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Parameters Affecting Water Hammer Wave Attenuation, Shape and Timing

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This paper investigates parameters that may affect water hammer wave attenuation, shape and timing. Possible sources that may affect the waveform predicted by the classical water hammer theory include unsteady friction, cavitation, and a number of fluid-structure interaction (FSI) effects. The discrepancies originate from assumptions in the development of the classical water hammer equations. Mathematical tools for modelling of unsteady friction effects, vaporous and gaseous cavitation, and fluid-structure interaction are presented. The method of characteristics is used as a basic tool. The paper concludes with a number of case studies showing how these parameters affect pressure traces in a simple reservoir-pipeline-valve system.

1. INTRODUCTION

Water hammer manifests itself as pressure fluctuations in piping systems, as rotational speed variations (overspeed, reverse rotation) in hydraulic machinery or as water level oscillations in surge tanks. The velocity profiles in rapid transients are significantly different from the “steady” profiles in slowly varying transients. Transients in pipelines may cause a drop in pressure large enough to invalidate the assumption of fluid homogeneity and continuity. Gas bubbles may be entrained in a liquid due to gas release during low-pressure transients, cavitation and/or column separation. The mechanical properties of pipe wall material and the rigidity of pipe supports may significantly influence the intensity of pressure oscillations. Undesirable water hammer effects may disturb overall operation of the hydraulic system (hydroelectric power plant, pumping system) and damage the system components; for example pipe displacement or rupture may occur. Water hammer loads can be kept within the prescribed limits by adequate control of operational regimes, installation of surge control devices or redesign of the original pipeline layout. Calibration and monitoring of hydraulic systems require detailed knowledge of water hammer wave attenuation, shape and timing. Engineers should be able to identify parameters that may violate the underlying assumptions in standard water hammer computer packages, i.e. the flow in the pipe is considered to be one-dimensional (cross-sectional averaged velocity and pressure distributions), the pressure is greater than the liquid vapour pressure, the pipe wall and liquid behave linearly elastically, unsteady friction losses are approximated as steady state losses, the amount of free gas in the liquid is negligible and fluid-structure coupling is weak (structure-induced pressure changes are much smaller than the water hammer pressures). The discrepancies between the computed and measured water hammer waves may also originate from discretization error in the numerical model, approximate description of boundary conditions, and uncertainties in measurement and input data.

The first part of the paper deals with mathematical tools for modelling unsteady friction, cavitation (vaporous and gaseous) and fluid-structure interaction. The method of characteristics transformation of the classical water hammer equations gives the standard water hammer solution procedure. The unsteady friction model is explicitly incorporated into the staggered grid of the method of characteristics. Incorporating discrete vapour cavities or discrete gas cavities with gas release into the standard water hammer model leads to the discrete cavity model. Coupling the classical water hammer equations with the radial-axial dynamic equations for the pipe wall (Poisson coupling) leads to a fluid-structure interaction model. Junction coupling is modeled through boundary conditions. Again, the method of characteristics transformation of the coupled system of equations is used. Unsteady friction, cavitation and FSI effects
may be combined into one general model. The paper concludes with a number of case studies showing how these parameters affect pressure traces in a simple reservoir-pipeline-valve system.

2. CLASSICAL WATER HAMMER THEORY

Water hammer equations are applied for calculation of the liquid unsteady pipe flow. The assumptions in the development of the water hammer equations are [1]:

1. Flow in the pipeline is considered to be one-dimensional with the velocity averaged and the pressure uniform at a section.
2. Unsteady friction losses are approximated as quasi-steady state losses.
3. The pipe is full and remains full during the transient.
4. There is no column separation during the transient event, i.e. the pressure is greater than the liquid vapour pressure.
5. Free gas content of the liquid is small such that the wave speed can be regarded as a constant.
6. The pipe wall and the liquid behave linearly elastically.
7. Structure-induced pressure changes are small compared to the water hammer pressure wave in the liquid.

