Non-interacting control by measurement feedback for "outputs complete" systems

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NON INTERACTING CONTROL
BY MEASUREMENT FEEDBACK
FOR "OUTPUTS COMPLETE" SYSTEMS

by

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ABSTRACT

In this paper we shall consider systems that in addition to a control input and a measurement output have two exogenous inputs and two exogenous outputs. Furthermore, we assume that the two exogenous outputs of these systems are complete, i.e. we assume that the state of these systems can be written as a linear combination of the two exogenous outputs.

For this kind of systems we shall solve the problem of non interacting control by measurement feedback. That is, we shall derive necessary and sufficient conditions for the existence of a measurement feedback compensator such that the transfer matrix of the resulting closed loop system, when partitioned according to exogenous inputs and exogenous outputs, has off-diagonal blocks equal to zero.

Keywords & Phrases
Non interacting control, measurement feedback, outputs complete, \((A,B)\)-compatibility, \((C,A)\)-compatibility.
1. Introduction

Consider a system, in state space representation, that in addition to a control input and a measurement output has two exogenous inputs and two exogenous outputs. Controlling such a system by means of a measurement feedback compensator results in a closed loop system that has two exogenous inputs and two exogenous outputs. Therefore, the transfer matrix of the closed loop system can be partitioned according to the dimensions of the exogenous inputs and outputs as a two by two block matrix. The problem that will be addressed in this paper can then be formulated as follows.

Given a system as described above, does there exist a measurement feedback compensator such that the transfer matrix of the closed loop system has off-diagonal blocks equal to zero? And if so, how can this compensator be computed?

The above problem formulation falls within the framework of non interacting control. The approach towards non interacting control as described in this paper is initiated in Willems [5] and is developed in Trentelman & Van der Woude [4]. In the latter paper also the distinction is made clear between the present approach and the point of view towards non interacting control as exposed in Morse & Wonham [2] and Hautus & Heyman [1]. The main contribution of this paper is that unlike Willems [5] and Trentelman & Van der Woude [4], where (dynamic) state feedback is required in the solution of the problem formulated above, in this paper we allow the problem to be solved by (dynamic) measurement feedback. In this context we also refer to Van der Woude [7] where measurement feedback is used to solve the almost version of the problem formulated above. In the present paper we shall be able to solve the problem formulated above for systems of which the two exogenous outputs are complete.

The outline of the paper is as follows. In Section 2 we shall give a mathematical formulation of the main problem of this paper. Furthermore we shall recall some well-known results coming from the geometric approach towards control theory. In Section 3 we shall derive some preliminary results. In fact, the main result of Section 3 consists of sufficient conditions for the solvability of the main problem of this paper even in the most general case that the two exogenous outputs are not complete. Necessary and sufficient conditions for the solvability of the main problem of this paper for systems with two exogenous outputs that are complete will be derived in Section 4. In Section 5 we shall state some remarks and conclusions. Furthermore, in Section 5 we shall give a conceptual algorithm, that, if it exists, provides a compensator that achieves non interaction.
2. Problem Formulation

Consider the finite-dimensional linear time-invariant system \( \Sigma \) given by

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) + G_1 v_1(t) + G_2 v_2(t), \\
y(t) &= C x(t), \\
z_1(t) &= H_1 x(t), \quad z_2(t) = H_2 x(t).
\end{align*}
\]

Here \( x(t) \in \mathbb{R}^n \) denotes the state of the system, \( u(t) \in \mathbb{R}^m \) the control input, \( v_1(t) \in \mathbb{R}^q_1 \), \( v_2(t) \in \mathbb{R}^q_2 \) the two exogenous inputs, \( y(t) \in \mathbb{R}^p \) the measurement output and \( z_1(t) \in \mathbb{R}^r_1 \), \( z_2(t) \in \mathbb{R}^r_2 \) the two exogenous outputs. \( A, B, C, G_1, G_2, H_1 \) and \( H_2 \) are real matrices of appropriate dimensions.

Assume that the system \( \Sigma \) is controlled by means of a measurement feedback compensator \( \Sigma_c \) described by

\[
\begin{align*}
\dot{w}(t) &= K w(t) + L y(t), \\
u(t) &= M w(t) + N y(t),
\end{align*}
\]

with \( w(t) \in \mathbb{R}^k \) the state of the compensator and \( K, L, M \) and \( N \) real matrices of appropriate dimensions.

Interconnection of the system \( \Sigma \) with the compensator \( \Sigma_c \) results in a closed loop system \( \Sigma_{cl} \) with two exogenous inputs \( v_1(t), v_2(t) \) and two exogenous outputs \( z_1(t), z_2(t) \). The closed loop system \( \Sigma_{cl} \) is described by

\[
\begin{align*}
\dot{x}_s(t) &= A_s x_s(t) + G_{1s} v_1(t) + G_{2s} v_2(t), \\
z_1(t) &= H_{1s} x_s(t), \quad z_2(t) = H_{2s} x_s(t),
\end{align*}
\]

where we have denoted

\[
x_s(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \quad A_s = \begin{bmatrix} A + BNC & BM \\ LC & K \end{bmatrix}, \quad G_{is} = \begin{bmatrix} G_i \\ 0 \end{bmatrix}, \quad (i=1,2),
\]

\[
H_{is} = [H_i, 0], \quad (i=1,2).
\]

Let \( T(s) \) be the transfer matrix of the closed loop system \( \Sigma_{cl} \). Then \( T(s) \) can be partitioned as

\[
T(s) = \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix}
\]

where \( T_{ij}(s) \) denotes the \( r_i \times q_j \) transfer matrix between the \( j \)-th exogenous input and the \( i \)-th exogenous output. It is clear that

\[
T_{ij}(s) = H_{is} (sI - A_s)^{-1} G_{js}.
\]

We are now able to give the following problem formulation.
Definition 2.1.
Let \( \Sigma \) be given. The non interacting control problem by measurement feedback (NICPM) consists of finding a measurement feedback compensator \( \Sigma_c \) such that in the closed loop system \( T_{12}(s) = 0 \) and \( T_{21}(s) = 0 \).

