Intelligent illuminance control in a dimmable LED lighting system

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The dimmability of light-emitting diodes (LEDs) offers lighting designers great flexibility in illuminating an indoor environment. However, even for experienced lighting designers, it is not easy to achieve both comfort and energy efficiency using large scale lighting systems. To improve this situation, in this paper we propose the use of a quantitative human perception model for an illuminance distribution in an indoor environment. Based on this model, we can optimize the dimming levels of the LED lighting system given the positions and preferences of users. Enhancing user satisfaction and reducing energy consumption are jointly taken into consideration.

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1. Introduction

As we know, the lighting quality of an office is able to affect the moods of its occupants, and therefore their comfort and productivity. It is reported that in the United States, a huge amount of potential productivity can be gained from direct improvements of the indoor environment.\textsuperscript{1,\,2}

Many papers claim that in an indoor environment, the light distribution can influence human beings. For example, it is summarized that human beings are influenced by light through two routes, i.e. the image forming pathway and the non-image forming
pathway. The former includes visual performance, visual perception, and visual comfort while the latter consists of the circadian effects.

The appropriate light setting is important for human beings. It is indicated that when the lighting environment is inappropriate, users will feel uncomfortable and are likely to underperform. Then, as the lighting environment gets better and better, the physical level, the functional level and the psychological level of user needs are satisfied one by one.

Hence, appropriate illumination is important for users in an office. However, different users often have different requirements and preferences for light. For a lighting system that only offers light for a small number of users, experienced light designers are capable to manually adjust the lighting system for every user. Yet for lighting systems in modern commercial buildings which have to satisfy the functional and psychological requirements of light for thousands of users, it is impractical to manually adjust the lighting system. As a compromise, most of the lighting systems tend to ignore the differences among individual preferences and offer them similar amounts of light according to a certain standard or recommendation.

As a first step towards large scale intelligent lighting systems, in this paper we propose and analyze a lighting control algorithm that is able to offer users personalized light levels. The biggest challenge here is that up to now, there is still a gap between current research results about the effects of indoor lighting and mathematical optimization. Most of the results use either psychological or biological measures, their statistical significance being based on analysis of variance. However, neither psychological nor biological measures can be directly used as input to an automated lighting control algorithm. In other words, we do not have a quantified model to describe how the illuminated environment influences the user. Hence a mathematical interpretation of available psychological and biological results is needed.

To our best knowledge, the only concrete mathematical model about user satisfaction that can be used in automated lighting control is a two-piece linear model, which will be discussed in detail in Section 4.1. With this model, many linear optimization methods can be applied to optimize the light distribution. However, this is very hard to generalize because many aspects in the lighting system and inside the human mind cannot be linearized.

As an improvement, in this paper, we introduce a flexible framework that is not only compatible with the linear model but also other realistic and practical user satisfaction models. Based on this framework, given the user preferences, a heuristic
algorithm is presented to optimize the light distribution which has many fewer limitations
than the linear optimization methods.

This paper is organized as follows. A mathematical formulation of a dimmable
LED lighting system is introduced in Section 2. Since increasing user comfort and reducing
energy consumption are usually two primary objectives of a lighting system, in Section 3,
we first briefly discuss the way to reduce energy consumption by dimming control. Then
the model of user satisfaction is described in Section 4. In Section 5, the dimming control
algorithm is presented. The performance of the algorithm is evaluated in Section 6.
Conclusions are drawn in Section 7.

2. The model of a dimmable LED lighting system

LEDs are widely used in modern lighting systems. Compared with traditional light sources,
such as incandescent lamps and fluorescent lamps, LEDs show superior performance in
many aspects such as higher energy efficiency, smaller size, faster on/off, longer lifetime,
and better dimmability.

Thus, in this paper we consider a dimmable LED lighting system which contains
$N_L$ ceiling-mounted dimmable LEDs. The dimming levels of these LEDs are hereinafter
referred to as the dimming vector of the lighting system and denoted$^1$ as
$w = [w_1, w_2, \ldots, w_{N_L}]^T$, where $w_i$ is the dimming level of the $i$ th LED
$(0 \leq w_i \leq 1, i = 1, 2, \ldots, N_L)$. Note that $w_i = 0$ means that the $i$ th LED is turned off while
$w_i = 1$ indicates that the LED emits light at its maximum power. Further, If not specified,
in this paper vectors and matrices are in bold lower cases and bold upper cases,
respectively.

