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Realization of NSHP-Filters

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Eindhoven, January 1978

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Abstract

In this note a class of 2-D transfer matrices is considered, much larger than the class of so-called causal transfer matrices. We will generalize the state space model of [1] such that also nonsymmetric-half-plane (NSHP) filters can be state space realized. Surprisingly enough this can be done using realization theory over a field.
1. Introduction

First of all we introduce some definitions and notations. \( \mathbb{R}[s] \) denotes the set of polynomials in \( s \) with real coefficients. \( \mathbb{R}[s,z] \) denotes the set of polynomials in two variables \( s \) and \( z \) with real coefficients. It is clear that \( \mathbb{R}[s,z] = \mathbb{R}[s][z] \), see also [1].

\( \mathbb{R}(s) \) denotes the set of real rational functions in \( s \). \( \mathbb{R}(s,z) \) denotes the set of real rational functions in two variables \( s \) and \( z \). Elements of \( \mathbb{R}(s,z) \) will be denoted by \( p/q \) with \( p \in \mathbb{R}[s,z] \) and \( q \in \mathbb{R}[s,z] \). For \( q \in \mathbb{R}[s,z] \) the degree of \( q \) in \( z \) will be denoted by \( \deg_z(q) \).

\( \mathbb{R}^{r \times m}(s,z) \) denotes the set of \( r \times m \) matrices with entries from \( \mathbb{R}[s,z] \).

\( \mathbb{R}^{r \times m}(s,z) \) denotes the set of \( r \times m \) matrices with entries from \( \mathbb{R}(s,z) \).

Let \( T \in \mathbb{R}^{r \times m}(s,z) \) then \( T \) can be written as \( P/q \) with \( q \) the LCM of the denominators of the entries of \( T \) and \( P \in \mathbb{R}^{r \times m}(s,z) \) and \( q \in \mathbb{R}[s][z] \). Now suppose that \( \deg_z(q) = n \). We then divide \( q \) and the entries of \( P \) by the coefficient of \( z^n \) thus making \( q \) a monic polynomial with coefficients from \( \mathbb{R}(s) \). Now it is clear that by this procedure we obtained an element of \( \mathbb{R}(s)^{r \times m}(z) \). We define the degree in \( z \) of \( P \) as the maximum of the degrees in \( z \) of the entries of \( P \).

We will now introduce the set of proper 2-D \( r \times m \)-transfer matrices \( \mathbb{R}^{r \times m}(s,z) \).

\[
\mathbb{R}^{r \times m}_{p}(s,z) = \{ T \in \mathbb{R}^{r \times m}(s,z) \mid T = P/q, \deg_z(q) > \deg_z(P) \}.
\]

\( T \in \mathbb{R}^{r \times m}_{p}(s,z) \) will be called strictly proper if we have \( \deg_z(q) > \deg_z(P) \).

This set will be denoted by \( \mathbb{R}^{r \times m}_{sp}(s,z) \). In this paper we will consider mainly strictly proper 2-D transfer matrices. Analogous results with only minor modifications can be obtained for the proper case.

Observe that we now have obtained a straightforward generalization of the 1-D notion of a proper transfer matrix. The coefficients of our 2-D transfer matrix are not real numbers as in the case of 1-D transfer matrices but they are rational functions (elements of the field \( \mathbb{R}(s) \)).
2. The realization procedure

For a transfer matrix \( T(s,z) \in \mathbb{R}^{r \times m}(s,z) \) there exists, as is well known, a minimal realization \((A(s),B(s),C(s))\) such that

\[
C(s) \in \mathbb{R}^{r \times n}(s), \quad A(s) \in \mathbb{R}^{n \times n}(s), \quad B(s) \in \mathbb{R}^{n \times m}(s)
\]

for some integer \( n \) such that

\[
T(s,z) = C(s)[zI - A(s)]^{-1}B(s) .
\]

Minimality is as usual equivalent to (see also \([1]\))

1) \((A(s),B(s))\) is a reachable pair.
2) \((C(s),A(s))\) is an observable pair.

Now we have the following theorem.

(2.1) **THEOREM.** \((A(s),B(s))\) is reachable iff \((A(s_0),B(s_0))\) is reachable for some complex number \( s_0 \).

\((C(s),A(s))\) is observable iff \((C(s_0),A(s_0))\) is observable for some complex number \( s_0 \).

**Proof.** By duality it is enough to prove only the reachability part. Suppose \((A(s),B(s))\) is a reachable pair thus by definition we have:

\[
[B(s)|A(s)B(s)|\ldots|A(s)^{n\times n-1}B(s)]
\]

has full rank and is therefore right invertible. So for every \( s_0 \in \mathbb{C} \) for which \( B(s_0) \) and \( A(s_0) \) are defined we have

\[
[B(s_0)|A(s_0)B(s_0)|\ldots|A(s_0)^{n\times n-1}B(s_0)]
\]

has full rank and therefore reachability of \((A(s_0),B(s_0))\). Now suppose \([B(s)|A(s)B(s)|\ldots|A(s)^{n\times n-1}B(s)]\) has not full rank then all \( n \times n \) minors are zero. Thus for every \( s_1 \in \mathbb{C} \) for which \( A(s_1) \) and \( B(s_1) \) are defined \((A(s_1),B(s_1))\) is not a reachable pair.

