappa_{\text{gr}}}$</td>
</tr>
</tbody>
</table>

5.3.1 Regenerator optimization

Optimized heat exchanger. In this way we show how our formalism can be applied in a practical situation.

The purpose of our investigation is to compare a pair of PTRs operating with regenerators to a pair of PTRs operating with a counterflow heat exchanger. Therefore, we will keep the flow area of the regenerator and the heat exchanger equal, so that we can interchange the regenerator and the heat exchanger, without changing other system parameters of the PTR. This means that

$$(1 - f_l) A_r = 2N A_{\text{hx}} = \frac{1}{2} N \pi d_1^2 = A_t.$$  (5.26)

For the heat exchanger we use a minimum wall thickness $\delta_w$ needed to stand the pressure

$$\frac{\delta_w}{d_1} = \frac{p}{p_c},$$  (5.27)

with $p_c$ the breaking stress of the material. The total area of the walls of the heat exchanger can be given as

$$2N A_{\text{hw}} = 2N \pi d_1 \delta_w = 4A_t \frac{\delta_w}{d_1} = 4A_t \frac{p}{p_c}.$$  (5.28)

We will minimize the irreversible entropy production rates for the regenerator by determining the optimum grain size. For the heat exchanger the minimum entropy production rates are found by determining the optimum tube diameter, both for laminar and turbulent flow in the tubes. The optimum situations of the regenerator and the heat exchanger will be compared.
5. Heat exchanger versus regenerator: a fundamental comparison

\[ \frac{d\dot{S}_{rt}}{dl} = \frac{a_1}{d^2_h} + b_1 d^2_h + c_1, \]  

(5.29)

with

\[ a_1 = C_z \frac{1 - f_t}{A_t} \eta m V_m^2 T, \]  

(5.30)

\[ b_1 = \frac{1}{12} \frac{1 - f_t}{fr} A_t^n \frac{C_p^2}{N_{ur} \kappa_{gr}} \frac{1}{T^2} \left( \frac{dT}{dl} \right)^2, \]  

(5.31)

\[ c_1 = \left[ (1 - f_t) C_h \kappa_{gr} + \frac{f_t}{1 - f_t} C_k \kappa_m \right] \frac{A_t}{T^2} \left( \frac{dT}{dl} \right)^2, \]  

(5.32)

due to flow resistance, heat exchange, and axial conduction respectively. The optimum value for the diameter is given by

\[ d_h^* = \frac{a_1}{b_1} = 1.92 f_t C_z N_{ur} \kappa_{gr} \eta T^3 \frac{p^2}{\tau} \left( \frac{dT}{dl} \right)^{-2}. \]  

(5.33)

This results in a minimum entropy production per unit length of

\[ \frac{d\dot{S}_{r0}}{dl} = 2 \sqrt{a_1 b_1 + c_1} = 1.44 \sqrt{C_z} \frac{1 - f_t}{\sqrt{f_t}} \sqrt{\frac{n^2}{N_{ur} \kappa_{gr}}} A_t \frac{R^2}{p \sqrt{T}} \frac{dT}{dl} + c_1. \]  

(5.34)

Typical numbers for the constants and the geometry of the system are given now. For the heat exchange we use a Nusselt number [6] \( N_{ur}=10 \). The constant used for the conductivity of the stainless steel matrix [6] is \( C_k=0.16 \). The constant for the flow impedance \( C_z \) has been obtained from measurements in our group and is equal to 1600 [7]. The constant for the axial conduction in the gas \( C_h=10 \), also obtained from measurements [7]. In our calculations we use \( p = 1.75 \) MPa, \( \dot{n} = 0.1 \) mol/s, \( f_t = 0.5 \), and \( A_t=10 \) cm\(^2\), which are typical values for CFPTR2 in which we want to compare the regenerator to the heat exchanger (see table 5.1). The entropy production rates depend on the temperature and the temperature gradient. In order to give an impression about how our relations work out in a practical case we substitute parameters for the temperature and temperature gradients as found experimentally in our laboratory.

The values are given table 5.3. The different components as well as the total entropy

<table>
<thead>
<tr>
<th>( T [K] )</th>
<th>( \kappa_m [W/Km] )</th>
<th>( \kappa_g [W/Km] )</th>
<th>( dT/dl [K/m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>15</td>
<td>0.13</td>
<td>200</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
<td>0.08</td>
<td>400</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>0.05</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 5.3: Thermal conductivity of stainless steel, helium, and the temperature gradient at different temperatures.
5. Heat exchanger versus regenerator: a fundamental comparison

Figure 5.3: Total entropy production rates per unit length (tot) and the contributions due to flow resistance (fr), heat exchange (hx), and conduction in the axial direction (c) for laminar flow at 200 K. Fig a: for the regenerator as functions of the diameter of the spheres. Fig b: for the heat exchanger as functions of the tube diameter.

production rates are given in figure 5.3a for a temperature of 200 K. It shows that there is a minimum entropy production rate for the optimum grain size.

### 5.3.2 Heat exchanger optimization; laminar flow

In the heat exchanger we can have laminar or turbulent flow. We will first discuss the situation where we have laminar flow. Later on we will discuss the situation with turbulent flow. Assuming laminar flow in the heat exchanger means for the friction factor [5]

\[
f_r = \frac{64}{Re}, \tag{5.35}\]

with \(Re\) the Reynolds number. If we are looking for the optimum tube diameter \(d_1\) the total entropy production can be written using the equations (5.7), (5.9), (5.14), and (5.25) as

\[
\frac{dS_{ht}}{dl} = \frac{a_2}{d_1^2} + b_2d_1^2 + c_2, \tag{5.36}
\]

with

\[
a_2 = \frac{128}{At} \eta n \frac{V^2}{m}, \tag{5.37}
\]

\[
b_2 = \frac{1}{At} \eta n \frac{C_p^2}{N_{uh}\kappa_g T^2} \frac{1}{\frac{d\theta}{dl}}^2, \tag{5.38}
\]

\[
c_2 = \left[ C_{\text{th}}\kappa_g + 4\frac{m_p}{p_c} \right] \frac{A_t}{T^2} \left( \frac{d\theta}{dl} \right)^2, \tag{5.39}
\]
due to flow resistance, heat exchange, and axial conduction respectively. The optimum value for the diameter is given by

\[ d_{10}^4 = \frac{a_2}{b_2} = \frac{512}{25} N_{\text{uh}} \kappa_g \eta \frac{T^3}{\rho^2} \left( \frac{dT}{dl} \right)^{-2}. \]  

(5.40)

This results in a minimum entropy production per unit length of

\[ \frac{dS_{\text{hx0}}}{dl} = 2\sqrt{a_2 b_2} + c_2 = 56.6 \sqrt{\frac{\eta \ast n^*}{N_{\text{uh}} \kappa_g}} A_t \frac{R^2}{p\sqrt{T}} \frac{dT}{dl} + c_2. \]  

(5.41)

Assuming fully developed laminar flow with uniform surface heat flux, the Nusselt number can be written as [5]

\[ N_{\text{uh}} = \frac{h d_1}{\kappa} = 4.36. \]  

(5.42)

The constant for axial conduction in the gas is obtained from measurements \( C_{fh} = 4.4 \) [7]. Using stainless steel tubes the breaking stress is \( p_c = 250 \) MPa. The total entropy production rates as well as the various components are given in figure 5.3b for a temperature of 200 K. The minimum entropy production rate is found at \( d_1 = 0.3 \) mm. This is a small value. However, the entropy flows in the heat exchanger are typically 1 W/K obtained from measurements. Increasing the diameter from 0.3 mm to 0.6 mm would only give an increase of the entropy flow to 5 mW/K, even if the heat exchanger would be one meter long. Therefore, we have some freedom to choose diameters which are more convenient than the small values found in the optimization.

5.3.3 Comparison

The total entropy production in the regenerator and the heat exchanger with laminar flow can be compared. For the ratio of the optimum diameters we can write

\[ \frac{d_{10}^4}{d_{10}^4} = \frac{48}{512} \frac{f_t C_z N_{\text{ar}}}{N_{\text{uh}}}. \]  

(5.43)

The ratio of the diameters is determined by the filling factor and the Nusselt numbers only. Using the constants as given before, we get

\[ \frac{d_{10}}{d_{10}} = 3.6. \]  

(5.44)

Introducing the molar flux \( j \)

\[ j = \frac{n^*}{A_t}, \]  

(5.45)
5. Heat exchanger versus regenerator: a fundamental comparison

and assuming all parameters to be constant except the molar flux, the ratio of the minimum total entropy production rates can be written as, using equation (5.34) and (5.41)

\[
\frac{d\dot{S}_{t0}}{dl} / d\dot{S}_{hxo} / dl = r_S = \frac{a + bj^2}{c + dj^2},
\]

with

\begin{align*}
  a &= \left[ (1 - ft) C_{fr}\kappa_g + \frac{ft}{1 - ft} C_k\kappa_m \right] \frac{dT}{dl}, \\
  b &= 1.44 \sqrt{C} \left( 1 - ft \right) \sqrt{\frac{\eta}{N_{ur}\kappa_g}} \frac{R^2 T^{3/2}}{p}, \\
  c &= \left[ C_{fr}\kappa_g + 4 \frac{p}{pc}\kappa_m \right] \frac{dT}{dl}, \\
  d &= 56.6 \sqrt{\frac{\eta}{N_{uh}\kappa_g}} \frac{R^2 T^{3/2}}{p}.
\end{align*}

The ratio of the total entropy production rate is also dependent on the system parameters \((j, p, T)\). From this equation it can be seen that there are two limiting situations. When \(j \to 0\) (meaning we have a small flow, or a large area, so heat conduction is dominant), the ratio is

\[r_S = \frac{a}{c} = 3.25.\]  \hfill (5.51)

using the constants for \(T=200\) K. The other limit situation is for \(j \to \infty\) (meaning we have a large flow, or a small area, so flow resistance is dominant), the ratio is

\[r_S = \frac{b}{d} = 0.475,\]  \hfill (5.52)

independent of the temperature. In figure 5.4 the ratio of entropy production rates are plotted as functions of \(j\) for three different temperatures (\(T=200\) K, \(T=100\) K and \(T=40\) K) with constant pressure. In figure 5.4 the transition value as well as the two limit situations for the molar flux can be seen. Below the transition value the regenerator has larger losses than the heat exchanger, above this value the regenerator has smaller losses than the heat exchanger. The transition molar flux decreases with decreasing temperature.

Figure 5.5 gives the relative contributions to the total entropy production rates due to the four loss mechanisms at three different temperatures \((A_t=10\) cm\(^2\), \(\tilde{n}=0.1\) mol/s, so \(j=100\) mol/sm\(^2\)). For the regenerator the conduction in the material causes most losses. The two conduction losses increase with decreasing temperature both for
5. Heat exchanger versus regenerator: a fundamental comparison

The regenerator and the heat exchanger. In the heat exchanger the conduction terms due to the gas and the material are of the same order.

5.3.4 Heat exchanger optimization: turbulent flow

After having discussed the optimization of the heat exchanger with laminar flow, we will now discuss the optimization of the heat exchanger with turbulent flow. Assuming fully developed turbulent flow in the heat exchanger ($Re \gg 10,000$) means for the friction factor [5], Nusselt number [8], and axial conduction constant $C_{fh}$ [7]

\begin{align*}
  f_t &= \frac{0.33}{Re^{1/4}} \quad (5.53) \\
  N_{uh} &= 0.0343Re^{3/4} \quad (5.54) \\
  C_{fh} &= 0.034Re^{3/4} \quad (5.55)
\end{align*}

Using the equations (5.53) to (5.55) in the formulae given in table 5.2, gives the total entropy production rate in the heat exchanger

\begin{equation}
  \frac{d\dot{S}_{tot}}{dl} = \frac{a_3}{d_1^{5/4}} + b_3d_1^{5/4} + e_3d_1^{3/4} + f_3, \quad (5.56)
\end{equation}

with
Figure 5.5: Four different entropy production terms as percentage of the total production, both for the regenerator (reg) and the heat exchanger (hx) at three different temperatures for the optimized situation at that temperature for laminar flow.

\[ a_3 = 1.11\eta^{1/4}M^{3/4}n^{11/4}\frac{1}{A_t^{7/4}}\frac{V_m^2}{T}, \]  
\[ b_3 = 16.9n^{5/4}\frac{C_f^2}{\mu_{\text{g}}}\left(\frac{\eta}{M}\right)^{3/4}\frac{1}{A_t^{1/4}T^2}\left(\frac{dT}{dl}\right)^2, \]  
\[ e_3 = 0.058\left(\frac{nM}{\eta}\right)^{3/4}A_t^{1/4}\frac{\kappa_{\text{g}}}{T^2}\left(\frac{dT}{dl}\right)^2, \]  
\[ f_3 = 4A_t\frac{p}{\rho_c}\frac{\kappa_{\text{m}}}{T^2}\left(\frac{dT}{dl}\right)^2, \]

for the flow resistance, heat exchange, axial conduction in the gas, and axial conduction in the material respectively. The total entropy production rates, as well as the different components are given as functions of the diameter \((A_t=10\text{ cm}^2,\) and \(n=0.1\text{ mol/s})\) in figure 5.6 for \(T=200\text{ K}\). In this figure we assume to have turbulent flow through the tubes, with a total flow of 0.1 mol/s.

As can be seen in figure 5.5, in the case of turbulent flow the optimum tube diameter is around \(d_1=30\mu\text{m}\). This means that around 500,000 tubes are needed to get the total area of 10 cm\(^2\). Apart from the fact that this is rather impractical, the flow is no longer turbulent at these small diameter values. The cross-over in this respect is at a tube diameter of about 1.5 mm, corresponding to 6 tubes in 10 cm\(^2\). In conclusion this
means that the optimum situation for the heat exchanger (lowest entropy production rate) is found for a situation with laminar flow.

### 5.4 Conclusion

The realistic optimum situation for the heat exchanger is a situation with laminar flow. Whether an optimized regenerator or heat exchanger is better depends on the system parameters \((j, p, T)\). Above a certain value of the molar flux, the heat exchanger has more entropy production than the regenerator. This value decreases with decreasing temperature. There are two limiting situations for the ratio of optimized entropy productions: heat conduction dominated and flow resistance dominated. For a heat conduction dominated situation the heat exchanger is more efficient than the regenerator. For a flow resistance dominated situation the heat exchanger is less efficient than the regenerator.

At temperatures far below 200 K the contributions of the flow resistance and the heat exchange become significantly less than the contributions of the heat conduction in the axial direction. Due to the weak dependence of the total entropy production rates on the diameter, one has some freedom to choose larger diameters than the small values found in the optimization procedure, which may be more convenient for applications.
Bibliography


Chapter 6

Analytical treatment of counterflow pulse-tube refrigerators

In the previous chapter we treated the various loss mechanisms in regenerators and heat exchangers. However, in that chapter some important system parameters, such as the temperature gradients in the CounterFlow Heat Exchanger (CFHX), have been obtained from experiments. In this chapter we derive a complete set of equations for the counterflow pulse-tube refrigerator from which all thermal parameters can be calculated in good approximation. Thus we obtain a powerful tool for the system analysis.

6.1 Idealizations

We assume that the two sides of the CFHX each consist of $N$ parallel circular capillaries. The capillaries are pairwise (one from the warm side and one from the cold side) thermally connected. The thermal resistance of the walls in the radial direction is assumed to be zero. The thermal conduction in the flow direction is neglected. The Reynolds numbers turn out to be so small that the flows are laminar (see chapter 5). The working fluid is helium which will be treated as an ideal gas. We will approximate the thermal conductivity of the gas by the relation

$$\kappa = \kappa_0 \sqrt{T}$$  (6.1)

where $\kappa_0$ is a constant. The viscosity we write as

$$\eta = \eta_0 \sqrt{T}$$  (6.2)

1 This chapter is based on an article [1], published in Cryogenics.
6. Analytical treatment of counterflow pulse-tube refrigerators

Figure 6.1: *Assumed time dependence of the pressure in one of the pulse tubes.*

with \( \eta_0 \) a constant. For helium in the temperature range between 40 K and room temperature equations (6.1) and (6.2), and the ideal-gas assumption are correct within 10% [2].

The compressor and the rotary valve generate pressure variations in the pulse tubes which will be approximated by a square-wave pattern between the high pressure \( p_H \) and the low pressure \( p_L \). It has a cycle time \( t_c \), as depicted in figure 6.1. For the pressure ratio we use the notation

\[
\pi_H = \frac{p_H}{p_L}.
\]  

(6.3)

As a result of the varying pressures the temperature profiles in the tubes vary in time. Figure 6.2 gives qualitative plots of these temperature profiles in situation \( \text{a} \) and in situation \( \text{c} \). The parameter on the horizontal axes is the reduced length \( l/L_t \), where \( l \) is the length coordinate along the pulse tube and \( L_t \) is the pulse-tube length. We assume that the processes in the pulse tubes are completely reversible and adiabatic. During the low-pressure phase \( \text{da} \) the gas enters the tube from the hot end with room temperature \( T_H \). The parameter \( \lambda \) gives the dimensionless length over which the gas has entered the tube in situation \( \text{a} \). The dotted lines schematically give the temperatures of the gas that never leaves the tube, the so-called gas piston. In situation \( \text{c} \) the profile at the cold end contains two branches. The curved part is due to the gas that entered the tube during phase \( \text{ab} \) (see equation (2.55)). The flat part is due to the gas that entered during phase \( \text{bc} \). In both cases the gas enters the pulse tube with the cold-end temperature \( T_L \).