We are mostly interested in the illuminance distribution on the task plane, which is
a horizontal plane with height 0.76 m$^6,7$ and divided into $N_T$ tiles. The illuminance
distribution over the task plane is denoted as $x = [x_1, x_2, \ldots, x_{N_T}]^T$, where $x_i$ represents the
illuminance at the centre of tile $i$. Note that in this paper, instead of the conventional
notation $E$, we use $x$ to indicate the illuminance distribution since $E$, is reserved for
energy consumption which will be discussed later.
Usually for the task plane of an office, there are three types of areas i.e., the task area, the surrounding area, and the other area. The task area is usually indicated as an area of about 1 m² in front of the user while the surrounding area is regarded as the area adjacent to task area with width 0.5 m. Two binary indicator vectors, \( \delta_T = [\delta_{t_1}, \delta_{t_2}, \ldots, \delta_{t_{N_T}}]^T \) and \( \delta_S = [\delta_{s_1}, \delta_{s_2}, \ldots, \delta_{s_{N_S}}]^T \) are used to indicate the property of a tile. For example, if tile \( j \) belongs to the task area of a user, then \( \delta_{t_j} = 1 \); otherwise \( \delta_{t_j} = 0 \). Note that different users may have different indicator vectors.

Moreover, for every tile, there is a minimum maintained illuminance requirement. For the entire task plane, this requirement is denoted as \( r = [r_1, r_2, \ldots, r_{N_r}]^T \). For example, for a typical office, the minimum maintained illuminance requirement for the task area, the surrounding area, and the other area are recommended to be 500 lx, 300 lx, and 60 lx (at least one fifth of the illuminance requirement for the surrounding area), respectively.

Assuming the dimmable LED lighting system is the only light source in the office, since light is additive, we have

\[
x = Iw,
\]

where \( I \) is the illuminance transfer matrix and denoted as

\[
I = \begin{bmatrix}
I_{1,1} & I_{1,2} & \cdots & I_{1,N_r} \\
I_{2,1} & I_{2,2} & \cdots & I_{2,N_r} \\
\vdots & \vdots & \ddots & \vdots \\
I_{N_r,1} & I_{N_r,2} & \cdots & I_{N_r,N_r}
\end{bmatrix},
\]

and \( I_{i,j} \) is the illuminance of the tile \( i \) when only the \( j \)th LED is fully turned on and the other LEDs are turned off. The illuminance transfer matrix \( I \) can be either acquired by lighting design software or by practical measurement.

If the office is equipped with calibrated sensors that are able to measure daylight distribution, we can add a term in equation (1) which represents the daylight distribution that is independent with \( w \). This will not influence the following optimization. The only consequence is the optimum \( w \) may have to be recalculated every a few minutes since daylight is dynamic.
3. Minimize energy consumption

As mentioned above, reducing energy consumption is one of the two most widely used objectives of a lighting system. Since the energy consumption of a LED has a linear relationship with its dimming level, the energy consumption of a lighting system can be formulated as a function of the dimming vector of the lighting system

\[ E(w) = \frac{p^T w + E_0}{p^T 1 + E_0}, \]  

where \( p = [p_1, p_2, \ldots, p_{N_L}]^T \) is the maximum power, i.e., undimmed power, of every LED in the lighting system; \( E_0 \) is the standby energy consumed by the lighting system such as power consumed by the drivers and sensors, which is independent of \( w \). To fit further optimization, the energy consumption is normalized in equation (2).

To minimize the energy consumption, the dimming vector is optimized by solving

\[ \arg \min_w E(w) \]

subject to \( 0 \leq w \leq 1, \quad Iw \geq r, \]  

where \( \leq \) and \( \geq \) represent the componentwise inequality. The first constraint is a physical requirement that the dimming level of every LED should be between 0 and 1. The second constraint is used to satisfy the minimum maintained illuminance requirement.

Since minimizing energy consumption is a simple convex optimization problem, it has been widely used as the objective functions in previous dimming optimization solutions. Moreover, equation (3) has also been discussed for a distributed LED lighting system already.

