Stirring turbulence with turbulence

Citation for published version (APA):

Cekli, H. E., Joosten, R., & van de Water, W. (2015). Stirring turbulence with turbulence. *Physics of Fluids*, 27(12), [125107]. <https://doi.org/10.1063/1.4937427>

DOI:

[10.1063/1.4937427](https://doi.org/10.1063/1.4937427)

Document status and date:

Published: 01/12/2015

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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- The final published version features the final layout of the paper including the volume, issue and page numbers.

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Citation: *Physics of Fluids* **27**, 125107 (2015); doi: 10.1063/1.4937427

View online: <http://dx.doi.org/10.1063/1.4937427>

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Stirring turbulence with turbulence

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(Received 8 June 2015; accepted 25 November 2015; published online 29 December 2015)

We stir wind-tunnel turbulence with an active grid that consists of rods with attached vanes. The time-varying angle of these rods is controlled by random numbers. We study the response of turbulence on the statistical properties of these random numbers. The random numbers are generated by the Gledzer-Ohkitani-Yamada shell model, which is a simple dynamical model of turbulence that produces a velocity field displaying inertial-range scaling behavior. The range of scales can be adjusted by selection of shells. We find that the largest energy input and the smallest anisotropy are reached when the time scale of the random numbers matches that of the largest eddies of the wind-tunnel turbulence. A large mismatch of these times creates a highly intermittent random flow with interesting but quite anomalous statistics. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4937427>]

I. INTRODUCTION

The standard way to stir turbulence in a wind tunnel is by passing a laminar wind through a grid that consists of a regular mesh of bars or rods. In this way, near-homogeneous and near-isotropic turbulence can be made with the largest eddy size set by the mesh size, but the maximum attainable turbulent Reynolds number is small.¹ The precise structure of such a grid should not matter much; according to Kolmogorov's theory, any stirred turbulent flow, if left to its own devices, becomes homogeneous and isotropic at small enough scales.^{2,3} However, in practical situations which involve finite Reynolds numbers, these scales may not be reached before viscosity starts to act. Therefore, the manner in which turbulence is stirred matters in practice, and grids which have a multiscale structure have recently been found to produce interesting flow statistics.^{4,5}

Much more vigorous turbulence can be stirred by so-called active grids that have moving elements. In this way, Taylor-based Reynolds numbers $Re_\lambda \approx 10^3$ can be achieved in flows that are homogeneous and near-isotropic. To stir turbulence, the space-time structure of active grids is controlled by random numbers. This paper addresses the question how these random numbers should be chosen to create the desired turbulence statistics. Rather than using a (pseudo-)random number generator, we will generate these random numbers with the help of a simple shell model of turbulence. These random numbers have spectral distributions that are similar to those of turbulence, while the statistical properties of these numbers can be quantified with turbulence parameters, such as integral length and time scales. Thus, we are stirring turbulence with turbulence. Other schemes to stir turbulence with a colored spectrum and spatial correlations might produce similar results, but they lack the guidance offered by the turbulence characterization.

Active grids, such as the one used in our experiment, were pioneered by Makita⁶ and consist of a grid of rods with attached vanes that can be rotated by servo motors. The properties of actively stirred turbulence were further investigated by Mydlarski and Warhaft⁷ and Poorte and Biesheuvel.⁸ A new development is to use active grids to tailor turbulence, for example, to create homogeneous

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shear turbulence, which has a constant gradient of the mean flow and a uniform fluctuating velocity, or to simulate the turbulent atmospheric boundary layer.⁹

For the active grid used in our experiments, the stirring protocol concerns the time-dependent angle of all axes. Several ways of random motion of the axes have been explored. In one, the axes are rotated with constant angular velocity, but the direction of rotation is switched at random times. In another protocol, also the velocity is picked randomly from a uniform distribution.⁸ In the protocol of Mydlarski and Warhaft,⁷ the axes are not rotated continuously, but their angles are flipped randomly. The influence of these simple protocols on the properties of the generated turbulence has been studied by Poorte and Biesheuvel.⁸ Protocols involving correlated random numbers to mimic the large-scale fluctuations of atmospheric turbulence were used by Knebel, Kittel, and Peinke.¹⁰

An active grid offers the possibility to locally act on the flow. The question is how to operate the grid with a time series of random numbers in order to achieve desired turbulence properties. Ideally, a turbulent flow could be generated numerically, and the outcome of the simulation could be fed to the grid as an initial condition to the turbulence created in the wind tunnel. As wind-tunnel experiments can achieve Reynolds numbers that are much higher than can be reached in a numerical simulation during times that stretch an extremely large number of large-eddy turnover times, direct numerical simulations cannot match the experimental requirement. Moreover, it is not yet clear how a dynamical grid influences the flow locally, and if the initial condition can be specified completely.

