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Dipole formation by two interacting shielded monopoles in a stratified fluid

M. Beckers, H. J. H. Clercx, and G. J. F. van Heijst

Fluid Dynamics Laboratory, Department of Physics, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

R. Verzicco

Politecnico di Bari, Dipartimento di Ingegneria Meccanica e Gestionale, Via Re David 200, 70125 Bari, Italy

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The interaction between two shielded monopolar vortices has been investigated experimentally in a nonrotating linearly stratified fluid and by full three-dimensional (3D) numerical simulations. The characteristic Reynolds and Froude numbers in the experiments are approximately $Re \approx 500$ – 1000 and $F \approx 0.2$ – 0.4 . In a first set of numerical simulations the flow is initialized with the experimentally obtained two-dimensional horizontal velocity field, which is measured in the horizontal symmetry plane of the vortices, and the lacking initial data such as the density perturbation and the full 3D vorticity field are provided by the so-called diffusion model [Beckers *et al.*, *J. Fluid Mech.* **433**, 1 (2001)]. A comparison of the experimental and numerical data shows that using the diffusion model to supply the full 3D initial data is reliable for the Reynolds and Froude numbers considered in present experiments. Conjecturing a wider range of applicability, a second set of numerical simulations has been performed using initial conditions for the flow field and the density perturbation entirely based on the diffusion model. These numerical simulations enable an investigation of the role of Reynolds numbers (up to $Re=10\,000$) and Froude numbers ($F=0.30$, 0.65 , and 1.0) which are outside the experimentally accessible range. These simulations provide a better understanding of the dipole formation process, the influence of the vortex shields and the density perturbation in this process, and of the 3D structure of the dipoles. Compactness of the dipole, entrainment of irrotational fluid from the rear, and tail formation behind the dipole have been discussed. Additionally, experimental and numerical results are reported on the interaction between two shielded monopoles of the same sign in a linearly stratified fluid. It is found that for the full range of Reynolds and Froude numbers mentioned above the presence of the vortex shields prevents the merging of these monopoles. © 2002 American Institute of Physics. [DOI: 10.1063/1.1431242]

I. INTRODUCTION

In many previous studies of vortices in a stably stratified fluid it was shown that these structures have a thin pancake-like shape, with a predominantly horizontal velocity distribution. Such vortices are abundant in the final stage of freely decaying turbulence in a stratified fluid, as illustrated in theoretical and numerical studies by, e.g., Riley *et al.*¹ and Lilly.² Many laboratory experiments of this so-called stratified turbulence have been performed in the past, for example, by pulling a grid through a stratified fluid (for an overview see, e.g., the review by Hopfinger³). Also the wake behind a bluff body, moving through a stratified environment, has attracted attention.^{4,5} In such a wake the vortices, arising at different levels in the fluid, usually have alternating positive and negative vertical vorticity distributions.⁶ It is still puzzling how these vortices interact and how vortex lines (lines following the local vorticity vector in the fluid) connect these many different vortices. For a single monopolar vortex in a stratified fluid the situation seems somewhat simpler. Recently, it was shown by Trieling and van Heijst⁷ and by Beckers *et al.*⁸ that such a single pancake-like vortex in a stratified fluid is always accompanied by a region of

oppositely signed vorticity. The appearance of this particular vorticity distribution is due to the fact that vortex lines need to make closed loops, because vortex lines can neither end at the free surface (no flow assumed near the surface), nor at the nonmoving no-slip tank bottom. A single vortex is therefore accompanied by a region of oppositely signed vertical vorticity, and in an (axisymmetric) monopole this region of oppositely signed vorticity has the shape of a circular ring around the core of the vortex. In this paper the interactions between two such shielded monopolar vortices and the resulting end products are discussed in more detail, and it will be shown that the region of oppositely signed vorticity plays an important role during the interaction process.

Freely evolving stratified turbulence has many similarities with purely two-dimensional (2D) turbulence, in which initially small structures continuously interact and combine into increasingly larger structures. Basically, two kinds of vortex interactions can be distinguished: either between two vortices with vorticity of the same sign or between vortices of opposite sign. In purely 2D flows such interactions usually take place between unshielded vortices. Oppositely signed vortices can form pairs which tend to travel through the do-

main, as both vortices induce a velocity in one another. An interaction between oppositely signed shielded vortices is a special case and only a few studies are reported in the literature.^{9–11} Merging, i.e., the coalescence of two monopoles of the same sign, has been described extensively in the literature.^{12,13} This process is vital for the formation of increasingly larger structures in the process of self-organization observed for evolving 2D turbulence in periodic or bounded domains.^{14–16} In general the interacting vortices will be single-signed, so without a region of oppositely signed vorticity. On the contrary, the influence of a shielding on 2D vortex merger was investigated by Carton.¹⁷

In the next section a short overview is presented of the experimental setup and the numerical method. Previous work regarding shielded vortices in a stratified fluid is briefly reviewed in Sec. III. In Sec. IV laboratory experiments and numerical simulations of interactions between two oppositely signed shielded monopoles are described. Section V describes similar interactions between two equally signed monopoles. A general numerical study of interacting oppositely or equally signed vortices (with idealized initial conditions) is described in Sec. VI, including an analysis of the 3D structure of a dipole in a stratified fluid. Finally, the results of this paper are summarized in Sec. VII.

