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A Novel Modeling Technique via Coupled Magnetic Equivalent Circuit with Vector Hysteresis Characteristics of Laminated Steels

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Abstract—This paper proposes a method to include the anisotropic hysteresis characteristics of soft-magnetic laminated steels in the magnetic equivalent circuit (MEC) modeling. The loop-based MEC formulation is improved to handle the non-linearity of the anisotropic magnetic hysteresis, including the dynamic classical eddy-current and excess fields. The developed MEC model is coupled with both the single-valued B-H curve (SVC) in magnetostatic and the dynamic vector hysteresis model (VHM) in transient analysis. Results with a single elementary MEC element show that an alternating magnetic field in a single direction with a peak value smaller than 300 A/m causes a discrepancy of more than 10% between the magnetic flux densities calculated by the VHM and SVC at 50 and 200 Hz excitation frequencies. Moreover, the proposed modeling technique is verified experimentally using the laminated transformer core of TEAM problem 32. The induced voltage calculated by the MEC model with the VHM demonstrates a good agreement with the measurements, while the MEC model with the SVC calculates inaccurate voltage waveforms. Lastly, the total iron loss dissipated in the transformer's iron core is investigated to verify the proposed technique under different excitation levels and frequencies up to 500 Hz. It is observed that the proposed MEC model with the vector hysteresis characteristics of laminated steels is able to calculate the iron loss accurately, while the conventional single-valued curve method fails to estimate the iron loss.

Index Terms—Ferromagnetic laminations, fixed-point method, iron loss estimation, magnetic equivalent circuit, magnetic vector hysteresis.

I. INTRODUCTION

The design of compact and efficient electromagnetic devices with laminated steel requires an electromagnetic simulation model coupled with the nonlinear magnetic characteristics of the soft-magnetic material as the authors discussed in [1]. This is commonly achieved by coupling the finite element method (FEM) with a nonlinear solver based on the single-valued magnetization curve (SVC), i.e. virgin or B-H curve. The main advantages of the SVC are its simple implementation and low computational cost. In [2], a nonlinear magnetostatic FEM model coupled with a SVC is applied to evaluate different rotor types of interior permanent-magnet synchronous machines. Moreover, the same approach is applied in [3] to investigate the influence of magnetic saturation on the control of permanent-magnet synchronous motors. The nonlinear FEM with a SVC is also employed in [4] for the analysis of magnetic material degradation due to punching, while it is used in [5] for the fault diagnosis of induction motors. Although the SVC exhibits a relatively low computational cost when it is coupled with a proper nonlinear solver, this approach requires determining the hysteresis, eddy-current, and excess loss coefficients of the Bertotti equation to estimate the iron loss in the post-processing as discussed in [6]. Prediction of the laminated steel losses in the post-processing is investigated in [7] and [8] using variable coefficients for different frequencies and induction levels. In addition to determining iron loss coefficients and extra post-processing for iron loss calculation, using the SVC also suffers from a low accuracy depending on the magnitude and frequency of the excitation. In particular, a magnetic field excitation that results in minor loops inside the major hysteresis loop of the soft-magnetic material decreases the accuracy of the SVC approximation as indicated in [9]. Therefore, researchers have been working on more sophisticated electromagnetic models coupled with the magnetic hysteresis property of laminated steels. Different dynamic hysteresis models are implemented with the finite element formulation in [10] and [11], where the convergence analysis of the coupled model is not presented. Moreover, a large number of iterations and unstable convergence are observed in [12] when the FEM model is coupled with the hysteresis B-H characteristics.