In this paper, we study the relation between the statistical properties of a model-turbulence signal that drives the grid and those of the generated wind-tunnel turbulence. We will find that isotropic turbulence results when matching the integral time scales, but also that wind-tunnel turbulence with unusual properties, such as highly intermittent large-scale statistics, can emerge from a gross mismatch of time scales. This is remarkable because intermittency, the non-Gaussian statistics of the velocity field, is a property of turbulent velocity differences measured over length scales much smaller than the stirring scale. This may find an application in wind-tunnel tests that seek to model the intermittent atmosphere.

In Section II, we will describe the experimental setup. A summary of the model used to produce the signal that drives the grid, the Gledzer–Ohkitani–Yamada (GOY) shell model,¹¹ is given in Section III. This model has been discussed extensively in the literature, but we find it useful and instructive to discuss the statistical properties of the generated velocity field. The way in which this velocity field is supplied to the active grid is discussed in Section IV. An important aspect is the choice of length and time scales, both in the simulation and in the experiment. In Section V, we show how the statistics of the stirred wind-tunnel turbulence depend on the match of time scales of the random numbers generated by the GOY model and the integral time scale of the experiment.

II. EXPERIMENTAL SETUP

A schematic drawing of the wind tunnel and a picture of the active grid are shown in Fig. 1. The active grid is placed in the 8 m long experimental section of a recirculating wind tunnel. Turbulent

FIG. 1. The active grid and a schematic drawing of the wind tunnel and the used coordinate system.

velocity fluctuations are measured at a distance 4.6 m downstream from the grid using a single x-wire anemometer; it is centered in the windtunnel cross section. Our grid has mesh size $M = 0.1$ m and consists of 17 axes whose instantaneous angles $\theta_i(t)$, $i = 1, \dots, 17$ are prescribed precisely using proportional-integral-derivative (PID) controllers. Angle information can be fed to the grid with a sampling time $\delta t_g = 10^{-2}$ s. The fastest changes that can be imposed on the flow are set both by the shortest angular period of the blades, 3×10^{-2} s, corresponding to the fastest rotation rate of 15 s^{-1} , and by the shortest time to respond to imposed velocity changes. From an experiment where we drive the grid with white noise, we estimate that the response time of the grid is $\approx 4 \times 10^{-3}$ s. This response time is limited by the inertia of the axes; it also limits the fastest turbulence time scale that can be directly imposed on the flow through the active grid. In a typical experiment, we feed a time series computed by the turbulence model to the grid and collect wind data during 450 s (4.5×10^3 large-eddy turnover times).

We measure the u, w velocity components of the flow, which allows an assessment of the flow isotropy. The locally manufactured hot-wire velocity probe had a $2.5 \text{ }\mu\text{m}$ diameter and a sensitive length of $400 \text{ }\mu\text{m}$, which is comparable to the typical smallest length scale of the flow in our experiments (the measured Kolmogorov scale is $\eta \approx 220 \text{ }\mu\text{m}$). The hot-wire probe was operated at constant temperature using computer controlled anemometers that were also developed locally. Each experiment was preceded by a calibration procedure.^{12,13} The signals captured by the sensors were sampled at 20 kHz, after being low-pass filtered at 10 kHz.

III. THE GOY SHELL MODEL

The turbulence model used for driving the grid is the GOY shell model. It is a dynamical model for the time dependent amplitudes $u_n(t)$ of Fourier modes at discrete wavenumbers k_n ,^{14–16}

$$\left(\frac{d}{dt} + \nu k_n^2\right) u_n = \iota (a_n u_{n+1} u_{n+2} + b_n u_{n-1} u_{n+1} + c_n u_{n-1} u_{n-2})^* + f_n, \quad (1)$$