If a measurement feedback compensator \( \Sigma_c \) is such that it solves (NICPM), then it is said that \( \Sigma_c \) achieves non interaction. As announced in the introduction, throughout this paper we shall assume that the two exogenous outputs of \( \Sigma \) are complete, i.e. \( \ker H_1 \cap \ker H_2 = \{0\} \). (See also Wonham [6]). Clearly, this means that the state \( x(t) \) can be written as a constant linear combination of the two exogenous outputs \( z_1(t) \) and \( z_2(t) \).

In Section 4 we shall derive necessary and sufficient conditions for the solvability of (NICPM) under the assumption of outputs complete. The conditions obtained will be stated in geometric terms. To that end we shall now recall some well-known concepts originating from the geometric approach towards control theory (cf. Wonham [6], Schumacher [3]).

Consider the dynamical system described by

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t)
\]

with state space \( \mathbb{R}^n \), control input space \( \mathbb{R}^m \), measurement output space \( \mathbb{R}^p \) and matrices \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m} \) and \( C \in \mathbb{R}^{p \times n} \). With respect to this system we now introduce the following.

A linear subspace \( V \) in \( \mathbb{R}^n \) is called an \((A,B)\)-invariant subspace if \( AV \subseteq V + \text{im} B \). It is well known, that this subspace inclusion is equivalent to the existence of a matrix \( F \in \mathbb{R}^{n \times n} \) such that \( (A + BF)V \subseteq V \).

Following Wonham [6] we call two \((A,B)\)-invariant subspaces \( V_1 \) and \( V_2 \) compatible with respect to the pair \((A,B)\) (or simply \((A,B)\)-compatible) if there exists a matrix \( F \in \mathbb{R}^{n \times n} \) such that both \((A + BF)V_1 \subseteq V_1 \) and \((A + BF)V_2 \subseteq V_2 \). It can be proved (see Wonham [6], Ex. 9.1) that two \((A,B)\)-invariant subspaces \( V_1 \) and \( V_2 \) are \((A,B)\)-compatible if and only if their intersection \( V_1 \cap V_2 \) is an \((A,B)\)-invariant subspace. If \( K \) is a linear subspace in \( \mathbb{R}^n \), then \( V^*(K) \) will denote the largest \((A,B)\)-invariant subspace contained in \( K \). \( V^*(K) \) can be calculated by means of the algorithm given in Wonham [6], Chapter 4.

Dualizing the concepts introduced above we obtain the following (cf. Schumacher [3]).

A linear subspace \( S \) in \( \mathbb{R}^n \) is called a \((C,A)\)-invariant subspace if \( A(S \cap \ker C) \subseteq S \). This subspace inclusion is equivalent to the existence of a matrix \( J \in \mathbb{R}^{n \times p} \) such that \( (A + JC)S \subseteq S \). Furthermore, two \((C,A)\)-invariant subspaces \( S_1 \) and \( S_2 \) are said to be compatible with respect to the pair \((C,A)\) (or simply \((C,A)\)-compatible) if there exists a matrix \( J \in \mathbb{R}^{n \times p} \) such that both \((A + JC)S_1 \subseteq S_1 \) and \((A + JC)S_2 \subseteq S_2 \). By the previous it is clear that two \((C,A)\)-invariant
subspaces $S_1$ and $S_2$ are $(C,A)$-compatible if and only if their sum $S_1 + S_2$ is a $(C,A)$-invariant subspace. If $L$ is a linear subspace in $\mathbb{R}^n$, then $S^*(L)$ will denote the smallest $(C,A)$-invariant subspace containing $L$. An algorithm to calculate $S^*(L)$ can be found in Schumacher [3]. The latter algorithm is in fact the dual of the algorithm mentioned previously for the determination of $V^*(K)$ with respect to a given linear subspace $K$. 
3. Sufficient Conditions

In this section we shall derive a preliminary result that we shall need in the proof of our main result. The result of this section provides sufficient conditions for the solvability of (NICPM). In order to establish these sufficient conditions, we shall make use of the following two results. The first result that we need is very general and is concerned with the existence of a common solution to a pair of linear matrix equations.

**Theorem 3.1.**

Let \( A_i \in \mathbb{R}^{i \times i}, B_i \in \mathbb{R}^{w \times i}, C_i \in \mathbb{R}^{i \times w} \) (\( i = 1, 2 \)) be given.

The following statements are equivalent.

1. There exists \( X \in \mathbb{R}^{w \times w} \) such that \( A_1 X B_1 = C_1 \) and \( A_2 X B_2 = C_2 \).

2. \( \text{Im} A_i \supseteq \text{Im} C_i \) (\( i = 1, 2 \)), \( \ker B_i \subseteq \ker C_i \) (\( i = 1, 2 \)) and

\[
\begin{bmatrix}
  C_1 & 0 \\
  0 & -C_2 \\
\end{bmatrix} \ker \left[ B_1, B_2 \right] \subseteq \text{Im} \begin{bmatrix}
  A_1 \\
  A_2 \\
\end{bmatrix}.
\]

**Proof.** See Van der Woude [7].

For the second result that we need here, we refer to the linear system \( \dot{x}(t) = Ax(t) + Bu(t), \) \( y(t) = Cx(t) \) as described in the previous section.

