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Joint Design of Optimum Partial Response Target and Equalizer for Recording Channels With Jitter Noise

Hongming Yang and George Mathew

Department of Electrical and Computer Engineering, National University of Singapore,
and Data Storage Institute, A*Star, Singapore 117576

We address joint design of optimum generalized partial response (GPR) target and equalizer for perpendicular recording channels with jitter noise. We develop a new cost function which accounts for the data-dependent nature of jitter noise based on the minimum mean square error (MMSE) criterion. Using the step-response-based channel model, we derive expressions for the statistics required to compute the optimum equalizer and target in the presence of jitter noise. We also derive a bit-response-based model for the jitter noise channel. We present an approach for doing simulations as well as analytical computations for the jitter noise channel without resorting to the widely used Taylor series approximations. Our computational and simulation results show that, while the targets designed without accounting for the jitter lead to error-floor effect in the bit-error-rate performance, the targets designed by our approach give significant performance improvement under high jitter conditions, with no sign of error-floor effect for the range of signal-to-noise ratios considered.

Index Terms—Media noise, partial response target design, perpendicular magnetic recording, transition jitter.

I. INTRODUCTION

PARTIAL RESPONSE (PR) equalization followed by Viterbi detector (VD) is the most commonly used approach, popularly known as partial response maximum likelihood (PRML), for data detection in magnetic recording channels. At high recording densities, however, the detection performance of PRML detectors suffer seriously due to media noise [1], which is the dominant disturbance in high-density magnetic recording channels. Unlike the most commonly used noise model of additive white Gaussian and data-independent electronics noise (AWGN), the media noise is correlated, data-dependent, nonstationary, and nonadditive in nature. In this paper, we address the problem of designing optimum equalizer and PR target for media noise dominated recording channels. We choose perpendicular recording channels for our investigation since perpendicular recording is the technology for next-generation disk drives.

Several approaches have been proposed in the literature to deal with the problem of degradation in detection performance due to media noise. One sort of approach is through designing optimum detectors. Moon and Park [2] proposed a pattern-dependent noise-predictive maximum-likelihood detector. By modeling the media noise as a finite-order Markov process [3], Kavčić and Moura [4] developed an optimum sequence detector using optimized branch metrics. Chen and Trachtenberg [5] proposed an approach that asymptotically becomes optimum maximum likelihood (ML). Since optimum detectors usually result in high computational complexity, there have been several proposals for suboptimum approaches that are simpler to implement. For example, modifications in the branch metrics of the VD were proposed by Zeng and Moon [6] to account for

the data-dependent variance of media noise and by Sun *et al.* [7] to account for the data-dependent mean of media noise.

Another sort of approach to deal with media noise is through designing generalized PR (GPR) target to minimize the noise correlation since the noise should be white Gaussian for VD to be optimum. This approach helps to improve the detector performance significantly compared to standard PR targets with integer coefficients. Caroselli *et al.* [8] was the first to employ data-dependent noise prediction for media noise channels. Moon and Zeng [9] studied the effect of different constraints in the design of GPR targets for channels with jitter noise. They reported the superiority of the so-called “monic constraint” (i.e., first tap of the target is set to 1) in dealing with noise correlation and getting performance close to that of the optimum target designed by maximizing the “effective detection signal-to-noise ratio” of VD. Oenning and Moon [10] presented an analytical solution for monic-constrained GPR targets for Lorentzian model based channels with media noise. An adaptive approach for designing monic-constrained GPR targets with data-dependent adaptation for compensating nonlinearities caused by medium and head was proposed by Zayed and Carley [11]. Whereas all the references quoted above deal with longitudinal recording, Sawaguchi *et al.* [12] and Kovintavewat *et al.* [13] presented target design approaches for perpendicular recording channels with jitter noise. While they also use the monic constraint, the targets of Sawaguchi *et al.* [12] incorporate dc suppression of varying degrees in view of the low-frequency disturbances (e.g., jitter noise, thermal asperity, etc.) in practical channels. Okamoto *et al.* [14] extended the data-dependent noise prediction approach to perpendicular recording to deal with jitter noise. The performance of several standard PR targets (with integer-valued coefficients) for perpendicular channels with jitter noise was studied by Okamoto *et al.* [15].

The most commonly used approach for modeling media noise is by incorporating position-jitter and width-variation in the step

response. However, to minimize the effort required in simulating the readback signal as well as to facilitate analytically feasible approaches for designing equalizer and target, it has been a common practice to approximate the step response containing position-jitter and width-variation by using the Taylor series expansion with terms up to first or second order [2], [10], [5]. This approximation turns out to be quite inaccurate at high recording densities and/or with large jitter and width-variation. On the other hand, without making these approximations, the problem of target design has never been addressed for media noise channels in the literature. Furthermore, even simulation of the media noise channel is a very tedious task without these approximations.