In figure 6.3 the nomenclature of the pressures, temperatures, and heat flows at various positions is indicated. The connection lines between the rotary valve and the cold ends of the pulse tubes are depicted as two lines in order to indicate the two temperatures of the gas flows, depending on the flow directions. We assume a steady
6. Analytical treatment of counterflow pulse-tube refrigerators

Figure 6.2: Two temperature profiles in the pulse tubes as functions of the reduced length $l/L_t$. One at situation a and one at situation c (see figure 6.1). The dotted lines represent the temperature profiles of the gas piston. They are for visual aid only. The profile at the hot end in situation a is flat. It is due to the gas that entered the tube from the hot end during phase da. The parameter $\lambda$ gives the dimensionless length over which the gas has entered the tube from the hot end in situation a. In situation c the profile at the cold end contains two branches. The curved part is due to the gas entering the tube during phase ab. The flat part is due to the gas entering during phase bc.

Figure 6.3: Schematic diagram of a counterflow pulse-tube refrigerator. In this figure various quantities, used in the text, are defined.
molar flow $\dot{n}$ in the CFHX. This is achieved by adding two buffer volumes, one at pressure $p_H$ and one at pressure $p_L$, in between the CFHX and the rotary valve.

### 6.2 Governing expressions

In Appendix A a complete set of relations is derived for the operation of the CFPTR. The derivation is based on the idealizations described above. In this section we summarize the results. The input parameters of the relations depend on the working fluid, the geometries of the pulse tube and the CFHX, and the compressor characteristics. The values of molar ideal-gas constant $R$, the molar heat capacity at constant pressure $C_p$, the viscosity at room temperature $\eta_H$, and the gas thermal conductivity at room temperature $\kappa_H$ are determined by the working fluid (helium). The values of the pulse-tube volume $V_t$, the operating frequency $\nu$, the total applied heating power $\dot{Q}_L$, and the parameter $\lambda$ (dependent on the orifice setting) are fixed by the system parameters. The values of the capillary diameters of the warm and the cold side $D_w$ and $D_c$, the length $L_X$, and the number of capillaries $N$, are properties of the CFHX. The compressor volume flow at the low-pressure side $V_{cL}$ and the compressor efficiency $\xi_c$ are compressor properties.

In our treatment we choose a certain value of $p_L$ and calculate the nine variables $f_1$, $T_L$, $\pi_H$, the effective temperature difference at the cold end of the pulse tube $\Delta T_L$, the temperature difference over the two channels in the CFHX $\Delta T$, the molar flow rate $\dot{n}$, the pressures at the high- and low-pressure side of the compressor ($p_{HC}$ and $p_{LC}$ respectively), and the temperature $T_{VH}$, from the following nine expressions:

1. the function $f_1$ is introduced to simplify the notation of some of the relations. It is a factor in the pressure dependence of the gas flow at the cold end and is given by equation (A.23)

$$f_1 = \frac{3}{5} (\pi_H - 1) + \frac{\lambda}{\pi_H^{2/5}}; \quad (6.4)$$

2. the average difference in temperature of the flow at the cold end of the pulse tube, given by equation (A.28)

$$\Delta T_L = \frac{\lambda}{f_1} \left( \frac{\pi_H^{2/5}}{\pi_H^{2/5}} - 1 \right) T_L; \quad (6.5)$$

3. the molar flow rate in the CFHX, equation (A.36)

$$\dot{n} = 2\nu f_1 \frac{V_t p_L}{RT_L}; \quad (6.6)$$
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4. the temperature difference over the CFHX, equation (A.35)

\[ \Delta T = \Delta T_L - \frac{\dot{Q}_L}{\bar{n}C_p}; \]  
(6.7)

5. the temperature of the gas flowing from the CFHX into the rotary valve, equation (A.38)

\[ T_{VH} = T_L + 2\Delta T; \]  
(6.8)

6. the length of the CFHX, equation (A.45)

\[ L_X = \frac{4\bar{n}C_pT_H}{N\pi H^2\Delta T} \left( 1 - \sqrt[3]{\frac{T_{VH}}{T_H}} \right); \]  
(6.9)

7. the volume flow at the low-pressure side of the compressor, equation (A.52)

\[ \dot{V}_{Lc} = \frac{\bar{n}RT_H}{p_{Lc}}; \]  
(6.10)

8. the pressure at the high-pressure side of the compressor, equation (A.50)

\[ p_{Hc}^2 = p_H^2 + 2\frac{128\eta_H}{N^2\pi H^2} \frac{n^2 C_p R}{N_H\pi H^2\Delta T} (T_H^2 - T_{VH}^2); \]  
(6.11)

9. the pressure at the low-pressure side of the compressor, equation (A.51)

\[ p_{Lc}^2 = p_L^2 - 2\frac{128\eta_H}{N^2\pi H^2} \frac{n^2 C_p R}{N_H\pi H^2\Delta T} (T_L^2 - T_{VH}^2). \]  
(6.12)

From equations (6.4) to (6.12) the seven parameters \( f_1, \Delta T_L, \Delta T, \bar{n}, T_{VH}, p_{Hc}, p_{Lc} \) can be eliminated which leaves two relations between \( T_L \) and \( \pi_H \). One is cubic and one is forth order in \( T_L \). The cubic equation can be solved analytically giving \( T_L \) as a function of \( \pi_H \) [3]. From the remaining relation \( \pi_H \) has to be solved numerically. Once \( \pi_H \) is known all other system parameters can be calculated with the analytical expressions given above.

6.3 Numerical results

In this section we present some calculated results. For the system components we use the values of CFPTR2 as given in table 6.1. Figure 6.4 is a plot of the minimum temperature \( T_L \) as a function of the frequency \( \nu \). The lowest temperature is reached at a frequency of 0.3 Hz. However, \( \dot{Q}_L=0 \) in these calculations. In reality there is
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Number of capillaries</td>
<td>N</td>
<td>331</td>
</tr>
<tr>
<td>Counterflow heat-exchanger length</td>
<td>L_X</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Diameter of capillary</td>
<td>D_c, D_w</td>
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</tr>
<tr>
<td>Frequency</td>
<td>( \nu )</td>
<td>1 Hz</td>
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<tr>
<td>Volume of one pulse tube</td>
<td>V_t</td>
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</tr>
<tr>
<td>Relative hot-end penetration length</td>
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</tr>
<tr>
<td>Room temperature</td>
<td>T_H</td>
<td>282 K</td>
</tr>
<tr>
<td>Low-pressure compressor volume flow</td>
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<tr>
<td>Low-pressure value</td>
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</tr>
<tr>
<td>Cooling power</td>
<td>Q_L</td>
<td>0 W</td>
</tr>
</tbody>
</table>

Table 6.1: Values of the system components as used in the calculations unless stated otherwise.

![Graph of T_L vs \( \nu \)](image1)

Figure 6.4: Calculated minimum temperature as a function of the operating frequency.

![Graph of T_L vs D](image2)

Figure 6.5: Calculated series of plots of \( T_L \) as functions of the capillary diameter \( D \) for various values of the length of the CPHX. The values of \( L_X \) are indicated in meters.
always a nonzero heat load. The model shows that the optimum frequency increases with the heat load.

Figure 6.5 is a series of plots of the minimum temperatures $T_L$ as functions of the capillary diameter for various values of the length of the CFHX. The number of capillaries is varied together with $D = D_c = D_w$ so that the area of the cross section is constant ($\frac{\pi}{4} ND^2 = 1.7 \text{ cm}^2$). Depending on the value of $L_X$ the minimum temperature is reached at different values of the diameter, but the value of $T_L$ at the minimum is independent of the length. The minimum temperature is reached with tube diameters of a few hundred micron, with a maximum length of 1 m, which is in agreement with the conclusion obtained in the previous chapter.

Figure 6.6 is a cooling power plot for various values of the low pressure $p_L$. The value of $p_L$ can be determined within a certain range by adding or removing helium from the system. For heating powers on the order of 20 W the best value is about 14 bar.

![Figure 6.6: Calculated cooling power plots for various values of $p_L$ indicated in bar.](image)

6.4 Validation of the model

The system used (CFPTR2) has the system parameters as given in table 6.1. The system is operated by a 7.5 kW compressor and a 'balanced contact' rotary valve (see chapter 7). The hot heat exchangers are water cooled. Both subsystems have a buffer volume, which is ten times larger than the volume of the pulse tube. The flow is measured in the supply leads from the compressor. The deviation between the model and the real system performance will be described first. Then we will verify some specific equations of the model. Afterwards, we compare the measurements to the complete model.
6. Analytical treatment of counterflow pulse-tube refrigerators

6.4.1 System description

First we look into the geometry of the CFHX. One side of the heat exchanger consists of \( N \) parallel circular capillaries. The capillaries are arranged in a regular triangular array with a free distance between the capillaries of 0.2 mm. They have an inner diameter of 0.8 mm and a wall thickness of 0.1 mm. In contrast to the model, where the gas on the other side of the heat exchanger is assumed to flow in \( N \) channels as well, this gas flows \textit{in between} the capillaries. A CFHX with 37 and one with 331 capillaries has been used. A schematic cross section of the CFHX with 37 capillaries is given in figure 6.7. The capillaries are fixed to the SS support flange by a special hard-soldering technique. The CFHX with 331 capillaries has an extra flow straightener in the cap to prevent uneven flow through the capillaries.

![Schematic cross section of the heat exchanger with 37 capillaries.](image)

In the model we assume a square-wave pattern for the pressure in the tubes as given in figure 6.1. In reality the pressure wave differs from the square wave. A typical pressure profile is given in figure 6.8. In order to compare the measurements with the calculations we approximated this profile by a square wave. The total areas below the real pressure profile and the square wave are the same, both for the high-pressure part and for the low-pressure part of the wave. The pressure \( p_L \) is 13 bar (see figure 6.8), with a heat load on the system of 20 W. According to the model (see figure 6.6) this is the best value for \( p_L \) at this \( \dot{Q}_L \).

The heat loads in the rotary valve and the pulse tubes (e.g. caused by conduction, heat shuttling, mechanical friction in the valve) are calculated from the net enthalpy flow in the heat exchanger \( \dot{H}_{\text{hx}} \).
Figure 6.8: Measured pressure profile in the tube. The dotted line gives the modified square-wave profile used in the calculations.

\[ \dot{H}_{\text{hx}} = \dot{n}C_p \Delta T, \]  

(6.13)

and the enthalpy flow in the pulse tubes \( \dot{H}_t \) (see equation (A.14)).

The last significant difference between the model and reality is that there are not always buffer volumes between the CFHX and the rotary valve. When there are no buffer volumes there is no constant flow in the heat exchanger. However, the rotary valve is only closed during a short period of time (10% of the cycle time), so the flow is only interrupted during a short time. The effect on the performance of the heat exchanger is negligible. This conclusion is supported by the observation with the CFHX with 331 capillaries where we really have installed buffer volumes \( V = 42 \text{ cm}^3 \) between the CFHX and the rotary valve.

6.4.2 Validation

We did a number of measurements for different orifice settings \( \lambda \), frequencies, heat loads, and pressures with both CFHXs. The \( \lambda \)-values are determined by using the equations (A.2) to (A.4). The CFHX with 37 capillaries consists of one single unit with \( L_X = 20 \text{ cm} \), the CFHX with 331 capillaries consists of two units of 10 cm each. As discussed in the section Governing Expressions we use four different kinds of input parameters. The relations depending on the working fluid are the same for all measurements. The geometry of the two CFHXs is the same apart from the value of \( N \). The pulse tube (frequency, heat load, and dimensionless hot-end length) and the compressor properties are obtained in the measurements. The results of these measurements are compared to the calculated model results.
Energy conservation implies that the temperature difference between the two gas flows is the same everywhere in the CFHX. The temperature is measured at all entrances and exits of the CFHX. A typical temperature profile of the CFHX with 331 capillaries, as given in figure 6.9, shows that we indeed have energy conservation.

Figure 6.9: Temperature profile of the CFHX with 331 capillaries. The temperature difference between the two gas flows is constant.

In the model the volume flow at the low-pressure side of the compressor is assumed to be constant and described by equation (6.10). Figure 6.10 shows the measured relation between the pressure at the low-pressure side of the compressor $p_{Lc}$ and the volume flow at the low-pressure side of the compressor $V_{Lc}$. It confirms equation (6.10) within 7%.

The molar flow rate in the CFHX is described by equation (6.6). For both CFHXs

Figure 6.10: Volume flow against the pressure, both at the low-pressure side of the compressor.
we measured and calculated $\dot{n}$ using the measured $p_H$, $p_L$, $\lambda$, and $T_L$ for different settings. These results are plotted in figure 6.11a. Theory and experiment agree within 5%. We can also calculate $\dot{n}$ from the calculated $p_H$, $p_L$, $\lambda$, and $T_L$ using equations (6.4) to (6.12). These results are given in figure 6.11b. Also in figure 6.11b theory and experiment agree nicely.

Another parameter that is both measured and calculated, is the pressure drop over the CFHX. The model predicts a very small pressure drop of 20 mbar. Unfortunately, we cannot measure such small pressure drops, because the calculated pressure drop is within the accuracy (100 mbar) of the sensors.

Now we compare the measured and calculated minimum temperature $T_L$ and the temperature difference between the hot and the cold gas in the CFHX. Measurements are done with different $\lambda$, $\dot{Q}_L$, $V_{Lc}$, and $p_L$. For 37 capillaries these results are shown in figure 6.12, for 331 capillaries in figure 6.13. For the CFHX with 37 capillaries the measured and modelled results agree within 10%. The errors in the heat load and the volume flow at the low-pressure side of the compressor affect the calculated $T_L$ and $\Delta T$ as can be seen in figure 6.13, around 10%.

For the CFHX with 331 capillaries there is a larger discrepancy between measurement and theory. One of the main reasons for this discrepancy is the effective length of the CFHX, which is smaller than the real length due to entrance effects for the gas flow. For practical reasons the CFHX with $N=331$ has a length of 10 cm. So, for this CFHX we have two components each with twice the entrance effects. Another reason for the discrepancy between the measured and calculated temperatures is the way the
Figure 6.12: Measured and calculated temperature difference (figure a) and lowest temperature (figure b) for the CFHX with 37 capillaries for different settings of the system. The lines give the one-to-one correspondence.

Figure 6.13: Measured and calculated lowest temperature and temperature difference for the CFHX with 331 capillaries for different settings of the system.
gas flow is distributed in between the capillaries. The gas flowing along the tubes first enters a chamber around the outside capillaries to distribute the gas. Afterwards, it spreads in between the capillaries and flows down. However, the hot incoming gas, is precooled in the chamber before it flows along the capillaries (see figure 6.14a).

![Figure 6.14: Schematic diagrams of the CFHX. The arrows indicate the flow directions. Figure a shows the entrance for the gas in between the capillaries. Inside the hexagon the capillaries are situated. The labels 1, 2, and 3 indicate the positions of the thermometers. Figure b shows a cross section of the heat exchanger. In the hatched areas the flow is less.](image_url)

Besides, the flow is not distributed homogeneously in between the capillaries. In the areas marked in figure 6.14b the flow is less. As a result of these two effects there is a temperature gradient over the heat exchanger in radial direction as can be seen in figure 6.15, which shows the measured $T$-inhomogenity in the horizontal plane of the CFHX.

### 6.5 Expansion of the model

This model describes the complete CFPTR. It is relatively simple to adjust it to other geometries. Here we will show the adjustments needed to the model to use it with extra heat exchangers and for the four-valve method. We will also use the model to show that we can reach 30 K with the CFPTR.

#### 6.5.1 Extra heat exchangers

Three step heat exchangers (for description see section 8.2.1) can be placed in front of the heat exchanger with $N=331$. This results in a larger pressure drop over the total heat exchanger (step heat exchangers and heat exchanger with capillaries together). However, the pressure drop is so small that we ignore this effect and still use the
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Figure 6.15: Temperature gradients along the CFHX. The positions on the x-axis refer to Fig.6.14a. At each position three temperatures are measured, at the top, the middle, and the bottom of the CFHX. The triangles give the temperature of the gas flowing inside the capillaries measured at the connection tubes. The temperature of the gas flowing in between the capillaries is measured at the connection tubes (squares) and at the outside wall of the CFHX (circles).

equations (6.11) and (6.12). The temperature of the gas flowing in the heat exchanger with \( N=331 \) at high pressure is changed from \( T_H \) to

\[
T_w = T_H - 3\Delta T. \tag{6.14}
\]

Using this in equations (6.4) to (6.12), the model predicts a 5 K lower \( T_L \) than for the system without the extra heat exchangers. This is in agreement with the measurements. Using just the heat exchanger with 331 capillaries gives a lowest temperature \( T_L=165 \) K. Including the step heat exchangers reduces the temperature to \( T_L=160 \) K. Measurements agree with the model with the same typical uncertainties as given in figure 6.13.