**Theorem 3.2.**

Let \( S_1, S_2 \) be \((C,A)\)-invariant subspaces and \( V_1, V_2 \) be \((A,B)\)-invariant subspaces in \( \mathbb{R}^w \) such that \( S_1 \subseteq V_1 \) and \( S_2 \subseteq V_2 \). Then there exists \( N \in \mathbb{R}^{m \times p} \) such that \( (A + BNC)S_1 \subseteq V_1 \) and \( (A + BNC)S_2 \subseteq V_2 \) if and only if

\[
\begin{bmatrix}
  A & 0 \\
  0 & -A \\
\end{bmatrix} ((S_1 \oplus S_2) \cap \ker [C, C]) \subseteq (V_1 \oplus V_2) + \text{Im} \begin{bmatrix}
  B \\
\end{bmatrix}.
\]

Here \( \oplus \) denotes the external direct sum, (cf. Wonham [6]).

**Proof.** Let \( \tilde{S}_1, \tilde{S}_2, \tilde{W}_1 \) and \( \tilde{W}_2 \) be matrices such that \( \text{Im} \tilde{S}_i = S_i \) (\( i = 1, 2 \)) and \( \ker \tilde{W}_i = V_i \) (\( i = 1, 2 \)). Then there exists \( N \in \mathbb{R}^{m \times p} \) such that \( (A + BNC)S_i \subseteq V_i \) (\( i = 1, 2 \)) if and only if there exists \( N \in \mathbb{R}^{m \times p} \) such that \( \tilde{W}_i A \tilde{S}_i + \tilde{W}_i BNC \tilde{S}_i = 0 \) (\( i = 1, 2 \)).

By Theorem 3.1 the latter is equivalent to:

\[
\text{Im} \tilde{W}_i B \supseteq \text{Im} \tilde{W}_i A \tilde{S}_i \ (i = 1, 2), \quad \ker C \tilde{S}_i \subseteq \ker \tilde{W}_i A \tilde{S}_i \ (i = 1, 2)
\]

and

\[
\begin{bmatrix}
  \tilde{W}_1 A \tilde{S}_1 & 0 \\
  0 & -\tilde{W}_2 A \tilde{S}_2 \\
\end{bmatrix} \ker [C \tilde{S}_1, C \tilde{S}_2] \subseteq \text{Im} \begin{bmatrix}
  \tilde{W}_1 B \\
  \tilde{W}_2 B \\
\end{bmatrix}.
\]

In its turn this is equivalent to:
A S_i \subseteq V_i + \text{im} B \quad (i = 1, 2), \quad A (S_i \cap \ker C) \subseteq V_i \quad (i = 1, 2)

and

\[
\begin{bmatrix}
A & 0 \\
0 & -A
\end{bmatrix}
\begin{bmatrix}
(S_1 \oplus S_2) \cap \ker [C, C] \subseteq (V_1 \oplus V_2) + \text{im} \begin{bmatrix} B \\ B \end{bmatrix}.
\end{bmatrix}
\]

The proof can now be completed using the observation that the conditions $A S_i \subseteq V_i + \text{im} B \quad (i = 1, 2)$ and $A (S_i \cap \ker C_i) \subseteq V_i \quad (i = 1, 2)$ are fulfilled trivially since $S_1$ and $S_2$ are $(C, A)$-invariant subspaces, $V_1$ and $V_2$ are $(A, B)$-invariant subspaces and $S_1 \subseteq V_1$ and $S_2 \subseteq V_2$.

Then the following theorem is the main result of this section.

**Theorem 3.3.**

Let the system $\Sigma$ be given. Let $S_1$, $S_2$ be $(C, A)$-invariant subspaces and let $V_1$, $V_2$ be $(A, B)$-invariant subspaces such that

(a) $\text{im} G_1 \subseteq S_1 \subseteq V_1 \subseteq \ker H_2$, \hspace{1em} $\text{im} G_2 \subseteq S_2 \subseteq V_2 \subseteq \ker H_1$,

(b) $V_1 \cap V_2$ is an $(A, B)$-invariant subspace,

(c) $S_1 + S_2$ is a $(C, A)$-invariant subspace and

(d) $\begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} ((S_1 \oplus S_2) \cap \ker [C, C]) \subseteq (V_1 \oplus V_2) + \text{im} \begin{bmatrix} B \\ B \end{bmatrix}.$

Then there exists a measurement feedback compensator $\Sigma_c$ such that in the closed loop system $T_{12}(s) = 0$ and $T_{21}(s) = 0$.

**Proof.** Because of (a), (d) and Theorem 3.2 there exists $N \in \mathbb{R}^{m \times p}$ such that $(A + BNC) S_i \subseteq V_i \quad (i = 1, 2)$. By (b) and (c) it follows that there exist $F \in \mathbb{R}^{m \times n}$ and $J \in \mathbb{R}^{n \times p}$ such that $(A + BF) V_i \subseteq V_i \quad (i = 1, 2)$ and $(A + JC) S_i \subseteq S_i \quad (i = 1, 2)$. Let $W_1$ and $W_2$ be linear subspaces in $\mathbb{R}^{2n}$ defined by

\[
W_i = \left\{ \begin{bmatrix} s \\ 0 \end{bmatrix} + \begin{bmatrix} \nu \\ \nu \end{bmatrix} \middle| s \in S_i, \nu \in V_i \right\} \quad (i = 1, 2).
\]

By (a) it is clear that

\[
\text{im} G_{1,e} \subseteq W_1 \subseteq \ker H_{2,e}, \quad \text{im} G_{2,e} \subseteq W_2 \subseteq \ker H_{1,e}.
\]

Now define the matrix $A_e \in \mathbb{R}^{2n \times 2n}$ by

\[
A_e = \begin{bmatrix}
A + BNC & BF - BNC \\
BNC - JC & A + BF + JC - BNC
\end{bmatrix}.
\]

The matrix $A_e$ can considered to be obtained by the interconnection of the system $\Sigma$ and the compensator given by
\[ \dot{w}(t) = (A + BF + JC - BNC)w(t) + (BN - J)y(t), \]

\[ u(t) = (F - NC)w(t) + Ny(t). \]