II. EXPERIMENTAL AND NUMERICAL SETUP

The laboratory experiments were carried out in a linearly stratified fluid. A linear stratification was established by applying the two-tank method.¹⁸ Vertical density measurements of the stratified fluid were performed by use of conductivity probes,¹⁹ and vortices were created by applying the tangential injection method introduced by Flór and van Heijst.²⁰ fluid with matched density is injected horizontally along the inner wall of a bottomless, thin-walled cylinder, which is positioned at a certain level in the stratification. An amount of fluid ΔV is injected during a period Δt at an injection rate defined by $Q = \Delta V / \Delta t$. The injection results in a circular flow inside the cylinder, and a monopolar vortex is formed after the cylinder is carefully removed. During this removal care is taken that the cylinder is moved vertically in order to prevent the just created vortex from becoming vertically sheared by motion induced at different levels. The removal should also be very slow, in order to prevent the generation of internal waves. To avoid these disturbing effects, the elevation speed should be no larger than approximately 1 cm s^{-1} .

To obtain quantitative information of the velocity field of the vortex, small polystyrene particles (of about 1 mm in diameter and with a density of approximately 1.04 g cm^{-3}) are added to the stratification and float at their neutrally buoyant level in the fluid. These tracer particles are illuminated from the side by a light sheet, produced by slide projectors. The thickness of this light sheet is typically of the order of 5 mm . Above the tank a video camera is mounted, that records the particle motions on video tape. After the experiment the tape can be processed with a particle tracking algorithm, DigImage,²¹ to determine the horizontal velocity field of the fluid at the level that has been illuminated by the

light sheet. From the 2D velocity distribution the vertical vorticity component ω_z can be calculated at the symmetry plane of the flow ($z=0$). In all experiments the time $\tau=0$ is defined as the moment when the fluid injection is stopped.

Numerical simulations have been performed with a finite-differences scheme of the time dependent incompressible 3D Navier–Stokes equations developed by Verzicco and Orlandi.^{22,23} This code has been adapted to enable the simulation of flows in a linearly stratified fluid. The equations of motion for the incompressible flow in a linearly stratified fluid are first written in the Boussinesq approximation. The fluid density and pressure are therefore decomposed in a part with a constant density, a part that represents a linear density profile and a perturbation, yielding

$$p(\mathbf{x}, t) = p_0(z) + \bar{p}(z) + \tilde{p}(\mathbf{x}, t), \quad (1)$$

$$\rho(\mathbf{x}, t) = \rho_0 + \bar{\rho}(z) + \tilde{\rho}(\mathbf{x}, t). \quad (2)$$

Here ρ_0 is the constant reference density, $p_0(z)$ the corresponding hydrostatic pressure distribution, and $\mathbf{x}=(x, y, z)$ the position in the fluid. The variables $\bar{p}(z)$ and $\bar{\rho}(z)$ thus represent the pressure and density differences between a linearly stratified fluid and the homogeneous fluid with density ρ_0 . The perturbations on the linear density profile are denoted by $\tilde{p}$ and $\tilde{\rho}$. The buoyancy frequency N is defined by $N^2 = -(g/\rho_0)(d\bar{\rho}/dz)$.

All the equations are nondimensionalized by a typical length scale L , a velocity scale V , and a time scale L/V . The density perturbation $\tilde{\rho}$ (with respect to the linear density profile), is scaled by the density difference: $\Delta\rho = N^2 L \rho_0 / g$. The perturbation pressure $\tilde{p}$ is scaled by $\rho_0 V^2$. This yields five nondimensional equations, for the five variables: $\mathbf{v}$, $\tilde{p}$, and $\tilde{\rho}$,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \tilde{p} - \frac{1}{F^2} \tilde{\rho} \mathbf{e}_z + \frac{1}{\text{Re}} \nabla^2 \mathbf{v}, \quad (3)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (4)$$

$$\frac{\partial \tilde{\rho}}{\partial t} + (\mathbf{v} \cdot \nabla) \tilde{\rho} - w = \frac{1}{\text{ScRe}} \nabla^2 \tilde{\rho}. \quad (5)$$

Three nondimensional numbers can be identified here: the Froude number F , the Reynolds number Re , and the Schmidt number Sc , which are defined as

$$F = \frac{V}{LN}, \quad \text{Re} = \frac{VL}{\nu}, \quad \text{Sc} = \frac{\nu}{\kappa}, \quad (6)$$

where ν is the kinematic viscosity of water and κ the diffusivity of salt in water. A realistic value of the Schmidt number for salt stratified water equals $\text{Sc} \approx 700$. Such a high Schmidt number requires a very fine spatial resolution in order to solve Eq. (5) accurately. However, in a previous study we have shown that for the particular case of the decay of a vortex in a linearly stratified fluid a Schmidt number of $\text{Sc} = 10$ is sufficient to include all essential flow phenomena in the numerical simulations, see Beckers *et al.*,⁸ and the present simulations also rely on that observation.

FIG. 1. (a) Graph showing a (scaled) cross section of the vertical vorticity at $z=0$ for a shielded monopolar vortex in a laboratory experiment. (b) Measured vertical density profile through the center of a similar vortex. Above the vortex symmetry plane it can be seen that isopycnals have been deflected downwards ($\tilde{\rho} < 0$), whereas below the vortex symmetry plane the isopycnals are deflected upwards ($\tilde{\rho} > 0$). Both graphs are taken from Beckers *et al.* (Ref. 8).