Water hammer equations include the continuity equation and the equation of motion [2]:

\[
\frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} - V \sin \theta + a^2 \frac{\partial V}{\partial x} = 0
\]
\[\frac{g}{\partial x} \left( \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{fV}{2D} \right) = 0
\]

Appendix II explains the symbols.

For most engineering applications, the convective terms \(V(\partial H/\partial x), V(\partial V/\partial x), \) and \(V \sin \theta,\) are very small compared to the other terms and may be neglected [3]. A simplified form of Eqs (1) and (2) using the discharge \(Q=VA\) instead of the flow velocity \(V\) is:

\[
\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0
\]
\[
\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{fQ|Q|}{2gDA^2} = 0
\]

The method of characteristics transformation of the simplified equations (3) and (4) produces the water hammer compatibility equations which are valid along the characteristic lines. The numerically stable water hammer compatibility equations, written in a finite-difference form for a computational section \(i,\) are [2]:

- along the \(C^+\) characteristic line (\(\Delta x/\Delta t = a\)):

\[
H_{i,j} - H_{i-1,j-\Delta t} + \frac{a}{gA} ((Q_u)_{i,j} - (Q_u)_{i-1,j-\Delta t}) + \frac{fA}{2gDA^2}(Q_u)_{i,j} \left| Q_{i-1,j-\Delta t} \right| = 0
\]

- along the \(C^-\) characteristic line (\(\Delta x/\Delta t = -a\)):

\[
H_{i,j} - H_{i+1,j-\Delta t} - \frac{a}{gA} (Q_u - (Q_u)_{i+1,j-\Delta t}) - \frac{fA}{2gDA^2} Q_{i,j} \left| Q_{i+1,j-\Delta t} \right| = 0
\]

The discharge at the upstream side of the computational section \(i ((Q_u)_{i})\) and the discharge at the downstream side of the section \((Q)\) are identical for the classical water hammer case. At a boundary (reservoir, valve), a device-specific equation replaces one of the water hammer compatibility equations. The staggered grid in applying the method of characteristics is used in this paper.
3. UNSTEADY FRICTION

The assumption of steady viscous losses may be satisfactory for slow transients where the wall shear stress has a quasi-steady behaviour. Previous investigations of the behaviour of steady friction models for rapid transients [4, 5] showed large discrepancies in attenuation, shape and timing of pressure traces when computational results were compared with measurements. The magnitude of the discrepancies is governed by the flow conditions (severity of transients; laminar, transitional or turbulent flow) and liquid properties (viscosity). Numerous unsteady friction models have been proposed to date [6] including one- (1D) and two-dimensional (2D) models. The 1D models approximate the actual cross-sectional velocity profile and corresponding viscous losses in different ways [6]. The two main approaches of 1D models are:

(1) Friction term is dependent on instantaneous mean flow velocity and instantaneous flow acceleration (local (temporal)) [7]; local and convective (temporal and spatial) [8]).
(2) Friction term is dependent on instantaneous mean flow velocity and weights for past velocity changes (as the result of a frequency-dependent relation between skin friction and flow acceleration) [9, 10].

On the contrary, the 2D models compute the actual cross-sectional profile continuously during the transient event. The drawbacks of the 2D models in comparison with 1D models are complexer modelling and larger CPU times. This paper deals with the Brunone unsteady friction model [8]. The Brunone model performs accurately in a number of flow situations including rapid valve closure in a water conveying system. The model is computationally effective.