For every \( x_t \in W_1 \) there exist \( s \in S_1 \) and \( v \in V_1 \) such that

\[ A_x x_t = A_x \begin{bmatrix} s \\ 0 \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} = \begin{bmatrix} (A + JC)s \\ 0 \end{bmatrix} - \begin{bmatrix} (A + JC)s \\ (A + JC)s \end{bmatrix} + \begin{bmatrix} (A + BNC)s \\ (A + BNC)s \end{bmatrix} + \begin{bmatrix} (A + BF)v \\ (A + BF)v \end{bmatrix}. \]

From this it is clear that if \( x_t \in W_1 \) also \( A_x x_t \in W_1 \). Hence, \( A_x W_1 \subseteq W_1 \). Analogously, we can prove that \( A_x \subseteq W_2 \). So we have

\[ A_x W_1 \subseteq W_1, \quad A_x W_2 \subseteq W_2. \]

From (5) and (6) it is easy to see that for any \( k \geq 0 \) \( H_{22} A^k G_{12} = 0 \) and \( H_{11} A^k G_{22} = 0 \), from which it is immediate that \( T_{21}(s) = H_{22}(sI - A)G_{12} = 0 \) and \( T_{12}(s) = H_{11}(sI - A)^{-1}G_{22} = 0 \).

The choice of the matrices \( K, L \) and \( M \) that together with \( N \) describe a compensator \( \Sigma_c \) that achieves non interaction is obvious from above.
4. Main Result

In this section we shall derive the main result of this paper. The result establishes necessary and sufficient conditions for the solvability of (NICPM) under the assumption that \( \ker H_1 \cap \ker H_2 = \{0\} \). However, before stating this result we have to point out the following.

Let \( Q_1 \in \mathbb{R}^{m \times n} \) be a matrix representing a map whose restriction to the subspace \( V^*(\ker H_1) + V^*(\ker H_2) \) is equal to the projection from \( V^*(\ker H_1) + V^*(\ker H_2) \) onto \( V^*(\ker H_1) \). Also, let \( -Q_2 \in \mathbb{R}^{m \times n} \) be a matrix representing a map whose restriction to the subspace \( V^*(\ker H_1) + V^*(\ker H_2) \) is equal to the projection from \( V^*(\ker H_1) + V^*(\ker H_2) \) onto \( V^*(\ker H_1) \) along \( V^*(\ker H_2) \). Since we assume that \( \ker H_1 \cap \ker H_2 = \{0\} \), it follows that \( V^*(\ker H_1) \) and \( V^*(\ker H_2) \) are linearly independent subspaces. Because of this linear independence, it is clear that \( Q_1 - Q_2 \) represents a linear map whose restriction to the subspace \( V^*(\ker H_1) + V^*(\ker H_2) \) is equal to the identity map restricted to \( V^*(\ker H_1) + V^*(\ker H_2) \).

Now, let \( \hat{S}_1 \) and \( \hat{S}_2 \) be linear subspaces in \( \mathbb{R}^{2n} \) defined by

\[
\hat{S}_i = \left\{ \begin{bmatrix} a \\ -b \end{bmatrix} \mid a \in S^*(\text{im} G_i), \ b \in S^*(\text{im} [G_1, G_2]) \right\} \quad (i = 1,2).
\]

The following theorem is the main result of this paper.

**Theorem 4.1.**

Let \( \Sigma \) be given and assume that \( \ker H_1 \cap \ker H_2 = \{0\} \). Then we have the following.

\( \text{(NICPM)} \) is solvable if and only if

\[
S^*(\text{im} G_1) \subseteq V^*(\ker H_2), \ S^*(\text{im} G_2) \subseteq V^*(\ker H_1), \ S^*(\text{im} [G_1, G_2]) \subseteq V^*(\ker H_1) + V^*(\ker H_2) \text{ and } \\
0 \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (\hat{S}_1 \oplus \hat{S}_2) \cap \ker \begin{bmatrix} C & 0 & C \\ 0 & I & 0 \end{bmatrix} \subseteq (V^*(\ker H_1) \oplus V^*(\ker H_2)) + \text{im} \begin{bmatrix} B \\ B \end{bmatrix}.
\]

Before proving this result we note that the conditions of the theorem can be checked constructively. Indeed, \( S^*(\text{im} G_1), S^*(\text{im} G_2), S^*(\text{im} [G_1, G_2]) \) and \( V^*(\ker H_1), V^*(\ker H_2) \) can be calculated using the algorithms described in Section 2. Next \( Q_1, Q_2 \) and \( \hat{S}_1, \hat{S}_2 \) can be determined after which the conditions of the theorem can be checked.

We now proceed with the proof of Theorem 4.1.

**Proof (if).**

Throughout the (if)-part of the proof we shall denote \( S_1^* = S^*(\text{im} G_1), S_2^* = S^*(\text{im} G_2), S^* = S^*(\text{im} [G_1, G_2]), V_1^* = V^*(\ker H_2) \) and \( V_2^* = V^*(\ker H_1) \). Consider the extended state space \( \mathbb{R}^{2n} \) together with the linear subspaces \( \hat{S}_1 \) and \( \hat{S}_2 \) as defined above. Furthermore, let \( \hat{V}_1 \) and \( \hat{V}_2 \) be linear subspaces in \( \mathbb{R}^{2n} \) defined by \( \hat{V}_i = V_i^* \oplus \mathbb{R}^n \quad (i = 1,2) \). Now consider the linear system obtained from \( \Sigma \) by adding to \( \Sigma \) a bank of \( n \) integrators \( \Sigma_a : x_a(t) = u_a(t), \ y_a(t) = x_a(t) \) with \( x_a(t), u_a(t), y_a(t) \in \mathbb{R}^n \).