Equations (3)–(5) are solved numerically, subject to a specific initial condition and with the appropriate boundary conditions. The initial state of the flow is defined by the three velocity components (u, v, w), and by the initial density distribution, prescribed in terms of the density perturbation $\tilde{\rho}(x, y, z)$. The flow is simulated with lateral stress-free walls, thus assuming that vortex-wall interactions are negligible. In the z direction periodic boundary conditions are used, which means that deviations of the linear density profile at the free surface and at the bottom of the tank are not taken into account. This actually means that the no-flux condition of salt at the bottom and the surface of the tank has been violated. It is assumed that this simplification of the boundary conditions in the vertical direction has no consequences for the flow evolution: the vortices in the numerical simulations will always be located at a sufficient distance from these upper and lower boundaries, as will also be the case for the vortices in the laboratory experiments.

III. VORTICES IN A STRATIFIED FLUID

Vortex structures in a stably stratified fluid have a disc-like shape.^{4,8,24} The fluid velocity in these vortices is mainly horizontal, concentrated in a thin layer of fluid. Above and below the symmetry plane the horizontal fluid velocity rapidly diminishes and the vertical variation of the velocity and vorticity field can then be described by a Gaussian function of the distance to the symmetry plane. These vortices are therefore usually called quasi two-dimensional and they show a strong decay due to vertical diffusion. The fluid motion is then smeared out in the vertical direction, the vortex thickens and the strength of the vortex in the horizontal symmetry plane decreases in time. Two other important features of a vortex in a stratified fluid are shown in Fig. 1, taken from Beckers *et al.*⁸ The first graph shows a cross section of the azimuthally averaged vertical component of the vorticity, ω_z , through the center of a circular symmetric vortex at its horizontal symmetry plane $z=0$. The vortex has a core of one-sign vorticity, surrounded by a ring of oppositely signed vorticity. The other graph, Fig. 1(b), shows a typical vertical density profile, measured by lowering a conductivity probe through the center of a vortex, just after the vortex was generated. Clearly the initial linear profile has been perturbed by the presence of the vortex. This perturbation can be identified as planes of isodensity being deflected towards the horizontal

symmetry plane of the vortex (downwards in the upper part and upwards in the lower part of the vortex).

In Beckers *et al.*⁸ a model describing the decay of such a single axisymmetrical vortex in a stratified fluid is derived, the so-called diffusion model. The model assumes a certain initial 3D structure of the velocity distribution inside the vortex and it describes how this vortex decays in time due to diffusion. To obtain a solution from this model the following initial velocity distribution is proposed:

$$v_\theta(r, z, 0) = \frac{1}{\Lambda \sqrt{2\pi}} e^{-1/2(z/\Lambda)^2} \times \frac{1}{2} r e^{-r^\alpha}, \quad (7)$$

together with $v_r = 0$ and $v_z = 0$. The initial velocity distribution $v_\theta(r, z, 0)$ consists of two parts, a Gaussian distribution in the z direction and a radial velocity distribution of a monopole with a shielded vorticity distribution as introduced by Carton *et al.*²⁵ The variable Λ defines the initial thickness of the vortex and α is the so-called steepness parameter, that determines the ratio between the vorticity values in the core and in the ring of opposite vorticity. The larger the value of α is, the deeper is the ring of opposite vorticity. This initial condition allows one to calculate the evolution of both $v_\theta(r, z, t)$ and the density perturbation $\tilde{\rho}(r, z, t)$.⁸

The model vortex and its evolution have been compared with vortices in laboratory experiments and a very close agreement was found for the range of Reynolds and Froude numbers characteristic for these experiments.⁸ This vortex model will therefore be used to show the basic features of the interaction between two shielded vortices. For present experiments and numerical simulations it suffices to confine ourselves to stating the two relations (for the maximum vorticity and maximum density perturbation) that illustrate the decay behavior of the vortex:

$$\omega_{z,\max}(t) = \frac{1}{\sqrt{\pi} \left(2\Lambda^2 + \frac{4}{\text{Re}} t \right)^{1/2} \left(1 + \frac{4}{\text{Re}} t \right)^2}, \quad (8)$$

$$\tilde{\rho}_{\max}(t) = \frac{F^2}{8\pi \sqrt{e} \left(2\Lambda^2 + \frac{4}{\text{Re}} t \right)^{3/2} \left(1 + \frac{4}{\text{Re}} t \right)^3}. \quad (9)$$

Applying the diffusion model assumes stability of the vortex and quasi-two-dimensionality of the flow. In Beckers

*et al.*⁸ it has been shown that certain combinations of F and Λ will result in an unstable density distribution. From the stability diagram shown in Fig. 7 of Ref. 8 it can be conjectured that for $\Lambda=0.3$ (a characteristic value measured in experiments) the vortex is baroclinically stable for $F \leq 1.1$. For two-dimensional inviscid flows the isolated vortices of the type discussed in this paper are azimuthally unstable for $\alpha > 1.85$.^{17,26} The role of the Reynolds and Froude numbers on the azimuthal instability has been investigated²⁷ and for the range of Re and F used in present (numerical) experiments the steepness parameter should be large ($\alpha \approx 4$) in order to develop azimuthal instabilities (see also Sec. VIC). A more detailed account on these issues can be found in Refs. 8 and 27. The role of the horizontal vorticity and vertical velocities is relatively unimportant for present range of Re , F , and Λ , as is shown in Ref. 8 where a detailed study on the quasi-two-dimensionality has been reported. Especially the choice of the initial density perturbation appears essential in order to reduce the amplitude of internal waves.