In this paper, we reinvestigate the GPR target design problem without approximating the channel model. We introduce a new mean square error (MSE) based cost function considering media noise, electronics noise, and intersymbol interference (ISI) for jointly designing the equalizer and target. This cost function accounts for the data-dependent nature of media noise. We also derive expressions for computing the statistics required for evaluating the coefficients of the optimum GPR target and linear equalizer. In this paper, we do not make any modifications in the VD. Simulation results show that our approach can provide significant gain in detection performance for channels with large jitter compared to the case where the equalizer and target are designed without considering the jitter noise effect. Further, compared to standard PR targets with integer-valued coefficients, our approach is indeed remarkably superior.

This paper is organized as follows. In Section II, we discuss a few different approaches for modeling high-density magnetic recording channels with media noise. Thereafter, we introduce a new cost function and derive an approach to compute optimum GPR target and equalizer in Section III. Computational and simulation results are presented in Section IV. Some additional comments, highlighting the distinctiveness of our work reported in this paper, are presented in Section V. Section VI concludes the paper.

II. MODELS FOR CHANNEL WITH MEDIA NOISE

The magnetic recording channels are usually modeled by bit response $h_b(t)$ or step response $h_s(t)$, where $h_b(t) = h_s(t) - h_s(t-T)$ and T is the bit period. Since the recording channel can be assumed to be band-limited at high densities, we use bit-rate sampled readback signal in this paper. The resulting readback signal samples with only electronics noise can be written as

$$\begin{aligned} r_n &= \sum_m b_m h_b(nT - mT) + v_n \\ &= \sum_m a_m h_s(nT - mT) + v_n \end{aligned} \quad (1)$$

where $\{b_n\}$ is the recorded bit sequence with $b_n \in \{-1, +1\}$, $\{a_n\}$ is the corresponding transition sequence with $a_n = b_n - b_{n-1}$ and $a_n \in \{+2, 0, -2\}$, and v_n is white Gaussian electronics noise with variance σ_v^2 .

Widely used channel models incorporating media noise include the microtrack model [16], the data-dependent autoregressive model [3], and the position-jitter and width-variation model

[10]. We focus on the position-jitter and width-variation model in this paper. Since position-jitter effect is the major media noise effect [17], we only consider position-jitter in this paper. Thus, the readback signal with media noise can be written as

$$r_n = \sum_m a_m h_s(nT - mT + \Delta_m) + v_n \quad (2)$$

where Δ_m is the jitter in the position of the transition corresponding to a_m and $\{\Delta_m\}$ are modeled as independent Gaussian random variables truncated to the range $(-T/2, T/2)$ with mean zero and variance σ_Δ^2 . Clearly, $\Delta_m \neq 0$ only if $a_m \neq 0$.

It is easy to note from (2) that the presence of jitter causes the durations of the bits recorded on the medium to be different from T . For example, duration of the bit b_m on the medium is given by

$$T_m = T + \Delta_m - \Delta_{m+1}. \quad (3)$$

Using this information, we can express the readback signal in terms of the bit response as

$$r_n = \sum_m b_m h_{b,m}(nT - mT + \Delta_m) + v_n \quad (4)$$

where

$$h_{b,m}(t) = h_s(t) - h_s(t - T_m) \quad (5)$$

is the bit response corresponding to the bit b_m of duration T_m . The model (4) is as accurate as (2) itself since we did not use any approximations in deriving (4).

For the sake of convenience in generating the readback signal, designing equalizer and doing analysis, Taylor expansion is usually used to simplify the channel model (2). For example, first-order approximation of the channel model with transition jitter is given by

$$\begin{aligned} r_n &\approx \sum_m a_m h_s(nT - mT) \\ &\quad + \sum_m a_m \Delta_m h_i(nT - mT) + v_n \end{aligned} \quad (6)$$

where $h_i(t) \triangleq (\partial h_s(t)/\partial t)$ is the impulse response of the channel. Thus, the media noise gets modeled as an additive data-dependent noise. Based on this simplification, analysis of linear equalization and optimum detection have been done in [10], [2].