Brunone Unsteady Friction Model

The friction factor, explicitly used in Eqs (5) and (6), is expressed as the sum of the quasi-steady part \( f_q \) and the unsteady part \( f_u \). The computation of the quasi-steady part \( f_q \) is straightforward, whereas the unsteady part \( f_u \) is related to the instantaneous local (temporal) acceleration \( 1/A(\partial Q/\partial t) \) and instantaneous convective (spatial) acceleration \( 1/A(\partial Q/\partial x) \) i.e.:

\[
f = f_q + \frac{kDA}{Q|Q|} \left( \frac{\partial Q}{\partial t} + a \text{sign}(Q) \frac{\partial Q}{\partial x} \right)
\]

in which \( \text{sign}(Q) = \{+1 \text{ for } Q \geq 0 \text{ and } -1 \text{ for } Q < 0 \} \). Equation (7) is Vítkovský’s formulation [11] of the original Brunone model [8]. The Brunone friction coefficient \( k \) can be predicted either empirically or analytically. Analytical definition of \( k \) using Vardy and Brown’s shear decay coefficient \( C^* \) [10] is used in this paper:

\[
k = \frac{\sqrt{C^*}}{2}; \quad C^* = \begin{cases} 
0.0476 & \text{for laminar flow} \\
7.41 & \text{for turbulent flow} \\
5.0 \log(14.5/R_e)^{0.05} & \end{cases}
\]

A first-order approximation for the friction term, i.e. \( \frac{fDA}{Q_{cav}} |Q_{cav}| \), is used in Eqs (5) and (6) when using the Brunone model.

4. CAVITATION

Cavitating flow usually occurs as a result of low pressures during transients. Cavitation significantly changes the water hammer waveform. Water hammer equations developed for pure liquid flow are not valid for the two-phase transient flow. There are two basic types of transient cavitating flow in piping systems [2]:

(1) One-component two-phase transient flow (vaporous cavitation; column separation).
(2) Two-component two-phase transient flow (gaseous cavitation; free gas in liquid flow).

4.1 Vaporous Cavitation

Vaporous cavitation (including column separation) occurs in pipelines when the liquid pressure drops to the vapour pressure of the liquid. The amount of free and/or released gas in the liquid is assumed small. This is usually the case in most parts of the hydroelectric power plant water conveying system (pressure tunnel, ...
penstock). The water hammer wave propagates at a constant speed as long as the pressure is above the vapour pressure. Cavitation may occur as a localized vapour cavity (large void fractions, often leading to a column separation) and/or as distributed vaporous cavitation (small void fractions). A localized (discrete) vapour cavity may form at a boundary (shut-off gate or valve, draft tube of a water turbine), at a high point along the pipe, or at an intermediate section of the pipe (intermediate cavity) if two low-pressure waves meet [12]. Distributed vaporous cavitation occurs when a rarefaction wave drops the liquid pressure in an extended section of the pipe to the vapour pressure of the liquid. Pressure waves do not propagate through an established mixture of liquid and vapour bubbles. The inability of pressure waves to propagate through a vapour bubble zone is a major feature distinguishing the flow with vaporous cavitation from the flow with gaseous cavitation. Both the collapse of a discrete vapour cavity and the movement of the shock wave front (interface separating the liquid and liquid-vapour mixture) into a vaporous cavitation zone condense the vapour phase back to the liquid phase. Column separation events in pipelines may be described by a set of one-dimensional equations representing a particular physical state of the fluid [1, 2, 12], i.e. water hammer equations for the liquid state, two-phase flow equations for a liquid-vapour mixture, shock equations for condensation of liquid-vapour mixture back to the liquid phase, and equations for a discrete vapour cavity.

A number of column separation models has been developed [2, 12, 13] including a discrete vapour cavity model and an interface vaporous cavitation model. A discrete vapour cavity model allows vapour cavities to form at computational sections in the method of characteristics. An interface model couples analytical and numerical methods for solving a complete set of column separation equations for all types of pipe configurations and various cases of interactions between the particular states of the fluid. This paper deals with a basic type of the discrete cavity model [2, 13].