Consider the NSHP \( S \) (see also \([5]\))

\[
S = \{(h,k) \in \mathbb{Z}^2 \mid h > 0 \text{ or } (k > 0 \text{ and } h = 0)\} .
\]

Now consider the I/O (input/output) description of a 2-D system

(2.2) \[
y_{hk} = \sum_{(i,j) \in S} T_{h-i,k-j} u_{ij} \quad (h,k) \in S
\]

where \( u_{ij} \in \mathbb{R}^m, \ y_{hk} \in \mathbb{R}^n, \ T_{pq} \) has its support in \( S \). This I/O system will be called a NSHP filter. Applying the 2-D Z-transform (or \((z,s)\) transform) (see \([1]\)) we obtain
\[
\hat{y}(s,z) = \sum_{(h,k) \in S} y_{hk} z^{s} s^{h} k
\]
\[
\tilde{y}(s,z) = \sum_{(h,k) \in S} u_{hk} z^{s} s^{h} k
\]
\[
T(s,z) = \sum_{(h,k) \in S} T_{hk} z^{s} s^{h} k
\]

and
\[
(2.3) \quad \hat{y}(s,z) = T(s,z) \tilde{u}(s,z).
\]

From now on we will assume that this \( T(s,z) \) is a strictly proper 2-D transfer matrix (for conditions see [1], [2]). We will now derive a state space description of this I/O map. Using (2.3) we obtain:

\[
(2.4) \quad \begin{align*}
\dot{x}_{h+1}(s) &= A(s)x_{h}(s) + B(s)u_{h}(s) \quad \dot{x}_{0}(s) = \delta \\
\dot{y}_{h}(s) &= C(s)x_{h}(s), \quad h = 0,1,\ldots
\end{align*}
\]

where \( x_{h}(s) \) is defined by
\[
x_{h}(s) = \sum_{k=\infty}^{\infty} x_{hk} s^{-k}
\]
and \( x_{hk} \in \mathbb{R}^{n} \), \( (h,k) \in S \).

\( u_{h}(s) \) and \( y_{h}(s) \) are defined analogously.

Remark. Observe that the state space is infinite dimensional. Writing the equations for every \( x_{hk} \), \( u_{hk} \), \( y_{hk} \) we obtain an infinite dimensional system where the system matrices are doubly infinite block-Toeplitz matrices [3]. This way of representation can be used in filtering [4].

Remark. (2.4) is a generalization of the first level realization defined in [1].

The generalized first level realization (2.4) can be realized itself to obtain a generalized second level realization. Compare also [2]. First of all we observe that there exists an integer \( d \) such that
\[
\tilde{A}(s) = A(s)/s^{d}, \quad \tilde{C}(s) = C(s)/s^{d}, \quad \tilde{B}(s) = B(s)/s^{d}
\]
are proper 1-D transfer matrices.

Now consider the equations:

\[
(2.5) \quad \begin{align*}
\dot{x}_{h+1}(s) &= \tilde{A}(s)x_{h}(s) + \tilde{B}(s)u_{h}(s) \quad \dot{x}_{0}(s) = \delta \\
\dot{y}_{h}(s) &= \tilde{C}(s)x_{h}(s), \quad h = 0,1,\ldots
\end{align*}
\]
Now take minimal realizations of $\tilde{A}(s)$, $\tilde{B}(s)$ and $\tilde{C}(s)$.

Let

$$\tilde{A}(s) = \tilde{AD} + \tilde{AC}[sI - \tilde{AA}]^{-1}\tilde{AB}$$

$$\tilde{B}(s) = \tilde{BD} + \tilde{BC}[sI - \tilde{BA}]^{-1}\tilde{BB}$$

$$\tilde{C}(s) = \tilde{CD} + \tilde{CC}[sI - \tilde{CA}]^{-1}\tilde{CB}$$

Then a generalized second level realization is:

$$b_{h,k+1} = \tilde{B}a_{h,k} + \tilde{B}u_h,k+d$$

$$a_{h,k+1} = \tilde{A}a_h,k + \tilde{AB}x_h,k+d$$

$$x_{h+1,k} = \tilde{A}x_h,k+d + \tilde{AC}a_h,k + \tilde{BD}u_h,k+d + \tilde{BC}b_h,k$$

$$c_{h,k+1} = \tilde{C}c_h,k + \tilde{CB}x_h,k+d$$

$$y_h,k = \tilde{CD}h,k+d + \tilde{CC}c_h,k$$

(2.6)

Initial conditions are zero, $b_{h,k}$, $a_{h,k}$, $c_{h,k}$ are intermediate state space variables of appropriate dimensions.

Remark. Observe that we have taken the number $d$ to be the same for $A(s)$, $B(s)$, $C(s)$. Of course this is not necessary but because of simplicity we have chosen to do so.

We will now assume that there exists a positive integer $I_u$ such that

$$u_{h,k} = 0 \text{ for } k < -I_u, (h,k) \in S$$

(2.7)

Introducing this condition it can easily be seen that equations (2.6) can be evaluated in a recursive way.

Remark. The condition (2.7) is not necessary but because of simplicity we have chosen this condition. Note also that (2.7) is a kind of uniformity condition.

Remark. For more details on equations like (2.6) see [2], where also 2-D systems with support in a certain subset of a half plane, not necessarily in the first and fourth quadrant, are taken into consideration. This is done by using spectral transformations.
3. Conclusion

In this note a generalized first level realization and a generalized second level realization of a (strictly) proper 2-D transfer matrix has been introduced. This approach leads to equations comparable to those of [2]. The advantage of this approach is that one can work over a field while in [2] one has to work over a ring. However [2] gives an a priori bound on the number $d$ (see (2.5)). It is still an open question how one can reduce the number $d$ by a state space isomorphism.
References