Magnetic equivalent circuit (MEC) modeling is a computationally cost-effective alternative to FEM for the analysis of electromagnetic devices with a soft-magnetic material [13]. MEC modeling works on a differential scheme that approximates the Taylor series expansion in terms of the difference between two point values, while FEM is an integral method that constructs the solution in each element from the basis functions. FEM is capable of handling extremely complicated, time-dependent geometries but requires a large number of elements for accurate calculations. Whereas the differential MEC can find the global solution with less number of elements than the integral FEM needs if the geometry of the electromagnetic problem can be represented with regular brick-shaped elements. [14] shows that MEC is more stable in the calculation of the 2-D motional eddy currents with regular elements in comparison with FEM. Due to its stable convergence and low computational time, the MEC is an attractive alternative to the FEM for modeling electromagnetic devices. Liu et. al. [15] proposed a module-based experimental scheme where the flux is estimated by a MEC coupled with
the SVC of the electrical steel without modeling the physical magnetic characteristics. Moreover, an equivalent magnetic circuit in [16] is integrated into the electrical equivalent circuit of a magnetic device used in the drive electronics in terms of lumped parameters. However, the magnetic reluctance is modeled using the measured major hysteresis loop of the soft-magnetic material without considering the minor loops or the anisotropy of the magnetic hysteresis. Moreover, a generalized simple equivalent electrical circuit is proposed to solve the Cauchy problem for the nonlinear hysteretic phenomenon in [17]. Equivalent circuit modeling of a hysteresis interior permanent magnet motor is investigated in [18], where the Elliptical modeling approximation is employed to model the major hysteresis loop. [19] recently proposed a Cauer’s equivalent circuit to present dynamic hysteresis characteristics instead of solving it with MEC. Finally, a significant contribution to modeling the magnetic hysteresis with MEC is presented in [20], where two nonlinear solvers are employed in series: the Newton-Raphson and fixed-point methods. The Newton-Raphson method calculates the magnetic flux density based on the SVC. Then, the calculated flux density distribution is updated with the fixed-point method using a Preisach hysteresis model. Although various authors have made significant progress in MEC modeling with soft-magnetic materials in the literature, a MEC model coupled with dynamic vector hysteresis behavior of laminated steels has not been proposed yet.

In this study, the formulation of the loop-based MEC model used in [21] is improved to couple the existing MEC model with a dynamic version of the anisotropic vector hysteresis model (VHM) presented in [22]. The proposed approach is able to simulate the anisotropic and dynamic magnetic characteristics of laminated steels, including the hysteresis, classical eddy-current and excess fields in transient instead of using the SVC in magnetostatic. Firstly, the system of equations of the coupled models with the SVC and VHM is solved iteratively using the fixed-point method for an elementary single MEC element under different magnetic field excitations [23]. Later, the proposed approach is extended to a real application with a three-limbed transformer using multiple MEC elements under the current excitation with different excitation levels and frequencies. The authors recently compared both models with the SVC and VHM for a single MEC element in [1]. This paper is the extended version of the previous work, where the authors present the experimental verification of the proposed model using transient simulations of the analyzed transformer. This paper has the following structure. In Section II, MEC modeling of laminated steels will be explained for both SVC and VHM. Section III will describe the transformer benchmark to verify the proposed method, while the results with SVC and VHM are compared in Section IV.

II. NONLINEAR MAGNETIC EQUIVALENT CIRCUIT MODEL OF LAMINATED STEELS

The loop-based (also known as mesh-based) MEC is employed for the magnetostatic and transient analysis in 2-D instead of the node-based MEC. A comparative study between both MEC techniques is presented in [24], where the loop-based model is found to be significantly better than the node-based model under nonlinear operation. The loop-based MEC method uses Kirchhoff’s voltage law for the magnetically linear MEC element in Fig. 1(a), where $R$, $\mathcal{F}$, and $\phi$ represent the reluctance, magnetomotive force (MMF) due to the excitation, and flux, respectively. When a MEC model is utilized for a soft-magnetic material, the magnetic saturation is modeled by introducing additional MMF sources as shown in Fig. 1(b) and (c), where the local magnetic saturation is modeled using the SVC and VHM, respectively. Both nonlinear models coupled with the SVC and VHM are solved iteratively using the fixed-point method as implemented in [25].

A. Coupling with the Single-valued B-H Curve

A first-order single-variable Taylor series expansion around the operating point $\vec{B}_0$ is used to model the SVC as

$$|\vec{H}| = f(|\vec{B}|) = f(|\vec{B}_0|) + f'(|\vec{B}_0|)(|\vec{B}| - |\vec{B}_0|),$$

Fig. 1. Single elementary magnetic equivalent circuit element with its lumped circuit parameters. (a) A linear magnetic equivalent circuit element. (b) A magnetic equivalent circuit element with the single-valued curve. (c) A magnetic equivalent circuit element with the vector hysteresis model.
The tangent line are determined iteratively based on the fixed-tangent line on the SVC, $\vec{H}$, hence the permeability and coercivity are obtained from the flux density, the magnetic saturation in each element depending on the magnitude of the magnetic field strength is defined as

$$\vec{B} = B_x \hat{x} + B_y \hat{y},$$

$$\vec{H} = H_x \hat{x} + H_y \hat{y},$$

respecting the cartesian coordinate system shown in Fig. 1(b). The operating point on the SVC, $\vec{B}_0$, is calculated by the MEC model. Then, (1) is re-written in a tangent line format

$$|\vec{H}| = f'(|\vec{B}_0|)|\vec{B}| - f'(|\vec{B}_0|)|\vec{B}_0|. \quad (4)$$

The slope and intercept of the tangent line of $f$ are called the reluctivity, which is one over the permeability ($\mu = \mu^{-1}$), and the coercivity ($H_c$)

