Squaring-down and the problems of almost-zeros for continuous-time systems
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Squaring-down and the problems of almost-zeros for continuous-time systems

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Abstract

In this paper we construct precompensators which square-down a system such that the resulting square system has the same unstable zeros and system gains. We derive the minimal gain of the precompensator to achieve such a square system. If the gain of this precompensator is too large then we derive an explicit trade-off between the gain of the precompensator and the number of unstable zeros we allow the precompensator to introduce in the system.

Keywords Nehari approximation, Almost-zeros, Squaring-down.

1 Introduction

When we have multi-input, multi-output systems with an unequal number of inputs and outputs, it is often desirable to 'square-down' the system via a suitable pre- or postcompensator in cascade with the plant. After this preliminary step, we obtain a control problem with a square system. Clearly, these pre- or postcompensator have to be chosen carefully. In general, they might introduce additional non-minimum-phase zeros or change the system gains in such a matter that it will severely limit the subsequent controller design.

In [4, 5], a method for squaring-down was presented which did not introduce additional unstable zeros. However, the gain of the pre- or postcompensator was not seriously studied. The main point of this paper is that in order to keep the gain of the pre- or postcompensator in check we sometimes need to introduce additional unstable zeros.

The reason behind the above problem is related to the concept of almost zeros (see e.g. [3, 2]). These are unstable points where the Rosenbrock system matrix almost loses rank (in the sense that the smallest singular value is very small but not identical to 0). A more precise definition can be found in [3, 2].

Whatever way we obtain our system, we always introduce inaccuracies. For non-square systems this often results in unstable zeros of the plant which are no longer zeros of the model. From there on, we can theoretically ignore this zero (since it is no longer there) but clearly for all practical purposes it is still present. Note that for square systems, small perturbations

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might slightly shift a zero but will not result in a change in the number of unstable zeros (excluding zeros on the boundary of the stability domain).

This paper will give explicit methods to construct these pre- or postcompensators while presenting an explicit trade-off between the gain of this pre- or postcompensator and the number of additional unstable zeros introduced by the pre- or postcompensator.

This paper will work with continuous-time systems and tall plants. A simple dualization argument will give the results for wide plants while the discrete-time analogue can be derived similarly.

The notation in this paper is quite standard. By $G^\sim$ we denote the transfer matrix described by $G^\sim(s) = G^T(-s)$. Moreover, for any matrix $H$ we denote by $\lambda_i(H)$ the eigenvalues of $H$.

If all eigenvalues are real we assume that the eigenvalues are ordered: $\lambda_1(H) \geq \ldots \lambda_i(H) \geq \lambda_{i+1}(H) \ldots \geq \lambda_n(H)$. By $\rho(H)$ we denote the spectral radius.

2 Inner-outer factorizations

In this section some results are recapitulated from [6]. It presents an extension of classical inner-outer factorizations in the sense that the inner factor is split in a square invertible inner system and a non-square inner system without any zeros.

Let a stable tall system $\Sigma$ be given with transfer matrix $G$ and state-space realization,

$$\Sigma: \begin{cases} \dot{x} = Ax + Bu, \\ z = Cx + Du, \end{cases} \tag{2.1}$$

where, for all $k$, $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$ and $z \in \mathbb{R}^p$. Here $A$ is stable and because we have a tall system $m \leq p$. We make the following assumption throughout this paper:

**Assumption 2.1** The system $\Sigma$ is stable, its direct feedthrough matrix is injective and the system has no invariant zeros on the imaginary axis.

In order to present inner-outer factorizations we first need some definitions.

**Definition 2.2** A system is inner if the system is stable and if for any input $u$ and output $z$ we have $\|z\|_2 = \|u\|_2$; equivalently a system is inner if it is stable and its transfer matrix $G$ satisfies $G^\sim G = I$. A system is called minimum-phase inner if the system is inner and has no invariant zeros.

A system is called outer if the system is stable and has a stable inverse.