We may also derive an approximate model for (4), however, using a different approach. The bit response in magnetic recording channels is the convolution of the channel impulse response $h_i(t)$ with the impulse response $\tilde{p}(t)$ of the write circuit, where $\tilde{p}(t)$ is an ideal rectangular pulse of duration T and amplitude 1.0. Therefore, we can express $h_{b,m}(t)$ as

$$h_{b,m}(t) = h_i(t) \otimes \tilde{p}_m(t) \quad (7)$$

where $\tilde{p}_m(t)$ is an ideal rectangular pulse of duration T_m (instead of T) and amplitude 1.0. Therefore, the Fourier transform of $h_{b,m}(t)$ can be expressed as

$$H_{b,m}(f) = H_i(f) \tilde{P}_m(f) \quad (8)$$

where $H_i(f)$ and $\tilde{P}_m(f)$ are the Fourier transforms of $h_i(t)$ and $\tilde{p}_m(t)$, respectively, with

$$\tilde{P}_m(f) = T_m \frac{\sin(\pi f T_m)}{\pi f T_m} e^{-j\pi f T_m}. \quad (9)$$

At high densities, most of the energy in $H_i(f)$ is contained well within the null-to-null bandwidth of $\tilde{P}_m(f)$. Therefore, we may approximate $H_{b,m}(f)$ as [18]

$$H_{b,m}(f) \approx T_m \frac{\sin(\pi f T)}{\pi f T} H_i(f) e^{-j\pi f T} \quad (10)$$

whose time-domain equivalent is

$$h_{b,m}(t) \approx \frac{T_m}{T} h_b(t). \quad (11)$$

Substituting (11) in (4), we obtain

$$r_n \approx \sum_m \tilde{b}_m h_b(nT - mT + \Delta_m) + v_n \quad (12)$$

where

$$\tilde{b}_m = b_m \frac{T_m}{T}. \quad (13)$$

Thus, the approximation amounts to a multiplicative change in the bit-amplitude and a shift in the bit-position, while the bit response is computed based on the bit-duration T . The model in (12) may be further approximated using Taylor expansion to derive a model similar to (6) as

$$r_n \approx \sum_m \tilde{b}_m h_b(nT - mT) + \sum_m \tilde{b}_m \Delta_m \tilde{h}_i(nT - mT) + v_n \quad (14)$$

where

$$\tilde{h}_i(t) = \frac{\partial h_b(t)}{\partial t} = h_i(t) - h_i(t - T). \quad (15)$$

Comparing the approximations made in deriving (6), (12), and (14), we can see that the approximate model (12) will be more accurate compared to (6) and (14) for large jitter.

The simplified models are acceptable only when the standard deviation σ_Δ of jitter is small enough compared to the bit duration T . But, the jitter becomes quite large at high densities, thus making the simplified models inaccurate. Hence, in this paper, we focus on the media noise problem based on the channel model (2), which gives a clearer and accurate picture of the effect of media noise on the readback signal without any approximations. Even though the exact models given by (2) and (4) are equivalent, we choose (2) for our study in this paper since it leads to some ease in theoretical analysis compared to (4), as explained at the end of Section III-A.

Fig. 1. Joint PR equalizer and target design for channels with jitter noise.

In this paper, we use the hyperbolic tangent function based perpendicular magnetic recording channel, whose step response can be written as

$$h_s(t) = \frac{A}{2} \tanh\left(\frac{\ln 3}{T_{50}} t\right) \quad (16)$$

where A is the peak-to-peak amplitude, and T_{50} is the time required for $h_s(t)$ to rise from $-A/4$ to $A/4$. We define the normalized density as $D_c = T_{50}/T$.

III. OPTIMUM JOINT TARGET AND EQUALIZER DESIGN

Fig. 1 shows the block schematic used for the joint design of the GPR target $G(z)$ and equalizer $W(z)$. For this design, we use the widely used minimum mean square error (MMSE) approach, that is, by minimizing $E[e_n^2]$ where $E[\cdot]$ denotes the expectation operator. The equalizer output y_n and target output d_n are given by

$$y_n = \mathbf{w}^T \mathbf{r}_n \quad \text{and} \quad d_n = \mathbf{g}^T \mathbf{b}_n \quad (17)$$

where superscript T denotes transpose, $\mathbf{w} = [w_{-K}, w_{-K+1}, \dots, w_{K-1}, w_K]^T$ is the equalizer with length $N_w = 2K + 1$, $\mathbf{g} = [g_0, g_1, \dots, g_{L-1}]^T$ is the target with length L , $\mathbf{r}_n = [r_{n+K}, r_{n+K-1}, \dots, r_{n-K}]^T$, and $\mathbf{b}_n = [b_n, b_{n-1}, \dots, b_{n-L+1}]^T$. First, we review the results for recording channels without media noise. Thereafter, we introduce a new cost function for channels with media noise and derive the expressions required for computing the statistics required for designing the optimum equalizer and target.