In experiments performed by Couder and Basdevant¹⁰ it was found that vortex couples could be formed by the interaction between two single monopoles. However, some dipoles were more compact and moved faster than others. It was found that in order to form compact vortex dipoles from two oppositely signed single monopoles a mechanism should be present that brings the vortices together. One of these mechanisms is the presence of a vorticity shielding around the two monopoles. Couder and Basdevant used such an initial vorticity distribution in a 2D numerical simulation. This simulation illustrated that during the interaction the shieldings are removed and due to this change in the vorticity distribution the two vortex cores are pushed together. As a result, the two cores formed a compact vortex dipole, that started to move by its self-propelling mechanism, whereas the two shields formed a weaker dipole that started to move in the opposite direction. Couder and Basdevant also showed that the interaction between two unshielded monopoles results in a dipole as well, but the distance between the two vorticity extrema then remains unchanged, because a mechanism for pushing the two vortices closer together is lacking.

Similar interactions between two oppositely signed vortices were studied in more detail by Schmidt *et al.*¹¹ The

interaction process as described for a purely 2D flow by Couder and Basdevant is also observed in the experiment reported by Schmidt *et al.*: the shields of opposite vorticity are shed off by the cores, which undergo a clear change in shape until they have formed a compact dipole. The shields eventually pair as well and form a much weaker dipole that propagates in the opposite direction. Mainly due to vertical diffusion the strength of all vortices decreases rapidly in time. Specifically this effect makes it difficult to compare laboratory experiments of vortices in a stratified fluid with purely 2D numerical simulations. Therefore, in this investigation we will use full 3D simulations to compare the interaction and the subsequent dipole formation process with our laboratory experiments. However, to proceed we first need to match the numerical simulation correctly with the results of a laboratory experiment, a procedure which is explained in detail in Sec. IV.

IV. NUMERICAL MODELING OF THE INTERACTION BETWEEN TWO MONOPOLES

Fully 3D numerical simulations of monopole interactions require the derivation of the appropriate initial velocity conditions from the experiments, the typical velocity and length scales V and L , the buoyancy frequency N (or alternatively, when V , L , and N are known, the Reynolds and Froude number of the flow), an estimated vortex thickness, and the steepness parameter α . These parameters can be extracted from the experimental data. Moreover, the vertical distribution of the initial vorticity field and a suitable initial density perturbation, which are not easily measured experimentally, are provided by the diffusion model. The validity of the latter assumption will be tested by performing numerical simulations with an experimentally determined initial horizontal velocity field, whereas the diffusion model provides the lacking initial data such as the vertical distribution of vorticity and the density distribution. An analysis of the results of the numerical simulations of the monopole interaction and subsequent dipole evolution for different Reynolds and Froude numbers, with initial velocity and density distributions fully determined by the diffusion model, is deferred to Sec. VI.

FIG. 2. Illustrations of the steps in the procedure used to match a numerical simulation with a laboratory experiment. (a) Measured velocity field for $z=0$. (b) Contours of the vertical vorticity ω_z at $z=0$ (only the first ten contours have been plotted). (c) Cross section of $\omega_z(y)$ through the vorticity extrema. The solid line represents the vorticity distribution associated with (10), but now for two monopoles with $\alpha=2.0$, and the dashed line with $\alpha=2.5$.

In Fig. 2(a) the velocity field at $z=0$ is shown from an early stage of an experiment on the interaction between two monopoles. The velocity distribution can easily be interpolated on a square grid of 65×65 grid points and on such a grid the distribution of the vertical vorticity can be calculated by finite differences. The vorticity distribution associated with the velocity field of the two monopoles is shown in Fig. 2(b), and indeed the vortices have a core of single-signed vorticity surrounded by a ring of oppositely signed vorticity. As a first step to obtain a nondimensional and 3D initial velocity field for the numerical simulation, the experimentally obtained 2D velocity field needs to be scaled by the typical velocity scale V and coordinates need to be scaled by L . The Reynolds number of the initial flow field can then be determined by $Re = VL/\nu$. Values for V and L that characterize the initial flow field can be found by considering the nondimensional velocity distribution, that has been used in the model for a single shielded vortex [Eq. (7)], with $\alpha=2$ for the steepness parameter. (By choosing $\alpha=2$ we already anticipate on the experimentally determined best estimate for α .) The associated (nondimensional) vertical vorticity distribution is given by

$$\omega_z(r, z) = \frac{1}{r} \frac{\partial(rv_\theta)}{\partial r} = \frac{1}{\Lambda \sqrt{2\pi}} (1-r^2) e^{-1/2(z/\Lambda)^2} e^{-r^2}. \quad (10)$$