4. Model of user satisfaction

Fundamentally speaking, the ultimate goal of a lighting system is to satisfy the needs of users, such as visibility; task performance; mood and atmosphere; visual comfort; aesthetic judgment; health, safety, and well-being; and social communication. However, since the user satisfaction is very subjective and can be influenced by many factors such as the task and the mood of the user, it is difficult to be quantified in a deterministic way.
As it is almost impossible to model and measure all such aspects, we use the acceptance probability to describe the user satisfaction $Q(x)$ of an illuminance distribution $x$.

$$Q(x) = \prod_{i=1}^{N_U} P_i(x),$$  \hspace{1cm} (4)

where $N_U$ is the number of occupants in the office; $P_i(x)$ is the probability that user $i$ accepts $x$. Thus, $Q(x)$ is the probability that all users accept $x$ simultaneously.

Note that in equation (4), a multiplicative model is used, which means that the optimization algorithm will try to avoid dissatisfying any user greatly. In practice, if a user is extremely dissatisfied, he will keep intervening with the system until his needs are satisfied. Thus, to actually maximize $Q(x)$ is equivalent to minimizing the probability that the lighting system is intervened.

Alternatively, an additive model which does not necessarily optimize the probability that all users accept the light distribution $x$ simultaneously has been proposed.\textsuperscript{5} Instead, it optimizes the expected number of users satisfied by $x$. These two different optimization objectives will result in different optimized light distributions. For instance, if the preference of one of the $N_U$ ($N_U \gg 1$) users is quite different from the preferences of the others, the objective of optimizing the expected number of satisfied users will ignore this user while the objective of optimizing the acceptance probability will make a compromise so that no user is extremely dissatisfied.

In order to model $P_i(x)$, we first define a term called satisfier, which is one feature of $x$ that influences the acceptance probability. Then $P_i(x)$ is determined by the weighted product of the appropriateness of all the satisfiers, i.e.,

$$P_i(x) = \prod_{j=1}^{N_s} S_{ij}^j (x, u_i),$$  \hspace{1cm} (5)

where $N_s$ is the number of satisfiers included in the model; $S_j(\cdot)$ is the $j$ th satisfier; $\tau_{i,j} \geq 0$ indicates the relative importance of the $j$ th satisfier for user $i$; $u_i$ is a vector that characterizes user $i$. In following subsections, every satisfier is introduced in a general way. If no confusion arises, the subscript of all user-related parameters are omitted, which simplifies the notation a lot.
There are many features that can be chosen as satisfiers, such as the illuminance of task area, the absence of glare, the spectrum of the light source, and the visual message of the light distribution. Some of these can be easily quantified while others cannot. As a first attempt, in this paper, we choose three satisfiers that are relatively easy to be quantified, namely the average illuminance of the task area, the relative illuminance distribution in the surrounding area, and the uniformity inside the task area. Other satisfiers will be included in further research.

4.1 The average illuminance of the task area

The average illuminance of the task area is the most important and the most well studied satisfier of the three. As a result, to increase user satisfaction, it is necessary to offer users personalized illuminance over their task areas. However, the preferred light levels vary widely among individuals, which depends on many factors such as gender, age, and culture. Thus, offering users the same amount of illuminance is not a good choice.

The user satisfaction was modeled as a two-piece linear function of the task area illuminance. As can be seen in Figure 1, every user is modeled to have a preferred light level, which will result in the maximum user satisfaction. As the light level deviates from the preferred one, the user satisfaction decreases linearly. The advantage of this linear model is that it can be integrated into equation (3) without breaking the convexity of the optimization problem.

However, there are still some drawbacks to this model. First of all, it is not smooth around the preferred light level, which is not consistent with our daily experience. Moreover, it is against the Fechner’s law, which points out that the subjective sensation of a human being is proportional to the logarithm of the stimulus intensity.