**Discrete Vapour Cavity Model (DVC M)**

The discrete vapour cavity model (DVM) allows vapour cavities to form at computational sections in the method of characteristics when the pressure drops to the liquid’s vapour pressure [2]. A liquid phase with a constant wave speed $a$ is assumed to occupy the reach between computational sections. The discrete vapour cavity is described by Eqs (5) and (6) with $H$ set to $H_v$, and the continuity equation for the vapour cavity volume [14]:

$$\forall_v = \int_{t_m}^{t} (Q - Q_a) dt$$  \hspace{1cm} (9)

The numerical solution of Eq. (9) within the staggered grid of the method of characteristics is [15]:

$$(\forall_v)_{i,j} = (\forall_v)_{i,j-2\Delta t} + ((1 - \psi)(Q_{i,j-2\Delta t} - (Q_v)_{i,j-2\Delta t}) + \psi(Q_{i,j} - (Q_v)_{i,j})) 2\Delta t$$  \hspace{1cm} (10)

The cavity collapses when the cumulative cavity volume becomes less than zero. The liquid phase is re-established and the water hammer solution using Eqs (5) and (6) is valid. The DVM model may generate unrealistic pressure pulses (spikes) due to collapse of multi-cavities. The model gives reasonably accurate results when the number of reaches is restricted. It is recommended that the maximum size of the discrete cavity at a section is less than 10% of the reach volume [13].

**4.2 Gaseous Cavitation**

Gaseous cavitation occurs in fluid flows when free gas is either distributed throughout a liquid (small void fraction) or trapped at distant positions along the pipe and at boundaries (large void fraction). Gas may be entrained in a liquid due to gas release during low-pressure transients, cavitation or column separation. Transient gaseous cavitation is associated with dispersive and shock waves. The pressure-dependent wave speed in a gas-liquid mixture is significantly reduced. Gas release takes several seconds whereas vapour release takes only a few milliseconds. The effect of gas release during transients is important in long pipelines in which the wave reflection time is in the order of several seconds. Methods for describing the amount of gas release were developed by Zielke and Perko [16].

Transient flow of a homogeneous gas-liquid mixture with a low gas fraction and with the liquid’s mass density may be described by classical water hammer equations (3) and (4) in which the liquid wave speed $a$ is replaced by the pressure-dependent gas-mixture wave speed $a_m$ [15]:
and in addition, the equation of the ideal gas assuming isothermal conditions:

\[
\forall \quad (H - z - h_e) = (H_0 - z - h_e) a_g A \Delta x
\]

The pressure-dependent wave speed \(a_m\) makes the system of equations highly non-linear. A number of numerical schemes including the method of characteristics have been used for solving the above set of equations [2, 17, 18]. These methods are complex and could not be easily incorporated into a standard water hammer code. Alternatively, the mass of distributed free gas can be lumped at computational sections leading to a discrete gas cavity model [15].

**Discrete Gas Cavity Model (DGCM)**

The discrete gas cavity model (DGCM) allows gas cavities to form at computational sections in the method of characteristics. As in DVCM, a liquid phase with a constant wave speed \(a\) is assumed to occupy the computational reach. The discrete gas cavity is described by the water hammer compatibility equations (5) and (6), the continuity equation for the gas cavity volume (10) (index \(g\) replaces \(v\)), and the equation of the ideal gas (12). The treatment of gas release by the DGCM is straightforward [19]. In addition, the DGCM model can be successfully used for simulation of vaporous cavitation by utilizing a low gas void fraction \((\alpha_g \leq 10^{-7})\) [13, 15]. In this case, when the discrete cavity volume calculated by the equation (10) is negative, then the cavity volume is recalculated by the equation (12).

5. FLUID-STRUCTURE INTERACTION

The quality of the classical water hammer equations (3) and (4) highly depends on a good estimate of the pressure wave speed \(a\). It is best to find \(a\) directly from measurements. Theoretical estimates of \(a\) are usually valid for thin-walled pipes, which experience either zero stress or zero strain in their axial direction. If non-zero dynamic axial stresses and strains in the pipe wall are taken into account, the following extended water hammer equations are obtained:

\[
\frac{\partial H}{\partial t} + \frac{a^2}{g A} \frac{\partial Q}{\partial x} = 2 \nu \frac{a^2}{g} \frac{\partial \dot{u}}{\partial z}
\]