Since the velocity field v_θ is made dimensionless by V and the coordinate r by L , the dimensional value of the vorticity is given by $V/L\omega_z$. One can also conclude from (10) that the vorticity changes sign for $r=1$, which means that L is the measured radius of the vortex. Furthermore, it can be seen that the maximum vorticity value is found at the vortex symmetry plane $z=0$ in $r=0$. Therefore, the velocity scale can be defined as $V = \sqrt{2\pi}\Lambda L\omega_{\max}$. The variable $\Lambda = \lambda/L$ is an estimate of the ratio between the initial thickness of the vortex and its radius and $\omega_{\max}$ is the measured extremum vorticity value (of either the positive or negative vortex). The thickness of the vortices in the present experiment could not be measured very well, but it is assumed to be of the same order as was found for a monopole created under similar conditions (see Ref. 8): $\Lambda = \lambda/L = 0.30 \pm 0.04$. The velocity distribution in Fig. 2(a) represents two equally strong (but oppositely signed) vortices with $|\omega_{\max}| = 0.84 \pm 0.04 \text{ s}^{-1}$ and a radius $L = 3.0 \pm 0.5 \text{ cm}$, yielding $V = 2.1 \pm 0.4 \text{ cm s}^{-1}$. The 2D velocity field is multiplied by a Gaussian distribution $(1/\Lambda \sqrt{2\pi}) e^{-1/2(z/\Lambda)^2}$ to expand the flow field in the vertical direction.

The choice for the value of the steepness parameter α is based on the measured vorticity profiles. In Fig. 2(c) a cross section of the nondimensional vorticity distribution through the two vortices is shown [where the y coordinate is scaled by L and where the nondimensional extremum vorticity value is $(1/\Lambda \sqrt{2\pi}) \approx 1.2$]. The lines that are drawn in this figure represent the vorticity distributions for two systems of two shielded monopoles, where for each the velocity profile is given by (7); the continuous line represents the case for two monopoles with $\alpha=2.0$, the dashed line represents a similar one, but now with $\alpha=2.5$. Apparently, in this experi-

ment the steepness parameter α has for both vortices a value close to $\alpha=2.0$, and hence this value has been used in the simulations. The relative distance between the core centers is approximately $d/L=3$. The density profile is measured just before the experiment took place. This profile can be used to calculate the buoyancy frequency of the fluid at the level where the vortices are formed; it yields $N=1.9 \text{ rad s}^{-1}$. Combining all these parameters gives values for the Reynolds and Froude number of the initial flow in the numerical simulation: $Re=630$ and $F=0.4$ (note that the error margin of these numbers is approximately 25%). Finally, also the dimensionless time needs to be defined by $t=(\tau-45) \times (V/L)$, thus correcting for the initial velocity field being obtained for $\tau=45 \text{ s}$.

The numerical simulation requires also the initial distribution for the density perturbation $\bar{\rho}(r, z)$. However, during the laboratory experiments it was not possible to make detailed 3D measurements of the density distribution, only a density profile could be taken before the experiment. It was therefore decided to use an analytical expression for the density distribution as initial condition. In Beckers *et al.*⁸ it was shown how for a single monopolar vortex the velocity and the associated density distribution could be constructed, given values for the typical velocity V , vortex radius L , its thickness Λ , the steepness parameter α , and the buoyancy frequency N . For the present experiment all these values have been determined as well as the positions of the vortices. Assuming that both shielded monopoles are circularly symmetric, an approximate density distribution corresponding to the experimentally obtained velocity field can be calculated by simply adding the distributions of $\bar{\rho}$ for the two separate monopoles. Above the symmetry plane of the vortices the isopycnals are deflected downwards ($\bar{\rho}<0$) and below the symmetry plane of the vortices isopycnals are deflected upwards ($\bar{\rho}>0$).

In Fig. 3 the results of both the laboratory experiment and the numerical simulation are shown, using the same intervals for the vorticity contours. Note that the initial vorticity distribution at $z=0$ is taken from the experiment and has been extended to $z \neq 0$ by applying the diffusion model. During the evolution of the vortices, some small errors in the measured velocity distribution may cause spurious vorticity contributions, giving the measured vorticity field a much more irregular appearance than the numerical one. However, apart from these irregularities the experimental and numerical results display an excellent agreement throughout the course of the dipole formation. The asymmetry of the resulting dipole from the simulation is due to the slightly asymmetric initial vorticity distribution at $z=0$. Figure 4 shows contour plots of the density perturbation $\bar{\rho}$ at $z=-0.16$, for which $\bar{\rho}$ initially had its maximum value. Although the initial density perturbation has been modelled, it appears to be a reasonable approximation, because no large adjustments take place after the start of the simulation such as the generation of internal waves. (For an illustration of the importance of the application of an initial distribution of the density perturbation for simulation of vortex flows in a stratified fluid, the reader is referred to Ref. 8.) During the dipole formation the positions of the maxima of the density perturbation move

FIG. 3. Comparison between the vorticity distributions at $z=0$ for the laboratory experiment (a)–(c) and for the 3D numerical simulation (d)–(f). The actual time (in seconds) is given by $\tau \approx (1.4t + 45)$ s. The contour interval is $\Delta\omega_z = 0.04$. The values for the parameters of the simulation are $Re = 630$, $F = 0.4$, $Sc = 10$, and $\Lambda = 0.3$. The injection rate in the experiment was $Q = 2.2 \text{ cm}^3 \text{ s}^{-1}$, the injected volume per vortex was $\Delta V = 50 \text{ cm}^3$ and $N = 1.9 \text{ rad s}^{-1}$.

towards each other, which is consistent with the formation of a compact dipole. The shape of the density perturbation for the dipole is not dramatically different from the initial condition, but while the dipole decreases in strength, the density perturbation also decays rapidly.