$$\mu = f'(|\vec{B}_0|)^{-1},$$

$$H_c = f(|\vec{B}_0|) - f'(|\vec{B}_0|)|\vec{B}_0|. \quad (5)$$

Hence, the permeability and coercivity are obtained from the tangent line on the SVC, $f$, for the magnetic flux density calculated by the MEC model, $\vec{B}_0$. The slope and intercept of the tangent line are determined iteratively based on the fixed-point algorithm. Although the MEC with an SVC uses only one scalar permeability, $\mu$, and one scalar coercivity, $H_c$, for each element depending on the magnitude of the magnetic flux density, the magnetic saturation in x- and y-directions is approximated as

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix} = \begin{bmatrix} \mu^{-1} & 0 \\ 0 & \mu^{-1} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix} + \frac{1}{|\vec{B}|} \begin{bmatrix} B_x H_c \\ B_y H_c \end{bmatrix}. \quad (7)$$

Then, both sides of (7) are multiplied by the length matrix $L$ to obtain an equation consisting of the lumped circuit parameters of the MEC element given in Fig. 1(b). The length matrix is defined as

$$L = \begin{bmatrix} l_x & 0 \\ 0 & l_y \end{bmatrix}, \quad (8)$$

where $l_x$ and $l_y$ are the lengths of the MEC element in x- and y-directions, respectively. The MEC element equation is derived as

$$\begin{bmatrix} H_{xl} \\ H_{yl} \end{bmatrix} = \begin{bmatrix} l_x (\mu l_y l_s)^{-1} & 0 \\ 0 & l_y (\mu l_x l_s)^{-1} \end{bmatrix} \begin{bmatrix} \phi_x \\ \phi_y \end{bmatrix} + \frac{1}{|\vec{B}|} \begin{bmatrix} B_x H_c l_x \\ B_y H_c l_y \end{bmatrix}, \quad (9)$$

where $l_s$ is the stack length of the MEC element. Then, (9) is re-written in terms of the lumped circuit parameters as

$$\begin{bmatrix} \mathcal{F}_{x+} \\ \mathcal{F}_{y+} \end{bmatrix} = \begin{bmatrix} R_x & 0 \\ 0 & R_y \end{bmatrix} \begin{bmatrix} \phi_x \\ \phi_y \end{bmatrix} + \begin{bmatrix} \mathcal{F}_{BH}^{x+} \\ \mathcal{F}_{BH}^{y+} \end{bmatrix}. \quad (10)$$

Fig. 1(b) shows the structure of a single MEC element with the SVC, $\mathcal{R}$, $\mathcal{F}_{x+}$, and $\mathcal{F}_{BH}$ refer to the reluctance, excitation related MMF, and magnetic saturation related MMF, respectively. Lumped parameters in the positive side of the x-axis of the MEC element with SVC are expressed using

$$\vec{H} = \vec{H}_h + \vec{H}_{el} + \vec{H}_{ex}, \quad (14)$$

where $f$ represents the SVC of the soft-magnetic material, which takes the magnitude of the magnetic flux density, $|\vec{B}|$, as input and gives the magnitude of the magnetic field strength, $|\vec{H}|$, as output. The magnetic flux density and field strength vectors are defined as

$$\vec{B} = B_x \hat{x} + B_y \hat{y},$$

$$\vec{H} = H_x \hat{x} + H_y \hat{y},$$

Fig. 2. Comparison of the vector hysteresis model with arbitrary uni-axial experimental data.

$$\mathcal{R}_{x+} = \frac{l_x}{\mu l_y l_s}, \quad (11)$$

$$\mathcal{F}_{x+}^{ex} = |\vec{H}_{ex}| \cos(\vec{H}_{ex}) l_{x+}, \quad (12)$$

$$\mathcal{F}_{BH}^{x+} = -\frac{B_x}{|\vec{B}|} H_c l_{x+}, \quad (13)$$

where $l_{x+}$ is the length of the positive part of the MEC element in x-direction, $\vec{H}_{ex}$ is the magnetic field excitation vector applied to the MEC element. Equations for other lumped parameters of the MEC element with SVC are derived with the same approach.