Note that a square minimum-phase inner system is necessarily a static unitary gain. Next we show that any system satisfying assumption 2.1 can be split into an inner and outer factor:

**Theorem 2.3** Let $\Sigma$ be given by (2.1). Then there exists a matrix $X \geq 0$ satisfying:

$$0 = A^T X + X A + C^T C - (X B + C^T D)(D^T D)^{-1}(B^T X + D^T C)$$

2
such that $A - BF$ is asymptotically stable where

$$F := (D^T D)^{-1}(B^T X + D^T C)$$

We can decompose the transfer matrix of $G = G_i G_o$ where $G_i$ is inner and $G_o$ is outer. One particular choice for $G_i$ and $G_o$ is given by

$$G_i(z) := [A - BF, B(D^T D)^{-1/2}, C - DF, D(D^T D)^{-1/2}]$$

$$G_o(z) := [A, B, (D^T D)^{1/2} F, (D^T D)^{1/2}]$$

We first recall the following a connection between the number of Hankel singular values and the number of zeros of an inner system.

**Lemma 2.4** : Let $\Sigma$ be given by (2.1). Let $P$ and $Q$ be defined by

$$QA + A^T Q + C^T C = 0$$

$$AP + PA^T + BB^T = 0$$

The Hankel singular values are defined by $\lambda_i^{1/2}(PQ)$. If $\Sigma$ is inner then all Hankel singular values are less than or equal to 1. In that case, the number of Hankel singular values equal to 1 is equal to the number of zeros of $\Sigma$ (counting multiplicity).

The next step involves splitting the inner part $G_i$ into a square inner system and a minimum-phase inner system. This is presented in the following theorem.

**Theorem 2.5** : Let $\Sigma$, as given by (2.1), be inner and assume that $\Sigma$ has no zeros at $\infty$. Moreover assume that $[A, B, C, D]$ is a balanced realization, i.e. there exists a diagonal matrix $P$ such that

$$PA + A^T P + C^T C = 0$$

$$AP + PA^T + BB^T = 0$$

where

$$P = \begin{pmatrix} I & 0 \\ 0 & P_1 \end{pmatrix}$$

with $P_1 < I$. If we decompose $A, B, C, D$ compatible with $P$, we get

$$A = \begin{pmatrix} A_{11} & 0 \\ -B_2 B_1^T & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

$$C = \begin{pmatrix} -DB_1^T & C_2 \end{pmatrix}.$$ 

We define $G_{mi}$ and $G_{si}$ by

$$G_{mi}(z) := [A_{22}, B_2, C_2, D]$$

$$G_{si}(z) := [A_{11}, B_1, -B_1^T, I]$$

Then $G = G_{mi} G_{si}$ where $G_{mi}$ is minimum-phase inner and $G_{si}$ is square and inner.
As noted before a minimum-phase inner system always has a stable left inverse. This is pointed out in the following lemma:

**Lemma 2.6:** Let the system $\Sigma$, as given by (2.1), be minimum-phase inner with observability gramian $Q$ and controllability gramian $P$. Then a stable left inverse $\Sigma_L$ of $\Sigma$ is given by

$$[A, (PC^T + BD^T), -D^T C (I - PQ)^{-1}, D^T]$$

A stable and co-inner left annihilator $\Sigma_A$ of $\Sigma$ is given by

$$[A, (PC^T + BD^T), -D^T A (I - PQ)^{-1}, D^T]$$

where $D_\perp$ is chosen such that $(D \ D_\perp)$ is square and unitary. □

The main point of this paper is that although the annihilator constructed in the above lemma is co-inner and therefore has a desirable gain, the same cannot be said of the stable left inverse as constructed above. If some of the Hankel singular values are close to 1 then the $H_\infty$ norm of $\Sigma_L$ will be very large. This case arises regularly in practical applications and in the next section we will study this problem. It seems counter-intuitive that an inner system (which, by definition, is norm-preserving) has a left inverse with such a large gain. However, this is closely related to the fact that we require a stable left-inverse.