6.5.2 Four-valve method

The so-called four-valve method can be used to increase the efficiency of pulse-tube refrigerators [4][5]. It uses four switching valves: two for the gas flow to the regenerator and two to control the gas flow to the hot end of the pulse tube. We look again at the time dependence of the pressure in the pulse tube as given in figure 6.1. In the four-valve method gas enters the pulse tube in phase \( ab \) via its hot end. During phase \( bc \) gas enters the pulse tube via the heat exchanger as usual. During phase \( cd \) gas leaves the pulse tube via its hot end. Phase \( da \) is unchanged; gas leaves the pulse tube via the heat exchanger. In this way the compressibility of the gas in the pulse tube is compensated by an extra gas input or output at the hot end of the tube (\( n_{h4} \)), instead
of by gas flowing through the heat exchanger. The gas flow \( \dot{n}_{h4} \) does not have to be cooled by the heat exchanger and is not causing dissipation in the heat exchanger. This will increase the performance of a PTR [6].

The model described mathematically by equations (6.4) to (6.12) can be modified to the four-valve method. In that case the flow \( \dot{n}_{h4} \) through the heat exchanger only consists of the gas entering the low-temperature side of the pulse tube during phase bc of figure 6.1. So

\[
\dot{n}_{h4} = 2\nu n_{Lbc} = 2\nu \frac{A_L L_{by} p_H}{R T_L} = 2\nu f_{4v} \frac{V_{pl} p_L}{R T_L},
\]

(6.15)

with

\[
f_{4v} = \lambda \pi^{2/5}. \tag{6.16}
\]

This function replaces \( f_1 \) in equations (6.5) and (6.6). It is the same as \( f_1 \) given by equation (6.4) except for the first term. In the expression for volume flow at the low-pressure side of the compressor (equation (6.10)) we need to replace \( \dot{n} \) by

\[
\dot{n} = \dot{n}_{h4} + \dot{n}_{h4},
\]

(6.17)

with

\[
\dot{n}_{h4} = \frac{6}{5} \nu \frac{V_{l} (p_H - p_L)}{R T_H}, \tag{6.18}
\]

which is calculated using an equation similar to equation (2.55).

Also in the compressor power (equation A.53)

\[
P = \frac{\dot{n} R T_H}{\kappa_c} \ln \frac{p_{hc}}{p_{lc}},
\]

(6.19)

we use \( \dot{n} \) as given in equation (6.17). Working out the new set of equations, describing the four-valve method, leads to two new relations between \( T_L \) and \( \pi_H \). Again one is cubic and one is forth order in \( T_L \).

As an example of the consequences we compare the performance of an orifice PTR to the same system working with the four-valve method. The input parameters as discussed in section 6.2 are the same in both cases. In figure 6.16 the minimum temperatures are given as functions of the frequency for \( \dot{Q}_L = 0 \) (solid line) and \( \dot{Q}_L = 10 \) W (dotted line). Using the four-valve method results in a lower temperature and more cooling power. However, the power needed to drive the compressor also increases (see figure 6.17a). In figure 6.17a the two components of the power, caused by the flow through the heat exchanger (\( \dot{n}_{h4} \)) and the flow to the hot end of the pulse tubes (\( \dot{n}_{h4} \)), are also given. In figure 6.17b the two flow components are given as functions of the frequency. The gas flowing to the hot end of the pulse tubes is just a small fraction of \( \dot{n}_{h4} \).
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Figure 6.16: Calculated lowest temperatures as functions of the frequency for $\dot{Q}_L=0$ (solid lines) and $\dot{Q}_L=10$ W (dotted lines), both for an orifice PTR (OPTR) and a system working with the four-valve method.

Figure 6.17: Comparison of the orifice PTR (OPTR) to a system using the four-valve method. In figure a the frequency dependent compressor power is given. The contributions to the total power, due to the flow through the heat exchanger ($l_4$) and due to the flow to the hot end of the pulse tubes ($h_4$), are also given. The two components of the flow in the four-valve method are given in figure b as functions of the frequency as well as the flow in the orifice pulse-tube refrigerator.
6.5.3 CFPTR at 30 K

In order to analyze the potential of the heat exchanger further we also apply the model to systems in which the lowest temperature gets below 30 K. Four possible geometries satisfying this goal are given in table 6.2. In the first column the present system, as described in table 6.1, is used. However, to reach temperatures below 30 K with this geometry, two heat exchangers with \( N = 1261 \) and \( L_x = 1 \) m in parallel are needed. If we change \( V_t, \nu, \) and \( \lambda, \) we can reach 30 K with much simpler systems. The geometry described in the second column still uses quite large pulse tube volumes. However, compared to the first column the heat exchanger needed is much simpler. Especially significantly decreasing the volume of the pulse tubes has a big influence on the performance. Looking at the last two columns, it can be seen that a simple heat exchanger (small amount of capillaries with reasonable diameters and lengths), can be used to reach 30 K if the volume of the pulse tubes is reduced to \( V_t = 0.5 \) cm\(^3\).

<table>
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<tr>
<th>heat exchanger</th>
<th>symbol</th>
<th>value</th>
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<td>( N )</td>
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<tr>
<td></td>
<td>( P ) (W)</td>
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<td>6</td>
<td>7</td>
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</tbody>
</table>

Table 6.2: System parameters resulting in a lowest temperature below 30 K.

6.6 Conclusion

We derived a complete set of relations for the operation of a CFPTR. The input parameters depend on the working fluid, the geometries of the pulse tube and the counterflow heat exchanger, and the compressor characteristics. It describes the performance of the system reasonably well. The calculated lowest temperature and temperature difference between the hot and cold gas in the CFHX agree within 10% with the measured data for the heat exchanger with \( N = 331 \). For the heat exchanger with \( N = 37 \) the trends of the measurement results and the experimental results differ. The calculated and measured molar flow agree within 5%. The model can be applied to a variety of other geometries. Adding extra heat exchangers or using the four-valve method increases the performance of the system. In most cases, treated in this chapter, the lowest temperatures are in the range of 100 K to 200 K. The model shows that, the
CFPTR can reach 30 K with simple heat exchangers for small pulse tubes volumes.
Bibliography


Chapter 7

Rotary valves

Most GM-type PTRs use a rotary valve to generate pressure oscillations in the system. In most commonly used valves the rotor is pressed tightly against the stator and large driving torques are needed. In this chapter we discuss two different types of valves, in which the forces on the rotor are balanced. The rotor of the first type makes no mechanical contact with the stator. We will call it the 'no-contact' valve. The second type is characterized by balanced forces but the stator and the rotor do make mechanical contact. We will call it the 'balanced contact' valve.

To operate a counterflow pulse-tube refrigerator a special rotary valve is needed. The principle is the same as for a conventional PTR, but the specific design differs [1], since in a CFPTR, two subsystems are used, operating in opposite phase. Therefore, extra connections are needed on the rotary valve. In this chapter we will describe these valves and discuss the advantages and the disadvantages.

7.1 Contact rotary valve: classical valve

To be able to make a fair comparison between the rotary valves with balanced forces and the classical valve where the rotor is pressed tightly against the stator, we first need to discuss the classical valve. This rotary valve, used in many systems [2][3], consists of a rotor and a stator placed in a housing. It is driven by a high-power motor. In the example in figure 7.1 one channel (1) in the stator leads to the low-pressure side of the compressor ($p_l$) and two channels (2) provide a connection to the regenerator of the PTR ($p_{reg}$). The valve drawn in figure 7.1 is used for the conventional PTR. The rotor has a slit (4) and two openings (3). A spring keeps the rotor in position. In figure 7.1a channels 1 and 2 are connected via 4, so the PTR is connected to the low-pressure side of the compressor. In figure 7.1b the rotor is turned 90°. Now the PTR is connected to the inside of the valve housing, which is under high pressure ($p_h$),
Figure 7.1: Schematic representation of a classical rotary valve consisting of the rotor, the stator, the housing, and the motor. In figure a the PTR is connected to the low-pressure side of the compressor; in figure b to the high-pressure side. Figure c gives the cross section of the stator. Figure d represents the cross section of the rotor.
via 2 and 3. During rotation of the valve, the channels 2 are alternatively connected to the high- and the low-pressure sides of the compressor.

The force, with which the rotor is pressed against the stator, is typically 100 N. A typical torque $\tau = 1.2 \text{ Nm}$ [1] is needed to keep the rotor in motion. These large forces make the valve liable to wear. In order to reduce the torques needed to rotate the valve, the 'no-contact' and 'balanced contact' rotary valves have been designed. They will be discussed below.

7.2 ‘No-contact’ valve

7.2.1 Design

The 'no-contact' rotary valve is drawn schematically in figure 7.2. It has been designed in collaboration with Sumitomo Heavy Industries. It consists of a rotor placed in a stator. In the stator connections are made to the high- and the low-pressure sides of the compressor ($p_h$ and $p_l$), and to the two subsystems of the CFPTR ($p_{t1}$ and $p_{t2}$). For each component two channels are made opposite to each other, so the forces on the rotor are balanced. In the rotor there are two types of channels: in the radial plane and in the axial direction. The channels in the radial plane provide the connections of the subsystems to the high- and low-pressure sides of the compressor respectively. The channel in the axial direction makes sure that the pressure in the housing is uniform. The width of the slit between the rotor and the stator, indicated in figure 7.2a with $\delta$, is typically 10-15 $\mu$m. The rotor is held in position by two stainless steel ball bearings and is driven by a small DC motor placed in the housing\(^1\). The maximum torque of the motor is 1 Nm.

7.2.2 Forces on the rotor

Up to first order the forces on the rotor are balanced in all directions. In the radial direction they are balanced due to the opposing connections to the high- and the low-pressure sides of the compressor and the two subsystems, and in the axial direction by the axial channel in the rotor. With perfectly balanced forces the net torque on the rotor should be zero. However, in practice there is a small nonzero torque.

The torque, needed to drive the rotor, can be estimated from the electrical current and the voltage of the motor. The current is proportional to the frequency because the friction is proportional to the angular velocity. The current also increases with increasing pressure amplitude. The voltage is constant and typically 2.2 V for a frequency of 1.1 Hz. A typical current profile is given in the upper part of figure 7.3.

\(^1\)Motor specifications: Maxon DC motor, RE 25, nr. 118745 and Maxon gearbox nr. 110396.
Figure 7.2: Schematic cross sections of the 'no-contact' rotary valve. The rotor is placed in the stator, without making mechanical contact to it. In the stator opposing connections are made to the high- and the low-pressure sides of the compressor, and to the two subsystems of the CFPTR. The channels in the rotor connect one subsystem to the high-pressure side, while the other subsystem is connected to the low-pressure side. The rotor is driven by a small DC motor. The axial channel makes sure that the pressure in the housing is uniform.
7. Rotary valves

Figure 7.3: Time dependences of the pressures and the current using the ‘no-contact’ rotary valve. The pressures at the high- and the low-pressure sides of the compressor \((p_h \text{ and } p_l)\), at the hot ends of the two subsystems \((p_{t1} \text{ and } p_{t2})\), and inside the housing of the rotary valve \((p_{hs})\) are given in the lower part. The current of the motor is given in the upper part. At \(a\) the valve starts to connect the subsystems to the compressor. At \(b\) the valve is completely open. At \(c\) the valve starts to close, and at \(d\) the valve is completely closed.

In the lower part of this figure the pressures at the high- and the low-pressure sides of the compressor \((p_h \text{ and } p_l)\), at the hot ends of the two subsystems \((p_{t1} \text{ and } p_{t2})\), and inside the housing of the rotary valve \((p_{hs})\) are given. The dashed vertical lines give the moments of opening and closing of the valve. At \(a\) the valve starts to open. The high pressure of the compressor starts to decrease and the low pressure increases. At \(b\) the valve is completely open. The high pressure of the compressor reaches a minimum and the low pressure a maximum. At \(c\) the valve starts to close. The high pressure of the compressor increases and the low pressure decreases. At \(d\) the valve is completely closed. The high and low pressures of the compressor are constant.

The current increases during the time period that the valve closes or is closed (from \(c\) to \(a\)). The torque on the rotor can be estimated from the power \(P\) consumed by the motor as follows. The electrical power is equal to the mechanical power \(P_m\) needed to keep the valve turning, so

\[
P = VI = P_m = \tau \omega. \tag{7.1}
\]

The base current is 17 mA (see figure 7.3), which is equal to the ‘no-load’ specifications of the motor. With \(V=2.2\ \text{V}\) and \(\omega=1.7\ \text{rad/s}\), the base torque is \(\tau=21.6\ \text{mNm}\) (millinewton meter). When the valve closes, the current increases to 22 mA. The torque at peak value is 28 mNm. The difference between the base torque and the peak
value is 6.4 mNm.

This torque on the rotor might be due to a change in momentum of the gas. In principle the flow phenomena in the slit and around the holes are very complicated as we are dealing with a compressible gas at velocities close to the speed of sound. Still we will make an effort to estimate this force by making some very crude assumptions. We consider the situation given in figure 7.4. The pressure in subsystem 1 is low and

\[ p_{t1} \]

Figure 7.4: Schematic drawing of the mass flow through the slit between the rotor and the stator in the 'no-contact' rotary valve at the moment the valve is about to connect subsystem 1 to the high-pressure side. Figure b gives a magnification of the part of the valve where the gas is flowing. The arrows represent the directions of the mass flow.

The valve is about to connect it to the high-pressure side. Gas is flowing from the compressor connections in the stator (see figure 7.4b) to subsystem 1 through the slit with velocities close to the speed of sound (c). The speed of sound is reached, because the ratio of the pressures in the two channels \( p_{t1} / p_h = 0.5 \) [4]. The mass flow can be estimated from

\[ \dot{m} = \rho \dot{V}, \quad (7.2) \]

with

\[ \rho = \frac{M_4 p_h}{RT}, \quad (7.3) \]

and

\[ \dot{V} = cd_c \delta. \quad (7.4) \]

In these formulae \( \rho \) is the density, \( p_h \) the pressure at the high-pressure side, \( \dot{V} \) the volume flow, \( M_4 \) the molar mass of helium, \( d_c \) the diameter of the rotor channel, and
Figure 7.5: Time dependences of the pressures (lower part) and the current (upper part) using the 'no-contact' rotary valve. The pressures at the high- and the low-pressure sides of the compressor ($p_h$ and $p_l$), at the hot ends of the two subsystems ($p_{t1}$ and $p_{t2}$), and inside the housing of the rotary valve ($p_{hs}$) are given in the lower part. The current of the motor is given in the upper part. The time interval with a high current is indicated with arrows.

The gas collides with the wall of the rotor channel and the direction of the velocity vector of the gas has to be changed towards the centre of the rotor channel. The force realizing this, is equal to

$$F = \dot{m} c = C_p \frac{p_h}{C_v} d_c \delta,$$

where we used

$$c^2 = \frac{C_p}{C_v} \frac{RT}{M_4}.$$ 

Substituting actual values ($\delta=12\, \mu m$, $d_c=4\, mm$, $p_h=16\, bar$) gives $F=0.12\, N$. As the valve is symmetrical in the radial direction, this force appears four times on the valve. The total reaction force is $0.48\, N$. Using $r=12.5\, mm$ gives a torque of $6\, mNm$. This value corresponds surprisingly well with the value of $6.4\, mNm$ found in the experiment as mentioned before.

In figure 7.5 the time dependence of the pressures and the current is given again, but now for a larger time interval. A system cycle is defined as the period in which both subsystems are connected once to $p_h$ and once to $p_l$. It has a time period $t_c$. 
Somewhat surprising the current shows a periodicity over two system cycles. This means a periodicity over half a turn of the rotor, for each complete turn results in four system cycles. This periodicity and the large increase in current which happens ones in this period, can be explained by the fact that both the rotor and the stator are oval shaped on the scale of the slit width (see figure 7.6). The deviation from a perfect circle has been measured. For the rotor it is 1.7 µm and for the stator it is 1 µm. This means that the width of the slit varies between 8 and 10 µm. When the rotor is in the position as given in figure 7.6a, the slit width between the rotor and the stator is at its minimum in the hatched areas, and it takes less effort to turn the rotor. This can be seen from equation (7.6), which shows that the force of the gas is proportional to the slit width $\delta$. So a smaller slit results in a smaller torque and less current is needed from the motor to keep the rotor turning. If the rotor continues to turn the slit becomes larger and more current is needed to drive the rotor. If the rotor is in the position as given in figure 7.6b, the slit width is at its maximum in the hatched areas and a large torque is needed to turn the rotor. This increase is indicated in figure 7.5 with the arrows. After half a turn the rotor is back in the position as given in figure 7.6a.