For the channel without media noise, the mean square error (MSE) at the equalizer output is given by [9]

$$E[e_n^2] = \mathbf{w}^T \mathbf{R} \mathbf{w} + \mathbf{g}^T \mathbf{A} \mathbf{g} - 2\mathbf{w}^T \mathbf{P} \mathbf{g} \quad (18)$$

where $\mathbf{R} = E[\mathbf{r}_n \mathbf{r}_n^T]$ is a $N_w \times N_w$ auto-correlation matrix of the channel output r_n , $\mathbf{A} = E[\mathbf{b}_n \mathbf{b}_n^T]$ is a $L \times L$ auto-correlation matrix of the input data b_n , and $\mathbf{P} = E[\mathbf{r}_n \mathbf{b}_n^T]$ is a $N_w \times L$ cross-correlation matrix between r_n and b_n .

To minimize the MSE in (18) while avoiding the trivial solution $\mathbf{g} = \mathbf{0}$ and $\mathbf{w} = \mathbf{0}$, some constraint needs to be imposed. The widely used monic-constraint (i.e., $g_0 = 1$) has noise whitening ability since it results in an equalizer that is equivalent to the forward equalizer of the MMSE solution of decision feedback equalization [11]. Hence, we focus on the design problem using monic-constraint in this paper.

The results of the monic constrained MMSE design for channels without media noise are given by [9]

$$\lambda = \frac{1}{\mathbf{i}^T (\mathbf{A} - \mathbf{P}^T \mathbf{R}^{-1} \mathbf{P})^{-1} \mathbf{i}} \quad (19)$$

$$\mathbf{g} = \lambda (\mathbf{A} - \mathbf{P}^T \mathbf{R}^{-1} \mathbf{P})^{-1} \mathbf{i} \quad (20)$$

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{P} \mathbf{g} \quad (21)$$

where λ is the Langrange multiplier and $\mathbf{i} = [1, 0, 0, \dots, 0]^T$ is a vector of length L .

Now we consider the channel model with jitter noise, as shown in Fig. 1. Since jitter noise is data-dependent and non-stationary, the optimum equalizer and target should also be data-dependent and hence time-variant, which may require unaffordable complexity. Also, any statistical computation done on the channel output will need to incorporate the data-dependence of media noise. On the other hand, prior to Viterbi detection, we have no idea about the recorded bit sequence and thus we cannot know what are the specific noise characteristics that must be used for designing the equalizer and target for each instant. Therefore, we use the following approach to circumvent these issues. We use the squared error averaged over all possible recorded sequences as our cost function to minimize, which is given by

$$\xi_{\text{MSE}} = \sum_{\bar{\mathbf{a}}} E [e_n^2 | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \quad (22)$$

where $\{\bar{\mathbf{a}}\}$ denotes any possible recorded sequence. Note that this cost function combines ISI, electronics noise, and media noise into one function. At the same time, this cost function accounts for the data-dependent nature of the media noise, with the sequences that result in more jitter noise (i.e., more transitions) receiving more weightage in the cost function, and vice versa. Thus, even though the cost function ξ_{MSE} and the resulting optimum equalizer and target are data-independent, the construction of the cost function ensures that the optimum solution is implicitly tuned to respond to the different data sequences in accordance with the amounts of jitter noise caused by these sequences.

Based on this new cost function, it is straightforward to see that the optimum equalizer and target are still given by (19)–(21) except that the matrices $\mathbf{R}$, $\mathbf{P}$, and $\mathbf{A}$ are now defined as

$$\bar{\mathbf{R}} = \sum_{\bar{\mathbf{a}}} E [\mathbf{r}_n \mathbf{r}_n^T | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \quad (23)$$

$$\bar{\mathbf{P}} = \sum_{\bar{\mathbf{a}}} E [\mathbf{r}_n \mathbf{b}_n^T | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \quad (24)$$

$$\bar{\mathbf{A}} = \sum_{\bar{\mathbf{a}}} E [\mathbf{b}_n \mathbf{b}_n^T | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}). \quad (25)$$

In this paper, we assume all the data patterns to be equally probable. Hence, we have $\bar{\mathbf{A}} = \mathbf{A}$. Computation of the conditional statistics required for $\bar{\mathbf{R}}$ and $\bar{\mathbf{P}}$ is not a trivial numerical problem and is the major task here. There arise some issues in this numerical computation process caused by the channel model in (2), which are discussed below.

A. Truncation of the Step Response

The step response of perpendicular recording channel has infinite length and, hence, we need to truncate it while doing sim-

ulations. When truncating the step response, we should ensure that there does not arise any instability or inaccuracy due to truncation. Since the step response given by (16) does not tend to zero at positions far from the transition position, we cannot simply truncate the step response to some range around $t = 0$.