As an improvement, a log-normal shaped function is proposed to describe the appropriateness as a function of the average illuminance of the task area. Over a practical range of illuminance, for a user \( u \), the appropriateness of the average task area illuminance can be modeled as

\[
S_t(x, u) = \exp\left\{-\frac{1}{2\sigma^2} (\ln \overline{x}_r - \ln \xi)^2\right\},
\]

where \( \xi \) and \( \sigma \) are the preferred illuminance and the tolerance of the user, respectively; \( \overline{x}_r \) is the average task area illuminance of the user, which can be derived as
\[
X_T = \frac{x^T \delta_T}{1^T \delta_T}.
\]  

(7)

**Figure 1.** Two user satisfaction models in the literature: The two-piece linear model\(^5\) (the dashed line) and the log-normal model\(^11\) (the solid line). The preferred illuminance is 500 lx.

### 4.2 The relative illuminance distribution in the surrounding area

In an office environment, it is energy efficient to make the illuminance of the surrounding area lower than the illuminance of the task area. However, a high light level immediately outside the task area is distracting and irritating.\(^7\) However, excessive illuminance differences between task area and its surroundings may lead to visual stress and eye strain.\(^18\) It is recommended that the minimum requirement for surrounding area illuminance is approximately 60% to 80% of the minimum requirement of the task area.\(^19\)

As a result, we model the second satisfier as a function of the minimum to maximum ratio between the average illuminance of the surrounding area (\(\overline{x_S}\)) and the average illuminance of the task area (\(\overline{x_T}\)). Similarly as \(\overline{x_T}\), \(\overline{x_S}\) can be derived as

\[
\overline{x_S} = \frac{x^T \delta_S}{1^T \delta_S}.
\]

(8)

The acceptance frequency was measured with different ratios in a windowless room.\(^7,20\) The experimental results are shown in Figure 2. According to these results, a linear model is used to describe the relationship between the acceptance frequency and the ratio
\( S_2(x, u) = \rho \frac{\min(x_S, x_T)}{\max(x_S, x_T)}. \) \hspace{1cm} (9)

**Figure 2.** The acceptance frequency as a function of the minimum to maximum ratio of \( x_S \) and \( x_T \).

It is worthwhile noting that mostly \( x_S \) is smaller than \( x_T \) because we want to reduce energy consumption as well. However, in very special situations \( x_S \) might be larger than \( x_T \). For example, as shown in Section 6, if there is a big difference between the preferences of two closely located users, \( x_S \) probably will be larger than \( x_T \) since in this case there is an overlap between the two task areas. This will be discussed in more detail in Section 6.

### 4.3 The uniformity inside the task area

Even if the average illuminance of task area is appropriate, large illuminance variations inside the task area should be avoided as it is uncomfortable. There are two commonly used methods to quantify the illuminance uniformity inside task area. The uniformity is quantified by the minimum-to-average illuminance ratio (\( \theta_a \)), and that ratio should not be less than 0.7 for common office work. In some of other literature, the uniformity is quantified by minimum-to-maximum illuminance ratio (\( \theta_m \)). Since we cannot find any publicly available experimental data where the uniformity is measured as \( \theta_a \), we model the third satisfier as a function of \( \theta_m \).
According to the experimental data,\textsuperscript{21} which is shown in Figure 3, $S_{i}(x,u)$ is modeled as

$$S_{i}(x,u) = 1 - \exp(\kappa \theta_{m}).$$

Figure 3. The percentage of users rating the illuminance uniformity of the task area (measured as $\theta_{m}$) as acceptable.\textsuperscript{21}

Note that in neither Figure 2 nor Figure 3, do we have any experimental data for extremely uncomfortable light distributions. However, due to the property of the multiplicative model, this will not affect the optimized light distribution significantly.

4.4 The minimum maintained illuminance requirement

For non-task related reasons, such as health considerations and safety (both perceived safety and real safety), complete darkness needs to be avoided in any part of the environment. Yet a low illuminance is adequate for these purposes. Typically the needed illuminance is one-fifth of the illuminance of the surrounding area. For a typical office, usually 60 to 100 lx is adequate.\textsuperscript{6} This amount of illuminance yields safety, is energy efficient, and does not require visual adaptation when entering and leaving the task area.\textsuperscript{6} If not specified, in this paper the minimum maintained illuminance requirement of all tiles are 60 lx.