The system made up of \( \Sigma \) and \( \Sigma_a \) is described by
\[ \dot{x}(t) = \hat{A} x(t) + \hat{B} u(t) + \hat{G}_1 v_1(t) + \hat{G}_2 v_2(t) . \] (7a)

\[ \dot{y}(t) = \hat{C} \dot{x}(t) , \] (7b)

\[ \dot{z}(t) = \hat{H}_1 \dot{x}(t) , \dot{z}_2(t) = \hat{H}_2 \dot{x}(t) \] (7c)

where we have denoted

\[ \hat{x}(t) = [x(t) \ x_a(t)] , \hat{u}(t) = [u(t) \ u_a(t)] , \hat{y}(t) = [y(t) \ y_a(t)] , \]

\[ \hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} , \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} , \hat{G}_i = \begin{bmatrix} G_i \\ 0 \end{bmatrix} (i=1,2) , \]

\[ \hat{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} , \hat{H}_i = [H_i,0] (i=1,2) . \]

In order to complete the (if)-part of the proof it suffices to show that system (7) together with the linear subspaces \( \hat{S}_1, \hat{S}_2, \hat{V}_1 \) and \( \hat{V}_2 \) satisfies the conditions of Theorem 3.3.

Indeed, if the conditions of Theorem 3.3 are fulfilled we may conclude that there exist matrices \( \hat{K} \in \mathbb{R}^{2n \times 2n} , \hat{L} \in \mathbb{R}^{2n \times (p+n)} , \hat{M} \in \mathbb{R}^{(m+n) \times 2n} \) and \( \hat{N} \in \mathbb{R}^{(m+n) \times (p+n)} \) such that for \((i,j) = (1,2),(2,1)\)

\[ 0 = [\hat{H}_1,0] (sl - \begin{bmatrix} \hat{A} + \hat{B} \hat{N} \hat{C} & \hat{B} \hat{M} \\ \hat{L} \hat{C} & \hat{K} \end{bmatrix})^{-1} \begin{bmatrix} \hat{G}_j \\ 0 \end{bmatrix} =
\]

\[ = [H_i,0,0,0] (sl - \begin{bmatrix} A + BN_{11}C & BN_{12} & BM_{11} & BM_{12} \\ N_{21}C & N_{22} & M_{21} & M_{22} \\ L_{11}C & L_{12} & K_{11} & K_{12} \\ L_{21}C & L_{22} & K_{21} & K_{22} \end{bmatrix})^{-1} \begin{bmatrix} G_j \\ 0 \\ 0 \\ 0 \end{bmatrix} . \]

In the latter we have partitioned

\[ \hat{K} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} , \hat{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} , \hat{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \text{ and } \hat{N} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} . \]

From this we may conclude that the compensator given by

\[ \begin{bmatrix} \dot{w}_1(t) \\ \dot{w}_2(t) \\ \dot{w}_3(t) \end{bmatrix} = \begin{bmatrix} N_{22} & M_{21} & M_{22} \\ L_{12} & K_{11} & K_{12} \\ L_{22} & K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} + \begin{bmatrix} N_{21} \\ L_{11} \\ L_{21} \end{bmatrix} y(t) , \]

\[ u(t) = [N_{12} M_{11} M_{12}] \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} + N_{11} y(t) \]

achieves non interaction and therefore solves (NICPM). So there remains to show that the system described by (7) together with the subspaces \( \hat{S}_1, \hat{S}_2, \hat{V}_1 \) and \( \hat{V}_2 \) satisfies the conditions of Theorem 3.3.
Note that \( \text{im} \hat{G}_1 \subseteq \hat{S}_1 \), \( \text{im} \hat{G}_2 \subseteq \hat{S}_2 \) and \( \hat{V}_1 \subseteq \ker \hat{H}_2 \), \( \hat{V}_2 \subseteq \ker \hat{H}_1 \). Furthermore, since \( S^* \subseteq V_1^* + V_2^* \) it follows that \( Q_1 S^* \subseteq V_1^* \) and \( Q_2 S^* \subseteq V_2^* \) from which it is immediate that \( \text{im} \hat{G}_1 \subseteq \hat{S}_1 \subseteq \hat{V}_1 \subseteq \ker \hat{H}_2 \) and \( \text{im} \hat{G}_2 \subseteq \hat{S}_2 \subseteq \hat{V}_2 \subseteq \ker \hat{H}_1 \). Because \( \ker H_1 \cap \ker H_2 = \{0\} \) also \( V_1^* \cap V_2^* = \{0\} \). Hence the subspaces \( V_1^* \) and \( V_2^* \) are \((A,B)\)-compatible. Therefore there exists a matrix \( F \in \mathbb{R}^{mxn} \) such that \( (A + BF)V_1^* \subseteq V_1^* \) and \( (A + BF)V_2^* \subseteq V_2^* \).

From this it is clear that with \( \hat{F} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix} \) also \( (\hat{A} + \hat{B} \hat{F})\hat{V}_1 \subseteq \hat{V}_1 \) and \( (\hat{A} + \hat{B} \hat{F})\hat{V}_2 \subseteq \hat{V}_2 \). Consequently the subspaces \( \hat{V}_1 \) and \( \hat{V}_2 \) are \((\hat{A},\hat{B})\)-invariant and even \((\hat{A},\hat{B})\)-compatible.

Note that \( \hat{A} (\hat{S}_1 \cap \ker \hat{C}) = \hat{A} ((S_1^* \cap \ker C) \oplus \{0\}) = (A (S_1^* \cap \ker C)) \oplus \{0\} \subseteq S_1^* \oplus \{0\} \subseteq \hat{S}_1 \) and analogously \( \hat{A} (\hat{S}_2 \cap \ker \hat{C}) \subseteq \hat{S}_2 \). Hence the subspaces \( \hat{S}_1 \) and \( \hat{S}_2 \) are \((C,A)\)-invariant.