**Example 2.7:** Consider the following system:

$$G(s) = \begin{pmatrix} s - 0.9 \\ s + 1 \\ s + 1 \end{pmatrix}$$

The inner factor of $G$ as derived above is in this case given by

$$G_i(z) = \begin{pmatrix} 0.71s - 0.64 \\ 0.71s - 0.71 \\ s + 0.95 \end{pmatrix}$$

which is already minimum-phase inner. It turns out that although $G_i$ is inner, the inverse derived in lemma 2.6 will have $H_\infty$ norm equal to 76.06. This might be an unacceptable gain for the precompensator. The problem is clearly associated to the fact that the components of the transfer matrix $G$ almost have an unstable zero in common. In the next section we will show that the minimal $H_\infty$ norm over all stable left-inverses of $G_i$ is in this case equal to 38.04. Finally in section 4 it is shown that via introduction of one unstable zero we can get the gain of the precompensator down to 1. □


3 Minimization of the $H_\infty$ norm of stable left inverses

In this section we will study the existence of stable left-inverses of a minimum-phase inner system. Clearly left inverses are not unique and we can try to construct the stable left inverse with minimal $H_\infty$ norm:

**Theorem 3.1**: Let the system $\Sigma$, as given by (2.1), be minimum-phase inner with observability gramian $Q$ and controllability gramian $P$. There exists a stable left inverse of $\Sigma$ with $H_\infty$ norm equal to $\rho[(I - PQ)^{-1/2}]$. Moreover, there does not exist a stable left inverse of $\Sigma$ with smaller $H_\infty$ norm.

The proof of the above theorem will be constructive and can hence be used to find the 'optimal' left inverse. Basically the proof reduces the problem to a Nehari approximation problem (see [1]).

**Proof**: Let $\Sigma_L$ and $\Sigma_A$ be as defined in lemma 2.6 with transfer matrices $G_L$ and $G_A$ respectively. Then the class of transfer matrices of stable left inverses of $\Sigma$ can be written as

$$\{ G_L + QG_A \mid Q \in H_\infty \}$$

Therefore we have to minimize the $H_\infty$ norm over all stable transfer matrices $Q$. We already noted in lemma 2.6 that $\Sigma_A$ is co-inner. Clearly for any $w \in L_2$ we can find $w_1, w_2$ such that $w = w_1 + w_2$ where $w_1 \in \text{Ker} \ G_A$ and $w_2 \in (\text{Ker} \ G_A)^\perp = \text{Im} \ G_A^\perp$. Using the properties derived above we have $w_1 = Gv_1$ and $w_2 = G_A^\perp v_2$ with $\|v_1\|_2 = \|w_1\|_2$ and $\|v_2\|_2 = \|w_2\|$. This implies that:

$$\|(G_L + QG_A)w\|_2^2 = \|v_1 + (G_LG_A^\perp + Q)v_2\|_\infty$$

This implies:

$$\|G_L + QG_A\|_\infty^2 = 1 + \|G_LG_A^\perp + Q\|_\infty^2$$

On the basis of our state space realization for $G_L$ and $G_A$ we can derive a state space realization for $G_LG_A^\perp$. We get

$$\Sigma_1 = [-A^T, (I - QP)^{-1}C^TD^\perp, B^T, 0] \quad (3.1)$$

Note that our realizations for $G_L$ and $G_A$ have the same state space dimension, say $n$, and hence we would have expected a realization for $G_LG_A^\perp$ of dimension $2n$ with $n$ stable and $n$ unstable poles. However, the stable poles turn out to be uncontrollable.

It is straightforward to check that the controllability and observability gramians of $\Sigma_1$ are $-Q(I - PQ)^{-1}$ and $-P$ respectively.

We are now faced with a classical Nehari approximation problem. We need to find the best (measured in the $L_\infty$ norm) stable approximation of an antistable system. According to [1], the minimal $L_\infty$ norm is equal to the square root of the spectral radius of the product of controllability and observability gramians. Moreover, this infimum is attained. This completes the proof of the theorem.
The above result shows that if we require a squared-down system with the same unstable systems and the same gains (the outer factor is preserved), then we need a precompensator of very high gain if the system has Hankel singular values close to 1. In the next section we show that by preserving the gain but by allowing the introduction of some additional unstable zeros we can reduce the gain of the precompensator dramatically.