Figure 5(a) shows how the extremum values of the vorticity of the dipole halves decrease in time for the laboratory experiment (represented by the markers), the numerical simulation (the solid line) and for the diffusion model for one single axisymmetric vortex, Eq. (8), with $Re=630$ (dotted line, which is nearly indistinguishable from the solid line). There is a close agreement, not only between the decay of the flow in the experiment and in the simulation, but also between the decay of a single monopole (described by the diffusion model) and that for each of the two vortices that form the dipole. Apparently, under the present flow conditions the diffusion model can perfectly be applied to describe the decay of $\omega_{z_{\max}}(t)$ and $\bar{\rho}_{\max}(t)$ for two interacting monopoles that eventually form the two halves of a dipole as well. In Fig. 5(b) the extremum values of the density perturbation during the simulation are given for both halves of the dipole and compared with the decay according to the diffusion model, see Eq. (9). This figure also indicates that no strong

adjustment process takes place initially, implying that the initial density perturbation indeed appears to be a reasonable approximation for the perturbation of the density distribution in the laboratory experiment.

V. INTERACTION BETWEEN TWO EQUALLY SIGNED SHIELDED MONOPOLES

The second type of vortex interactions that has been studied is that between two equally signed shielded monopoles. Figure 6 shows the results of a dye visualization experiment, performed under the same conditions (N , Q , and ΔV) as the dipole formation in Figs. 3(a)–3(c). The photographs illustrate that the vortices here also transform into two dipoles that start to move away from each other. The dipoles are formed, because apparently oppositely signed vorticity in the shieldings becomes concentrated in two patches on either side of the vortex cores, see Figs. 6(b) and 6(c). Then these vorticity patches roll up into two vortices [Fig. 6(d)] that pair with the initial cores forming two dipoles [Figs. 6(e) and 6(f)]. These dipoles are asymmetric, because they are each composed of a (strong) core vortex and a weaker vorticity blob originating from the shielding. This interaction scenario

FIG. 4. Contour plots of $\bar{\rho}$ at $z = -0.16$ obtained from the numerical simulation for the same times as shown in Fig. 3. The increments in the contours are $\Delta\bar{\rho} = 0.001$.

FIG. 5. (a) Graph showing the decay of the extremum vorticity values of the dipole in the laboratory experiment (circles for the positive vortex core, triangles for the negative core) and the numerical simulation (solid line). The dashed line represents the decay of a monopole under the same conditions ($Re=630$ and $\Lambda=0.3$), according to the diffusion model (8). (b) Decay of the maximum values of the density perturbation in the positive vortex (continuous line) and the negative one (dotted line) obtained from the numerical simulation. The dashed line represents the decay according to the diffusion model (9) for $F=0.4$, $\Lambda=0.3$, and $Re=630$.

is remarkable, because it is entirely opposite to the case of two nonshielded monopoles. It has been shown that, provided the separation distance between the vortices is smaller than a certain critical value, such interactions result in the merger of the two vortices.^{12,13} The presence of a vorticity shielding around the two interacting monopoles appears to inhibit such a merger.

This repulsive interaction of two shielded monopoles was first discussed in a numerical study on the merger of shielded vortices with piecewise-constant vorticity by Carton.¹⁷ In that paper it was observed how the two vortices first merged, formed an intermediate tripolar structure and then broke up into two dipoles. Later Valcke and Verron²⁸ numerically studied the interaction between two shielded baroclinic vortices. Although in general these authors considered so-called two-layer baroclinic vortices, they also presented two cases of interactions between shielded barotropic (i.e., 2D) vortices. These simulations showed the same kind

of interaction as the laboratory experiment in Fig. 6. It was concluded that the merging of shielded vortices is very unlikely. The present results thus show that merging of shielded vortices is also unlikely in linearly stratified fluids. These observations indicate that vortex merging, which represents a crude and phenomenological picture of the inverse energy cascade, in quasi-2D turbulence in stratified fluids is a more complex phenomenon than suggested by simulations of freely evolving 2D turbulence.¹⁴

We have performed several experiments with different configurations of the cylinders, and all interactions indeed resulted in a separation of the two vortex cores, as shown in Fig. 6. Figure 7 shows the evolution of the vorticity distribution at $z=0$ for one of the experiments (a)–(c) and for a corresponding 3D numerical simulation (d)–(f). The initialization procedure of the numerical simulation was similar to the case described in the preceding section: the initial flow

FIG. 6. Photographs of a dye visualization experiment of the interaction between two equally signed shielded monopoles (with counter clockwise flow in both cores). The injection rate was $Q=2.2 \text{ cm}^3 \text{ s}^{-1}$ and the injected volume per vortex was $\Delta V=50 \text{ cm}^3$ and $N=1.9 \text{ rad s}^{-1}$.