where ν is the kinematic viscosity, k_n is an exponentially spaced grid of scalar wavenumbers, $k_n = 2^{n-4}$, $n = 1, \dots, 22$, and where the factors $a_n = k_n$, $b_n = -k_{n-1}/2$, $c_n = -k_{n-2}/2$ ensure conservation of energy, enstrophy, and helicity in the inviscid ($\nu = 0$) and unforced ($f_n = 0$) system. The model mimics the Navier–Stokes equation in wavenumber space, but as the wavenumbers are scalar, it lacks the flow structures typical for turbulence, such as vortices. Further, the nonlinear advection term which in the Navier–Stokes equation couples all wavenumbers to all others is here restricted to interactions between neighboring wavenumbers. The sweeping of the small scales by large ones is absent, and the time dependence of the velocity field is that of a Lagrangian one.¹⁷ In our simulations, we followed the standard numerical approach of Pisarenko *et al.*,¹⁸ took $\nu = 10^{-7}$ and $k_0 = 1/16$, and forced the first shell, $f_n = 5 \times 10^{-3} \delta_{n,1} (1 + \iota)$, with ι the imaginary unit.

In our experiment, we modulate turbulence by passing a laminar wind with velocity $U \approx 10 \text{ ms}^{-1}$ through the active grid. Because the mean velocity is much larger than the fluctuating velocity, we supply Eulerian velocity fields, $u(x, t)$, with $x = Ut$. Strictly, therefore, we stir a Eulerian field with Lagrangian velocities.

Let us now summarize the statistical properties of the shell-model velocity field. From the complex shell velocities, we compute the space-time turbulent velocity field

$$u(x, t) = \Re \sum_{n=1}^{22} e^{ik_n x} u_n(t). \quad (2)$$

The energy dissipation computed from the energy input in the forced shell, $\epsilon = \Re \langle u_1^* f_1 \rangle = 3.7 \times 10^{-3}$, defines the Kolmogorov length scale $\eta = (\nu^3/\epsilon)^{1/4} = 2.3 \times 10^{-5}$, Kolmogorov time scale $\tau_\eta = (\nu/\epsilon)^{1/2} = 5.2 \times 10^{-3}$, and the Taylor Reynolds number $\text{Re}_\lambda = u^2(\nu\epsilon)^{-1/2} = 5.4 \times 10^3$, with u the turbulent velocity, $u^2 = \langle u^2(0, t) - \langle u(0, t) \rangle^2 \rangle$, $u = 0.32$. All quantities are in dimensionless (computational) units.

The wavenumber spectrum is determined by the shell amplitudes, $E(k_n) = k_n^{-1} \langle |u_n|^2 \rangle$, while the frequency spectrum follows from the Fourier transform of $u(x = 0, t)$. The integral length and time

scales L and T are computed from these spectra, $L = \pi E(k_1)/2u^2 = 7.5$ and $T = \pi E(f = 0)/2u^2 = 46$, respectively.

The dynamic range of length and time scales of the active grid is limited, and therefore, we define a filtered version of the velocity field by restricting the sum in Eq. (2) to the first N shells,¹⁹

$$u^{(N)}(x, t) = \Re \sum_{n=1}^N e^{ik_n x} u_n(t), \quad (3)$$

with the associated frequency spectra

$$E_N(f) = \left\langle \left| \int u^{(N)}(x = 0, t) e^{-2\pi i f t} dt \right|^2 \right\rangle. \quad (4)$$

The associated cut-off wavenumber k_N corresponds to a spatial scale $l_N = 2\pi/k_N$. Because the velocity fluctuations at scale l are $u(l) \propto l^{1/3}$, the corresponding cut-off frequency f_N scales with the cut-off wavenumber as $f_N \propto k_N^{2/3} \propto 2^{2N/3}$.

Time traces of shell-model velocities, with both the velocity field restricted to $N = 2$ and the fully resolved velocity field ($N = 22$), are shown in Fig. 2. The time axes of the two figures are chosen such as to show all details of the velocity signal. The time scale of the $N = 2$ truncation is a factor $2^{20/3} \approx 10^2$ larger than that of the $N = 22$ truncation. The resolved velocity is characterized by long episodes of relatively calm, interspersed with short bursts of turbulent activity. This highly intermittent character is in accordance with the very large Reynolds number, $\text{Re}_\lambda = 5.4 \times 10^3$, and has been noticed before.¹⁵

The statistical properties of the simulated turbulent velocity are illustrated in Fig. 3. The spatial spectrum shows inertial-range behavior, $E(k) = C_K \epsilon^{2/3} k^\alpha$, with $\alpha = -1.74$, which is close to $-5/3$, and with Kolmogorov constant $C_K \approx 1.5$. The frequency spectrum $E_{22}(f)$ shows the expected inertial-range behavior $E(f) \sim f^{-2}$ of a Lagrangian field,¹⁷ while the cut-off frequencies f_N of the filtered spectra $E_N(f)$ scale as $f_N \propto 2^{2N/3}$. The probability density function (PDF) $P(\Delta u)$ of temporal velocity increments evolves from Gaussian at large time delays, to highly intermittent stretched exponential ($P(\Delta u) \sim \exp(-\beta|\Delta u|^\alpha)$, $\alpha < 1$) for the shortest time delays.