B. Coupling with the Dynamic Vector Hysteresis Model

The anisotropic congruency-based static VHM developed by the authors in [22] is employed in this study to be coupled with the loop-based MEC formulation. The VHM model is constructed by applying Maytergozys vector generalization method to an anisotropic scalar congruency-based hysteresis model. This anisotropic model is developed by a trigonometric interpolation of two scalar congruency-based models implemented using the first-order reversal curves measured in parallel and perpendicular directions with respect to the material rolling direction. The measured data are filtered and processed to remove the measurement noise. The filtered data is interpolated using B-spline functions to obtain the static version of the hysteresis model. The proposed congruency rule in [22] is applied to any approximated second or higher-order reversal curve using the measured first order reversal curves. Fig. 2 compares the final vector hysteresis model of NO27 laminated electrical steel with arbitrary uni-axial experimental data measured using the Epstein frame, where the hysteresis model exhibits a good correlation with the measurement. In order to include the dynamic field components, eddy-current and excess fields, in the MEC model, the magnetic field strength is defined by the sum of three components as discussed in [26]
where $\vec{H}_d$, $\vec{H}_{ex}$, and $\vec{H}_{ed}$ are the hysteresis, classical eddy-current, and excess fields, respectively. The hysteresis field is obtained from the VHM as elaborated in detail in [22]. The eddy-current field is calculated using the classical eddy-current approximation as

$$\vec{H}_d = \frac{\sigma d^2}{12} \frac{d\vec{B}}{dt},$$

(15)

where $\sigma$ and $d$ are material conductivity and lamination thickness, respectively. Furthermore, Mayergoyz vector generalization is applied to derive the vector excess field as implemented in [27]

$$\vec{H}_{ex} = k_{ex}^e \int \frac{1}{2} \vec{e}_\phi \delta^e \left| \frac{dB_\phi}{dt} \right| \delta^e \phi,$$

(16)

where

$$\vec{e}_\phi = \cos(\phi) \hat{x} + \sin(\phi) \hat{y},$$

(17)

$$B_\phi = \vec{e}_\phi \cdot \vec{B}.$$  

(18)

In (16), $k_{ex}^e$ and $\alpha^e$ are vector excess field coefficients identified through an optimization procedure by comparing the dynamic hysteresis model results with experimental data measured at various frequencies. Also, $\delta^e$ is defined as

$$\delta^e = \text{sgn} \left( \frac{dB_\phi}{dt} \right),$$

(19)

where sgn is the signum function. Moreover, the magnetic flux density and field strength are defined as in (2) and (3), respecting the coordinate system in Fig. 1(c). Unlike (1) of the SVC, the magnetic saturation in the model with VHM is expressed separately in both directions as

$$H_x = f_1(B_x, B_y),$$

(20)

$$H_y = f_2(B_x, B_y).$$

(21)

Therefore, it requires to apply two separate first-order two-variable Taylor series expansions for $x$- and $y$-directions

$$H_x = f_1(B_{x0}, B_{y0}) + \frac{\partial f_1(B_{x0}, B_{y0})}{\partial x}(B_x - B_{x0}) + \frac{\partial f_1(B_{x0}, B_{y0})}{\partial y}(B_y - B_{y0}),$$

(22)

$$H_y = f_2(B_{x0}, B_{y0}) + \frac{\partial f_2(B_{x0}, B_{y0})}{\partial x}(B_x - B_{x0}) + \frac{\partial f_2(B_{x0}, B_{y0})}{\partial y}(B_y - B_{y0}).$$

(23)

Then, the Taylor expansions are modified to represent the tangent lines as implemented for the SVC as

$$H_x = \frac{\partial f_1}{\partial x} B_x + \frac{\partial f_1}{\partial y} B_y - \frac{\partial f_1}{\partial x} B_{x0} - \frac{\partial f_1}{\partial y} B_{y0} + f_1(B_{x0}, B_{y0}),$$

(24)

$$H_y = \frac{\partial f_2}{\partial x} B_x + \frac{\partial f_2}{\partial y} B_y - \frac{\partial f_2}{\partial x} B_{x0} - \frac{\partial f_2}{\partial y} B_{y0} + f_2(B_{x0}, B_{y0}).$$

(25)

The slopes of the tangent lines in (24) and (25) include two self-permeabilities ($\mu_{xx}$, $\mu_{yy}$), and two mutual permeabilities ($\mu_{xy}$, $\mu_{yx}$) while the intercepts define the coercivities in both directions, $H_{cx}$ and $H_{cy}$. Therefore, (24) and (25) are combined to get

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix} = \begin{bmatrix} \mu_{xx}^{-1} & \mu_{xy}^{-1} \\ \mu_{yx}^{-1} & \mu_{yy}^{-1} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix} + \begin{bmatrix} H_{cx} \\ H_{cy} \end{bmatrix}.$$  

(26)