### 7.2.3 Leak rate

The rotor and the stator are separated by a slit, through which gas leaks. This leak flow does not reach the pulse tubes and therefore does not contribute to the cooling power. However, if the valve is at low temperatures such as in CFPTR2, it flows through the heat exchanger. So it contributes to the losses in the heat exchanger. Besides, it causes a heat load on the rotary valve as will be discussed in the next section.
We will now estimate this leak rate through the valve. Consider two surfaces at a distance $\delta$. They are separated by a slit with a typical width $d_c$ and length $L$ (see figure 7.4). The molar flow $\dot{n}_{lk}$ from a high pressure $p_h$ to a low pressure $p_l$ through the slit can be calculated using

$$\dot{n}_{lk} = \frac{p_h^2 - p_l^2}{2\eta Z RT} \quad (7.8)$$

with $Z$ a flow impedance given by

$$Z = \frac{12L}{d_c \delta^3} \quad (7.9)$$

with $\eta$ the viscosity. These expressions are valid for compressible laminar flows at conditions far from 'choking', i.e. the exit Mach number should be much smaller than unity. They cannot be used to describe the gas flow at the moment of opening the valve, because then the gas velocities are close to the speed of sound. However, during most of the cycle the Mach numbers are around 0.03, and these expressions can be used. We operate the system CFPTR2 with the orifice closed (so in basic mode). Using the pressures as given in figure 7.7 and $L=1.6$ mm, the calculated total leak rate is $\dot{n}_{lk}=0.023$ mol/s. The total flow going into the valve is 0.087 mol/s. This means that 25% of the total flow leaks away in the rotary valve.

The leak flow can also be estimated from the time dependence of $p_h$ when the valve and the orifice are closed.

Figure 7.7: Pressure profiles in the system with the 'no-contact' rotary valve for the high- and low-pressure sides of the compressor and the two pulse tubes. The dotted lines give the moments the valve starts to close (c) and starts to open (a).
7. Rotary valves

\[ *n_{lk} = \frac{C_V}{C_p} \frac{d \left( V_t p_t / RT \right)}{d\ell}, \]  

(7.10)

with \( T \) the average temperature in the tube which is 250 K in this measurement and \( V_t \) the volume of the pulse tube. The pressure derivative is calculated between the dotted lines in figure 7.7, which indicate the moments the valve starts to close (c) and starts to open (a). Then the leak rate is \( *n_{lk} = 0.029 \text{ mol/s} \). So the leak rate in the rotary valve is around 30% of the total molar flow.

7.2.4 Heat load

Using the rotary valve at low temperatures, there are three parasitic energy flows in the valve, causing a heat load. The dissipated motor input power can be calculated from equation (2.41) and is \( P = 0.5 \text{ W} \). The energy flow due to the heat-shuttle effect can be calculated from equation (2.43) and is \( \dot{Q}_{hsv} = 2.7 \text{ W} \). The third heat load is due to the internal gas leak in the rotary valve. The associated energy flow can be estimated from

\[ \dot{Q}_{lk} = C_p *n_{lk} \Delta T \]  

(7.11)

with

\[ *n_{lk} = \frac{p_1 p_0 Z}{RT_v}. \]  

(7.12)

In this equation \( Z \) is the resistance for the flow through the slit, which is of the form as described in equation (7.9) and \( T_v \) the temperature of the rotary valve. The energy flow due to the gas leak is \( \dot{Q}_{lk} = 17 \text{ W} \). This adds up to a total heat load of 20 W.

7.2.5 Discussion

The 'no-contact' rotary valve has been designed to reduce the pressure losses and the forces on the valve. The observed torque of 28 mNm in the 'no-contact' valve is much smaller than the 1.2 Nm torque in the classical valves [1][2][3]. Due to the low friction hardly any maintenance is needed on the valve. The leak rate of the gas flow between the rotor and the stator is around 30%. If the valve is operated at low temperatures this causes a heat load of 17 W. Other heat loads are caused by the input power of the motor (\( P = 0.5 \text{ W} \)) and the heat-shuttle effect in the motor housing (\( \dot{Q}_{hsv} = 2.7 \text{ W} \)). The heat load due to the leak rate can be reduced by decreasing the slit width to values below 10 \( \mu \text{m} \). However, our current design with the 'off the shaft' ball bearings does not allow this.
7. Rotary valves

The valve can get blocked by particles larger than the slit size. However, our experience in the lab with this type of valve has shown that the chances of the valve getting stuck are very small. It happened once during one year of operation.

7.3 'Balanced contact' valve

7.3.1 Design

The ‘balanced contact’ rotary valve has been designed in order to overcome the risk of blockages of the ‘no-contact’ valves, and to decrease the leak rate and the dissipation in the valve. The ‘balanced contact’ valve is schematically drawn in figure 7.8. The main parts are the rotor and the stator, which do make mechanical contact. In the rotor (figure 7.8b) the number of oval-shaped channels is equal to four. In the stator (figure 7.8c) connections are made to the high- and the low-pressure sides of the compressor and to the two subsystems. The major point of the ‘balanced contact’ valve is that

![Diagram of 'balanced contact' valve](image)

Figure 7.8: Schematic representation of the ‘balanced contact’ rotary valve. The rotor and the stator do make mechanical contact. Four oval-shaped channels are made in the rotor (figure b). In the stator (figure c) connections are made to the high- and the low-pressure sides of the compressor and to the two subsystems. This configuration provides that the forces on the rotor are balanced. The rotor is driven by a small DC motor.
the areas of the channels to \( p_l \) and \( p_h \) in the contact area between the rotor and the stator are equal, while the classical valve, shown in figure 7.1, only had a connection to \( p_l \) in the contact area [1][2][3]. In figure 7.8a, in the stator, only the connection to the high-pressure side of the compressor is drawn. The other connections in the stator are made in the same way, but the horizontal channels are positioned at different heights. The rotor is made of rulon. The stator is made of brass. The surface of the stator has been smoothened and chromed. The rotor is driven by a small DC motor.

### 7.3.2 Forces on the rotor

The pressure in the housing around the rotor is \( p_{hs} \). The spring on the axis of the rotor is only to keep the rotor in place during mounting and demounting and gives a negligible force on the rotor. The force, pressing the rotor against the stator, can be calculated using equation (7.13)

\[
F_i = \int (p_{hs} - p(\vec{r}, t)) \, dA. \tag{7.13}
\]

The integrand is nonzero only in the contact area. The pressure in the contact area is \( p(\vec{r}, t) \) and varies in position and time. The position vector of surface element \( dA \) is \( \vec{r} \). The torque, needed to keep the rotor in steady rotation, is proportional to the pressing force \( F_i \).

The ideal pressure of the housing gives \( F_i = 0 \) during the whole cycle, meaning that there is no net force on the rotor. This ideal pressure is around the average of the high and low pressures. However, during the cycle, the pressure distribution \( p(\vec{r}, t) \) changes. If, at some time in the cycle, the pressure distribution results in a negative force, the rotor will be lifted a bit. Gas will flow from the high-pressure side of the compressor to the housing. The pressure in the housing will rise until it is high enough to result in a positive \( F_i \) during the whole cycle.

A typical current profile is given in the lower part of figure 7.9. In the upper part of this figure the pressures are given for the high- and the low-pressure sides of the compressor (\( p_h \) and \( p_l \)), at the hot ends of the pulse tubes of the two subsystems (\( p_{h1} \) and \( p_{h2} \)), and inside the housing (\( p_{hs} \)). The variations of \( p_{hs} \) have been multiplied by a factor ten. The dashed lines give the moments of opening and closing of the valve as in figure 7.3. The current is high when the valve closes or is closed (between \( c \) and \( a \)). The maximum torque needed to rotate the rotor can be calculated from the so-called torque constant \( \tau_c = 2.9 \) Nm/A. The voltage of the motor is constant and 4.6 V for a system frequency of 0.9 Hz. The torque \( \tau = 0.29 \) Nm. This is a factor ten higher.

\footnote{Motor specifications: Maxon DC motor, RE 25, nr. 118745 and Maxon gearbox, nr. 110396.}
Figure 7.9: The upper part gives the pressures in the CFPR using the 'balanced contact' rotary valve. The pressures at the high- and the low-pressure sides of the compressor (\(p_h\) and \(p_l\)), at the hot ends of the two pulse tubes (\(p_{t1}\) and \(p_{t2}\)), and inside the housing (\(p_{hs}\)) are given. The variations of the pressure in the housing have been multiplied by a factor ten. The current of the motor is given in the lower part. At a the valve starts to open, at b the valve is completely open, at c the valve starts to close, and at d the valve is completely closed.

than the torque needed for the 'no-contact' valve, but still a factor four lower than the torque needed for the classical valve.

The pressure in the housing \(p_{hs}\) can be changed manually via a valve which is not shown in figure 7.8 (it is shown in figure 8.1). It increases when the housing is connected to the high-pressure side of the compressor, and decreases when it is connected to the low-pressure side of the compressor. If we increase \(p_{hs}\), the torque increases.

For a torque of 0.29 Nm the typical leak rate, calculated with equation (7.10), is 1% of the total flow. The leak rate through the valve decreases with increasing \(p_{hs}\). Increasing the pressure amplitude in the system increases the torque of the motor as well.

### 7.3.3 Influence pressure profile

The shape of the pressure profile is influenced by the dimensions and the shape of the holes in the rotor and the stator [5][6][7]. In the experiments reported here, both the length \(l_r\) and the width \(d_r\) of the holes in the rotor (indicated in figure 7.8) are varied. The connections in the stator have the constant dimensions \(d_s=3\) mm, \(l_s=6\) mm. The rotor has a diameter of 32 mm. The total cycle time of the system is \(t_c\). The waiting time \(t_w\) is the time during which the valve is closed. The various widths, lengths, and
### Table 7.1

<table>
<thead>
<tr>
<th>number</th>
<th>$l_r$ (mm)</th>
<th>$d_r$ (mm)</th>
<th>$t_w/t_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.9-11.9</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>9.6</td>
<td>3</td>
<td>0.57</td>
</tr>
<tr>
<td>3</td>
<td>11.5</td>
<td>3</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>11.5</td>
<td>6</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>14.3</td>
<td>3</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>15.6</td>
<td>3</td>
<td>-0.15</td>
</tr>
<tr>
<td>7</td>
<td>12.9</td>
<td>3</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 7.1: Different widths and lengths of the holes in the rotor used for the pressure wave-shape experiments. The corresponding ratios of the waiting times to the cycle time are given as well.

### Figure 7.10

The different rotors used in the experiments. The numbers refer to table 7.1.

Relative waiting times are given in table 7.1. Pictures of the rotors with corresponding number are given in figure 7.10. In rotor 1 the slits are made at different diameters. Rotor 2 has the same waiting time as rotor 1, but it is symmetrical. Rotor 3 is also symmetrical, but has a smaller waiting time than rotor 2. Rotor 4 has the same waiting time as rotor 3, but now the contact area between the rotor and the stator is decreased by enlarging $d_r$ and making extra holes in the rotor (see figure 7.11). The idea is that for rotor 4, the pressure distribution is controlled in a larger part of the contact area and the torque is smaller. However, the leak rate is larger. The rotors 5 to 7 have again the same design principle as rotor 3. Out of curiosity we designed rotor 6, which has a negative $t_w$, meaning that the high- and low-pressure sides of the compressor are connected for some time during the cycle.

To compare the performance of the systems with the different rotors we mounted them in CFPTTR2 with the three-staged step heat exchanger (see section 8.2.1). Two different types of pulse tubes are used. Due to some design error pulse tubes 1 do
not have good flow straighteners. Pulse tubes 2 do have good flow straighteners (see section 8.4.2). Also two types of motors are used: a small and a large one. The various orifices have been adjusted in each experiment to reach the lowest temperature. For the different rotors the minimum temperatures and the peak-to-peak amplitude $\Delta p_{\text{hs}}$ of the oscillations of $p_{\text{hs}}$ can be seen in figure 7.12. The measurements with a waiting time larger than 0.27 of the cycle time are done with a small motor and pulse tubes 1. There is a heat load on the lowest temperature, produced by turbulence in the pulse tubes. The small motor is barely strong enough to keep the system going. It gives an irregular pressure profile (in time), which might cause the larger $\Delta p_{\text{hs}}$.

At $t_w/t_c=0.27$ we see three different points for rotor 3. The largest $T_L$ and $\Delta p_{\text{hs}}$ are measured with the small motor and pulse tubes 1. By increasing the power of the motor, $T_L$ and $\Delta p_{\text{hs}}$ decrease because the pressure profile becomes more regular in time. Afterwards pulse tubes with better flow straighteners were introduced (pulse tubes 2), and $T_L$ and $\Delta p_{\text{hs}}$ decreased again because there is less turbulence in the tubes. With pulse tubes 2 the optimal frequency is higher and $p_1$ is smaller. Both these effects lead to a lower leak rate in the valve and a lower $T_L$ as well.

The measurements with $t_w/t_c < 0.27$ have been done with the larger motor and pulse tubes 2. Rotor 4, gives a higher $T_L$ and $\Delta p_{\text{hs}}$ than rotor 3 although $t_w/t_c$ is almost the same. The reduction in contact area between the rotor and the stator increased the leak rate between the housing and the channels in the stator (leading to an increase of $\Delta p_{\text{hs}}$) and probably also to an increase in the leak rate between the channels. Both these effects lead to an increase of the dissipation in the valve and result in a higher $T_L$.

Decreasing $t_w/t_c$ more results in an increase in $T_L$ and $\Delta p_{\text{hs}}$. By making the slots in the rotor larger the leak rate and dissipation increase and $T_L$ increases. The lowest temperature with the smallest $\Delta p_{\text{hs}}$ is found at $t_w/t_c=0.27$. 

Figure 7.11: *Schematic representations of rotor 3 and rotor 4. The dotted lines give the holes in the stator as reference.*
Figure 7.12: Lowest temperature (upper curve) and amplitude of the pressure oscillation inside the valve housing (lower curve) for the different rotors as function of relative waiting time. The rotor numbers are indicated. Three different system configurations are used. The points have been connected for visual aid only.

### 7.3.4 Heat load on the valve

This valve can operate at room temperature and at low temperatures. When operating at low temperatures, it introduces a heat load. Again, this heat load consists of three contributions: the dissipated motor input power $P=1$ W, the heat load due to the leak flow $\dot{Q}_{lk}=1.5$ W, and the heat load due to the heat-shuttle effect $\dot{Q}_{hsv}=2$ W. The total heat load is 4.5 W.

### 7.3.5 Discussion

Advantages of the ‘balanced contact’ valves are the small torques, the easy manufacturing, adjustable valve timing, small dissipation, and no danger of blockages of the valve. Since the rotor and the stator make contact, the valve needs maintenance and small particles might get into the system. The total heat load on this valve is a factor of four smaller than on the ‘no-contact’ valve mainly due to the lower leak rate. The pressure in the contact area between the rotor and the stator is of key importance for this valve. It depends strongly on the detailed structure of the rotor and the stator surfaces on the microscopic scale, and therefore it is difficult to control.
7. Rotary valves

7.4 Conclusions

In this chapter two types of rotary valves have been discussed. The valves have several advantages. Both of them are based on the principle that the forces on the rotor are balanced, making sure that the driving torques are small. This results in less power needed to operate the valve. The timing of the valves can be chosen by changing the width of the rotor channels.

The 'no-contact' valve has as the extra advantage that the rotor and the stator make no contact, so it is not liable to wear and no maintenance is needed. Up till now one of our systems with 15 µm slits is working with this type of valve without any problems for over one year. However, the leak rate in the slit between the rotor and stator is fairly large which is not acceptable for CFPTDR2.

In the 'balanced contact' valve the rotor and the stator make mechanical contact. This type of valve has a small internal leak, is relatively easy to fabricate, and does not get stuck easily. The lowest temperature is found at $t_w/t_c=0.27$. 


Bibliography


Chapter 8

Single-stage counterflow systems

In this chapter the single-stage counterflow pulse-tube refrigerators are discussed. They are operated with a rotary valve at room temperature (CFPTR1) or at low temperatures (CFPTR2). Measurements are done with three different types of counterflow heat exchangers. The measurement results will be discussed and compared with theory.

8.1 Design

The two systems described in this chapter are based on a single set-up in which the rotary valve can be positioned either before or after the counterflow heat exchanger. Placing the rotary valve before the heat exchanger results in an oscillating flow through the heat exchanger (CFPTR1, as schematically given in figure 2.9a). Placing the rotary valve after the heat exchanger gives a steady flow in the heat exchanger (CFPTR2, as schematically given in figure 2.9b). Figure 8.1 is a more detailed schematic diagram of CFPTR2.

The system is operated by a 7.5 kW compressor and a 'balanced contact' rotary valve. The motor of the rotary valve is positioned at room temperature, even with the valve itself being at low temperatures. The pressure in the rotary valve housing can be controlled by two orifices, which are connected to the high- and the low-pressure sides of the compressor.

To make sure that the gas, flowing into the heat exchanger, is at room temperature, two water-cooled heat exchangers are used. A spiraled copper tube for the gas flow is inserted into a small container, through which the cooling water flows.