IV. CONTROLLING THE GRID

The GOY model produces a space-time signal $u^{(N)}(x, t)$ [Eq. (3)] which we translate to axis-angle signals $\theta_i(t)$ to drive the grid. There is not a simple relation between $\theta_i(t)$ and the flow velocity. It appears that only at very low grid frequencies, the mean flow approximately follows the time-varying

FIG. 2. Time series simulated by shell model Eq. (1). In computing the velocity signal $u^{(N)}(x, t)$ [Eq. (3)], we have truncated the shell contributions at $N = 2$ and $N = 22$ for (a) and (b), respectively. The time axes of the two figures are chosen such as to show all details of the velocity signal.

FIG. 3. Spectra and probability density functions of the simulated shell model which is used to drive the active grid. (a) The full line is the normalized spectrum $E(k_n)/(C_K \epsilon^{2/3})$, with $C_K = 1.5$; the dashed line is a fit $E(k) \sim k^{-1.74}$, which is close to the Kolmogorov prediction $k^{-5/3}$. (b) Frequency spectra $E_N(f)$ for $N = 2, 6, 10$, and 22 , respectively [Eq. (4)]. In the inertial range, the frequency cutoff f_N scales with the wavenumber cutoff k_N as $f_N \sim k_N^{2/3}$, which amounts to a factor $2^{8/3} \approx 6.35$ between the $N = 2, 6$, and 10 cutoffs. The dashed line is a fit $E_{22}(f) \sim f^{-2.0}$. (c) PDF of time-wise velocity increments $\Delta u(\tau) = u(t + \tau) - u(t)$ at $\tau / \tau_\eta = 4.3$ and $\tau / \tau_\eta = 1.2 \times 10^4$; the dashed line is a Gaussian fit.

transparency of the grid.²⁰ In the present paper, we use the simple approach to set the v and w components of the velocity proportional to the angles of the vertical and horizontal axes. Thus, we assume that the statistics of the fluctuating velocities at the location of the grid are related to the statistics of the grid angles.

The quantities of the shell model are dimensionless and the art is to match them to physical units of space, time, and velocity. To this aim, we must scale the dimensionless units of the GOY model,

$$u^{(N)}(x, t) = C_u \Re \sum_{n=1}^N e^{ik_n C_x x} u_n(C_t t), \quad (5)$$

so that x and t are now in physical units. We take the mesh grid size M as the smallest length scale we can impose onto the flow and we associate it with the smallest length scale $2\pi/k_N$ of the driving signal $u^{(N)}(x, t)$, from which the spatial scaling factor C_x (which has dimension m^{-1}) follows

$$C_x k_N M = 2\pi.$$

The grid moves spatially incoherently at this scale, but is correlated at larger scales $> 2\pi/k_N$.

The possibilities of true spatial modulation are restricted by the small number of (independent) grid cells, but a much larger dynamical range of time scales can be imposed through temporal modulation. The fastest time scale that can be used to drive the grid is the grid sample time $\delta t_g = 10^{-2}$ s, which is comparable to its dynamical response time; there is no bound on the slowest time scale.

Let us define a reference time scale T^e in the windtunnel as the mesh size divided by the typical turbulent velocity, $u = 1$ m/s, so that $T^e = M/u = 0.1$ s. This time should be comparable to the integral time of the windtunnel turbulence if the stationary grid is open. The temporal scaling factor

FIG. 4. (a) Dissipation rate ϵ of windtunnel turbulence as a function of T/T^g for various shell filter settings N . It is maximal at $N = 4$ and $T/T^g \approx 1$. (b) Large-scale anisotropy u/w of the turbulent flow as a function of T/T^g . For $N = 4$, which corresponds to the strongest turbulence, it is minimal at $T/T^g \approx 1$.

C_t in Eq. (5) defines a (dimensionless) grid time T^g as $C_t = T^g/T^e$. If T^g is chosen such that it equals the integral time T of the GOY model, the integral times of the driving signal, and the reference time T^e of the windtunnel match. Therefore, the ratio T/T^g is a measure of the mismatch between these time scales. We anticipate that the “resonance condition” $T/T^g = 1$ is special, but also that interesting random flows result when T^g is chosen very different from the GOY model integral time T .