By comparing the step response and bit response, we can solve the truncation problem. When we use truncated bit response for simulating perpendicular magnetic recording channels, it works fine because the tails of bit response tend to zero at positions far from the bit position. But, this is equivalent to the assumption that the input bits before the first bit and after the last bit are 0. This means that we should have transitions with level +1 or -1 at the starting bit position and the ending bit position of the data pattern, whereas the transition levels inside the data pattern are +2, -2, or 0. Therefore, if we add two extra transitions with levels +1 or -1 at the two ends of the transition sequence according to the bits recorded, we can use step response to do simulations instead of bit response. With this change, we can now truncate the step response such that it reaches saturation on both sides.

In the absence of jitter noise, it can be shown that the truncation of step response is equivalent to truncation of bit response, and hence either step response or bit response can be used in simulations and computations. When jitter noise is present, we prefer to use step response instead of bit response because the responses due to different transitions are independent given a data pattern while this property does not hold for bit response [see (2)–(5)].

B. Computation of Matrices $\bar{\mathbf{R}}$ and $\bar{\mathbf{P}}$

To get the optimum solution, we need to compute the correlation matrices $\bar{\mathbf{R}}$ and $\bar{\mathbf{P}}$ given by (23)–(24). Clearly these can only be obtained through numerical computations since there are no closed-form expressions.

In jitter noise dominated high-density perpendicular recording channels, the matrix $\bar{\mathbf{R}}$ is found to be highly ill-conditioned with some eigenvalues close to zero. Hence, it is important to avoid numerical inaccuracy as much as possible. Therefore, instead of resorting to data-averaging to estimate the conditional correlations, we develop a rigorous analytical approach and the solution of which can be computed numerically. The details are as follows.

First of all, we note that by defining the cost function ξ_{MSE} in (22) as the averaged squared error over all possible data patterns, the data-dependence and associated nonstationarity caused by jitter noise are taken care of. In other words, the data-dependent averaging converts the underlying nonstationary problem into a wide-sense stationary problem.¹ As a result, $\bar{\mathbf{R}}$ is a toeplitz symmetric matrix. Therefore, to compute $\bar{\mathbf{R}}$, we only need to develop the expression for $E[r_k r_{k-n}]$, where $\{r_n\}$ is the equalizer input given by (2). From (23), we get

$$E[r_k r_{k-n}] = \sum_{\bar{\mathbf{a}}} E[r_k r_{k-n} | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}). \quad (26)$$

For the sake of convenience, we shall rewrite (2) as

$$r_k = \tilde{r}_k + v_k = \sum_i a_i h(i, k) + v_k \quad (27)$$

¹This statement is true only if the data $\{b_n\}$ is wide-sense stationary.

where $\hat{r}_k$ denotes the signal without electronics noise and $h(i, k) \triangleq h_s(kT - iT + \Delta_i)$ denotes the step response at position kT due to the transition at position iT . Therefore, we get

$$E[r_k r_{k-n}] = \tilde{\phi}_{k,n} + \sigma_v^2 \delta_n \quad (28)$$

where

$$\begin{aligned} \tilde{\phi}_{k,n} &= E[\tilde{r}_k \tilde{r}_{k-n}] \\ &= \sum_{\bar{\mathbf{a}}} \sum_{i,j} E[a_i h(i, k) a_j h(j, k-n) | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \end{aligned} \quad (29)$$

with $\delta_k = 1$ if $k = 0$ and $\delta_k = 0$ if $k \neq 0$.

When $i \neq j$, $h(i, k)$ and $h(j, k-n)$ are independent since the transition jitters at different positions are independent. Thus, we need to compute $E[h(i, k) | \bar{\mathbf{a}}]$ and $E[h(j, k-n) | \bar{\mathbf{a}}]$ using the probability density function (pdf) of the transition jitter. The effect of conditioning with data pattern $\bar{\mathbf{a}}$ in $E[h(i, k) | \bar{\mathbf{a}}]$ is to know whether or not there is a transition at position iT . When $i = j$, $h(i, k)$ and $h(j, k-n)$ are dependent since they contain the same transition jitter at position iT . Then we need to compute $E[h(i, k)h(i, k-n) | \bar{\mathbf{a}}]$. Since we assumed that each data pattern is equally probable, we obtain

$$\tilde{\phi}_{k,n} = \frac{1}{2^{L_d-1}} \sum_{\bar{\mathbf{a}}} \sum_{i,j} E[a_i h(i, k) a_j h(j, k-n) | \bar{\mathbf{a}}] \quad (30)$$

where L_d is the length of transition sequences considered.