The two pulse-tube subsystems are identical. The dimensions of the pulse tubes are given in table 8.1. In the pulse tubes two different types of flow straighteners have been used. In pulse tubes 1, seven screens of mesh\(^1\) # 200 are used. However,

\(^1\)Mesh size is expressed in wires per inch.
Figure 8.1: Detailed schematic diagram of CF PTR2. 1-compressor, 2-rotary valve, 3,6,7-water-cooled heat exchangers, 4-counterflow heat exchanger, 5-pulse tubes, 8-orifices, 9-buffers, 10-rotary valve motor, 11,12-rotary valve orifices, 13-vacuum chamber, 14-compressor shunt valve. The pressure sensors are indicated with $p_i$. The most important temperatures $T_i$ are given. Two flowmeters are indicated by $f_1$ and $f_2$.

<table>
<thead>
<tr>
<th>inner diameter (mm)</th>
<th>24.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>outer diameter (mm)</td>
<td>25.4</td>
</tr>
<tr>
<td>length (mm)</td>
<td>275</td>
</tr>
<tr>
<td>volume (cm$^3$)</td>
<td>124.4</td>
</tr>
</tbody>
</table>

Table 8.1: Dimensions of the pulse tubes of CF PTR1 and CF PTR2.
due to the continuously changing pressure during operation of the system, the screens were tilted. Therefore, we made a new design (pulse tubes 2) in which the flow straighteners are mechanically fixed to the pulse tubes. The results of both types of pulse tubes are reported here, to show the difference between pulse tubes with and without proper flow straighteners. In pulse tubes 2, sintered phosphor bronze plates with particle size $d_p=100$ µm are used. Using equation (2.31) the minimum thickness for one plate is $s=5.1$ mm. However, in experiments we saw that a flow straightener performs better if it consists of stages of a few plates, separated by rings, instead of one thick layer. The gas flow redistributes after each step, which results in a nicely distributed flow in the tubes. In pulse tubes 2, a three-stage flow straightener is used. The three layers of phosphor bronze plates have a thickness of 1.7 mm each. The total thickness of phosphor bronze is 5.1 mm as calculated. The pressure drop over the flow straighteners can be calculated using equation (2.27) and typically is 300 Pa.

At the hot end, the pulse tubes are cooled by two different types of heat exchangers. One type (indicated with 6 in figure 8.1) is identical to the water-cooled heat exchangers described before. During the first experiments it turned out that the hot end of the tubes became too hot, especially during cool down. Therefore, an extra heat exchanger was installed (indicated with 7 in figure 8.1). It consists of a water-cooled copper block, which is placed around the hot ends of the pulse tubes, to extract the heat directly.

Both systems have a first orifice, and a buffer volume of 1 l. The temperatures are measured with calibrated resistors. For pressure measurements differential pressure sensors are used. The flows are measured with flow meters. Two flow meters are used. One measures the total flow supplied by the compressor. The other measures the flow in the system, which can be different from the compressor flow due to the compressor shunt valve (indicated with 14 in figure 8.1).

A picture of the low-temperature part of the system, which is normally placed in the vacuum chamber, is given in figure 8.2. Before the picture is taken, the system is run in open air for several minutes. Ice is formed at the coldest part of the system as can be seen in the picture.

### 8.2 Heat exchangers

The designs of the three types of counterflow heat exchangers, used in this investigation, will be discussed here. We used step heat exchangers, capillary heat exchangers, and a tube-in-tube heat exchanger. In all heat exchangers we assume equal $m\cdot c_p$ in the high- and low-pressure sides of the heat exchanger.
8. Single-stage counterflow systems

8.2.1 Step heat exchanger

The first heat exchanger tested is a three-stage step heat exchanger (as shown in figure 8.2). Ideally a step heat exchanger has a temperature distribution as given in figure 8.3. The essential feature is that the temperature of the gas varies in steps rather than continuously as in the heat exchangers to be discussed later. In the ideal case the temperatures of the gas flows leaving the heat exchanger are equal to that of the heat exchanger body $T_b$, so

$$T_b = \frac{1}{2} \left( T_{hi} + T_{ci} \right). \quad (8.1)$$

The efficiency, using equation (2.21), is

$$\varepsilon = \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} = 0.5. \quad (8.2)$$
The efficiency of a single step heat exchanger is too low to lead to a good performance of the CFPTR. Using a few step heat exchangers in series rapidly increases the efficiency. With three step heat exchangers $\varepsilon=0.75$. Advantage of the step heat exchanger is that the temperatures of the heat exchanger body and the gases leaving the heat exchanger are well defined. We tested the step heat exchanger just for scientific purposes, to improve our knowledge of the CFPTR.

The heat exchanger used is a three-stage step heat exchanger. Every stage is made of a copper body with two copper tubes wound around it. The tubes have an inner diameter $d_1=2$ mm and an outer diameter of 4 mm. The 3.5 turns are made on a coil with diameter 35 mm. The length $L_X$ of each copper tube used in one stage is 0.5 m. A picture of one stage of the step heat exchanger is given in figure 8.4.

Figure 8.4: Picture of one stage of the step heat exchanger. It shows the copper tubing before it is soldered to the copper body.

The actual temperature of the gas flow leaving the heat exchanger can be calculated from the energy balance. We look at one channel. The temperature of the wall $T_b$ is uniform. The characteristic symbols are given in figure 8.5. The energy balance gives

$$d\dot{Q} = -\dot{n}C_p dT = Nu\kappa g \pi (T - T_b) dl.$$  

(8.3)

Introducing the characteristic length $L_c$

$$L_c \equiv \frac{\dot{n}C_p}{Nu\kappa g \pi},$$

(8.4)

gives

$$(T - T_b) = (T_{hi} - T_b) \exp \left(-l/L_c\right).$$

(8.5)
8. Single-stage counterflow systems

$$T_{ho} = 0.0155 (T_{hi} - T_b) \approx T_b.$$

(8.6)

8.2.2 Capillary heat exchangers

According to chapter 5, heat exchangers with many capillaries in parallel should give a good performance. We made three heat exchangers with a bundle of capillaries in one big hexagonal tube. The capillaries are arranged in a regular triangular array as given in figure 8.6. Three parameters are defined in figure 8.6; the outer diameter of the tubes $D$; the distance between the tube centres (pitch) $P$; and the distance from...
Figure 8.7: Schematic cross section of the support structure of the heat exchanger with 37 capillaries.

the wall $W$. There is one channel in the middle with $n$ hexagonal rows of capillaries around it. The total number of capillaries $N$ is [1]

$$N = 1 + 3n(n + 1). \quad (8.7)$$

One gas flow passes through the capillaries while the other gas flow passes along the capillaries. The area in between the capillaries is divided into three kinds of regular subchannels, indicated with hatched areas in figure 8.6. For each subchannel the formulae for the flow area, wetted perimeter, and friction factor as functions of $P$, $D$, and $W$ are given in Appendix B.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$N$</th>
<th>$d_1$</th>
<th>$D$</th>
<th>$W$</th>
<th>$P$</th>
<th>$L_X$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1.0</td>
<td>1.2</td>
<td>1.34</td>
<td>1.34</td>
<td>200</td>
<td>0.683</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>0.8</td>
<td>1.0</td>
<td>1.16</td>
<td>1.16</td>
<td>200</td>
<td>0.785</td>
</tr>
<tr>
<td>10</td>
<td>331</td>
<td>0.8</td>
<td>1.0</td>
<td>1.20</td>
<td>1.20</td>
<td>200</td>
<td>0.972</td>
</tr>
</tbody>
</table>

Table 8.2: Dimensions of the capillary heat exchangers used. The efficiency is calculated at $T=200$ K, $p=20$ bar and $n^*=0.1$ mol/s. The dimensions are in mm.

In this investigation we have built three heat exchangers, based on this principle. The dimensions are given in table 8.2. The inner diameter of the capillaries is $d_1$. The efficiency, given in table 8.2, is calculated using equation (2.22) at $T=200$ K, $n^*=0.1$ mol/s, and $p=20$ bar. A schematic cross section of the support structure of the heat exchanger with 37 capillaries is given in figure 8.7. The capillaries (not shown in the figure) are fixed to the SS support flange by a hard-soldering technique. This technique is developed especially for this application by the coworkers of the central workshop of the Eindhoven University of Technology (GTD). The heat exchanger with
8. Single-stage counterflow systems

331 capillaries has flow straighteners in the cap to prevent uneven flow through the capillaries. The gas that has to flow along the capillaries first enters a ring around the capillaries (see figure 8.7). Here the flow can spread. Afterwards, it flows along the capillaries. The heat exchangers with 7 and 37 capillaries consist of one single unit with $L_X=20$ cm. The soldering of the caps took place in a vacuum-oven, which was too small to place the heat exchanger upright inside. Therefore, the heat exchanger was lying inside. When the capillaries were heated they expanded and due to gravity they were bent. The stress on the capillaries was so large that they did not recover to their original position when they were cooled down. They are touching on one side of the heat exchanger cross section, while there are no capillaries at all on the other side. Exaggerated it would look somewhat like figure 8.8. To prevent bending of the capillaries for $N=331$ this heat exchanger consists of two parts of 10 cm each, which are soldered standing upright inside the vacuum-oven.

![Figure 8.8](image)

Figure 8.8: *Cross section of the capillary heat exchanger with uneven distribution of the capillaries.*

8.2.3 Tube-in-tube heat exchanger

A fairly simple continuous heat exchanger is a coiled tube-in-tube heat exchanger. The tubes are made of stainless steel with the dimensions as given in table 8.3. The efficiency can be calculated using equation (2.22). At $T=200$ K, $p=20$ bar, and $\dot{n}=0.1$ mol/s the efficiency is $\varepsilon=0.90$.

<table>
<thead>
<tr>
<th></th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>$D$ (mm)</th>
<th>$L_X$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner tube</td>
<td>3.7</td>
<td>4.0</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>outer tube</td>
<td>5.0</td>
<td>6.0</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>coil</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Table 8.3: *Dimensions of the coiled tube-in-tube heat exchanger with $d_i$ the inner diameter, $d_o$ the outer diameter, and $D$ the coil diameter.*
8. Single-stage counterflow systems

8.3 Results

The results of CFPTR1 and CFPTR2 with the three different types of heat exchangers will be described and compared to the theory. We will start with the set-up of the three-stage step heat exchangers.

8.3.1 Step heat exchangers in CFPTR1

When the step heat exchangers are used with the rotary valve at room temperature (so in CFPTR1), they will work as regenerators because the large mass of the heat exchanger cannot oscillate in temperature. The temperatures of the gas leaving a specific block are equal to the temperature of the block (see figure 8.3). So the temperature of the gas varies in steps. In fact we have a so-called step regenerator. Based on energy conservation there should be constant temperature differences between the blocks. This is confirmed by experiment as shown in figure 8.9. The lowest temperature reached with this configuration is 199 K.

![Figure 8.9: Temperature distribution of the step heat exchangers used in CFPTR1 with \(n=0.14\) mol/s. The arrows indicate that the flow is oscillating in counterphase in both channels. The numbers are temperatures in K.](image)

8.3.2 Step heat exchangers in CFPTR2

When the step heat exchangers are used in CFPTR2 one channel is always connected to the high-pressure side while the other channel is connected to the low-pressure side. Based on energy conservation the temperature differences between gas at the high- and low-pressure sides should be constant. Figure 8.10 gives a typical measured temperature profile. It can be seen that the temperature differences indeed are the same everywhere. The efficiency of each heat exchanger block is 0.5 as predicted with equation (8.2), so the total efficiency is indeed 0.75. The lowest temperature reached with the step heat exchangers in CFPTR2 is \(T_L=202\) K.

Operating the system with the step heat exchangers, two different types of flow straighteners in the pulse tubes have been tested as discussed in section 8.1. The
8. Single-stage counterflow systems

$$\begin{align*}
291.5 & \quad 275.1 & \quad 259.1 & \quad 243.0 \\
\Delta=16.3 & \quad \Delta=16.1 & \quad \Delta=16.1 & \quad \Delta=16.0 \\
275.2 & \quad 259.0 & \quad 243.0 & \quad 227.0
\end{align*}$$

Figure 8.10: Temperature distribution of the step heat exchangers used in CFPTR2 with $n=0.14$ mol/s. The flow directions are indicated with the arrows. The temperature differences between the gas flows are given with $\Delta$. The numbers are the temperatures in K.

Figure 8.11: Temperature distribution along the pulse tubes for two different types of flow straighteners. The position $l=0$ is at the cold end of the pulse tubes.

Temperature profiles of the pulse tubes, measured on the pulse-tube walls, are given in figure 8.11. The position $l=0$ is at the cold end of the pulse tubes. Both pulse tubes have water-cooled hot heat exchangers, which are positioned at $l=27.5$ cm. The sintered plates in pulse tubes 2, have such a high heat capacity that they act as regenerators. Therefore, the lowest temperature of pulse tubes 2, is not at the bottom of the pulse tubes, but just above the flow straighteners. The pulse tubes with the sintered plates (pulse tubes 2) reach the lowest temperature. This is caused by the fact that the screens in pulse tubes 1, are tilted as discussed in section 8.1. This causes turbulence in the pulse tubes and a heat load on the lowest temperature. In all the other experiments described in this chapter pulse tubes 2, are used.

The energy balance is analyzed using the equations given in section 2.4. In the no-load situation the calculated net cooling power of the system $\dot{Q}_{Ln}$ should be zero (see equation (2.32)). For this situation the ideal cooling power $\dot{Q}_L$ and the various losses (enthalpy flow in the heat exchanger $\dot{H}_{hx}$, losses in the rotary valve, heat conduction
in the pulse tube, and heat shuttle effect in the pulse tube \( \dot{Q}_{\text{hs}} \)) are calculated for a number of pressure amplitudes in the pulse tubes and given in figure 8.12. In this figure, the enthalpy flows due to the non-ideality of \(^4\text{He} \), the two losses in the rotary valve, and the heat flow in the pulse tubes walls are added and indicated as \( \dot{Q}_{\text{loss}} \). The enthalpy flow in the heat exchanger is indicated with \( H_{\text{hx}} \), the heat shuttle-effect in the pulse tubes as \( \dot{Q}_{\text{hs}} \). The calculated net cooling power \( \dot{Q}_{\text{Ln}} \), should be zero in this no-load situation. The lines are for visual aid only.

8.3.3 Capillary heat exchangers in CFPR2

Heat exchangers with \( N=7 \) and \( N=37 \)

Now we will describe the results of the capillary heat exchangers with 7 and 37 capillaries. Typical temperature distribution of these heat exchangers are given in figure 8.13.

The measured and calculated efficiencies for different flows, using equations (B.1) to (B.7), are given in figure 8.14. For the heat exchanger with \( N=7 \) the measured and calculated efficiency agree nicely. For the heat exchanger with 37 capillaries the measured efficiency is 10% lower than the calculated values. It is likely that this is caused by the fact that the capillaries are bent. This results in an uneven flow distribution around the capillaries and reduced heat exchange. The lowest temperature reached with the heat exchanger with \( N=7 \) is \( T_\text{L}=218 \) K, while with \( N=37 \) \( T_\text{L}=214 \)
Figure 8.13: Typical temperature distributions of the capillary heat exchangers with $N=7$ (figure a) and $N=37$ (figure b), with $\dot{n}=0.14$ mol/s.

Figure 8.14: Calculated and measured efficiencies for the capillary heat exchangers with $N=7$ and $N=37$ used in CFPT2. The full lines connect the measured points. The dotted lines connect the calculated points.
Figure 8.15: Calculated and measured pressure drops for flow through and along the capillaries in the heat exchangers with $N=7$ and $N=37$ measured in CFPTR2. The dotted line gives the one-to-one correspondence.

K is obtained.

The pressure drops for the flows through and along the capillaries are measured and calculated as given in figure 8.15. In the calculations the equations (B.1) to (B.14) are used. Within about 100 mbar both heat exchangers agree with the theoretical values. As a general conclusion we can say that both heat exchangers perform as expected, only the thermal performance of $N=37$ is too low.

**Heat exchanger with $N=331$**

Now the results, obtained with the heat exchanger with $N=331$, will be discussed. In order to investigate entrance effects, this heat exchanger has two opposite entrances for the gas flow passing along the capillaries, instead of one, as indicated in figure 8.7. The flow going along the capillaries has been connected in three different ways as shown in figure 8.16. First we used configuration a. It turned out that the performance of the heat exchanger did not meet our expectations. The measured efficiency was almost 10% too low (see figure 8.17). Then we tested configuration b. The performance was better than with configuration a, but still the measured $\varepsilon$ were lower than the calculated values (see figure 8.17). The efficiency is higher because the flow is forced to pass the capillaries in the radial direction in configuration b (see the dotted lines in figure 8.16), increasing the heat exchange. This difference in performance is also seen in the lowest temperature, which is for configuration a $T_L=172$ K, while configuration b reaches $T_L=165$ K. For both configurations it turned out that the temperature in the horizontal plane is not constant as would be expected for an even distribution of
Figure 8.16: Schematic diagrams of the three different configurations for the flow along the capillaries of the heat exchanger with \( N = 331 \). Figure a has all connections on one side. Figure b has connections on opposing sides. Figure c shows the case with asymmetric connections: it has two opposing connections between the two heat exchanger components and one connection at the top and the bottom of the total heat exchanger. The temperatures of the in- and outgoing gas flows are indicated. The arrows indicate the flow directions. The dotted lines give extreme gas flow paths. The grey areas mark the lee-sides of the system in which the gas flow is reduced.