(13)

\[
\frac{\partial H}{\partial x} + \frac{1}{g A} \frac{\partial Q}{\partial t} + \frac{f Q_0 |Q_0|}{4 g A^2} = 0
\]

(14)

The right-hand term in Eq. (13) is the time derivative of the axial strain, which is equal to the space derivative of the axial velocity \(\dot{u}\), multiplied with two times the Poisson ratio \(\nu\) and \(a^2 / g\). To find the pipe velocity \(v\), two additional equations have to be solved. These could be named extended steel hammer equations, because they are mathematically equivalent to the Eqs (13) and (14):

\[
\frac{\partial \sigma}{\partial t} - \rho_s a_s^2 \frac{\partial \dot{u}}{\partial x} = \nu \rho g \frac{R}{e} \frac{\partial H}{\partial t}
\]

(15)

\[
\frac{\partial \sigma}{\partial x} - \rho_s \frac{\partial \dot{u}}{\partial t} + \frac{f \rho Q_0 |Q_0|}{8 e A^2} - \rho_s g \sin \theta = 0
\]

(16)

The classical pressure and stress wave speeds are defined by

\[
a = \left( \frac{K}{\rho} \right)^{\frac{1}{2}} \left\{ 1 + \frac{2 R K}{e E} (1 - \nu^2) \right\}^{-\frac{1}{2}} \quad \text{and} \quad a_s = \left( \frac{E}{\rho_s} \right)^{\frac{1}{2}}
\]

(17, 18)
The MOC transforms the four coupled Eqs (13-16) to compatibility equations which, disregarding skin friction and gravity, and in terms of pressures, stresses and velocities, read:

\[
\frac{d p}{dt} \pm \rho \tilde{a} \frac{d V}{dt} + G_f \left\{ \frac{d \sigma}{dt} \mp \rho_s \tilde{a} \frac{d u}{dt} \right\} = 0
\]

(19)

\[
\frac{d \sigma}{dt} \mp \rho_s \tilde{a}_s \frac{d u}{dt} + G_s \left\{ \frac{d p}{dt} \pm \rho \tilde{a}_s \frac{d V}{dt} \right\} = 0
\]

(20)

Eqs (19) and (20) are valid along characteristic lines with \( dx/dt = \pm \tilde{a} \) and \( dx/dt = \pm \tilde{a}_s \), respectively. The Poisson coupling factors \( G_f \) and \( G_s \) are:

\[
G_f = -2 \frac{\nu}{\rho_s} \left\{ \left( \frac{\tilde{a}_s}{\tilde{a}} \right)^2 - 1 \right\}^{-1} \quad \text{and} \quad G_s = \frac{R}{e} \left\{ \left( \frac{\tilde{a}_s}{\tilde{a}} \right)^2 - 1 \right\}^{-1}
\]

(21, 22)

The modified wave speeds \( \tilde{a} \) and \( \tilde{a}_s \) follow directly from the characteristic equation of the coupled system (13-16). In water-filled steel pipes, they slightly differ from \( a \) and \( a_s \) [20, 21]. Each jump in pressure travelling at speed \( \tilde{a} \) is now accompanied with a jump in axial pipe stress according to [22]

\[
\Delta \sigma = -G_s \Delta p
\]

(23)

Similarly, each jump in axial stress travelling at speed \( \tilde{a}_s \) is accompanied with a jump in pressure (precursor) according to

\[
\Delta p = -G_f \Delta \sigma
\]

(24)

The relations (23, 24), together with the definitions (21, 22), say something about the importance of distributed FSI.

Local FSI occurs at valves, orifices, expansions, contractions, elbows, bends and branches, noting that under severe transients all these pipe components will vibrate to a certain extent. The dynamic interaction of a local component with fluid unsteadiness is called junction coupling. The simplest example is the closed free end where fluid and structural velocities, and pressures and stresses, are proportional to each other:

\[
V = \dot{u} \quad \text{and} \quad A \cdot p = A_s \sigma
\]

(25, 26)

More information on the subject can be found in reviews by Wiggert [23], Tijsseling [24] and Wiggert and Tijsseling [25].