Since each recorded bit is equally probable to be +1 or -1, we can compute the probability distributions of $\{a_i\}$. It can be easily shown that

$$\Pr[a_i = 2] = \Pr[a_i = -2] = 1/4 \quad (31)$$

$$\Pr[a_i = 0] = 1/2 \quad (32)$$

$$\Pr[a_{i+1} | a_i = 0] = \Pr[a_{i+1}] \quad (33)$$

$$\Pr[a_{i-1} | a_i = 0] = \Pr[a_{i-1}] \quad (34)$$

$$\Pr[a_{i+1} = -a_i | a_i \neq 0] = \Pr[a_{i+1} = 0 | a_i \neq 0] = 1/2 \quad (35)$$

$$\Pr[a_{i-1} | a_i \neq 0] = \Pr[a_{i+1} | a_i \neq 0] \quad (36)$$

and the transitions which are not next to each other are independent. Hence, substituting (31)–(36) in (30), we get

$$\begin{aligned} \tilde{\phi}_{k,n} &= \frac{1}{2^{L_d-1}} \sum_{\bar{\mathbf{a}}} \sum_i \sum_{j=i-1}^{i+1} E[a_i a_j h(i, k) h(j, k-n) | \bar{\mathbf{a}}] \\ &= \frac{1}{2^{L_d-1}} \sum_{\bar{\mathbf{a}}} \sum_i \sum_{j=i-1}^{i+1} a_i a_j E[h(i, k) h(j, k-n) | \bar{\mathbf{a}}] \\ &= 2 \sum_i E[h(i, k) h(i, k-n)] \\ &\quad - \sum_i E[h(i, k)] E[h(i-1, k-n)] \\ &\quad - \sum_i E[h(i, k)] E[h(i+1, k-n)]. \end{aligned} \quad (37)$$

Thus, we see from (37) that we do not need to compute the auto-correlation matrices for each data pattern and sum them up. Instead, we only need to compute $E[h(i, k)]$ and $E[h(i, k)h(i, k-n)]$ for the transition at position iT . Thus, possible numerical inaccuracy is highly minimized. Note that $E[h(i, k)]$ is only a function of $k-i$, and $E[h(i, k)h(i, k-n)]$ is a function of $k-i$ and $k-n-i$.

Another thing to note is that each data pattern must start and end with transitions with amplitude +1 or -1. In other words, a_1 and a_{L_d} are equally probable to be +1 or -1. Thus, the summations in (37) will be modified slightly when i is equal to 1 and L_d . Doing this, we get

$$\begin{aligned} \tilde{\phi}_{k,n} &= E[h(1, k)h(1, k-n)] \\ &\quad + E[h(L_d, k)h(L_d, k-n)] \\ &\quad + 2 \sum_{i=2}^{L_d-1} E[h(i, k)h(i, k-n)] \\ &\quad - \sum_{i=2}^{L_d} E[h(i, k)] E[h(i-1, k-n)] \\ &\quad - \sum_{i=1}^{L_d-1} E[h(i, k)] E[h(i+1, k-n)]. \end{aligned} \quad (38)$$

Similarly, we can obtain the (p, q) th element of $\bar{\mathbf{P}}$ as

$$\begin{aligned} \bar{P}_{p,q} &= \sum_{\bar{\mathbf{a}}} E[r_{n+K-p} b_{n-q} | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \\ &= \sum_{\bar{\mathbf{a}}} \sum_i E[h(i, n+K-p) a_i b_{n-q} | \bar{\mathbf{a}}] \Pr(\bar{\mathbf{a}}) \\ &= E[h(n-q, n+K-p)] \\ &\quad - E[h(n-q+1, n+K-p)]. \end{aligned} \quad (39)$$

Thus, given the channel step response, jitter pdf, and electronics noise power, we can accurately compute the correlation matrices $\bar{\mathbf{R}}$ and $\bar{\mathbf{P}}$.

IV. COMPUTATIONAL AND SIMULATION RESULTS

As an example, we show here the computational results under the following conditions: Perpendicular magnetic recording, effective length of truncated step response is 40 bits, equalizer length is $N_w = 15$, target length is $L = 5$, linear density is $D_c = 2.0$, monic-constraint for target response, range of transition jitter is from $\sigma_\Delta/T = 0\%$ to $\sigma_\Delta/T = 10\%$, and at least 1000 error bits are collected for every estimate of the bit-error rate (BER). Conventional Viterbi detector (VD) with no modifications is used for data detection. The strength of the electronics noise (AWGN) v_n is chosen according to the signal-to-noise ratio (SNR) defined as

$$\text{SNR(dB)} = 10 \log_{10} \left(\frac{V_{op}^2}{\sigma_v^2} \right) \quad (40)$$

where σ_v^2 is the variance of AWGN and $V_{op} = 2A$. Thus, our SNR definition includes only the electronics noise.