Figure 8.17: Measured and calculated efficiencies of the heat exchanger with 331 capillaries for the configurations a and b measured in CFTR2. The dotted line gives the theoretical prediction.
The temperature distributions of the three configurations are studied in more detail. A diagram of one heat exchanger component with the positions of the thermometers is given in figure 8.18. The temperatures, indicated with $T_A$ to $T_F$, are measured on the outside of the heat exchanger so they correspond with the temperatures of the gas flowing along the capillaries. The temperatures $T_1$ to $T_4$ are measured on the connection tubes.

![Diagram of heat exchanger component](image)

Figure 8.18: Schematic diagram of one heat exchanger component. The positions of the thermometers are indicated.

The measured temperature profiles are given in figure 8.19 for the situation that warm (high-pressure) gas is flowing through the capillaries and the cold (low-pressure) gas along the capillaries. In the ideal case the temperature in the horizontal plane of the heat exchanger should be constant. However, this is not the case in any of the configurations. Temperature differences as large as 15 K are observed. The discrepancy is caused by two effects. The gas flowing along the capillaries first enters a ring around the capillaries, to distribute the gas flow evenly along the capillaries (see figure 8.7). However, the hot incoming gas is precooled in this ring before it flows along the capillaries. This introduces an asymmetry in the cooling of the gas flow. Furthermore, the flow is not distributed homogeneously in between the capillaries. In the lee-areas, marked in figure 8.16 by the grey zones, there is less flow. These two effects result in the temperature gradients in the horizontal plane. As the heat exchanger with $N=331$ is made of two components, the flow is distributed inhomogeneously twice. Therefore, the measured efficiencies of the heat exchanger are significantly smaller than the calculated values (see figure 8.17).

In all the calculations we assume a flow in the heat exchanger, which is constant in time. However, the rotary valve is closed during a short period of time (10% of the cycle time), so the flow is interrupted periodically. In a separate experiment we inserted two buffer volumes ($V=42$ cm$^3$) between the heat exchanger and the rotary
Figure 8.19: Temperature profiles of the three different configurations as described in figure 8.16. The closed points are the temperatures measured on the outside of the heat exchanger body. The open points are measured on the connection tubes. The dotted lines connect the temperatures measured at the same vertical level.

As said before (see section 2.2) the two pulse tubes are operated in opposite phase. The hot ends of the pulse tubes can be connected directly with an orifice. The buffers at room temperature can be omitted. This reduces the total system volume with 2 l. The lowest temperature, reached with the hot ends connected, is 170 K. This is 5 K higher than when the buffer volumes are used. However, the buffers are only omitted in a few measurements to prove that the CFPTR still performs well without them. In most measurements the buffer volumes are used, because the pressures in the buffers give valuable information on system characteristics. The difference in lowest temperatures for the CFPTR might be caused by incomplete optimization of the system.
8. Single-stage counterflow systems

8.3.4 Capillary heat exchanger in CFPTR1

From the capillary heat exchangers only the one with \( N = 331 \) is tested in CFPTR1. Five types of experiments are compared here, which are described below.

1. The measurements, as described in the previous section, will be presented here as well (so measurements of CFPTR2). The rotary valve is at low temperatures and there is a steady (DC) flow in the heat exchanger. The heat exchanger performs based on the DC heat-exchange effect.

2. In these measurements the rotary valve is at room temperature. There is an oscillating (AC) flow in the heat exchanger. Both subsystems are connected, so the gas flow is passing both through and along the capillaries, but always in counterphase. In this case we look at the temperature of the subsystem that is connected to the gas flow passing along the capillaries. The performance is based on a combination of the AC heat-exchange effect and the regenerative effect.

3. In this case the same measurements are done as in case 2. However, we now look at the temperature of the subsystem that is connected to the gas flow passing through the capillaries. The performance is based on a combination of the AC heat-exchange effect and the regenerative effect.

4. In these measurements the rotary valve is also at room temperature, but now only one of the subsystems is connected. There is only oscillating gas flow along the capillaries. So there is no heat exchange. The performance is based on the regenerative effect only.

5. In this case the same measurements are done as in case 4. However, we now look at the temperature of the subsystem, which is connected to the gas flow passing through the capillaries. The performance is based on the regenerative effect only.

For all cases the valve settings are optimized for different frequencies. The results are given in figure 8.20. To have a fair comparison, the results of case 1, where the rotary valve is at low temperatures, are corrected for the heat load caused by the rotary valve. From figure 8.20 it can be seen that the system performs the worst if there is only DC heat exchange (case 1).

Now we will compare the cases 2 to 5, which are measured with the rotary valve at room temperature. The lowest temperatures are reached in the cases 3 and 5, where the performance is given of the subsystem that is connected to the gas flow passing through the capillaries. This might be due to the fact that the flow in the capillaries
Figure 8.20: Lowest temperatures reached with the capillary heat exchanger with \( N = 331 \), as functions of the frequencies. Five different cases are shown. 1-DC heat-exchange effect, 2-AC heat-exchange effect and regenerative effect for the subsystem connected to the gas flow passing along the capillaries. 3-AC heat exchange effect and regenerative effect for the subsystem connected to the gas flow passing through the capillaries. 4-Regenerative effect for the subsystem connected to the gas flow passing along the capillaries. 5-Regenerative effect for the subsystem connected to the gas flow passing through the capillaries.

is nicely distributed, while the gas flowing along the capillaries is not (see previous section). Comparing the cases 4 and 2 (both for the subsystems connected to the gas flow passing along the capillaries), it can be seen that case 2 gives lower temperatures. The only difference between the cases 4 and 2, is that in case 2 we have besides the regenerative effect also the AC heat-exchange effect, which apparently improves the performance. However, this positive effect for the subsystem connected to the gas flow passing along the capillaries, is a negative effect for the subsystem connected to the gas flow passing through the capillaries. This is seen when comparing the cases 3 and 5. The only difference between them, is the extra AC heat-exchange effect which we have in case 3. This extra effect increases the lowest temperature reached.

The AC heat-exchange effect in the heat exchanger, can be estimated using equation (2.12). For the wall of the capillaries we can calculate \( \tau \) as given in figure 8.21 for \( \nu = 1 \) Hz. Figure 8.21 shows that the heat exchanger will start working as a good heat exchanger if \( T < 30 \) K. For the operation temperature region of CFPTI (\( T > 150 \) K) the heat exchange is poor.
8. Single-stage counterflow systems

Figure 8.21: Ratio of the amplitudes of the gas and the wall temperatures $\tau$ as function of the temperature for the capillary heat exchanger with $N=331$ for $\nu=1$ Hz.

8.3.5 Tube-in-tube heat exchanger in CFPR2

The performance of CFPR2 is optimized by changing the frequency and the valve settings, using the tube-in-tube heat exchanger as described in section 8.2.3. The optimum temperatures of both pulse tubes are given in figure 8.22. The optimum temperature reached with this configuration is $T_L=184.6$ K, which is 20 K higher than the lowest temperature of the heat exchanger with 331 capillaries.

Figure 8.22: Frequency dependence of the optimum temperatures for the tube-in-tube heat exchanger with CFPR2.

In figure 8.23 the measured and calculated efficiencies (using equations (2.16) to (2.22)) of the heat exchanger are given. The measured efficiency is 3% lower than the
calculated $\varepsilon$. This is caused by the fact that the inner tube is not concentric with the outer tube. At most places the inner tube will touch the outer tube. The effective heat exchange area is smaller, resulting in a lower efficiency. This heat exchanger has a 5% lower thermal efficiency than the heat exchanger with 331 capillaries. However, due to the definition of the efficiency (see section 2.3.2), this 5% difference makes a huge difference in the performance of the heat exchanger.

Finally, we compare the measured and calculated pressure drops using the equations (4.11) to (4.21) over the inner and outer tube, in figure 8.24. The pressure drop over the inner tube is much lower than over the outer tube due to the larger hydraulic diameter. The measured values of the inner tube are higher than the calculated ones because of the difficult geometry of the entrance of the gas flow, causing extra entrance
effects, which are not taken in to account. Looking at the outer tube we see that the measured pressure losses are slightly lower than the calculated ones. Again this might be due to the fact that the inner tube touches the outer tube.

8.4 Conclusions

We successfully built a CFPTR that operates from room temperature to 150 K. Three different types of heat exchangers are tested to understand and optimize the system. A three-stage step heat exchanger, a coiled tube-in-tube heat exchanger, and three heat exchangers with 7, 37, or 331 capillaries have been used. The overall best results are summarized in figure 8.25, where we plotted the lowest temperatures and the corresponding efficiencies for all the heat exchangers. Due to the definition of the efficiency this cannot be extrapolated to a $T_L$ at $\varepsilon=1$.

![Figure 8.25: Summary of the overall best performances of the heat exchangers.](image)

Both for the system with AC flow in the heat exchanger (CFPTR1) as for the system with DC flow in the heat exchanger (CFPTR2) the measurements and the calculations agree with each other. The different loss mechanisms of the system have been quantified. The capillary heat exchanger with $N=331$ gives the best performance. The temperature range of the system can be increased further by increasing the length of the heat exchanger (as discussed in section 6.6.3) or by reducing the negative entrance effects.
Bibliography

Chapter 9

Hybrid system

In this research we show that temperatures below 50 K are needed for an optimal performance of the counterflow heat exchanger. Therefore, we built a hybrid counterflow pulse-tube refrigerator (CFPTR3). The hybrid system consists of a CFPTR precooled by a conventional PTR. Hybrid systems are used very commonly nowadays (for example in [1] [2]) and they are very successful. However, the performance of these hybrid systems is not well understood. In this chapter we will describe the first preliminary experimental results of CFPTR3.

9.1 CFPTR3

The chapters 2, 5, 6, and 8 show that, from a theoretical and experimental point of view, heat exchangers can be used to replace regenerators in the high-temperature region. However, it is also shown in these chapters, that the counterflow heat exchangers for large systems, designed to reach temperatures below 50 K starting from 300 K, would be very complicated and expensive. Therefore, we have chosen to build a hybrid system that uses precooling and regenerators at the high-temperature part of the cooler and counterflow heat exchangers at the low-temperature part (CFPTR3).

A schematic diagram of CFPTR3 is given in figure 9.1. It consists of a precooling stage, which is a conventional PTR (the first stage), and a CFPTR operating at low temperatures (the second stage). The two stages have separate gas systems. Both are operated by 6 kW compressors and 'no-contact' rotary valves. Each pulse tube has an orifice, a double-inlet orifice, and a minor orifice to optimize the performance. The buffer volumes are typically ten times the volumes of the pulse tubes.

The dimensions of the components are given in table 9.1. The two subsystems of the second stage are identical, as usual. The flow straighteners in the pulse tubes are made of sintered phosphor bronze plates with particle size $d_p=100 \ \mu m$. Using
Figure 9.1: Schematic diagram of CFPTR3. 1-compressors, 2-rotary valves, 3-first-stage regenerator, 4-second-stage regenerators, 5-precooling heat exchanger, 6-counterflow heat exchanger, 7-first-stage pulse tube, 8-second-stage pulse tubes, 9-hot heat exchangers, 10,11-buffers, 12,13-first orifices, 14,15-double-inlet orifices, 16,17-minor orifices, 18-heat shield, 19-vacuum chamber. The most important temperatures $T_i$ are given. The pressure sensors are indicated with p.

<table>
<thead>
<tr>
<th>stage</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>pulse tube</td>
<td>inner diameter (mm)</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>outer diameter (mm)</td>
<td>36.1</td>
</tr>
<tr>
<td></td>
<td>length (mm)</td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>volume (cm$^3$)</td>
<td>212</td>
</tr>
<tr>
<td>regenerator</td>
<td>inner diameter (mm)</td>
<td>56.3</td>
</tr>
<tr>
<td></td>
<td>outer diameter (mm)</td>
<td>57.7</td>
</tr>
<tr>
<td></td>
<td>length (mm)</td>
<td>221.5</td>
</tr>
<tr>
<td></td>
<td>volume (cm$^3$)</td>
<td>551</td>
</tr>
</tbody>
</table>

Table 9.1: Dimensions of the pulse tubes and the regenerators of CFPTR3.
equation (2.31) the minimum thickness for the plates is \( s = 5.1 \) mm. A three-stage flow straightener is used as described in section 8.1. The pressure drop over the flow straighteners is calculated with equation (2.27) and is typically 300 Pa.

The composition of the regenerators is given in table 9.2. To get a good distribution

<table>
<thead>
<tr>
<th></th>
<th>material</th>
<th>size</th>
<th>mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) stage</td>
<td>phosphor bronze screens</td>
<td>mesh # 200</td>
<td>1581 g</td>
</tr>
<tr>
<td>2(^{nd}) stage</td>
<td>sintered stainless steel</td>
<td>porosity 51%</td>
<td>243 g</td>
</tr>
<tr>
<td></td>
<td>sintered stainless steel</td>
<td>porosity 46%</td>
<td>268 g</td>
</tr>
</tbody>
</table>

Table 9.2: Composition of the regenerators of CFPTR3.

of the gas flow in the first stage regenerator several screens with mesh size # 10 are placed at the top and the bottom of the regenerator. The high-temperature parts of the second-stage regenerators are filled with material with a high porosity, while the other halves are filled with material with a lower porosity. This is done because of the density change of the gas with temperature.

Through the precooling heat exchanger the first stage extracts heat from the second stage. The precooling heat exchanger consists of four units made of copper blocks with lengths of 33 mm with three channels (see figure 9.2). In the channel in the middle

![Figure 9.2: Schematic diagram of a unit of the precooling heat exchanger.](image)

(indicated with 1) flows gas of the first stage. The diameter of this channel is varied in experiments. In the channels on the side (indicated with 2a and 2b) flows gas of the two subsystems of the second stage. The diameter of these channels is \( d = 1.5 \) mm.

The units can be arranged in series or in parallel to compose different precooling heat exchangers.

The counterflow heat exchanger has a coiled tube-in-tube configuration. The tubes are made of stainless steel with dimensions as given in table 9.3.

The coldest parts of the first and second stages are placed inside a heat shield, which is precooled by the first stage. A picture of the low-temperature part of the system, which is normally inside the vacuum chamber is given in figure 9.3.

To measure temperatures above 50 K calibrated resistors are used. Temperatures below 50 K are measured with calibrated diode thermometers. For pressure measurements differential pressure sensors are used.
Table 9.3: Dimensions of the counterflow heat exchanger used in CFPTR3, with $d_i$ the inner diameter, $d_o$ the outer diameter and $D$ the coil diameter.

<table>
<thead>
<tr>
<th></th>
<th>$d_i$ (mm)</th>
<th>$d_o$ (mm)</th>
<th>$D$ (mm)</th>
<th>$L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner tube</td>
<td>3.7</td>
<td>4.0</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>outer tube</td>
<td>5.0</td>
<td>6.0</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>coil diameter</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 9.3: Picture of the low-temperature part of configuration b of CFPTR3.
9. Hybrid system

9.2 Preliminary results

9.2.1 Stand alone systems

Before we tested the hybrid system, we first tested the first and the second stages separately. The precooling heat exchanger is absent. The optimized first stage reached $T_{L1}=38.5$ K in the no-load situation and 44 K @ 20 W applied heat. The second stage, operated without the first stage, reached $T_{L2}=114$ K in the no-load situation.

9.2.2 Hybrid system

Here, the first results of the hybrid system will be described. The system will be tested more in further research that is not included in this thesis.

Precooling heat exchanger

The experiments have shown that the flow resistance of the precooling heat exchanger plays a crucial role in the performance of the system. Therefore, four different configurations have been tested (see table 9.4). We also included the results of the standalone systems with the label \( a \) in this table. The minimum temperatures are given. In this table $\Delta p_1$ is the pressure loss over the combination of the regenerator and the precooling heat exchanger for the first stage. The pressure loss $\Delta p_2$ over the combination of the regenerator, the precooling heat exchanger, and the inner tube of the counterflow heat exchanger for one subsystem of the second stage is given as well. The characteristic lengths, according to equation (8.4), and the actual lengths $L_m$ of the precooling heat exchanger are also given. Note that configuration \( b \) has a much higher temperature than the other cases. Therefore, the pressure losses are higher and the characteristic length is different.