In the following lemma we give an explicit formula for a suboptimal Hankel norm approximation of a antistable system:

**Lemma 3.2**: Let a tall system \( \Sigma \), as given by (2.1), be anti-stable with observability gramian \( Q \) and controllability gramian \( P \). Suppose \( \gamma^2 > \rho(PQ) \). Then there exists a stable system \( \Sigma_a \) such that \( \gamma^{-1}(\Sigma + \Sigma_a) \) is all-pass. One such system \( \Sigma_a \) has a realization \([K, L, M, N]\) where:

\[
K = -A^T + C^T M \\
L = QB + C^T N \\
M = (CP + NB^T)(\gamma^2 I - QP)^{-1}
\]

and \( N \) is any matrix such that \( N^TN = \gamma^2 I \). \( \square \)

In [1] also an optimal Hankel norm approximation is derived but our next objective is to study pole-zero cancellations and then the above formulas are easier to work with. Note that the stable left-inverse \( \Sigma_L \) of \( \Sigma \) had the same McMillan degree as \( \Sigma \). However the left-inverse derived in theorem 3.1 is of the form \( G_L + QG_A \) and is of potentially much higher McMillan degree. However, the following lemma derives an explicit formula for a stable left-inverse whose McMillan degree is equal to the McMillan degree of \( \Sigma \) and whose \( H_\infty \) norm can be arbitrarily close to the optimal value derived in lemma 3.1.

**Lemma 3.3**: Let the system \( \Sigma \), as given by (2.1), be minimum-phase inner with observability gramian \( Q \) and controllability gramian \( P \). For any \( \gamma \) such that \( \gamma^2 > \rho[(I - PQ)^{-1}] \), we define \( \Sigma_\gamma \) by

\[
[A - BB^TQR^{-1}, -\gamma^2(PC^T + BD^T) + BD^T, D^TCR^{-1}, D^T]
\]  

where \( R = \gamma^2(I - PQ) - I \). Then \( \Sigma_\gamma \) is a stable left inverse of \( \Sigma \) and its \( H_\infty \) norm is less than or equal to \( \gamma \). \( \square \)

**Proof**: We will follow the construction as used in the proof of theorem 3.1. We have to find a stable \( Q \) which minimizes the \( H_\infty \) norm of \( \Sigma_1 + Q \). It is easily seen, since \( D \) is an inner matrix, that if we find \( Q \) which minimizes the \( H_\infty \) norm of \( D\Sigma_1 + Q_1 \) and define \( Q := D^TQ_1 \) then \( Q \) minimizes the \( H_\infty \) norm of \( \Sigma + Q \). We know that the observability and controllability gramians of \( D\Sigma_1 \) are \(-Q(I - PQ)^{-1} \) and \(-P \) respectively. We have:

\[
\rho[PQ(I - PQ)^{-1}] = \rho[I - (I - PQ)^{-1}] = \rho(I - PQ)^{-1} - 1 < \gamma^2 - 1
\]
Using lemma 3.2 we find stable $Q_1$ such that $(\gamma^2 - 1)^{-1/2}(D\Sigma_1 + Q_1)$ is all-pass where:

$$Q_1 = [A - BB^TQ^{-1}, -P(I - PQ)^{-1}, CR^{-1}, (\gamma^2 - 1)^{1/2}D]$$

We have:

$$\gamma^2 = \|D\Sigma_1 + Q_1\|_\infty + 1 \geq \|\Sigma_1 + D^TQ_1\|_\infty + 1 = \|\Sigma_L + D^TQ_1\Sigma_A\|_\infty.$$  

Moreover $\Sigma_\gamma := \Sigma_L + D^TQ_1\Sigma_A$ is a stable left-inverse of $\Sigma$. Using some standard algebraic manipulations, we find that (3.2) is a minimal realization of $\Sigma_\gamma$.

4 Squaring-down with the introduction of additional unstable zeros

In the previous section, we saw the possibility of unacceptable high gain in the precompensator. This section presents a method to avoid this problem. We have:

**Theorem 4.1**: Let the system $\Sigma$, as given by (2.1), be minimum-phase inner with observability gramian $Q$ and controllability gramian $P$. Suppose

$$\lambda_i[(I - PQ)^{-1}] > \lambda_{i+1}[(I - PQ)^{-1}],$$

then there exists a stable system $\Sigma_S$ with $H_\infty$ norm equal to $\lambda_{i+1}[(I - PQ)^{-1/2}]$ such that $\Sigma_S\Sigma$ is square and inner and has at most $i$ unstable zeros. Moreover, there does not exist a stable system $\Sigma_S$ with $H_\infty$ norm smaller than $\lambda_{i+1}[(I - PQ)^{-1/2}]$ such that $\Sigma_S\Sigma$ is inner and has at most $i$ unstable zeros.