6. CASE STUDIES

A number of case studies are presented to show how the effects of unsteady friction, cavitation and fluid-structure interaction change the water hammer waveform in a simple reservoir-pipeline-valve system. Each case is simulated by a standard water hammer model based on Eqs (5) and (6), and by a corresponding MOC numerical model that incorporates unsteady friction, cavitation, fluid-structure interaction, or a combination of these. The standard water hammer model is herein referred to as the 'reference model'. In addition, the numerical results from unsteady friction and cavitation models are compared with the results of measurements in a laboratory apparatus from the literature.

6.1 Unsteady Friction

The effect of unsteady friction on water hammer wave forms is investigated for the case of rapid closure of the downstream end valve in a 37.2 m long upward sloping pipe of 22 mm internal diameter and 1.6 mm wall thickness (Fig. 1). The experimental apparatus is fully described by Bergant et al. [6].
Numerical results from the 'reference model' (steady friction) and the unsteady friction model (modified Brunone et al. model (Eq. 7)) are compared with the measured results from the experimental run with the initial flow velocity $V_0 = 0.2 \text{ m/s}$, static head in the tank 2, $H_T = 32 \text{ m}$, valve closure time $t_c = 0.009 \text{ s}$, and water hammer wave speed $a = 1319 \text{ m/s}$ [6]. The number of reaches for each computational run is $N = 32$. The results are compared at the valve ($H_{ve}$) and at the midpoint ($H_{mp}$) and are presented in Fig. 2.

Fig. 2 Comparison of Heads at the Valve ($H_{ve}$) and at the Midpoint ($H_{mp}$); $V_0 = 0.2 \text{ m/s}$.

Computational results obtained by the 'reference model' (Figs 2(a) and 2(b)) clearly exhibit the discrepancies in water hammer wave attenuation, shape and timing when compared to measured results. On the contrary, the results from the unsteady friction model show only minor discrepancies for later times of transient event (Figs 2(c) and 2(d)). In this case, large discrepancies generated by the 'reference model' are attributed to the unsteady friction effects (non-uniform velocity profile).

### 6.2 Cavitation

The effect of vaporous cavitation on the transient wave form is presented for the case of rapid closure of the valve positioned at the downstream end of the pipeline apparatus as shown in Fig. 1. Computational and measured results are presented for the run with the initial flow velocity $V_0 = 0.3 \text{ m/s}$, static head in tank 2, $H_T = 22 \text{ m}$, valve closure time $t_c = 0.009 \text{ s}$, and water hammer wave speed $a = 1319 \text{ m/s}$ [12]. The number of reaches
for each computational run is \( N = 32 \); a weighting factor of \( \psi = 1.0 \) is used in Eq. (10) of the DVCM model. Computational results from the 'reference model' and the DVCM model are compared with measured results at the valve \((H_{ve})\) and at the midpoint \((H_{mp})\) and are depicted in Fig. 3. The 'reference model' results (Figs 3(a) and 3(b)) significantly differ from the measured results. The actual flow situation represents a column separation case with a maximum head significantly larger than the water hammer head (at the valve - 'reference model': \(H_{ve,max} = 62.3\) m, measurement: \(H_{ve,max} = 95.6\) m). The minimum computed pressure head is well below the liquid vapour pressure head. The assumption of fluid homogeneity and continuity in the 'reference model' is violated when the pressure drops to the liquid vapour pressure. The classical water hammer theory fails to predict the column separation event; the column separation model should be used in this case. Figs 3(c) and 3(d) show the results obtained by the DVCM model. There is a reasonable agreement between the DVCM model results and the measurements. The maximum short-duration pressure pulse at the valve is slightly overpredicted \((H_{ve,max} = 102.7\) m). The DVCM model generates faster transients in later times.