4.5 A simple example for two users

According to the user satisfaction model introduced above, here a simple example is given to show how the illuminance distribution affects user satisfaction. Suppose an
office has been illuminated to \( I_x \) lx, ideally uniformly, i.e. \( \forall i, x_i = x \). Assuming \( \forall i, j, i \neq j \), then according to equation (4), the optimum illuminance \( x^* \) can be calculated as

\[
x^* = \arg \max_x \mathcal{Q}(x)
\]

\[
= \arg \max_x \prod_{i=1}^{N_U} \mathcal{S}_i(x, u_i)
\]  

(11)

\[
= \exp \left( \sum_{i=1}^{N_U} \ln \frac{\sigma_i}{\sigma_i^*} \sum_{i=1}^{N_U} \frac{1}{\sigma_i^*} \right).
\]

As can be seen from equation (11), the optimum illuminance is a weighted logarithmic average of every user's preferred illuminance. The more tolerant a user is, the less his/her preference influences the compromised illuminance, which is consistent with our intuition.

We compare this with the two-piece linear model:

\[
\mathcal{S}_i(x, u_i) = \max \left( \min \left( k_{D_i} (x - \xi_i) + 1, -k_{B_i} (x - \xi_i) + 1 \right), 0 \right),
\]

(12)

where \( k_{D_i} \) and \( k_{B_i} \) are both positive constants that describe the tolerance of user \( i \) for darkness and brightness, respectively.

For \( N_U = 2 \) (we assume \( \xi_1 \leq \xi_2 \)), the optimum illuminance given by the liner model \( x_i^* \) can be expressed as

\[
x_i^* = \max(\min(x_i, x_H), x_L),
\]

(13)

where

\[
x_i = \frac{1}{2} (\xi_1 + \xi_2) + \frac{1}{2} \left( \frac{1}{k_{B_i}} - \frac{1}{k_{D_i}} \right),
\]

(14)

\[
x_H = \min \left( \xi_2, \xi_1 + \frac{1}{k_{B_i}} \right),
\]

(15)

and

\[
x_L = \max \left( \frac{1}{k_{B_i}} - \xi_2, \xi_1 \right).
\]

(16)
From equations (13) to (16) we can see that $x_i^*$ is very sensitive to $k_{B_i}$ and $k_{D_i}$. Since they are both very small positive numbers, the absolute value of the difference between their reciprocals might be very large, which will lead to $x_i^* = x_L$ or $x_i^* = x_H$.

In Figure 4, we illustrate a simple example with $N_U = 2$, $\xi_1 = 300$ lx, $\xi_2 = 800$ lx, $\sigma_{i_1}^2 = 0.8$, $\sigma_{i_2}^2 = 0.1$, $k_{B_1} = 1/400$, $k_{D_1} = 1/500$, $k_{B_2} = 1/1150$, and $k_{D_2} = 1/850$. The thin dashed lines are the two-piece linear model of $x_S$ while the thin solid lines are the log-normal model of $S$. The thick curves are the product of corresponding $S_i(x, u_i)$ and $S_1(x, u_2)$. The optima locate at $x^* = 717$ lx and $x_i^* = 800$ lx under the log-normal model and linear model, respectively. It is can be seen that in this particular example, $x_i^* = x_H$.

**Figure 4.** A simple example for the case of two users. The thick dashed lines are the result based on the two-piece linear model of $S$ while the thick solid lines are the result based on the log-normal model of $S$.

### 4.6 Acquiring the user preference

As discussed above, in the user satisfaction model $Q(\cdot)$, a user $u$ is specified by seven parameters; the preferred illuminance $\xi$, the tolerance $\sigma$, the two uniformity sensitivity parameters $\rho$ and $\kappa$, and the relative importance for the three satisfiers. Furthermore, as the number of satisfiers included in the model increases, the number of parameters needed to specify a user also increases.
Before optimizing the dimming vector $\mathbf{w}$, all the parameters need to be acquired by the lighting system. For the lighting system, there are two main approaches to get these parameters, either by inquiring of the user directly or by learning from the user adaptively.

However, except for the preferred illuminance, the other parameters are not easy to be self-evaluated by the user. Thus, the lighting system has to learn or estimate those user-related parameters. For example, a naive way to estimate the preferred light level and the tolerance of a user is to let the user choose his/her preferred light level multiple times and use the mean and variance as estimations of $\zeta$ and $\sigma^2$, respectively.