Feeding the turbulence-like random time series of the GOY model to the grid controller still involves a few practical issues. The time series resulting from integration of the GOY model, which consists of a velocity $u^{(N)}(x_j, t_i)$ at each integration time step t_i , has a too fine temporal resolution. For example, for the case of matched integral time scales T and T^e , we can only supply $T^e/\delta t_g = 10$ samples to the grid in a model integral time T .

Therefore, the numerically computed time series must be low-pass filtered and downsampled. Most of the required filtering is already implicit in the computed time series, as the integration time step is much smaller than the Kolmogorov time τ_η . The signal produced by Eq. (1) is smooth at the Kolmogorov scale τ_η . Further, the time series $u^{(N)}(x, t)$ of velocities is already low-pass filtered at the cut-off frequency f_N of shell k_N . The only way to further reduce the time scale of the generated time series is to explicitly low-pass filter the computed velocity signal $u^{(N)}(x, t)$ with the inevitable loss of information. For this, we used a simple binomial filter. In merely subsampling without filtering, this information would appear aliased at lower frequencies.

For example, at $N = 2$, with the simulated velocity signal limited to the first two shells, the fastest signal that was fed to the grid has a time scale $158\tau_\eta$, with consequently the ratio of the integral time scales $T/T^g = 5.62$. For the integral time scales of the driving signal to match that of the turbulence, this signal must be sped up by downsampling and filtering. For increasing N , the driving signal contains more and more small-scale fluctuations. At $N = 10$, the smallest time step supplied is approximately the Kolmogorov time, and an extreme mismatch between the integral scales of the driving and that of the wind-tunnel turbulence results. It is as if we try to impose the intermittent viscous scale statistics of the GOY model at integral scales of the experiment.

Finally, because we supply instantaneous axis angles to the grid, we associate the left-hand side $u^{(N)}(x, t)$ of Eq. (5) with angles, choosing $C_u = \pi/3u$. Our grid has 17 axes, which are made to correspond to discrete values of x in Eq. (5), $x_i = (i - 1)M$. In summary, the angular signal fed to the axis i is $u^{(N)}(x_i, t)$, $i = 1, \dots, 17$, where the indices $i = 1, \dots, 7$ correspond to the vertical axes and $i = 8, \dots, 17$ to the horizontal ones.

The key point of our experiment is the variation of the spatial and temporal matches of the supplied turbulent-like random numbers to those of the windtunnel turbulence. To that aim, we will start from a GOY time series at $N = 2, 4, 6, 8$, and 10, where the truncation N sets the spatial coherence of the grid. At each of those N , we will then vary the relative time scale T/T^g and measure the properties of the turbulence generated.

FIG. 5. (a) and (b) Full lines show probability density functions of longitudinal velocity increments at integral scale $r/\eta = 1200$ and dissipative scale $r/\eta = 8$. The dashed lines are Gaussians $P(x) = \pi^{-1/2} \exp(-x^2)$, with $x = \Delta u / \Delta u_{rms}$. (a) For time series I at $T/T^g = 1.4$ and (b) for time series II at $T/T^g = 45$. (c) Third-order structure function for cases I and II. For case I, it is compared to the Kolmogorov prediction $G_3 = -(4/5)\epsilon r$ (dashed line).

V. RESULTS

In our experiment, we will vary T/T^g from a value larger than 1 to values much smaller than unity. When $T/T^g \gg 1$, the signal supplied is too fast compared to the large-eddy turnover rate of windtunnel turbulence, and in the other limit, it is too slow. We characterize the generated wind-tunnel turbulence by its anisotropy and the dissipation rate ϵ . The (pseudo-)energy dissipation rate ϵ was inferred from a single derivative, $\epsilon = 15\nu \langle (\partial u / \partial x)^2 \rangle$, with ν the kinematic viscosity. Separations x followed from time delays t , $x = Ut$, through invocation of Taylor's frozen turbulence hypothesis. In isotropic turbulence, $\epsilon = (15/2)\nu \langle (\partial w / \partial x)^2 \rangle$, but the latter version gave smaller values; however, both versions exhibited a similar dependence on T/T^g . The isotropy is quantified by the ratio of turbulent velocities u/w .