To find the expressions for the lumped circuit parameters of the MEC element given in Fig. 1(c), both sides of (26) are multiplied by the length matrix in (8), which results in

$$\begin{bmatrix} H_x \\ H_y \end{bmatrix} = \begin{bmatrix} l_x \mu_{xx} l_x^{-1} & l_x \mu_{xy} l_x^{-1} \\ l_y \mu_{yx} l_y^{-1} & l_y \mu_{yy} l_y^{-1} \end{bmatrix} \begin{bmatrix} \phi_x \\ \phi_y \end{bmatrix} + \begin{bmatrix} l_x H_{cx} \\ l_y H_{cy} \end{bmatrix}.$$  

(27)

To be used in Kirchhoff’s law, (27) is re-written in terms of the reluctance, flux, and MMF such as

$$\begin{bmatrix} \frac{F_{xx}}{R_{xx}} \\ \frac{F_{xy}}{R_{xy}} \end{bmatrix} = \begin{bmatrix} \frac{\phi_x}{R_{xx}} \\ \frac{\phi_y}{R_{xy}} \end{bmatrix} + \begin{bmatrix} \frac{F_{BH}}{R_{xx}} \\ \frac{F_{BH}}{R_{xy}} \end{bmatrix}.$$  

(28)

The main difference between two MEC elements modeled with SVC and VHM in Fig. 1 is the red colored flux-dependent MMF sources which represent the mutual saturation. Lumped circuit parameters in the positive side of the $x$-axis of the MEC element with the VHM are expressed using

$$R_{xx+} = \frac{l_x}{l_y l_x \mu_{xx}},$$

(29)

$$F_{xx+} = -H_{cx} l_x,$$

(30)

$$F_{yx+} = -R_{xy+} \phi_y,$$

(31)

where the mutual reluctance, $R_{xy+}$, and the magnetic flux in $y$-direction, $\phi_y$, are calculated using

$$R_{xy+} = \frac{l_x}{l_y l_x \mu_{xy}},$$

(32)

$$\phi_y = \frac{\phi_y + \phi_y}{2}.$$  

(33)

Other lumped circuit parameters of the MEC element coupled with the VHM are derived using the same approach. Then, the fixed-point method is applied to solve nonlinear equations iteratively for both SVC and VHM. In the developed fixed-point algorithm, the loop-based MEC calculates the magnetic flux density which is used by the nonlinear magnetic characteristics of the material (SVC or VHM) to calculate the permeability and the coercivity. In the next fixed-point iteration, the calculated permeability and coercivity values update the related lumped circuit parameters. The convergence is determined by the relative error in the modulus of the magnetic flux density between the successive iterations.

### III. Analyzed Laminated Benchmarks

The proposed modeling technique is tested with two different geometries: a single elementary MEC element presented in Fig. 1 and a three-limbed transformer illustrated in Fig. 3.
A. Single Magnetic Equivalent Circuit Element

A single 2-D MEC element is investigated to compare both models due to its simplicity in modeling. The analyzed MEC element exhibits a square shape with 1 mm edge and stack lengths. All four branches of the MEC element are connected to the ground as in Fig. 1. The material of the MEC element is selected as NO27 electrical steel. The SVC of NO27 is obtained from quasi-static measurements for different excitation directions. The average of the measured curves is applied to the MEC element in Fig. 1(b) as the SVC of the soft-magnetic material. Moreover, a finite set of first-order reversal curves of NO27 are measured by the Epstein frame under quasi-static conditions to develop the VHM for Fig. 1(c).

B. Three-limbed Transformer

A three-limbed transformer is modeled in 2-D using both SVC and VHM for the experimental verification of the proposed model with the VHM. The analyzed transformer benchmark shown in Fig. 3 consists of an iron core made of NO27 electrical steel and two copper coils wound around the left and right legs of the transformer in the same direction. The transformer core benchmark is similar to the proposed experimental setup in [28], also called TEAM problem 32, for verifying the vector hysteresis analysis of laminated steels. This specific experimental setup, TEAM problem 32, is used to validate magnetic field analysis with vector hysteresis characteristics of laminated steels.

A. Single Magnetic Equivalent Circuit Element

The MEC model with the SVC is simulated under different excitation levels to verify the developed loop-based MEC model and the nonlinear solver with the fixed-point algorithm. Fig. 4(a) shows that simulation results of the model agree with the measured SVC of the soft-magnetic material. After the verification of the MEC model and its nonlinear solver, both rotating and alternating magnetic fields are applied to both models with the SVC and VHM as

\[ \vec{H}_e = |\vec{H}_e|\angle(2\pi ft), \]  

(34)

### Table I

<table>
<thead>
<tr>
<th>Dimensions of the Three-limbed Transformer</th>
</tr>
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<tbody>
<tr>
<td>( h_t ) [mm]</td>
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<tr>
<td>180</td>
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</table>

Fig. 3. Analyzed three-limbed transformer benchmark.