Let \( \hat{x} \in (\hat{S}_1 + \hat{S}_2) \cap \ker \hat{C} \). Since \( \hat{x} \in \hat{S}_1 + \hat{S}_2 \) we can write \( \hat{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \) with \( x_1 = a_1 + Q_1 b_1 + a_2 + Q_2 b_2 \) and \( x_2 = b_1 + b_2 \) where \( a_1 \in S_1^* \), \( a_2 \in S_2^* \) and \( b_1, b_2 \in S^* \). But since also \( \hat{x} \in \ker \hat{C} \) it follows that \( C x_1 = 0 \) and \( x_2 = 0 \). So \( b_1 = -b_2 \) and \( x_1 = a_1 + a_2 + (Q_1 - Q_2) b_1 = a_1 + a_2 + b_1 \), where the latter equality follows from properties of \( Q_1 \), \( Q_2 \) and \( (Q_1 - Q_2) \).

Therefore, any \( \hat{x} \in (\hat{S}_1 + \hat{S}_2) \cap \ker \hat{C} \) can be written as \( \hat{x} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix} \) with \( x_1 \in S^* \cap \ker C \), from which it is immediate that \( \hat{A} \hat{x} = \hat{A} \begin{bmatrix} x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} y \\ 0 \end{bmatrix} \) with \( y \in S^* \).

Note that

\[
\begin{bmatrix} y \\ 0 \end{bmatrix} = \begin{bmatrix} (Q_1 - Q_2)y \\ 0 \end{bmatrix} = \begin{bmatrix} Q_1 y \\ 0 \end{bmatrix} - \begin{bmatrix} Q_2 y \\ 0 \end{bmatrix}
\]

which implies that \( \hat{A} \hat{x} \in \hat{S}_1 + \hat{S}_2 \). Consequently, \( \hat{A} ((\hat{S}_1 + \hat{S}_2) \cap \ker \hat{C}) \subseteq \hat{S}_1 \) and \( \hat{S}_2 \).

Finally we note that from the last condition mentioned in the theorem it follows that

\[
\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -A & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{S}_1 \oplus \hat{S}_2 \end{bmatrix} \cap \ker [\hat{C}, \hat{C}] \subseteq \begin{bmatrix} (V_1^* \oplus \{0\}) \oplus (V_2^* \oplus \{0\}) \end{bmatrix} + \text{im} \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}
\]

from which it is immediate that

\[
\begin{bmatrix} \hat{A} & 0 \\ 0 & -\hat{A} \end{bmatrix} \begin{bmatrix} \hat{S}_1 \oplus \hat{S}_2 \end{bmatrix} \cap \ker [\hat{C}, \hat{C}] \subseteq \langle \hat{V}_1 \oplus \hat{V}_2 \rangle + \text{im} \begin{bmatrix} \hat{B} \\ \hat{B} \end{bmatrix}.
\]

Now the conditions of Theorem 3.3 are fulfilled and the proof of the (if)-part is completed.

(Only if)

Assume there exists a compensator \( \Sigma_c \) such that \( T_{12}(s) = 0 \) and \( T_{21}(s) = 0 \). Note that

\[
T_{ij}(s) = H_{i,s}(sI - A_s)^{-1} G_{j,s} = \sum_{k=0}^{\infty} (H_{i,s} A_s^k G_{j,s}) s^{-k+1},
\]

from which it is clear that for all \( k \geq 0 \)
\[ H_{1,s} A^k_s G_{2,s} = 0 \text{ and } H_{2,s} A^k_s G_{1,s} = 0. \] The latter implies that for all \( k \geq 0 \) \( A^k_s \text{im} G_{2,s} \subseteq \text{ker} H_{1,s} \) and \( A^k_s \text{im} G_{1,s} \subseteq \text{ker} H_{2,s}. \) Now define the linear subspaces \( W_1 \) and \( W_2 \) in \( R^{**} \) by \( W_i = \sum_{k=0}^\infty A^k_s \text{im} G_{i,s} \) \( (i = 1,2). \)

It is clear that \( \text{im} G_{1,s} \subseteq W_1 \subseteq \text{ker} H_{2,s}, \) \( \text{im} G_{2,s} \subseteq W_2 \subseteq \text{ker} H_{1,s}, \) \( A_s W_1 \subseteq W_1 \) and \( A_s W_2 \subseteq W_2. \) Let \( S_1, S_2, S, V_1 \) and \( V_2 \) be linear subspaces in \( R^a \) defined by

\[
S_i = \left\{ x \in R^a \mid \begin{bmatrix} x \\ 0 \end{bmatrix} \in W_i \right\} \quad (i = 1,2),
\]

\[
S = \left\{ x \in R^a \mid \begin{bmatrix} x \\ 0 \end{bmatrix} \in W_1 + W_2 \right\}
\]

and

\[
V_i = \left\{ x \in R^a \mid \text{there exists } w \in R^k \text{ such that } \begin{bmatrix} x \\ w \end{bmatrix} \in W_i \right\} \quad (i = 1,2).
\]

Note that \( S_i \) is the intersection of \( W_i \) with \( R^a \) \( (i = 1,2) \), \( S \) is the intersection of \( W_1 + W_2 \) with \( R^a \) and \( V_i \) is the projection of \( W_i \) onto \( R^a \) along \( R^k \) \( (i = 1,2). \)