**Proof**: Let $\Sigma_L$ and $\Sigma_A$ be as defined in lemma 2.6 with transfer matrices $G_L$ and $G_A$ respectively.

We have to find $G_S$ such that $G_SG =: G_R$ is inner and has at most $i$ unstable zeros. However, this implies that $G_R^TG_S$ is a left-inverse of $G$ with $i$ unstable poles.

Conversely if $G_T$ is a left inverse of $G$ with no more than $i$ unstable poles, then we can always factorize $G = G_vG_S$ where $G_v$ is inner and square while $G_S$ is stable. But then $G_SG$ is inner and has at most $i$ unstable zeros.

The above argument implies that we need to search for a left-inverse of $G$ with at most $i$ unstable zeros and minimal $L_\infty$ norm.

The class of transfer matrices of all left inverses of $\Sigma$ (in $L_\infty$) can be written as

$$\{ G_L + QG_A \mid Q \in L_\infty \}$$

We can show that $G_A$ is minimum-phase. This excludes unstable pole-zero cancellations. Therefore, the number of unstable poles of the left-inverse is equal to the number of unstable poles of $Q$.

Hence we have to minimize the $L_\infty$ norm over all transfer matrices $Q$ with at most $i$ unstable zeros. As already noted in the proof of theorem 3.1, we have:

$$\|G_L + QG_A\|^2_\infty = 1 + \|G_LG_A^\sim + Q\|^2_\infty$$
Also, we already have a state space realization for $G_L G_A^\gamma$: given by (3.1) and its controllability and observability gramians are $-Q(I - PQ)^{-1}$ and $-P$ respectively.

We are now faced with a classical Nehari approximation problem. We need to find the best (measured in the $L_\infty$ norm) approximation of an antistable system. The difference with theorem 3.1 is that we allow for $i$ unstable zeros. According to [1], the minimal $L_\infty$ norm is equal to the $i + 1$-largest eigenvalue of the product of controllability and observability gramians. Moreover, this infimum is attained. This completes the proof of the theorem.

Note that lemma 3.2 can be extended to find an approximation with at most $i$ unstable poles of a given anti-stable system:

**Lemma 4.2**: Let a tall system $\Sigma$, as given by (2.1), be anti-stable with observability gramian $Q$ and controllability gramian $P$. Suppose

$$\lambda_i(PQ) > \gamma^2 > \lambda_{i+1}(PQ).$$

Then there exists a system $\Sigma_x$ with $i$ unstable poles such that $\gamma^{-1}(\Sigma + \Sigma_x)$ is all-pass. One such system $\Sigma_x$ has a realization $[K, L, M, N]$ where:

\[
K = -A^T + C^T M \\
L = QB + C^T N \\
M = (CP + NB^T)(\gamma^2 I - QP)^{-1}
\]

and $N$ is any matrix such that $N^T N = \gamma^2 I$.

In the proof of theorem 4.1 we derived a left-inverse of $\Sigma$ with at most $i$ unstable poles and which was of the form $G_L + QG_A$. As we did in lemma 3.3 we can construct a left-inverse with at most $i$ unstable poles whose McMillan degree is equal to the McMillan degree of $\Sigma$ and whose $H_\infty$ norm can be arbitrarily close to the optimal value derived in the proof of theorem 4.1:

**Lemma 4.3**: Let the system $\Sigma$, as given by (2.1), be minimum-phase inner with observability gramian $Q$ and controllability gramian $P$. For any $\gamma$ such that

$$\lambda_i[(I - PQ)^{-1}] > \gamma^2 > \lambda_{i+1}[(I - PQ)^{-1}],$$

we define $\Sigma_\gamma$ by

$$[A - BB^T QR^{-1}, -\gamma^2(PT^T + BD^T) + BD^T, D^T CR^{-1}, D^T]$$

where $R = \gamma^2(I - PQ) - I$. Then $\Sigma_\gamma$ has at most $i$ unstable poles and is a left inverse of $\Sigma$ and its $H_\infty$ norm is less than or equal to $\gamma$.