FIG. 7. Comparison between the vorticity distributions at $z=0$ for the laboratory experiment (a)–(c) and for the 3D numerical simulation (d)–(f) of the interaction between two equally signed shielded monopoles. The actual time (in seconds) is given by $\tau \approx (1.8t + 45)$ s. The contour interval is $\Delta\omega_z = 0.04$. The values for the parameters of the simulation are in this case $Re=500$, $F=0.3$, $Sc=10$, and $\Lambda=0.3$. The injection rate in the experiment was $Q=2.2 \text{ cm}^3 \text{ s}^{-1}$, the injected volume per vortex was $\Delta V = 50 \text{ cm}^3$ and $N=1.9 \text{ rad s}^{-1}$.

field consists of a combination of the experimentally obtained vorticity field at the symmetry plane $z=0$ with the density perturbation and the 3D vorticity field obtained from the diffusion model. For the present simulation values for the Reynolds and Froude number were found to be $Re=500$ and $F=0.3$ (also in this case the error margin is approximately 25%), based on the experimentally measured flow field at $\tau=45$ s. The experiment indeed shows, see Fig. 7(b), that vorticity from the shields is advected by the cores to two opposite sides of the vortex system. Two asymmetric dipoles are then formed and start to move in opposite directions. As the distance between the two cores increases, a patch of oppositely signed vorticity from the shieldings is advected into the region between the cores. Eventually, the cores become circular again (c).

The vorticity contours in Figs. 7(d)–7(f) from the numerical simulation again show an excellent qualitative agreement with the experiment. For this experiment also a comparison was made between the decay of the vorticity maximum of the two vortex cores in the experiment, in the simulation and according to the diffusion model [as in Fig. 5(a)], and it was found that all three curves compared very well.

VI. NUMERICAL INVESTIGATIONS OF MONOPOLE INTERACTIONS

The interactions between oppositely or equally signed monopoles have been studied in more detail by using numerical simulations similar to those of which results are shown in Figs. 3 and 7, but now for idealized initial conditions from the diffusion model. These simulations facilitate the investigation of the influence of several parameters on the dipole formation process, such as the initial separation distance d/L of the vortices, the Reynolds and Froude number of the flow and the steepness parameter α . Moreover, it enables a study of the 3D structure of the dipole that is

formed by the interaction between two shielded monopoles. The role of the different parameters on the dipole formation process will be illustrated with a few examples. At the end of this section we present an overview of data from a large set of numerical simulations of the interaction between oppositely signed and between equally signed monopoles. These data support the conclusions drawn from some exemplary cases below.

A. The initial separation distance

In the paper by Schmidt *et al.*¹¹ a relationship was derived between the formation time of the dipole and the initial separation distance between the monopoles for the case of a purely 2D flow. The formation time T of the dipole could be defined in two ways: either by the time when the distance between the cores has become a minimum or by the moment when the dipole has attained its largest translation speed. Both definitions appear to yield similar formation times. It appeared that for small separations (i.e., $d/L < 4.8$) the formation time is approximately an exponential function of the relative distance. For $d/L > 4.8$ an exponential function was found as well, but in that case with a much larger time constant. Such a systematic investigation of the formation time is not repeated for the present 3D vortices, because it is impossible to carry out the necessary long-time integrations to observe the dipole formation process for $d/L > 4.0$. The simulations discussed in Sec. VIE (with $2.0 \leq d/L \leq 4.0$, $500 \leq Re \leq 10\,000$, and $0.30 \leq F \leq 1.0$) show qualitatively similar behavior as in Ref. 11: the formation time of the dipole is approximately an exponential function of the initial relative distance of the vortices. Furthermore, it is found that the formation time decreases with increasing Reynolds number (keeping F constant), and increases with increasing Froude number (keeping Re constant).

The present comparison concentrates only on the differences between two numerical simulations. The initial flow

conditions for both simulations are defined by $Re=2000$, $F=0.3$, $\Lambda=0.3$, $Sc=10$, and $\alpha=2$, but two different values for the initial separation distance are used. The results of an interaction between these two monopoles (with an initial separation distance $d/L=2.5$) are illustrated in Fig. 8, where horizontal cross sections of the vorticity distribution ω_z at $z=0$ are shown. The interaction scenario is not very different from the one shown in the laboratory experiment (Fig. 3); the rings of opposite vorticity are shed off from the cores. This brings the cores closer to each other, so that they form a dipole that starts to move in one way, while the rings form a (much weaker) dipole that moves in the opposite direction. Similar pictures can be obtained for various initial core separations. We already mentioned that the time it takes for the dipole to be formed increases with increasing separation distance. Figures 9(a) and 9(b) illustrate this fact with the evolution of the distance Δy between the vortex cores and the x positions of the vortices, respectively, for $d/L=2.5$ and 3.0 . It can be seen that a smaller distance between the monopoles indeed results in a faster dipole formation. Moreover, vertical diffusion causes an overall decay of the vortices so that the longer it takes for the dipole to be formed, the slower will be its translation (once formed). Consequently, the maximum velocity of the dipole is smaller for the case with the larger initial vortex separation. The smallest distance between the two dipole halves, however, is equal to 1.6 ± 0.1 , and in this particular case thus approximately independent of the initial

separation between the two monopoles. (The data summarized in Table II are based on a more accurate measurement of the minimum separation.)