An attempt was made to improve the DVCM results. A modified Brunone et al. unsteady friction model (Eq. 7) with a Reynolds dependent friction coefficient \( k \) (Eq. 8) was incorporated in the DVCM model. The results from the DVCM model with unsteady friction are presented in Fig. 4. The inclusion of unsteady friction generates a significant damping of the largest short duration pressure pulse \((H_{ve,max} = 94.1\) m) and of the pressure spikes. There is a slight improvement in timing for later times. A comprehensive study of the DVCM model with unsteady friction is needed to test the performance of modified DVCM for a broad range of parameters.
6.3 Fluid-Structure Interaction

The test system shown in Fig. 1 is used to theoretically study the possible effects of FSI. Therefore, the 37.2 m long straight copper pipe is assumed not to be restrained against axial motion along its entire length. The tank at its upstream end is a fixed point, and the massless valve downstream is either fixed or free to move, depending on the type of FSI under investigation. The pressure and axial stress wave speeds are taken as $\bar{a} = 1316.5$ m/s and $\bar{a}_s = 3686.2$ m/s, so that their ratio is 5/14. A non-staggered computational grid with $5 \times 32 = 160$ reaches, but with $\Delta t = 5 \Delta x / \bar{a}$, allows standard MOC calculation without interpolation (except for the boundary values needed at intervals) [26]. The effects of the different FSI mechanisms on the classical water hammer waveform are demonstrated in the four cases below.

![Graphs showing pressure heads at different coupling types](image)

Fig. 5 Comparison of Heads at the Valve ($H_{ve}$) and at the Midpoint ($H_{mp}$); $V_0 = 0.2$ m/s.

**Poisson Coupling**

To solely study the effects of Poisson coupling, the valve is fixed (immovable). Valve closure generates a travelling pressure rise, which radially expands the pipe wall. The radial expansion is accompanied with an axial contraction, which sends out a stress wave and an associated pressure change in the fluid (precursor). These effects are initially very small as can be seen from the pressure heads in Fig. 5(a,b): the first period of water hammer is not much affected by FSI. However, Fig. 5(a,b) also shows that the effects cumulate to such an extent that unrealistically high and low
pressures result. There is a continuous exchange and redistribution of energy between the water hammer wave and the axially vibrating pipe. A beat phenomenon develops \[27\]. Precursor waves have been observed in laboratory experiments, but as far as the authors know, Poisson-coupling beat has not (although Fig. 4(a) of Budny et al. \[28\] and Fig. 16 of Vennaturo \[29\] exhibit cumulative Poisson effects). Three obvious reasons are (i) unsteady friction and rubbing at pipe supports dampen the transient event, (ii) pipe anchors never are entirely stiff or entirely inert when impact loaded, and (iii) pipes like to vibrate in their lowest and hence flexural mode, which is made possible through axial-lateral coupling at supports, and through sagging of pipes.

**Junction Coupling**

To solely study the effects of junction coupling, Poisson's ratio is set equal to zero so that the mechanisms described in the previous paragraph do not exist. The valve is free to move now; it follows the pipe vibration. The pressure rise generated by valve closure pushes the valve in downstream direction, thereby creating additional storage for the fluid and as a result a lower initial pressure rise, Fig 5(c,d). The fluid is not brought entirely to rest; it has the velocity of the valve. The axial stress wave generated by the motion of the valve travels to and from the upstream tank and at its return, after time \(2L/\alpha_s\), it pulls the valve back. This "pumping" action explains the second pressure rise in Fig. 5(c,d). In contrast to the Poisson-coupling case, Fig. 5(a,b), junction coupling makes the pressure out of phase with classical water hammer, Fig. 5(c,d). The system becomes slower than the classical \(4L/\alpha\) system, possibly because of the increased storage capability at the valve when the pressure is high.