The above lemma is still valid for $i = n$ where (4.1) is replaced by $\lambda_n[(I - PQ)^{-1}] > \gamma^2 \geq 1$. An interesting case we obtain for $\gamma = 1$. In that case $\Sigma_0$ is nothing else than $G^\gamma$.

The proof of the above lemma goes along the same lines as the proof of lemma 3.3. Lemma 4.3 can be used to construct a system satisfying the requirements of theorem 4.1 with McMillan degree $n - i$ where $n$ is the McMillan degree of $\Sigma$:

Partition $\Sigma_\gamma = \Sigma_3 \Sigma_4$ where $\Sigma_4$ is square, anti-stable and all-pass while $\Sigma_3$ is stable. Then $\Sigma_3$ is stable and $\Sigma_4 \Sigma$ is square and inner. Finally $\Sigma_3$ has McMillan degree $n - i$ and its $H_\infty$ norm is less than or equal to $\gamma$. 

8
5 Almost zeros

In the previous section we saw that the gain of the precompensator of strongly related to
the number of Hankel singular values of the minimum-phase system close to 1. Since Hankel
singular values equal to 1 correspond to zeros of the inner system (see lemma 2.4), it is natural
to expect that these Hankel singular values close to 1 in some way correspond to almost-zeros.
Almost zeros were introduced in [3, 2]. There definition was more or less that almost zeros were
the local minima of the smallest singular value. They showed that this definition has some
desirable invariance properties. However if the smallest singular value has a local minimum
of 0.5 then it is hardly justified to call this an almost zero. Moreover, it turns out to be
extremely difficult to actually calculate these almost zeros.

We only have partial results on the correspondence between almost-zeros and the gains of the
squaring-down precompensators but we nevertheless feel they are enlightening.

Suppose we have a minimum-phase inner system $\Sigma$ with transfer matrix $G$ and $m$ inputs.
Define $\nu$ by:

$$
\nu := \inf_{s \in \mathbb{C}^+} \sigma_m[G(s)] > 0
$$

where $\mathbb{C}^+$ denotes the right half plane. We have the following (straightforward) result:

**Lemma 5.1** : Any stable left inverse $\Sigma_L$ of $\Sigma$ satisfies $\|\Sigma_L\|_\infty > \nu^{-1}$.

However, if we compare with example 2.7 then we note that the minimal achievable $H_\infty$ norm
of the left inverse was 38.04. On the other hand $\nu^{-1}$ can be easily checked to be larger than
27.58.

Our claim is that if $\nu$ is large then there will exists a stable left-inverse with a reasonable
$H_\infty$ norm. However, we did not succeed in finding a proof of this result.

Generically a non-square system has no zeros. This implies that small perturbations of the
system parameters will generically remove all zeros. For this perturbed system we can then
find a stable inverse. But because the system is so close to a system with zeros it is intuitively
clear that the system is very difficult to invert. We claim that this is the main cause of the
problems identified in this paper.

6 Conclusion

In this paper we have derived explicit expressions for the gain of the precompensator which
achieves the squaring-down of the system. If this gain was unacceptably high then we showed
that by allowing the precompensator to introduce a number of unstable zeros we obtained
an explicit trade-off between the number of zeros introduced and the minimal gain of the
precompensator.

The main restrictions of this paper is that we do not allow any changes in the unstable zeros
already present in the system. Nor do we allow any changes in the gains of the system. This
can be seen since we first split our system into a minimum-phase inner, a square inner and an
outer factor. We only square-down the minimum-phase inner part and leave the square-inner
and outer factors unchanged. A subject of current research is whether incorporating the
square-inner or outer factors in the design might reduce the gain of the precompensator while being able to guarantee that all design limitations of the non-square system are preserved in the squared-down system.

References


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<td>A.W.J. Kolen, A.H.G. Rinnooy Kan, C.P.M. van Hoesel, A.P.M. Wagelmans</td>
<td>Sensitivity analysis of list scheduling heuristics</td>
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<tr>
<td>93-11</td>
<td>March</td>
<td>A.A. Stoorvogel, J.H.A. Ludlage</td>
<td>Squaring-down and the problems of almost-zeros for continuous-time systems</td>
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