Now it can be shown (cf. Schumacher [3]), that \( S_1, S_2 \) and \( S \) are \((C,A)-\)invariant subspaces and that \( V_1, V_2 \) are \((A,B)-\)invariant subspaces. Furthermore, it is clear that \( \text{im} G_{1} \subseteq S_1 \subseteq V_1 \subseteq \text{ker} H_{2} \), \( \text{im} G_{2} \subseteq S_2 \subseteq V_2 \subseteq \text{ker} H_{1} \) and \( \text{im} [G_{1},G_{2}] \subseteq S \subseteq V_1 + V_2 \) from which it follows that \( S^*(\text{im} G_{1}) \subseteq V^*(\text{ker} H_{2}), \) \( S^*(\text{im} G_{2}) \subseteq V^*(\text{ker} H_{1}) \) and \( S^*(\text{im} [G_{1},G_{2}]) \subseteq V^*(\text{ker} H_{1}) + V^*(\text{ker} H_{2}). \)

Let \( x \in S_1 \), then \( \begin{bmatrix} x \\ 0 \end{bmatrix} \in W_1. \) By the \( A_s \)-invariance of \( W_1 \) it follows that \( A_s \begin{bmatrix} x \\ 0 \end{bmatrix} \in W_1 \) and more specific \((A+BNC)x \in V_1. \) Therefore, \((A+BNC)S_1 \subseteq V_1 \) and \((A+BNC)S_2 \subseteq V_2. \)

Now let \( Q_1 \) and \( Q_2 \) be matrices as described in the beginning of this section and let \( x \in S. \)

Since \( S \subseteq V_1 + V_2, V_1 \subseteq \text{ker} H_{2}, V_2 \subseteq \text{ker} H_{1} \) and \( \text{ker} H_{1} \cap \text{ker} H_{2} = \{0\} \) the vector \( x \) can be decomposed uniquely as \( x = x_1 - x_2 \) with \( x_1 \in V_1 \) and \( x_2 \in V_2. \) In particular \( x_1 = Q_1 x \) and \( x_2 = Q_2 x. \) Because \( x \in S \) we have \( \begin{bmatrix} x \\ 0 \end{bmatrix} \in W_1 + W_2. \) Therefore, there exist \( \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \in W_1 \) and \( \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \in W_2 \) such that \( \begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} \) with \( a_1 \in V_1, a_2 \in V_2 \) and \( b_1, b_2 \in R^k. \) It is now clear that \( a_1 = x_1 = Q_1 x, a_2 = -x_2 = -Q_2 x \) and \( b_1 = -b_2. \) So if \( x \in S \) then \( \begin{bmatrix} x \\ 0 \end{bmatrix} \) can be written as \( \begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} Q_1 x \\ y \end{bmatrix} - \begin{bmatrix} Q_2 x \\ y \end{bmatrix} \) with \( y \in R^k \) such that \( \begin{bmatrix} Q_1 x \\ y \end{bmatrix} \in W_1 \) and \( \begin{bmatrix} Q_2 x \\ y \end{bmatrix} \in W_2. \) Because \( W_1 \) and \( W_2 \) are \( A_s \)-invariant subspaces we have \( \begin{bmatrix} A+BNC & BM \\ LC & K \end{bmatrix} \begin{bmatrix} Q_i x \\ y \end{bmatrix} \in W_i \) \( (i = 1,2) \) and in particular we have \( ((A+BNC)Q_i x + BM y) \in V_i \) \( (i = 1,2). \)
Let \( \{x_1, x_2, \ldots, x_t\} \) be a basis for \( S \). By the previous it follows that for every \( j \in \{1, 2, \ldots, t\} \) there exists \( y_j \in \mathbb{R}^k \) such that \( ((A + BN'C)Q_j + BM_j y_j) \in V_1 \) and \( ((A + BN'C)Q_2 x_j + BM_j y_j) \in V_2 \). Now define matrices \( N' \in \mathbb{R}^{m \times p} \) and \( M' \in \mathbb{R}^{m \times n} \) by \( N' = N \) and \( M'(x_1, x_2, \ldots, x_t) = (M y_1, M y_2, \ldots, M y_t) \). Then it follows that \((A + BN'C)S_1 \subseteq V_1\), \((A + BN'C)S_2 \subseteq V_2\), \((A + BN'C)Q_1 + BM'M' \subseteq V_1\) and \((A + BN'C)Q_2 + BM'M' \subseteq V_2\).

Define the linear subspaces \( \tilde{S}_1 \) and \( \tilde{S}_2 \) in \( \mathbb{R}^{2n} \) by

\[
\tilde{S}_i = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} Q_i b \\ b \end{bmatrix} \middle| a \in S_1, b \in S \right\} \quad (i = 1, 2)
\]

By the previous it is clear that \([A + BN'C, BM']S_1 \subseteq V_1\) and \([A + BN'C, BM']\tilde{S}_2 \subseteq V_2\).

Now let \( \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \in \tilde{S}_1 \) and \( \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \in \tilde{S}_2 \) be such that \( \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = 0 \).

There exist \( v_1 \in V_1 \) and \( v_2 \in V_2 \) such that \((A + BN'C)a_1 + BM'b_1 = v_1\) and \((A + BN'C)a_2 + BM'b_2 = -v_2\).

Denote

\[
\begin{bmatrix} y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = -\begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}
\]

then it follows that

\[
\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0 & -A & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} B \\ \end{bmatrix}[N', M'] \begin{bmatrix} y_1 \\ z_1 \end{bmatrix} \in (V_1 \oplus V_2) + \text{im} \begin{bmatrix} B \\ \end{bmatrix}.
\]

Hence,

\[
\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & 0 & -A & 0 \end{bmatrix} \begin{bmatrix} \tilde{S}_1 \oplus \tilde{S}_2 \end{bmatrix} \cap \ker \begin{bmatrix} C & 0 & C & 0 \\ 0 & I & 0 & I \end{bmatrix} \subseteq (V_1 \oplus V_2) + \text{im} \begin{bmatrix} B \\ \end{bmatrix}.
\]

The (only if)-part is now proved by the observation that \( \tilde{S}_1 \subseteq \tilde{S}_1, \tilde{S}_2 \subseteq \tilde{S}_2, V_1 \subseteq V^*(\ker H_2) \) and \( V_2 \subseteq V^*(\ker H_1) \).

\[\square\]
5. Conclusions and Remarks

5.1. In this paper we studied systems that apart from a control input and a measurement output have two exogenous inputs and two exogenous outputs. Furthermore, we assumed that the two exogenous outputs of the systems that we consider are complete. This assumption is expressed in a formula by \( \text{ker} H_1 \cap \text{ker} H_2 = \{0\} \).