**Poisson and Junction Coupling**

Figure 5(e,f) shows the combined effects of Poisson and junction coupling. Readers with good eyes may spot the precursor wave travelling ahead of the main water hammer wave in Fig 5(f). It is evident that FSI, when compared with the reference model, causes larger extreme pressures, high-frequency fluctuations, and a phase shift. It is noted that in multi-pipe systems with unrestrained elbows FSI may also cause, after one water hammer period, a profound damping of the pressure wave as a consequence of waterhammer-induced flexural pipe vibration \[30, 31\]. FSI will not cause damping in unrestrained single-pipe systems.

**FSI and Cavitation**

Figure 6 shows the combined effects of Poisson coupling, junction coupling and cavitation (DVCM). A large phase shift and high-frequency fluctuations are the most striking features. The subject of FSI and cavitation has been extensively dealt with in reference \[22\].

![Fig. 6 Comparison of Heads at the Valve (H_{ve}) and at the Midpoint (H_{mp}); V_0 = 0.3 m/s.](image)

7. CONCLUSIONS

This paper intends to display a state of the art portrait; the advances of unsteady friction, cavitation and FSI with respect to classical water hammer are examined. Unsteady friction is the most likely explanation of the higher than anticipated damping in many experimental results. Both cavitation, in particular column separation, and FSI may lead to transient pressures larger than classical water hammer theory lets us believe. The example presented shows that an unsteady friction model is able to predict the damping and dispersion observed in laboratory experiments in non-cavitating flow. For cavitating flows the widely used discrete cavity models give workable results, but unsteady
friction gives also here improvement. The mechanisms causing FSI are well understood these days, and models combining FSI with cavitation and unsteady friction have been developed and become available to practical engineers.

APPENDIX I. - REFERENCES


APPENDIX II. - NOTATION

The following symbols are used in this paper:

- \( A \) = pipe area
- \( a \) = water hammer (pressure) wave speed
- \( a_m \) = liquid-gas mixture wave speed
- \( a_s \) = stress wave speed
- \( \tilde{a} \) = modified FSI wave speed (pressure, stress)
- \( C^* \) = Vardy shear decay coefficient
- \( D \) = pipe diameter
- \( e \) = pipe wall thickness
- \( f \) = Darcy-Weisbach friction factor
- \( G \) = Poisson coupling factor
- \( g \) = gravitational acceleration
- \( H \) = piezometric head (head)
- \( h_v \) = gauge vapour pressure head
- \( K \) = fluid bulk modulus
- \( k \) = Brunone friction coefficient
- \( L \) = pipe length
- \( N \) = number of computational reaches
- \( p \) = pressure
- \( Q \) = discharge or node downstream-end discharge
- \( Q_u \) = node upstream-end discharge

Subscripts:
- \( g \) = gas
- \( i \) = node number
- \( mp \) = midpoint
- \( q \) = quasi-steady part
- \( r \) = relative to pipe wall
- \( s \) = structure, solid, stress

Abbreviations:
- DGCM = discrete gas cavity model
- DVCM = discrete vapour cavity model
- FSI = fluid-structure interaction
- MOC = method of characteristics

\( R \) = inner pipe radius
\( Re \) = Reynolds number = \( \frac{VD}{\nu} \)
\( t \) = time
\( t_c \) = valve closure time
\( t_{vc} \) = time of cavitation inception
\( \dot{u} \) = axial pipe velocity
\( V \) = flow velocity
\( x \) = distance along pipe
\( z \) = pipeline elevation
\( \alpha_g \) = gas void fraction
\( \Delta p \) = jump in pressure
\( \Delta t \) = time step
\( \Delta L \) = reach length
\( \Delta \sigma \) = jump in axial stress
\( \theta \) = pipe slope
\( \nu \) = kinematic viscosity, Poisson ratio
\( \rho \) = mass density
\( \sigma \) = axial stress
\( \psi \) = weighting factor
\( \forall \) = discrete cavity volume
\( N \) = number of computational reaches
\( \psi \) = node downstream-end discharge
\( Q_u \) = node upstream-end discharge

Abbreviations:
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\( V \) = flow velocity