The optimum GPR targets designed under different jitter variances are shown in Fig. 2. The SNR used in this design is 35 dB for all cases. Observe that the effective target length decreases with increase in jitter. In fact, with 10% jitter, the optimum target

Fig. 2. Optimal target responses obtained for recording channel with jitter.

Fig. 3. Normalized MMSE obtained for recording channel with jitter.

uses only four out of the five taps provided. Further, the slopes of the target at the sampling points are seen to progressively decrease with increase in jitter percentage. These characteristics are very desirable to minimize the amount as well as effect of jitter noise at detector input. Similar observations were reported earlier for longitudinal recording in [9], [10]. It is also important to note that the target energy keeps decreasing with increase in jitter, thereby predicting severe degradation in the performance of VD.

In the MMSE and BER performances given in Figs. 3–5, the equalizer and target are reoptimized for each SNR. Fig. 3 shows the MMSE, normalized by the target energy, for different amounts of jitter. Also shown, for the sake of comparison, is the normalized MSE computed for 5% jitter channel using the equalizer and target designed for jitter-free case. This curve lies below the MMSE curve corresponding to 5% jitter channel with corresponding optimum equalizer and target since the target energy for the jitter-free channel is much higher than the target energy for 5% jitter channel,² as can be seen from Fig. 2. What is important to note is that the target optimized for jitter-free case results in error-floor in the MSE curve when used on jitter channels. This will lead to similar error-floor effect

²If we examine the unnormalized MSE plots (not shown here) corresponding to Fig. 3, we will find that the MSE computed for 5% jitter channel using equalizer and target optimized for jitter-free case is the highest for all SNRs, which is quite expected. Furthermore, this MSE has its minimum at SNR = 44 dB and increases monotonically as SNR is further reduced or increased from 44 dB.

Fig. 4. BER comparison between designed target responses and fixed target [1 2 3 2 1] for 3% jitter.

Fig. 5. BER comparison between designed target responses and fixed target [1 2 3 2 1] for 5% jitter.

in BER performance too. On the other hand, targets optimized for jitter channels result in monotonically decreasing MMSE, thereby implying that these targets help to delay the onset of error-floor to much higher SNR values. As the slopes of these MMSE curves approach zero, we can expect the error-floor to manifest.

To have a clear idea of the effectiveness of the joint target and equalizer design, we need to check the BER performance of VD using the targets and equalizers obtained. Fig. 4 shows the BER performances using different target responses and linear equalizers for 3% jitter. Since [1 2 3 2 1] is a widely used standard target for perpendicular channels [15], we also include the performance of VD with this fixed target for the purpose of comparison. The detection performance of VD on a jitter-free channel using a monic-constrained GPR target designed for 0% jitter is included in Fig. 4 to serve as a reference for assessing the performance for other targets. First of all, we note that the performance of fixed target remains the same whether or not jitter noise is accounted for in the equalizer design. This shows that the choice of target is highly critical in media noise channels, even as was observed in [10]. It is also clear from Fig. 4 that the BER plots for the fixed target are showing indications of error-floor even as early as 36–38 dB. Compared to this, the monic-constrained GPR targets improve the performance significantly for channels

with jitter, and that too with no indication of error-floor. However, we note that the performance of the GPR target which is optimized for the jitter channel is not significantly better than that of the GPR target which is optimized for jitter-free channel, the gain being just about 0.1–0.2 dB at a BER of 10^{-6} . This is mainly because 3% jitter is not large enough to make the overall channel noise strongly data-dependent in nature, for the range of SNRs considered here. We can expect the effect of jitter to show up if we do the simulation at much higher SNRs and/or high jitter. The simulation conditions chosen for Fig. 5 help to verify this conjecture.

Fig. 5 shows the BER performance comparison results for channels with 5% jitter. The advantage of the proposed joint design approach is clearly manifest in this figure. In particular, the region of interest to us in this paper is the very high SNR region where jitter noise is dominant. In this region, the proposed design approach provides significant improvement in performance while the other targets exhibit error-floor effect. While the fixed target $[1 \ 2 \ 3 \ 2 \ 1]$ is not even able to reach a BER of 10^{-4} , the GPR target designed without considering jitter seems to level off to a BER of 10^{-5} . On the other hand, even at 40 dB, there is not yet an indication of the onset of error-floor for the proposed approach. Thus, we see that the onset of error-floor is much delayed by the GPR target whose design incorporates the jitter, compared to all the other targets.