For this kind of systems we derived necessary and sufficient conditions for the existence of a measurement feedback compensator such that the resulting closed loop system has off-diagonal blocks equal to zero.

Of course analogous results may be derived for systems of which the two exogenous inputs have a property \( \text{im} G_1 + \text{im} G_2 = \mathbb{R}^n \) dual to the previous mentioned property of outputs complete. An interesting problem for future research will be the above problem for systems where no additional assumptions are put upon the two exogenous inputs or the two exogenous outputs. In this context we want to refer to Van der Woude [7]. In the latter paper it is argued that the solvability of this general problem is equivalent to the existence of a proper rational matrix \( X \) such that \( A_1 X B_1 = C_1 \) and \( A_2 X B_2 = C_2 \) where \( A_1, A_2, B_1, B_2, C_1 \) and \( C_2 \) are all strictly proper rational matrices. It is clear that the latter requires a generalization of Theorem 3.1 to matrices over the ring of proper rational functions.

5.2. To end this section we shall give a conceptual algorithm that, if it exists, determines a compensator \( \Sigma \) that achieves noninteraction. Therefore, let the system \( \Sigma \) with the additional property of outputs given and assume that (NICPM) is solvable. Hence, assume that the conditions of Theorem 4.1 are fulfilled. Then the algorithm proceeds through the following steps.

1. Let \( \hat{A}, \hat{B}, \hat{C}, \hat{G}_1, \hat{G}_2, \hat{H}_1 \) and \( \hat{H}_2 \) be matrices and let \( \hat{S}_1, \hat{S}_2, \hat{V}_1 \) and \( \hat{V}_2 \) be linear subspaces in \( \mathbb{R}^{2n} \) as defined in the (if)-part of the proof of Theorem 4.1.

2. Compute matrices \( \hat{F} \in \mathbb{R}^{(n+m) \times 2n}, \hat{T} \in \mathbb{R}^{2n \times (n+p)} \) and \( \hat{N} \in \mathbb{R}^{(n+m) \times (n+p)} \) such that \( (\hat{A} + \hat{B} \hat{F}) \hat{V}_i \subseteq \hat{V}_i \) \( (i=1,2) \), \( (\hat{A} + \hat{T} \hat{C}) \hat{S}_i \subseteq \hat{S}_i \) \( (i=1,2) \) and \( (\hat{A} + \hat{B} \hat{N} \hat{C}) \hat{S}_i \subseteq \hat{V}_i \) \( (i=1,2) \). The existence of \( \hat{F}, \hat{T} \) and \( \hat{N} \) is proved in the (if)-part of the proof of Theorem 4.1.

The computation of for instance \( \hat{N} \) can be performed as follows. Let \( \bar{S}_i \) \( (i=1,2) \) and \( \bar{W}_i \) \( (i=1,2) \) be matrices such that \( \text{im} \bar{S}_i = \hat{S}_i \) \( (i=1,2) \) and \( \ker \bar{W}_i = \hat{V}_i \) \( (i=1,2) \). Then it suffices to compute \( \hat{N} \) such that \( \bar{W}_i \hat{A} \bar{S}_i + \bar{W}_i \hat{B} \hat{N} \hat{C} \bar{S}_i = 0 \) \( (i=1,2) \). Using Kronecker products the latter two linear matrix equations can be written as one linear matrix equation which can be solved using standard techniques, whereupon rearrangement of the obtained solution provides \( \hat{N} \).

Similar remarks can be made with respect to the computation of \( \hat{F} \) and \( \hat{T} \).

3. Define matrices \( \hat{K} \in \mathbb{R}^{2n \times 2n}, \hat{L} \in \mathbb{R}^{2n \times (n+p)} \) and \( \hat{M} \in \mathbb{R}^{(n+m) \times 2n} \) by \( \hat{K} = \hat{A} + \hat{B} \hat{F} + \hat{T} \hat{C} - \hat{B} \hat{W} \hat{C}, \hat{L} = \hat{B} \hat{W} - \hat{T}, \hat{M} = \hat{F} - \hat{W} \hat{C} \).

Now decompose...
\[ \hat{N} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}, \quad \hat{M} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \quad \hat{L} = [L_1, L_2] \]

with \( N_{11} \in \mathbb{R}^{m \times p}, N_{21} \in \mathbb{R}^{n \times p}, \)

\( N_{12} \in \mathbb{R}^{m \times q}, N_{22} \in \mathbb{R}^{n \times q}, M_1 \in \mathbb{R}^{m \times 2n}, M_2 \in \mathbb{R}^{n \times 2n}, L_1 \in \mathbb{R}^{2n \times p}, L_2 \in \mathbb{R}^{2n \times q}, \)

and define matrices \( K \in \mathbb{R}^{3n \times 3n}, L \in \mathbb{R}^{3n \times p}, M \in \mathbb{R}^{m \times 3n} \) and \( N \in \mathbb{R}^{m \times p} \) by

\[ K = \begin{bmatrix} N_{22} & M_2 \\ L_2 & \hat{K} \end{bmatrix}, \quad L = \begin{bmatrix} N_{21} \\ L_1 \end{bmatrix}, \quad M = [N_{12} \quad M_1] \quad \text{and} \quad N = N_{11}. \]

Then the matrices \( K, L, M \) and \( N \) constitute a measurement feedback compensator that achieves non interaction.
References