Fig. 4. (a) Verification of the magnetic equivalent circuit model with the measured single-valued curve. (b) Convergence comparison between models with the single-valued curve and vector hysteresis model. (c) Convergence of the model with the vector hysteresis model for different excitation angles.
Fig. 5. Comparison of the magnetic flux densities by the models with the single-valued curve and vector hysteresis model: (a) $B_x$. (b) $B_y$. (c) $|\vec{B}|$.

where $|\vec{H}_e|$ is taken as a constant for the rotational magnetic field, and it is defined as a time-varying sinusoidal signal for the alternating magnetic field. Moreover, the angle of the excitation is linearly dependent on time for the rotating magnetic field, while it is set to zero for the alternating field excitation. Firstly, a rotating magnetic field excitation is applied with 200 A/m magnitude and 50 Hz frequency. The nonlinear solver is employed for each time step, while one electrical period is divided into 100 time steps. The comparison between convergence behaviors of models with the SVC and VHM for a random time instant is presented in Fig. 4(b). Finite accuracy on calculating the weight function of the VHM makes the convergence of VHM slower than SVC. Also, any error from the previous time step propagates to the current time step in the VHM, while time steps are independent of each other for the model with SVC. Although the convergence of the SVC is similar for each time step for the rotating magnetic field excitation with a constant magnitude, the angle of the applied field affects the convergence of the VHM as shown in Fig. 4(c). Bi-directional excitations such as 45 and 135 degrees decrease the convergence rate of the nonlinear solver with the VHM.

Moreover, magnetic flux density outputs of the single MEC element models with both magnetic saturation modeling techniques are compared under the magnetic field excitation. Fig. 5 shows the variation of $B_x$, $B_y$, and $|\vec{B}|$ with respect to the excitation angle for the same rotating excitation given in (34). It is observed that $B_x$ and $B_y$ calculated by the SVC and VHM have different peak values and phase angles. The phase shift between the outputs of both modeling techniques is found to be approximately 15 degrees, while the difference between peak values of the outputs is calculated as 0.1 T. However, the phase and magnitude discrepancies between the outputs of both models strongly depend on the excitation. Fig. 5(c) shows that the SVC gives a constant magnitude of the magnetic flux density due to the constant magnitude of the applied rotating magnetic field. However, the anisotropic magnetic saturation results in varying flux density magnitude for the MEC model with the VHM. In addition to the rotating magnetic field case, the alternating magnetic field is also applied to the MEC element. In Fig. 6, the flux density variations calculated by two methods are compared for alternating magnetic field excitations in $x$-direction with 100, 300, and 1000 A/m peak values; and 50 and 100 Hz frequencies. It is indicated that applying a larger magnetic field magnitude decreases the difference between the results calculated by the SVC and VHM. Moreover, increasing the alternating frequency of the excitation widens the operating hysteresis loop.

The root-mean-square (RMS) discrepancy in $|\vec{B}|$ between the models with SCV and VHM is calculated for both rotating and alternating cases. The RMS discrepancy in percentage (RMSDP) is obtained using

$$\text{RMSDP} = 100 \times \frac{(B_{\text{VHM}} - B_{\text{SVC}})_{\text{RMS}}}{(B_{\text{VHM}})_{\text{RMS}}},$$

where

Fig. 6. Comparison of the magnetic flux density variations calculated by the models with the single-valued curve and vector hysteresis model for the alternating excitation with different magnitudes and frequencies: (a) $H_{ex} = 100 \text{ A/m}$. (b) $H_{ex} = 300 \text{ A/m}$. (c) $H_{ex} = 1000 \text{ A/m}$. 

where RMS is determined for an excitation period. Fig. 7(a) presents the percentage discrepancy between the methods for the rotating magnetic field with different magnitudes, while Fig. 7(b) shows the same comparison for the alternating magnetic field with a fixed angle and different peak values. It is observed that the modeling of the dynamic vector hysteresis characteristics is more critical for the alternating magnetic field than the rotating one since the discrepancy is generally more significant for the alternating excitation, which is usually present in a tooth body of electrical machines. Moreover, Fig. 7(a) and (b) show that applying an excitation with a smaller frequency decreases the accuracy of the flux density calculation with the SVC. Although using the SVC is a valid approximation for the accurate flux density calculation under deep magnetic saturation, SVC is not able to estimate the iron loss. The total iron loss density of the MEC element is calculated by the model with the VHM using

\[ P_{VHM} = \frac{f A}{\rho}, \]

where \( A \) is the area inside the hysteresis loops computed by the VHM, and \( \rho \) is the mass density of the material which is 7650 kg/m³ for NO27. The total iron loss by the dynamic VHM consists of the hysteresis, eddy current, and excess losses. The iron loss density is also computed by the model with the SVC using the statistical iron loss separation theory of Bertotti [29]

\[ P_{SVC} = P_{SVC,h} + P_{SVC,el} + P_{SVC,ex}. \]