V. ADDITIONAL COMMENTS

We shall now comment on some specific aspects of the work reported in this paper, with the objective of highlighting the distinctiveness of our work from already published work.

First, we comment on the importance of numerical accuracy in the computation of $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$ in Section III-B. Actually, $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$ can also be estimated through data-averaging over a sufficiently large sequence of channel output samples $\{r_n\}$. But, the matrix $\hat{\mathbf{R}}$ for jitter dominated perpendicular recording channel turns out to be highly ill-conditioned. Therefore, since the design equations (19)–(21) include matrix inversion operations, any small inaccuracy in $\hat{\mathbf{R}}$ tends to result in serious impact on the performance. Consequently, the number of samples of r_n needed to keep the estimation error very small becomes excessively large to be of practical use. On the other hand, by using the expressions developed in this paper, we can accurately compute $\hat{\mathbf{R}}$ with very minimum computational and memory requirements. We may also point out that the use of first-order model given in (6) does not lead to an ill-conditioned $\hat{\mathbf{R}}$. Since all the existing studies are based on models of the type (6), the problem of numerical inaccuracy in the estimation of $\hat{\mathbf{R}}$ was not encountered.

We choose to use the channel model (2) instead of (6) in our computations and simulations in this paper, since (2) is more accurate for high-density recording with transition jitter in the range of 5% to 10%. In fact, for such high jitter, the first-order model in (6) can be grossly in error and the conclusions drawn based on studies conducted using this model can be misleading. Based on the channel model (2), we also give a rigorous approach to do computations and simulations using step response instead of bit response. To our knowledge, no work has been reported in the past on deriving analytical expressions for computing the correlations using the exact model (2). There have

also been no remarks in existing publications on how to do simulation using the exact model (2) for perpendicular recording channels where the underlying step response is of infinite duration. We also developed a bit-response-based exact model given by (4) for simulating jitter noise channel. This model makes the simulation effort even easier compared to the step-response-based model in (2). Further, the approximate model (12), which we developed based on (4), is more accurate than the first-order model in (6).

The cost function (22) used in this paper accounts for the data-dependence of media noise in an implicit way even though the expression is data-independent. It can be seen that sequences with more transitions, which result in more significant jitter noise effect, will have larger weightage in the cost function, and vice versa. Thus, the proposed cost function results in a good compromise between hardware complexity and performance, as an explicitly data-dependent equalizer-target approach would result in unaffordable complexity levels.

There have been publications on analytical approaches for the design of GPR target [10], [12]. But, these approaches are based on infinite length equalizers (i.e., joint design of infinite-length equalizer and finite-length target), even though these may be replaced/approximated by finite-length equalizers after getting the target. On the other hand, our proposal is for joint design of equalizer and GPR target where both the equalizer and target are of finite length.

In this paper, the SNR definition (40) includes only the electronics noise, and the effect of transition jitter is measured by the ratio σ_{Δ}/T . This is similar to the approach used in [13], [11]. On the other hand, several papers use a different SNR definition where the noise power is taken as the sum of the powers of electronics noise and jitter noise, with the jitter noise power computed based on either single transition or multiple transitions [12], [15], [19]. Since jitter noise and electronics noise have very different effect on the detection performance, defining the SNR by adding the powers of these noises may lead to inconsistent results. For example, the detection performance for a given SNR could be quite different for two different compositions (e.g., 25:75, 75:25) of the two noises. For this reason, we choose to use only the electronics noise in defining the SNR.

Finally, note that our proposal in this paper is only for designing optimum equalizer and target. We did not propose any modifications to the detector. On the other hand, there have been several proposals for developing optimum or suboptimum detectors for jitter noise channels, as we reviewed in Section I. Our method is complementary to these proposals for detector modifications. Therefore, the equalizer and target designed using our approach can be combined with any of these detection approaches to result in even better performance for data detection in the presence of jitter noise.

VI. CONCLUSION

In this paper, we consider high-density perpendicular magnetic recording channels with jitter noise and propose a novel approach to jointly design optimum generalized partial response target and linear equalizer. We investigate the problem using a step-response-based channel model without making any approximations and derive analytical expressions for the statistics required to obtain the optimum target response and equalizer

through minimizing a new MMSE-based cost function. The resulting GPR targets provide significant gain over targets which are designed without considering jitter noise. In particular, the GPR targets designed using the proposed cost function significantly delay the onset of error-floor in the detection performance compared to other targets.

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Manuscript received July 28, 2005; revised October 16, 2005 (e-mail: elemg@nus.edu.sg).