From configurations \( b \) to \( e \) the diameter of the channels is increased, less units are used in series, and the flow is divided over units in parallel. The flow resistance of the

<table>
<thead>
<tr>
<th>label</th>
<th>configuration</th>
<th>$d_1$</th>
<th>$T_{L1}$</th>
<th>$T_{L2}$</th>
<th>$\Delta p_1$</th>
<th>$\Delta p_2$</th>
<th>$L_c$</th>
<th>$L_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>no precooling heat exchanger</td>
<td>1.0</td>
<td>249</td>
<td>211</td>
<td>2.0</td>
<td>0.3</td>
<td>4.1</td>
<td>13.2</td>
</tr>
<tr>
<td>b</td>
<td>4 units in series</td>
<td>1.5</td>
<td>73.4</td>
<td>53.1</td>
<td>1.3</td>
<td>1.8</td>
<td>8.2</td>
<td>6.6</td>
</tr>
<tr>
<td>c</td>
<td>4 units in series</td>
<td>1.5</td>
<td>73.4</td>
<td>53.1</td>
<td>1.3</td>
<td>1.8</td>
<td>8.2</td>
<td>6.6</td>
</tr>
<tr>
<td>d</td>
<td>2 units in series</td>
<td>1.5</td>
<td>67.7</td>
<td>48.7</td>
<td>1.2</td>
<td>1.5</td>
<td>4.5</td>
<td>6.6</td>
</tr>
<tr>
<td>e</td>
<td>2 units in series in parallel</td>
<td>1.5</td>
<td>67.7</td>
<td>48.7</td>
<td>1.2</td>
<td>1.5</td>
<td>4.5</td>
<td>6.6</td>
</tr>
</tbody>
</table>

Table 9.4: Lowest temperatures (in K) of both stages for five different configurations of the precooling heat exchanger. The pressure losses (in bar), the characteristic length $L_c$, and the actual length $L_m$ of the precooling heat exchanger (in cm) are also given. The inner diameter of channel $d_1$ is given in mm.
precooling heat exchanger decreases each step and the lowest temperatures of both stages go down. To show that the measured pressure losses are in agreement with the calculated losses we compare configurations c and d. With equations (4.2) and (4.5), combined with the irreversible loss coefficient of 90° bends in square ducts ($K=1.40$\cite[p.63]{3}), we can calculate the pressure losses for 4 units in series and 2 units in series, including all bends, for $\dot{n}=0.4 \text{ mol/s}$. The calculated difference between configurations c and d is $\Delta p=0.45 \text{ bar}$. In the measurements we found $\Delta p=1.7-1.3=0.4 \text{ bar}$ (see table 9.4). In the same way configurations d and e are compared, and also here the measurements are in agreement with the theory (0.1 bar and 0.14 bar respectively).

Configuration a, the standalone systems, has the lowest temperature of the first stage.

<table>
<thead>
<tr>
<th>configuration c</th>
<th>configuration d</th>
</tr>
</thead>
<tbody>
<tr>
<td>to regenerators</td>
<td>to regenerators</td>
</tr>
<tr>
<td>90.1</td>
<td>73.7</td>
</tr>
<tr>
<td>112</td>
<td>102</td>
</tr>
<tr>
<td>117</td>
<td>70.4</td>
</tr>
<tr>
<td>86.5</td>
<td>70.9</td>
</tr>
<tr>
<td>85.1</td>
<td>83.6</td>
</tr>
<tr>
<td>81.6</td>
<td>71.8</td>
</tr>
<tr>
<td>2a 1 2b</td>
<td>2a 1 2b</td>
</tr>
<tr>
<td>to pulse tubes</td>
<td>to pulse tubes</td>
</tr>
</tbody>
</table>

Figure 9.4: Temperature distributions around the precooling heat exchanger for configurations c and d. For configuration d the flow to pulse tube 1 cannot be measured, due to the equipment used.

The temperature distributions around the precooling heat exchanger in the configurations c and d are given in figure 9.4. In configuration c the temperatures of the gas going to the second stage pulse tubes (85.1 K and 83.6 K), are lower than the temperature of the last unit of the precooling heat exchanger (86.5 K). In configuration c, $L_m > L_c$ (see table 9.4), and we have a step heat exchanger. However, the measured temperature in configuration c is not the same as the temperature of the unit, but lower. This is caused by the oscillating flow. We measure the average temperature of the gas flowing to and from the pulse tube. If the flow comes from the pulse tube, the temperature is lower than the temperature of the unit. So the average temperature is smaller than $T_{phx}$.

In configuration d the temperatures of the channels, connected to the second stage pulse tubes (71.8 K and 70.4 K), are higher or the same as the temperature of the last unit of the precooling heat exchanger (70.9 K). This might be caused by the fact that in configuration d, $L_m < L_c$ (see table 9.4). In that case the heat exchanger does not
work as an ideal step heat exchanger. The temperature of the gas flow leaving the unit will be higher than the temperature of the unit, instead of being equal to it.

**Heating power - temperature relation first stage**

For configuration d we measured the relation between the heating power supplied to the precooling heat exchanger, and the temperatures of the precooling heat exchanger and the regenerator. The results are given in figure 9.5. The precooling heat exchanger reaches 48 K when 20 W of heating power is supplied.

Comparing configuration e to a (see table 9.4) it can be seen that \( T_{L1} \) has gone up and \( T_{L2} \) has gone down. Indeed the first stage is precooling the second stage. So heat from the second stage is removed (\( T_{L2} \) goes down) and taken up by the first stage (\( T_{L1} \) goes up). The amount of exchanged heat can be calculated from equation (2.39) using \( \Delta T \) as the temperature difference between the cold end of the second-stage regenerator (\( T_{r2a} \) and \( T_{r2b} \)) and the precooling heat exchanger (\( T_{phx} \)). With \( \Delta T=27 \) K and \( \dot{n}=0.26 \) mol/s this results in a heat load on the first stage of \( \dot{Q}_{L1}=130 \) W.

If the second stage is switched on, \( T_{L1} \) increases from 43 K to 70 K. So the increase is 27 K. Assuming a linear relation between the temperature and the heating power, as given in figure 9.5, gives an increase in temperature of 32 K @ 130 W. This is of the same order as the 27 K as mentioned before.

**Counterflow heat exchanger**

Another important component of the CFPTR is the counterflow heat exchanger (see figure 9.1). A typical temperature distribution over this heat exchanger is given in
Figure 9.6: Typical temperature distribution over the counterflow heat exchanger in CFPTR3.

In figure 2.8 the ratios of the amplitudes of the gas- and the wall-temperature oscillations for the inner wall are calculated with equation (2.12) as functions of the temperature for laminar and turbulent flow. In the heat exchanger typically $R_e=70,000$, so the flow is highly turbulent. From figure 2.8 it can be seen that already at room temperature, but certainly below 70 K, the temperature oscillations in the wall should follow the oscillations in the gas very well.

9.3 Conclusion and suggestions

We have built a hybrid system which is a CFPTR precooled by a conventional PTR. The experiments have shown that the flow resistance of the precooling heat exchanger plays a crucial role in the system performance. With two units in series in parallel, we have a small flow resistance combined with good heat exchange.

The hybrid system is a powerful tool for further investigation of the interplay between the heat exchange and regenerative effect. Extra regenerators filled with lead, can be placed between the precooling heat exchanger and the counterflow heat exchanger to lower the base temperature of the second stage ($T_{phx}$). Furthermore, capillary counterflow heat exchangers, as discussed in section 8.2.2, can be installed to enhance the performance.
Bibliography


Chapter 10

Summary

Pulse-tube refrigerators are new cryocoolers with no moving parts at low temperatures. So far all pulse-tube refrigerators have regenerators in which heat is stored and released during the cooling cycle. This research focuses on the basic concept of CounterFlow Pulse-Tube Refrigerators (CFPTRs). This special type of pulse-tube refrigerators consists of two identical pulse tubes operating in opposite phase, while the two regenerators are replaced by one counterflow heat exchanger. In this thesis special attention is paid to the different basic design options of the CFPTR, to the fundamental comparison of heat exchangers to regenerators, to the development of an analytical model of the CFPTR, and to the theoretical analysis of ideal pulse-tube refrigerators with real gases.

The main emphasis in this research has been on the question whether heat exchangers can be advantageous above regenerators. Regenerators and heat exchangers are compared from a fundamental point of view. The analysis is based on the various irreversible processes causing entropy production. It shows that for each operating temperature one can choose the system parameters in such a way that heat exchangers are better than regenerators.

Three different types of experimental CFPTRs have been developed (CFPTR1-3). In all systems different types of heat exchangers have been tested based on tube-in-tube designs. This concept is chosen because it allows high pressures with small pressure losses. The pressure losses in oscillating flow up to 20 Hz show no difference from the steady flow results. For higher frequencies there is a tendency for an increase in the pressure drop.

CFPTR1 and CFPTR2 are single-stage machines and have no regenerators at all. CFPTR1 has an oscillating flow in the heat exchanger and therefore cooling due to the regenerative and the heat-exchange effect. At temperatures above 70 K the heat-exchange effect is small, which is seen both in the measurements and in the theoretical
analysis. CFPTR2 has a steady flow in the heat exchanger, which is realized by placing the rotary valve after the heat exchanger. The rotary valve is specially designed to have balanced forces on the rotor. In this way the torque needed to drive the rotor and the friction heat are small. In this research a complete set of relations is derived, that describes the behavior of the system very well. The lowest temperature reached with CFPTR2 is 165 K, which is in agreement with the model.

CFPTR3 is a two-stage machine, which has regenerators but only at the high-temperature part. The second stage uses regenerators and heat exchangers. Both stages have their own compressor and rotary valve. They are connected only thermally. In this thesis the preliminary results are given.

In the theoretical analysis of pulse-tube refrigerators it is usually assumed that the working fluid is an ideal gas. However, the behavior of real systems depends on the real gas properties. It is shown that thermodynamic properties of the nonideal working fluid have a profound influence on the performance of the pulse-tube refrigerator at very low temperature. This explains one order of magnitude of the observed factor of one hundred discrepancy between experiment and theory. The analysis also shows that the real gas properties of the working fluid sometimes can result in a performance that is even better than in the ideal-gas case. This opens new ways for improvements.
Appendix A

Derivation expressions chapter 6

In this appendix we give a detailed derivation of the expressions governing the system performance as described in chapter 6. The derivation is based on the general assumptions given in the section Idealizations of chapter 6. The various subsystems are the pulse tubes with its heat exchangers, the rotary valve, the counterflow heat exchanger, and the compressor.

A.1 Pulse-tube expressions

In this subsection we derive relations for the molar flows and the energy flows in the pulse tubes as functions of $p_L$ and $p_H$, the orifice setting, and the tube dimensions.

A.1.1 Hot end

The gas, entering the tube from the hot end during the low-pressure phase of the cycle (phase da in figure 6.1), has temperature $T_H$. We introduce a dimensionless parameter $\lambda$, which is determined by the orifice setting,

$$\lambda = \frac{L_a}{L_t}$$

(A.1)

where $L_a$ is the maximum length over which the gas plug penetrates the pulse tube from the hot end (situation a in figure 6.1) and $L_t$ the length of the pulse tubes. We can also write

$$\lambda = C_0 \left[ \left( \frac{p_H}{p_L} \right)^2 - 1 \right],$$

(A.2)

with $C_0$ the dimensionless conductance of the orifice. For a square-wave pressure profile in the pulse tube we can write

\[1\] This appendix is based on an article [1], published in cryogenics.
A. Derivation expressions chapter 6

\[ C_0 = \frac{\pi C_V (p_H - p_L)}{\alpha C_p} \left[ \left( \frac{p_H}{p_L} \right)^2 - 1 \right]^{-1}, \]  
(A.3)

with

\[ \frac{1}{\alpha} = \frac{1}{\pi \nu (p_H - p_L)} \frac{V_B}{\dot{p}_B} \frac{d\dot{p}_B}{dt}. \]  
(A.4)

The amount of gas is

\[ n_H = \frac{L_a A_t p_L}{RT_H} = \lambda V_i p_L \]  
(A.5)

Here \( R \) is the molar ideal gas constant and \( V_i \) is the pulse-tube volume given by

\[ V_i = A_t L_t, \]  
(A.6)

where \( A_t \) is the cross-sectional area. After compression from \( p_L \) to \( p_H \) (situation \( b \) in figure 6.1) the temperature of this gas at the hot end rises to the temperature

\[ T_{Hb} = \pi_T T_H \]  
(A.7)

with, for helium,

\[ \pi_T = \left( \frac{p_H}{p_L} \right)^{2/5}. \]  
(A.8)

The length of the gas plug changes from \( L_a \) to

\[ L_b = L_a \left( \frac{p_L}{p_H} \right)^{3/5}. \]  
(A.9)

During the high-pressure phase \( bc \) the gas flows through the hot heat exchanger and the orifice into the buffer. The isobaric cooling of \( n_H \) moles per cycle from \( T_{Hb} \) to \( T_H \) gives a heat power release at one hot heat exchanger of

\[ \dot{Q}_{H1} = \nu n_H C_p (T_{Hb} - T_H) \]  
(A.10)

Here \( C_p \) is the molar heat capacity at constant pressure and \( \nu \) is the frequency given by

\[ \nu = \frac{1}{t_c}. \]  
(A.11)

The heat flow \( \dot{Q}_{H1} \) is equal to the average enthalpy flow in one pulse tube

\[ \overline{H_{t1}} = \dot{Q}_{H1}. \]  
(A.12)

With equations (A.5), (A.7), and (A.1)

\[ \dot{Q}_{H1} = \frac{C_p}{R} \nu V_i p_L \lambda (\pi_T - 1). \]  
(A.13)

The total average enthalpy flow in the two tubes is

\[ \overline{H_t} = 2 \frac{C_p}{R} \nu V_i p_L \lambda (\pi_T - 1). \]  
(A.14)
A. Derivation expressions chapter 6

A.1.2 Cold end

During the phase $ab$ the pressure is increased from $p_L$ to $p_H$. The length of the gas column, which is inside the tube in situation $a$, will be reduced from $L_t$ to $L_t/\pi_H^{3/5}$ in situation $b$. The temperature profile of the gas that has flown into the tube from the cold end during step $ab$ is given by

$$ T(l) = T_L \left( \frac{L_t}{L_t - l} \right)^{2/3} \text{ for } 0 \leq l \leq L_c $$  \hspace{1cm} (A.15)

with

$$ L_c = L_t - \frac{L_t}{\pi_H^{3/5}}. $$  \hspace{1cm} (A.16)

Relation (A.15) can be derived as follows: consider a gas element that enters the pulse tube ($l = 0$) when the pressure is $p$. Its temperature is $T_L$. When the pressure is equal to $p_i$ the position of this gas element is

$$ l_i = L_t - L_t \left( \frac{p}{p_i} \right)^{3/5} $$  \hspace{1cm} (A.17)

and the temperature

$$ T_i = T_L \left( \frac{p_i}{p} \right)^{2/5}. $$  \hspace{1cm} (A.18)

Eliminating $p/p_i$ from equations (A.17) and (A.18) gives equation (A.15).

The amount of gas, entering the tube from the cold end during phase $ab$ is, with equation (A.15),

$$ n_{Lab} = \frac{A_t p_H}{RT_L L_t^{2/3}} \int_0^{L_c} (L_t - l)^{2/3} \, dl $$  \hspace{1cm} (A.19)

or

$$ n_{Lab} = 3 \frac{V_t (p_H - p_L)}{5 RT_L}. $$  \hspace{1cm} (A.20)

During the phase $bc$ the pressure is constant. The temperature profile inside the pulse tube moves towards the hot end over a distance $L_b$. The amount of gas that flows into the tube from the cold end is

$$ n_{Lbc} = \frac{A_t L_b p_H}{RT_L}. $$  \hspace{1cm} (A.21)

The total amount of gas flowing into the tube at the cold end during phases $ab$ and $bc$ is

$$ n_L = n_{Lab} + n_{Lbc}. $$  \hspace{1cm} (A.22)

We introduce a function $f_1$ of the pressure ratio $\pi_H$ which helps simplifying the notation

$$ f_1 = \frac{3}{5} (\pi_H - 1) + \lambda \pi_T. $$  \hspace{1cm} (A.23)
With equations (A.23), (A.9), (A.20), and (A.21)

\[ n_L = f_1 \frac{p_L V_L}{RT_L}. \]  

(A.24)

The gas given by equation (A.24) flows into the tube with temperature \( T_L \). When it returns to the cold heat exchanger during the process steps \( cd \) and \( da \) it has an average temperature \( T_{Le} \) which is lower than \( T_L \). The value of \( T_{Le} \) can be derived from the enthalpy flow in one pulse tube as follows: the average enthalpy flow in one tube is given by

\[ \overline{\dot{H}_{11}} = \nu n_L C_p \left( T_L - T_{Le} \right) \]  

(A.25)

and by equation (A.12) so

\[ n_L (T_L - T_{Le}) = n_H (T_{Hb} - T_H). \]  

(A.26)

With

\[ \overline{\Delta T_L} = T_L - T_{Le} \]  

(A.27)

and equations (A.5), (A.7), and (A.24) we get an expression for the average temperature difference of the gas near the cold end of the tube

\[ \overline{\Delta T_L} = \frac{\lambda}{f_1} (\pi_T - 1) T_L. \]  

(A.28)

This important relation is the driving temperature difference behind the cooling of PTRs.