The hysteresis component of the iron loss density of the SVC is calculated using

\[ P_{SVC,h} = W_h(|\vec{B}|) f, \]

where the induction dependent parameter \( W_h \) represents the energy enclosed by the hysteresis loops and it is obtained from quasi-static measurements. The eddy current loss density is estimated by

\[ P_{SVC,el} = \frac{1}{T \rho} \int_0^T \left[ \frac{\sigma d^2 |\vec{B}|}{12} \right] \frac{d|\vec{B}|}{dt} dt, \]

where \( T \) is the electrical period. The eddy-current field in (39) is the scalar version of the expression given in (15). Lastly, the iron loss density due to the scalar excess field is calculated using

\[ P_{SVC,ex} = \frac{1}{T \rho} \int_0^T \left[ k_{ex}^s \delta^s \right] \frac{d|\vec{B}|}{dt} dt, \]

where the scalar excess field coefficients, \( k_{ex}^s \) and \( \delta^s \), are identified using the iron loss data provided by material manufacturers. Also, \( \delta^s \) is calculated using

\[ \delta^s = \text{sgn} \left( \frac{d|\vec{B}|}{dt} \right). \]

Fig. 7(c) presents the variation of the iron loss density by the SVC and VHM with respect to the peak value of the alternating magnetic field for two different frequencies. It is observed that the discrepancy between both models increases with large frequencies and excitation levels under the alternating magnetic field excitation, which results in only major hysteresis loops without minor loops. Therefore, the SVC approximation cannot estimate the iron loss causing an inaccurate efficiency computation although it provides accurate flux density calculation under magnetic saturation.

### B. Three-limbed Transformer

The three-limbed transformer presented in Fig. 3 is employed for the experimental verification of the proposed modeling technique with the dynamic vector hysteresis characteristics of laminated steels. The transformer is modeled using the loop-based MEC in 2-D with 1296 square MEC elements, each of which has 5 mm edge length. The MEC model of the transformer is coupled with both SVC and VHM of the NO27 electrical steel. The current excitation is applied to both the left leg coil \( I_1 \) as in Fig. 8(a), while the current of the right leg coil \( I_2 \) is set to zero. The flux linkage variations of both coils with respect to the time are calculated as in Fig. 8(b) using the flux density obtained from both models with SVC and VHM. Phase shift and non-identical waveforms are observed in Fig. 8(b). Moreover, the self, \( L_{inc} \), and mutual, \( M_{inc} \), incremental inductances are calculated as

\[ \begin{bmatrix} L_{inc} & M_{inc} \end{bmatrix} = \begin{bmatrix} \frac{\partial \lambda_1}{\partial t} & \frac{\partial \lambda_2}{\partial t} \end{bmatrix}. \]
Fig. 8. Comparison between both transformer models with single excited coil. (a) Applied currents. (b) Flux linkages. (c) Self and mutual incremental inductances.

Fig. 9. Comparison between both transformer models with two excited coils. (a) Applied currents. (b) Flux linkages.

Fig. 10. Comparison between the magnetic flux density distributions in the transformer benchmark calculated by two magnetic saturation modeling methods. (a) The single-valued curve (b) The vector hysteresis model (c) The discrepancy between both techniques.

Fig. 8(c) shows the variation of the self and mutual incremental inductances. In addition to the phase shift, a significant discrepancy is observed between RMS and peak values of the incremental inductances calculated by the SVC and VHM, which is critical for accurate control of electromechanical systems. In addition to the comparison of the self and mutual incremental inductances, the experimental setup of the three-limbed transformer is employed to verify the proposed MEC model with the VHM when both left and right coils are excited, as given in Fig. 9(a). Peak values of both left and right leg coil currents are set to 0.5 A, while the excitation frequency is 100 Hz. Moreover, the number of turns of each coil is set to 75. One electrical period is divided into 100 time steps in the simulation models. The magnetic flux density distributions calculated by MEC models with the SVC and VHM are compared in Fig. 10 for a random time instant. Fig. 10(c) shows that the absolute difference between two distributions reaches 0.7 T peak discrepancy. Additionally, the computation times of one electrical period of the models with the SVC and VHM are found to be 123 s and 201 s, respectively. Then, flux linkages of both coils are calculated as in Fig. 9(b) using magnetic flux density distributions by the models with SVC and VHM under the current excitation in Fig. 9(a). A phase shift is observed between the flux linkage
Fig. 11. Experimental verification of the proposed magnetic equivalent circuit model coupled with the vector hysteresis model for different excitation levels at 100 Hz excitation frequency. (a) $I=0.5$ A. (b) $I=1$ A. (c) $I=1.5$ A.