Moreover, if no information or only partial information about a user is known, we can build the model conditioned on the known information. For instance, if only some categorical information is known such as the gender and the age of the user, the average parameters of users belong to the same category can be used. In the worst situation, if nothing is known about a user, we can use the average parameters of all users to build the user satisfaction model. The more we know about the user, the more accurate the model will be.

Fortunately, several advanced machine learning techniques have been developed to learn the latent preference parameters of users. Since this is beyond the scope of this paper, for more details about those preference learning approaches, we refer readers to those papers.

5. The dimming vector optimization

The previous sections mathematically model a dimmable LED lighting system and the user satisfaction. In this section, the dimming vector is optimized to make the output illuminance distribution both energy efficient and comfortable. However, the two objectives, reducing energy consumption and increasing user satisfaction, usually cannot be achieved at the same time. Thus we need to make a trade-off between those two objectives.

As shown in equation (17), a parameter $\gamma (0 \leq \gamma \leq 1)$ is used to tune the relative importance of these two objectives. Then the optimum dimming vector $\mathbf{w}^*$ can be acquired by solving
arg min_w \gamma \mathcal{E}(w) - (1 - \gamma) \mathcal{Q}(\mathbf{I}w)
\text{subject to } 0 \leq w \leq 1, \quad \mathbf{I}w \geq \mathbf{r}.

When \gamma = 1, the optimization problem reduces to minimizing the energy consumption of a lighting system.\textsuperscript{12,13}

Considering the number of LEDs in the lighting system and the objective function in equation (17), optimizing the dimming vector is equivalent to finding a global optimum of a non-convex function in a constrained high dimensional space. It is very hard, or even impossible to find a closed expression of \( \mathbf{w}^* \).

Thus, to solve this constrained non-convex optimization problem, a heuristic iterative optimization algorithm called simulated annealing is used, which is inspired from the annealing process in metallurgy.\textsuperscript{27,28} Generally speaking, every iteration consists of two steps. First a neighbour state of the current state is generated. Then we calculate the acceptance probability of the new generated state.

For this particular minimization problem, as can be seen from the pseudo-code in Algorithm 1, every state is a dimming vector. To generate a neighbour dimming vector, we sequentially add a small perturbation to every dimension.

\begin{algorithm}
\caption{The Dimming Vector Optimization Algorithm}
\begin{algorithmic}
\State \textbf{Initialization}
\State \textbf{while} Not Stop \textbf{do}
\State \hspace{10pt} \textbf{for} \ i = 1 \ \textbf{to} \ N_c \ \textbf{do}
\State \hspace{20pt} Generate a neighbor dimming vector \( \mathbf{w}' \) by adding a small perturbation at dimension \( i \)
\State \hspace{20pt} Calculate the acceptance probability of \( \mathbf{w}' \)
\State \hspace{20pt} \textbf{if} \ \mathbf{w}' \ \textbf{is accepted} \ \textbf{then}
\State \hspace{30pt} Set the current state as \( \mathbf{w}' \)
\State \hspace{20pt} \textbf{end if}
\State \hspace{10pt} \textbf{end for}
\State \textbf{end while}
\end{algorithmic}
\end{algorithm}
end for

Decrease the temperature

Check whether the stop criteria is met

end while

Output the optimum dimming vector $w^*$

---

6. The simulation results

In this section, as an example, the dimming control algorithm is applied to a virtual office which is shown in Figure 5. This is because the essence of the problem can be intuitively understood better when $N_u$ is small. The way of modeling the lighting system and optimizing the dimming vector, nevertheless, is generic so that it can be applied to an office with more users. Moreover, for a large building that contains multiple offices, we could optimize the light distribution for every office independently.
The two users locate in the office are chosen from the six virtual users listed in Table 1. Moreover, we let $\rho = 0.96$, $\kappa = 3.9$, $\tau_{\cdot 1} = 1$, $\tau_{\cdot 2} = 0.125$ and $\tau_{\cdot 3} = 0.1$ for all virtual users. Here the values for $\rho$ and $\kappa$ are the regressed results of Figure 2 and Figure 3, which represent the behaviour of average users.