The dependence of the dissipation rate ϵ on the relative time scale T/T^g is shown in Fig. 4(a). Clearly, the optimum energy input into turbulence is reached when the time scale of the random time series from the GOY model matches the experimental integral time, $T/T^g = 1$. This "resonance" phenomenon resembles the case of purely periodic forcing where for some periodic modes, the dissipation displays a resonant enhancement when the stirring frequency matches the large-eddy turnover rate.²⁰ It is remarkable that resonant enhancement not only applies to *periodic* but also to *random* driving of an active grid.

The maximum of the energy dissipation as a function of the relative times T/T^g is largest for a cut-off shell index $N = 4$. This behavior can be understood from the spectral energy cascade. At $N = 4$, grid cells are incoherent on a scale k_4 , which implies approximate coherence on a scale k_2 as the energy in this scale is a factor $4^{5/3} \approx 10$ larger than that in scale k_4 . This implies that two halves in one direction of the grid are moving incoherently, resulting in shear layers that have the extent of a

TABLE I. Statistics of the turbulent flow for cases I and II. The flow of case II is very anisotropic, while its large u_{rms} gives rise to a large turbulence Reynolds number.

Case	N	T/T^g	u_{rms} (m/s)	v_{rms} (m/s)	U (m/s)	ϵ (m ² /s ³)	η (m)	Re_λ
I	2	1.4	1.0	0.66	9.1	1.4	2.2×10^{-4}	8.9×10^2
II	10	45	2.4	0.65	8.9	1.5	2.2×10^{-4}	4.9×10^3

few times the integral length scale and a large energy input. At smaller N , these shear layers are too small, while at larger N , the entire grid moves coherently, with small variations at the higher shell numbers. No shear layers result and now the grid is choking the entire wind-tunnel flow at random times.

Driving the grid with matched time scales, $T/T^g \approx 1$ also results in turbulence with the smallest anisotropy for $N = 4$ as shown in Fig. 4(b). At a large mismatch, also the anisotropy becomes large. Similar behavior is found for the small-scale anisotropy inferred from the energy spectra of the u, w velocity components.

A question is how the statistics of the stirred turbulence is affected by a large mismatch with the stirring scale, in particular if we could endow large-scale turbulence with small-scale statistics. The remarkable answer to this question is illustrated in Figs. 5(a) and 5(b) which show probability density functions of longitudinal velocity increments at integral-scale separation, $r/\eta = 1200$ and dissipative scale $r/\eta = 8$ for two stirring time series I ($N = 2, T/T^g = 1.4$) and II ($N = 10, T/T^g = 45$). At the largest separation $r/\eta = 1200$, the PDFs are compared to Gaussians with the same $\Delta u_{\text{rms}} = \langle (\Delta u)^2 \rangle^{1/2}$. At $T/T^g = 45$, the integral-scale PDF strongly deviates from a Gaussian and displays the near-exponential tails that are characteristic for inertial-range intermittency. Thus, we have endowed large-scale motion with intermittent behavior.

Isotropic turbulence at large Reynolds numbers should satisfy the exact relation for the third-order structure function $G_3(r) = -\frac{4}{5}\epsilon r$. As Fig. 5(c) illustrates, this relation holds well for the time series I, with nearly matched integral time scales, while the extreme case II does not even show an inertial range where $G_3(r) \propto r$ approximately. The turbulence statistical properties of the two cases are summarized in Table I. The random and strongly intermittent flow of case II is very anisotropic, with the large value of the turbulent Reynolds number $\text{Re}_\lambda = u^2(\nu\epsilon)^{-1/2}$ owing to the anomalously large streamwise velocity fluctuations.

VI. CONCLUSION

We conclude that in stirring turbulence with an active grid, not only the distribution of the random numbers that drive the grid matters, but also their correlation properties. The GOY model allowed us to produce random numbers with properties that can be expressed in terms of turbulence quantities, such as the integral and dissipative time scales. Turbulence needs to be stirred with random numbers whose integral time scale matches that of the wind-tunnel flow. A large mismatch leads to interesting statistics of the velocity increments in the windtunnel, but the small-scale velocity statistics no longer satisfy the fundamental Kolmogorov relation: turbulence cannot be fooled.

Being able to specify the correlation properties of the random signal driving the grid is an essential refinement of the control of active-grid motion. The number of independent grid cells in our grid limits the dynamic range of spatial scales; however, active grids are now underway that have a much larger number of cells, each of which can be addressed individually.

ACKNOWLEDGMENTS

This work is part of the research programme of the “Stichting voor Fundamenteel Onderzoek der Materie (FOM),” which is financially supported by the “Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).” We thank Freek van Uittert, Gerald Oerlemans, and Ad Holten for technical assistance.

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