### A.1.3 Cold heat exchangers

Now we will take the energy balance of the cold heat exchangers into consideration. We assume that the valve is an ideal step heat exchanger. This means that gas, leaving the valve, always has a temperature which is equal to the valve temperature \( T_V \).\(^2\) If the heat load on one cold heat exchanger is \( \dot{Q}_{L1} \) the energy balance over one cold heat exchanger is

\[ \nu n_L C_p \left( T_V - T_L \right) + \dot{Q}_{L1} = \nu n_L C_p \Delta T_L. \]  

(A.29)

So the temperature difference between the valve and the cold heat exchanger of the tube is

\[ \Delta T = \Delta T_L - \frac{\dot{Q}_{L1}}{\nu n_L C_p}. \]  

(A.30)

With

\[ \dot{Q}_{L1} = \frac{1}{2} \dot{Q}_L, \]  

(A.31)

\(^2\)This is not an essential assumption. If the valve is not an ideal step heat exchanger the temperature reduction is just a fraction of \( \Delta T \).
where $\dot{Q}_L$ is the total heat load on the cold heat exchangers
\[ \dot{Q}_L = \dot{Q}_{L1} + \dot{Q}_{L2} \]  
(A.32)

we finally get
\[ \Delta T = \Delta T_L - \frac{\dot{Q}_L}{2\nu n_L C_p}. \]  
(A.33)

The average gas flow in the CFHX is
\[ \bar{n} = 2\nu n_L \]  
(A.34)

(in each pulse tube $n_L$ so twice $n_L$ per cycle) so
\[ \Delta T = \Delta T_L - \frac{\dot{Q}_L}{\bar{n} C_p}. \]  
(A.35)

With equation (A.24) equation (A.34) gives
\[ \bar{n} = 2\nu f^1 V_t p_L \frac{R T_L}{N u \pi \kappa}. \]  
(A.36)

**A.2 Valve**

The gas enters the valve at the high-pressure side with temperature $T_{VH}$. The enthalpy balance of the valve gives
\[ T_{VH} - T_V = T_V - T_L = \Delta T. \]  
(A.37)

For later use we express
\[ T_{VH} = T_L + 2\Delta T. \]  
(A.38)

**A.3 Counterflow heat exchanger**

In this subsection we derive the relations for the temperature profile and the pressure drops in the CFHX. Due to the simplified analytical relations (6.1) and (6.2), powerful analytical relations can be obtained which describe the behavior quite well.

**A.3.1 Temperature profile**

The molar flow in the CFHX is given by equation (A.36). The temperature difference $\Delta T$ between the two gas flows is obtained from equation (A.33). Energy conservation requires that this temperature difference is the same everywhere in the CFHX. The temperature gradient can be obtained from energy conservation and is given by
\[ \frac{\bar{n}}{N C_p} \frac{dT}{dz} = -N_u \pi \kappa \Delta T_w, \]  
(A.39)
here $\Delta T_w$ is the temperature difference between the gas and the wall and $z$ the length coordinate of the CFHX ($z = 0$ at the hot end). For laminar flow in circular tubes, as considered in chapter 6, the Nusselt number is \[ N_u = \frac{48}{T}. \] (A.40)

In a symmetrical situation
\[ \Delta T_w = \frac{1}{2} \Delta T. \] (A.41)

As a result, equation (A.39) with equation (6.1) gives
\[ \frac{dT}{dz} = -\frac{N_u \pi \kappa_0 \Delta T}{2\hat{n}C_p} \sqrt{T}. \] (A.42)

For the temperature $T_w$ of the gas in the warm side, integration of equation (A.42) gives
\[ T_w = \left( \sqrt{T_H} - \frac{N_u \pi \kappa_0 \Delta T}{4\hat{n}C_p} z \right)^2, \] (A.43)

where $T_H$ is room temperature. The temperature in the warm channel at the cold end of the CFHX ($z = L_X$ where $L_X$ is the length of the heat exchanger) is the same as the temperature of the gas entering the valve
\[ \sqrt{T_{VH}} = \sqrt{T_H} \left( 1 - \frac{N_u \pi \kappa_H \Delta T}{4\hat{n}C_p} L_X \right). \] (A.44)

So $L_X$ is given by
\[ L_X = \frac{4\hat{n}C_p T_H}{N_u \pi \kappa_H \Delta T} \left( 1 - \sqrt{\frac{T_{VH}}{T_H}} \right). \] (A.45)

For the low-temperature side the temperature profile is obtained directly from
\[ T_c (z) = T_w (z) - \Delta T \] (A.46)

with equation (A.43)
\[ T_c = T_H \left( 1 - \frac{N_u \pi \kappa_H \Delta T}{4\hat{n}C_p} z \right)^2 - \Delta T. \] (A.47)

### A.3.2 Pressure drop

The pressure drop in the CFHX, still assuming laminar flow, is given by \[ \frac{dp}{dz} = -\frac{128}{\pi D^6} \frac{V}{\eta N}. \] (A.48)
with \( \dot{V}/N \) the volume flow per channel. With

\[
\dot{V} = \frac{\dot{n}RT}{p} \tag{A.49}
\]

and equations (A.43), (6.1), (6.2), and (A.44) we get for the square of the pressure at the high-pressure side of the compressor

\[
p_{Hc}^2 = p_H^2 + 2 \frac{128\eta_H^2}{N^2\pi D_w^4} \frac{n^2 C_p R}{N u \nu_H \Delta T} \left( T_H^2 - T_{VH}^2 \right). \tag{A.50}
\]

The temperature at the low-pressure side is given by equation (A.47). However, the presence of the extra \( \Delta T \) term gives nasty mathematical formulae while in fact \( \Delta T \ll T \) so it gives only a small correction for the pressure dependence. So, for the square of the pressure at the low-pressure side of the compressor, we can write

\[
p_{Lc}^2 = p_L^2 - 2 \frac{128\eta_H^2}{N^2\pi D_c^4} \frac{n C_p R}{N u \nu_H \Delta T} \left( T_H^2 - T_{VH}^2 \right). \tag{A.51}
\]

### A.4 Compressor

Finally we give the relations characterizing the compressor. The volume flow at the low-pressure side of the compressor is given by

\[
\dot{V}_{Lc} = \frac{\dot{n}RT_H}{p_{Lc}}. \tag{A.52}
\]

For a given compressor this has a fixed value. The power \( P \) needed to compress the gas from \( p_{Lc} \) to \( p_{Hc} \) is

\[
P = \frac{\dot{n}RT_H}{\xi_c} \ln \frac{p_{Hc}}{p_{Lc}}. \tag{A.53}
\]

with \( \xi_c \) the efficiency of the compressor, which has a typical value of about 0.5.
Bibliography


Appendix B

Capillary heat exchanger: formulae

The capillaries are arranged in a regular triangular array as given in figure B.1. Three parameters are defined in figure B.1; the outer diameter of the tubes $D$; the distance between the tube centres (pitch) $P$; and the distance from the wall $W$. The area in between the capillaries is divided into three kinds of regular subchannels. For each subchannel the formulae for the flow area $A_i$, wetted perimeter $P_{wi}$, and friction factor $f_r$ for laminar flow can be given. The formulae are obtained from [1]. For the triangular central subchannel holds

$$ f_r R_e = 147 \left( \frac{P}{D} - 1 \right)^{0.24} \quad \text{for } 1.12 \leq \frac{P}{D} < 1.6 $$

(B.1)

$$ A_i = \frac{\sqrt{3}}{4} P^2 - \frac{\pi}{8} D^2 $$

(B.2)

$$ P_{wi} = \frac{\pi}{2} D, $$

(B.3)

for the triangular corner subchannel

$$ f_r R_e = 99 + 39 \left( \frac{W}{D} - 1 \right) - 55 \left( \frac{W}{D} - 1 \right)^2 \quad \text{for } 1.1 \leq \frac{W}{D} \leq 1.5 $$

(B.4)

$$ A_i = \frac{1}{\sqrt{3}} \left( \frac{W - D}{2} \right)^2 - \frac{\pi}{24} D^2 $$

(B.5)

$$ P_{wi} = \frac{\pi}{6} D + \frac{2}{\sqrt{3}} \left( \frac{W - D}{2} \right). $$

(B.6)

for the wall subchannel
Figure B.1: Schematic diagram of the regular triangular array used for the capillary heat exchangers for the case \( n = 2 \). The definitions of the parameters \( P \), \( D \), and \( W \) are given in the text [1].

\[
f_t R_e = 44 + 257 \left( \frac{P}{D} - 1 \right) - 268 \left( \frac{P}{D} - 1 \right)^2 \quad \text{for } 1.1 \leq \frac{P}{D} \leq 1.5 \tag{B.7}
\]

\[
A_i = \left( W - \frac{D}{2} \right) P - \frac{\pi}{8} D^2 \tag{B.8}
\]

\[
P_{wi} = \frac{\pi}{2} D + P \tag{B.9}
\]

For turbulent flow only the friction factor in the triangular central subchannel is known

\[
f_t = A_1 R_e^{-0.25} \quad \text{for } 10^4 \leq R_e \leq 5 \times 10^4 \tag{B.10}
\]

\[
f_t = A_1 R_e^{-0.2} \quad \text{for } 5 \times 10^4 \leq R_e \leq 2 \times 10^5 \tag{B.11}
\]

with

\[
A_1 = 0.171 + 0.012 \frac{P}{D} - 0.07 \exp^{-50(\frac{P}{D} - 1)} \quad \text{for } 1 \leq \frac{P}{D} \leq 2. \tag{B.12}
\]

The Nusselt number in the triangular corner subchannel and the wall subchannel are not known. For the triangular central subchannel we found formulae for the laminar and turbulent case, both with an accuracy of 3% for \( 1 \leq \frac{P}{D} \leq 2 \).

\[
N_u = 7.55 \frac{P}{D} - \frac{6.3}{\left( \frac{P}{D} \right)^{17/16(\frac{P}{D} - 0.81)}} \approx 6.4 \text{ laminar flow} \tag{B.13}
\]

\[
\frac{N_u}{N_{ut}} = 0.018 R_e^{0.79} \left( \frac{P}{D} \right)^{0.17} \left( 1.05 \frac{P}{D} + 1 \right)^{0.21} \text{ turbulent flow} \tag{B.14}
\]
Bibliography

Appendix C

Samenvatting

Lagetemperatuur-supergeleiders, gekoelde magneten voor MRI scanners, het vloeibaar maken van gassen zoals helium en waterstof: allemaal onderwerpen van vandaag en morgen, waarbij een betrouwbare machine die kan koelen tot enkele graden kelvin, van cruciaal belang is. Op het ogenblik wordt er veel onderzoek gedaan naar koelen met behulp van pulsbuiskoelers. Het voordeel van pulsbuiskoelers boven andere koelmachines is dat ze geen bewegende onderdelen hebben in het koude gedeelte van het systeem. Hierdoor zijn ze goedkoper, betrouwbaarder en veroorzaken ze minder mechanische trillingen dan andere koelmachines. Een pulsbuiskoeler bestaat uit een compressor, een roterende kraan, een regenerator (buis gevuld met poreus materiaal met een hoge warmtecapaciteit), een warmtewisselaar, een (lege) buis, nog een warmtewisselaar, een kraantje en een buffer (zie figuur C.1).

![Diagram van een pulsbuiskoeler](image)

Figure C.1: Schematische weergave van een pulsbuiskoeler. De warmtewisselaars zijn aangegeven met ‘ww’. Het koudste punt van de pulsbuiskoeler is aangegeven met $T_L$.

Hoe werkt zo’n pulsbuiskoeler? De compressor en de roterende kraan zorgen ervoor dat het systeem achter de roterende kraan wisselend wordt aangesloten op hoge- en lagedruk. Als de hogedruk wordt aangesloten, vindt er compressie plaats en beweegt het gas naar rechts. In de regenerator geeft het gas warmte af aan het regeneratorma-
C. Samenvatting

teriaal. Tijdens de expansiefase (als de lagedruk is aangesloten) neemt het gas weer warmte op van het regeneratormateriaal.

Van belang is de temperatuur van het gas dat heen en weer stroomt in de buurt van de koude warmtewisselaar. Door de aanwezigheid van het kraantje en de buffer stroomt dit namelijk met een temperatuur $T_L$ naar rechts maar komt tijdens een andere fase van de cyclus met een temperatuur terug die lager is dan $T_L$. Dit zorgt ervoor dat er koeling plaatsvindt.

Een van de belangrijkste beperkende factoren van een pulsbuiskoeler is dat de warmtecapaciteit van de meeste vaste stoffen erg klein wordt als de temperatuur onder de 15 K komt. Hierdoor kan het regeneratormateriaal geen warmte meer opnemen en stopt de koeling van de pulsbuiskoeler. Speciale materialen die wel een hoge warmtecapaciteit hebben bij deze lage temperatuur, zijn erg duur en vaak erg zwaar.

In plaats van het gebruik van speciale materialen zouden de regeneratoren wellicht vervangen kunnen worden door eenvoudige, goedkope en lichte warmtewisselaars. Het onderzoek dat in dit proefschrift beschreven wordt, richt zich op de basisprincipes van een speciaal type pulsbuiskoeler, waarin warmtewisselaars worden gebruikt in plaats van regeneratoren. Dit type koeler wordt de tegenstroompulsbuiskoeler (CFPTR) genoemd. Hij bestaat uit twee identieke pulsbuizen die in tegenfase werken. Dit betekent dat als de ene pulsbuis is aangesloten op hogedruk, de andere is aangesloten op lagedruk, en een halve periode later andersom. De twee regeneratoren zijn vervangen door één tegenstroomwarmtewisselaar. De nadruk heeft in dit onderzoek gelegen op het beantwoorden van de vraag of warmtewisselaars voordelen hebben boven regeneratoren.

Regeneratoren en warmtewisselaars zijn vanuit fundamenteel oogpunt vergelijkbaar. De analyse is gebaseerd op entropieproductie ten gevolge van verschillende irreversibele processen. Dit betekent dat er berekend is hoeveel energie er verloren gaat aan niet te voorkomen onomkeerbare processen. De analyse laat zien dat voor alle mogelijke temperatuurintervallen de systeemparameters zo gekozen kunnen worden dat warmtewisselaars beter zijn dan regeneratoren.

Tijdens dit onderzoek zijn drie verschillende experimentele tegenstroompuliskoelers ontwikkeld (CFPTR1-3). In alle systemen zijn verschillende warmtewisselaars getest die gebaseerd zijn op buis-in-buis ontwerpen. Voor dit concept is gekozen omdat daarbij hoge drukken en kleine drukverliezen mogelijk zijn. Gebleken is dat in de systemen die in dit onderzoek gebruikt zijn, de drukverliezen in oscillerende stromingen tot 20 Hz gelijk zijn aan die van constante stroming. Voor hogere frequenties ontstaan verschillen.

CFPTR1 en CFPTR2 zijn enkeltrapsmachines, die helemaal geen regenerator hebben, ook niet bij hoge temperaturen. In de warmtewisselaar van CFPTR1 heerst een
oscillerende stroming. Hierdoor is de werking van de warmtewisselaar gebaseerd op een combinatie van het regeneratieve en het warmtewisselingseffect. De metingen en de theoretische analyse laten zien dat bij temperaturen boven 70 K het warmtewisselingseffect klein is. In de warmtewisselaar van CFPTR2 heerst een constante stroming, doordat de roterende kraan na de warmtewisselaar is geplaatst (in het gedeelte met lage temperatuur). De roterende kraan is zo ontworpen dat de krachten op de rotor in balans zijn. Hierdoor zijn het benodigde draaimoment en de wrijvingswarmte klein.

In dit onderzoek is een compleet stelsel vergelijkingen afgeleid dat het gedrag van het systeem goed blijkt te beschrijven. De laagste temperatuur die met CFPTR2 gehaald is, is 165 K. Dit is in overeenstemming met het resultaat van de berekeningen.


Gewoonlijk wordt in de theoretische analyse van pulsbuiskoelers aangenomen dat het systeem is gevuld met een ideaal gas. Het gedrag van werkelijke systemen hangt echter af van de werkelijke gaseigenschappen. In dit proefschrift wordt aangetoond dat het rendement van de pulsbuiskoeler voor een groot deel wordt bepaald door de thermodynamische eigenschappen van het niet-ideale gas. Hiermee is de discrepantie in rendement tussen experiment en theorie tot 1/10 gereduceerd. De analyse toont verrassenderwijs ook aan dat de werkelijke gaseigenschappen soms kunnen resulteren in een _beter_ werking dan het geval is bij een ideaal gas. Dit opent nieuwe mogelijkheden voor verbeteringen.
Appendix D

Dankwoord

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Manon
Appendix E

Curriculum Vitae

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Afstuderen bij de vakgroep Lage Temperaturen (Prof.dr. A.T.A.M. de Waele)

Various aspects of pulse tube refrigerators

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Counterflow pulse-tube refrigerators