The technical parameters of the lighting system used in the simulation are illustrated in Table 2. Note that the following results have to be interpreted with caution because these results are determined by many practical factors such as layout of the office, the parameters and the arrangement of the luminaires.
6.1 A simple example

Initially we start with a simple example, the dimming control algorithm is applied to the office which accommodates User C and F. In this simulation, $\gamma = 0.5$, which means increasing user satisfaction is given equal priority with reducing energy consumption. The LED grid size is 0.4 m and the distance between the two users is 3 m.

As can be seen from Figure 6, the task areas are illuminated more brightly than the non-task area. The numerical results are shown in Table 3. From this example, we can see that the dimming optimization algorithm is able to offer users a well illuminated environment. The average task area illuminance of the two users are very close to their preferences.

![Figure 5](image)

**Figure 5.** The optimized output illuminance distribution (in lx) of the LED lighting system. The position of the two users are (2m, 1.5m) and (5m, 1.5m), respectively.

6.2 The optimum distance between two users and the optimum LED grid size

To find the optimum distance between two adjacent users, we consider four different pairs of users, as listed in Table 4. For each pair, we plot the minimum value of the objective function as a function of the distance between the two users (Figure 7). Note that for all the pairs, the sum of preferred illuminances are the same. Thus, the amount of energy needed to satisfy both users are approximately equal.
Figure 6. The minimum value of the objective function as a function of the distance between adjacent users for different user combinations. The LED grid size is 40 cm. The trade-off parameter $\gamma$ is taken as 0.5.

From Figure 7, we can see that when the distance between the two users is more than two metres, there is no significant difference among the four pairs. This is also consistent with our daily experience that in most open offices, the distance between adjacent users is two metres. The reason is two-fold. First, a distance of two metres offers the lighting system enough flexibility to satisfy all kinds of user combinations. Actually as the distance between the two users increases, the overlapping area gradually diminishes. This trend is summarized in Table 5.

The other reason is that with the two metre distance, the energy is more effectively used. To illuminate the task area of one user, part of the surrounding area of the other user is illuminated at the same time. Since in this simulation, the average illuminance of task area is much more important than the other two satisfiers, this is a smart way to save energy without hurting user satisfaction. Actually this is also the reason why all the four tails are slightly tilted up (especially for the pair User E and F).

Another result that can be seen from Figure 7 is that if practically possible, it is better to arrange users with similar preferences next to each other. This makes it relatively easier for the lighting system to satisfy the users and save energy. This also confirms the intuition that if the preference difference between two adjacent users is large, probably they cannot be satisfied at the same time.
Now we fix the distance between two users as two metres and start changing the LED grid size. Generally speaking, the smaller the LED grid size, the more freedom the system has. However, as shown in Figure 8, if the two users are difficult to satisfy at the same time, the LED grid size cannot be too large. Otherwise at least one of the users cannot be satisfied. Considering the installation cost, 40 to 50 centimetres is a good choice for the LED grid size of an office.

Figure 7. The minimum value of the objective function as a function of the LED grid size for different pairs of users. The distance between the two users is two metres. The trade-off parameter $\gamma$ is taken as 0.5.

Note that the results illustrated in Figure 7 and Figure 8 are only used to show the behaviour of the dimming control algorithm. In practice, many other aspects have to be taken into consideration in determining the LED grid size and the distances between users.

6.3 Trade-off between user satisfaction and energy consumption

To explore the relationship between user satisfaction and energy consumption, we want to acquire the maximum achievable user satisfaction given the maximum allowed energy consumption. Then the original optimization problem (equation 17) is slightly modified into
\[
\begin{align*}
\max_w & \quad Q(Iw) \\
\text{subject to} & \quad 0 \leq w \leq 1, \\
& \quad E(w) \leq E_L, \\
& \quad Iw \geq r.
\end{align*}
\] (18)

where \( E_L \) is the given energy consumption limitation.

Figure 8. The curves show the trade-off between user satisfaction and energy consumption. The distance between the two users and the LED grid size are 2 m and 0.5 m, respectively.

Figure 9 illustrates the results of equation (18) for the four pairs of users. For all the four pairs, increasing the given energy consumption always improves the achievable user satisfaction. When \( E_L \) is below 0.25, increasing the given energy consumption could efficiently enhances the achievable user satisfaction. Yet above \( E_L \approx 0.25 \), further increasing the allowed energy consumption cannot improve satisfaction much.