Fig. 12. Comparison of the iron loss measurements with simulation results by the single-valued curve and vector hysteresis model. (a) $\tilde{I}=0.5$ A. (b) $\tilde{I}=1$ A.

waveforms as indicated in Fig. 8(b). Additionally, induced voltages of both coils are obtained by taking the time derivative of the flux linkages calculated by the simulation models. The coil voltage is the sum of the induced voltage and resistive voltage drop of the coils

$$V = \frac{d\lambda}{dt} + IR, \quad (43)$$

where $V$ is the coil voltage, $\lambda$ is the flux linkage, $I$ is the coil current, and $R$ is the coil resistance. The coil resistance is measured as 0.15 $\Omega$ for both coils. The calculated coil voltages are compared with the measured data in Fig. 11(a), where light red and blue colors represent measurements, dashed lines are the SVC, and solid lines are the VHM. A good agreement between the measurements and the model with the VHM is observed, while the SVC approximation results in inaccurate voltage prediction. In addition, simulations and experiments are repeated with increased excitation levels: $I=1$ A and $I=1.5$ A as in Fig. 11(b) and Fig. 11(c). It is observed that the VHM provides much more realistic results than the SVC for all investigated excitation levels. The slight discrepancy between measurements and the proposed modeling technique with the VHM is explained by the neglected end-winding effect and flux leakage due to the 2-D modeling.

Lastly, the total iron loss dissipated in the core of the transformer is computed by both models using (36) and (37). The iron loss is also obtained experimentally from voltage and current measurements by subtracting the total copper loss from the total input power of the transformer core, since the output power of the system is equal to zero. The iron loss comparison between simulation models and measurements are presented in Fig. 12(a) and (b) for 0.5 and 1 A peak current values, respectively. The results are compared up to 500 Hz excitation frequency. It is observed that the total iron loss can be estimated accurately using the proposed model by including the dynamic vector hysteresis characteristics of laminated steels. However, the SVC fails to calculate the iron loss accurately especially for large frequencies.

V. CONCLUSION

A new modeling technique to include the anisotropic dynamic vector hysteresis characteristics of laminated steels in the magnetic equivalent circuit modeling is presented in this study. The proposed vector hysteresis modeling technique coupled with an equivalent magnetic circuit improves the accuracy of electromagnetic simulations with laminated steels compared to the single-valued magnetization curve. In contrast, the single-valued curve exhibits faster convergence and reduced computation time. The discrepancy between the magnetic flux densities calculated by the single-valued curve and vector hysteresis model exceeds 55% and 100% with the alternating and rotating magnetic field excitations at 50 Hz, respectively.
respectively. It is also concluded that the significance of the vector hysteresis model in accurately calculating the magnetic flux density increases for relatively low excitation levels, which prevents magnetic saturation. However, electromagnetic problems containing multiple laminated steel MEC elements, such as the analyzed transformer core, should be solved considering the magnetic vector hysteresis characteristics under any magnetization level. The reason is that not every MEC element operates under magnetic saturation when a high excitation level is applied to the windings of the analyzed benchmark. The proposed magnetic equivalent circuit model with the vector hysteresis characteristics of laminated steels is experimentally verified for various excitation levels up to 500 Hz using a transformer core setup. Since the proposed modeling technique is developed in 2-D using the classical eddy current approximation, its computational cost is comparable with the model coupled to the single-valued curve. However, the maximum excitation frequency that provides accurate simulation results is found to be 500 Hz due to the classical eddy current approximation. Extending the current modeling technique to 3-D to include diffusion equations for calculating an accurate eddy current distribution at high frequencies remains to be future work. Moreover, it is demonstrated that the induced coil voltage calculated by the proposed coupled model exhibits good agreement with measurements. At the same time, the conventional single-valued curve magnetic saturation approximation results in unrealistic voltage waveforms. In addition, it is observed that the single-valued curve obtained from quasi-static measurements cannot estimate the iron loss accurately, especially under a low level of excitations with large frequencies. The proposed modeling method with the dynamic vector hysteresis characteristics of laminated steels provides more accurate estimations of the flux density, flux linkage, inductance, and voltage for electrical machines. Therefore, the proposed method exhibits considerable potential for designing, analyzing, and controlling various electromechanical systems with laminated steels.

REFERENCES