Considering the difference among the four pairs, we can see that above the saturation point (around 0.25), the lighting system could offer almost perfect light distributions for Users E and F while for Users A and C, the maximum achievable user satisfaction is relatively low because of the large preference difference and the low tolerances.
Note that there is a very interesting result that when the given energy consumption is far below saturation, $E_L = 0.15$ for instance. Then, the maximum achievable user satisfaction of Users E and F is less than that of Users A(B) and D. This is because in this simulation, the distance between the two users is set to be 2 metres, which will result in an overlap between one's task area and the other's surrounding area. In this particular case, the illuminance of the task area of User D contributes a lot in illuminating the surrounding area of the other user, which is efficient when the energy limitation is tight.

Further, as can be seen in Figure 9, the saturation point is around $E_L \approx 0.25$. This means that on average, the power needed is much lower than installed. In the other words, for this particular office, the undimmed power of each LED should be halved or even less.

From those simulation results we can see that in principle, the quantitative framework can be used in light design. However, when applying it to real lighting systems, many recommendations and regulations have to be taken into consideration.

7. Conclusion

This paper refines a mathematical model of a dimmable LED lighting system that consists of a central controller and several dimmable LEDs. The central controller is able to control the dimming levels of all LEDs. The target of the lighting system is to offer an appropriate illuminance distribution which not only saves energy but also attempts to satisfy the users.

Based on several papers on human perception and experience of light, we propose a generic mathematical framework that is capable of quantitatively evaluating user satisfaction for an illuminance distribution. In this framework, three satisfiers are included, namely the average illuminance of the task area, the relative illuminance distribution in the surrounding area, and the uniformity inside the task area. Each of the three satisfiers is expressed as a feature that can influence the probability of accepting the illuminance distribution.

Compared with a previously proposed two-piece linear user satisfaction model, our framework has a better compatibility and extensibility. More satisfiers could be included to make the framework more practical and realistic. Based on this framework, a heuristic algorithm is presented to optimize the dimming vector so that a balance between enhancing user satisfaction and decreasing energy consumption can be achieved.
The results show that using the optimization algorithm, the lighting system is able to offer every user an illuminance that is close to that preferred by the user. It is shown, based on our model and the chosen parameters, that two metres is a reasonable distance between two adjacent users and 40 to 50 centimetres is an appropriate choice for the LED grid size for an office. Moreover, if practically possible, it is beneficial to arrange users with similar preferences next to each other.

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References


**Table 1.** The technical parameters of the lighting system. Most of the LED parameters are based on the Philips LED Spot family.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The undimmed power of each LED</td>
<td>6W</td>
</tr>
<tr>
<td>The semi-angle of the light beam at half power of each LED</td>
<td>12.3°</td>
</tr>
<tr>
<td>The luminous flux of each LED</td>
<td>300 lm</td>
</tr>
<tr>
<td>The standby power of the lighting system</td>
<td>5W</td>
</tr>
</tbody>
</table>
Table 2. Numerical simulation results of the simple example.

<table>
<thead>
<tr>
<th>Satisfier</th>
<th>User C</th>
<th>User F</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average illuminance of the task area</td>
<td>996 lx</td>
<td>700 lx</td>
</tr>
<tr>
<td>The relative illuminance distribution in the surrounding area</td>
<td>0.67</td>
<td>0.88</td>
</tr>
<tr>
<td>The uniformity inside the task area</td>
<td>0.74</td>
<td>0.75</td>
</tr>
<tr>
<td>Pair</td>
<td>Preference difference</td>
<td>Tolerances</td>
</tr>
<tr>
<td>------------</td>
<td>-----------------------</td>
<td>------------</td>
</tr>
<tr>
<td>User A + C</td>
<td>Large</td>
<td>Low + Low</td>
</tr>
<tr>
<td>User A + D</td>
<td>Large</td>
<td>Low + High</td>
</tr>
<tr>
<td>User B + D</td>
<td>Large</td>
<td>High + High</td>
</tr>
<tr>
<td>User E + F</td>
<td>Small</td>
<td>Low + Low</td>
</tr>
</tbody>
</table>
## Table 4. Overlaps of various distances between adjacent users.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlap between task areas</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Overlap between task area and surrounding area</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Overlap between surrounding areas</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>