In Figs. 9(c) and 9(d) the evolutions of $\omega_{z_{\max}}$ and $\tilde{\rho}_{\max}$ are plotted. Each of these figures shows four lines representing the decay for (1) a single monopole according to the diffusion model, (2) a single monopole in a 3D numerical simulation (for $Re=2000$, $F=0.3$, and $\Lambda=0.3$), (3) one of the dipole halves formed during the dipole formation process with $d/L=2.5$, and (4) similar as (3), but now for $d/L=3.0$. Two aspects attract the attention: in the first place, the decay of the vorticity extrema for each of the dipole halves appears to be somewhat stronger than for a single monopole, and second, the dipole formation causes a temporal increase (due to the overall decay of the vortex this increase means in fact a slower decay) in the density perturbation. This combination of an enhanced decay of the vorticity and a slower decay of the density perturbation suggests that during the dipole formation potential energy is generated at the cost of kinetic energy of the flow. A possible explanation for this effect is that, when the shields around the cores are removed, the cores are allowed to expand in the lateral direction more easily. Conservation of mass then implies a slight compression of the vortex, and, consequently, an increase of the density perturbation.⁸

FIG. 8. Results of a 3D numerical simulation of the dipole formation. The contour plots represent horizontal sections at $z=0$. The contour increments are $\Delta\omega_z=0.04$. The parameters of the simulations are $Re=2000$, $F=0.3$, $\Lambda=0.3$, $d/L=2.5$, and $\alpha=2$. Note, that the computational domain is twice as large in the x direction as the domain shown here.

FIG. 9. (a) Evolution of the distance between the two vortex cores for two different initial distances (the asterisks and the boxes denote the data for $d/L = 2.5$ and 3.0 , respectively). (b) Positions along the x axis of the vorticity extrema for the same two cases. (c) Decay of the maximum value of the vorticity and (d) of the maximum density perturbation. The meaning of the four lines is explained in the text.

B. The Reynolds and Froude number dependence

To investigate the effect of the initial Reynolds number on the dipole formation process, the results of three simulations with $Re = 500$, 2000 , and 5000 , respectively, are compared in Figs. 10 and 11. The Froude number, the initial vortex thickness and the initial core separation are both kept constant ($F = 0.3$, $\Lambda = 0.3$, and $d/L = 2.5$). In all three cases dipoles were formed and for $t = 100$ the (ω, ψ) -scatter plots of these dipoles are shown in Figs. 10(a)–10(c). Scales of both ω and ψ have been adjusted to obtain approximately same-sized graphs. Note that the streamfunction Ψ is obtained by solving $\omega_z = -\nabla^2 \Psi$. In a frame comoving with the dipole the streamfunction transforms into $\psi = \Psi - Uy$, where U is the velocity of the dipole, moving in the x direction. The dashed lines in Figs. 10(a)–10(c) are drawn to illustrate the (non)linearity of the (ω, ψ) relationship. These scatter plots show that for $Re = 500$ there is an approximately linear relationship between ω and ψ , similar to what could be seen for the laboratory experiments (see Fig. 4 in Ref. 11). By com-

paring the resulting dipoles from the three typical numerical simulations ($Re = 500$, 2000 , and 5000) and from the laboratory experiments, four remarkable features of the dipoles were observed. (1) For a larger Reynolds number the (ω, ψ) relationship appears to consist of two branches that are slightly shifted with respect to each other. The nonlinearity around the origin of the (ω, ψ) -scatter plot suggests that specifically along the symmetry axis of the dipole the vorticity distribution deviates from the distribution according to the Lamb dipole model, for which the (ω, ψ) relationship is linear. (2) The dye visualization experiments shown by Schmidt *et al.*¹¹ indicate that after a compact dipole was formed it starts to grow again by entraining undyed fluid from behind. An example is given in Fig. 11(a). (3) At the same time contour plots of the vorticity distribution, see, e.g., Fig. 11(b), illustrate that (rather on the contrary) a tail-like structure is formed behind the dipole. This tail, however, was only found for the lower Reynolds number flows ($Re \leq 2000$). (These differences could be observed by using the same rela-

FIG. 10. (a)–(c) (ω, ψ) scatter plots of three dipoles generated by the interaction between two shielded monopoles with $d/L = 2.5$ ($F = 0.3$ and $\Lambda = 0.3$). The streamfunction ψ has been corrected for the translation of the dipoles with velocities $U_{(a)} = 0.028$, $U_{(b)} = 0.064$, and $U_{(c)} = 0.080$.

FIG. 11. (a) Dye visualization illustrating the advection of undyed fluid into the dipole. (b) Vorticity contours for the dipoles formed for $Re=2000$ ($\Delta\omega=0.02\omega_{\max}$) and (c) typical cross section of $\omega_z(y)$ through the vorticity extrema for the dipoles formed with $Re=500$ (continuous line) and with $Re=5000$ (dashed line).

tive values for the vorticity contours, like $\Delta\omega_z=0.02\omega_{z_{\max}}$, because the absolute vorticity values are much higher in the vortex with $Re=5000$ compared to the one for $Re=2000$.) (4) Finally, cross sections of the vorticity distribution through the two cores of the dipoles showed that in particular for high Reynolds number dipoles the vorticity distribution between the two cores is not straight; it shows a kink around the dipole's center. This is clearly illustrated in Fig. 11(c) where cross sections of the vorticity distribution inside the dipole are shown for $Re=500$ and 5000 .

The formation process of dipoles and their compactness [and thus the nonlinearity of the (ω, ψ) relationship] are determined by several parameters such as Re , F , d/L , etc. However, only the Reynolds number is varied in the present set of simulations (the variation of the Froude number, while keeping the Reynolds number constant, is discussed later on), and the deviation of these dipoles from Lamb's dipole model can be explained by the effects of viscosity only. It is therefore reasonable to assume that the entrainment of fluid into the dipole and the formation of a tail in the vorticity distribution is due to lateral diffusion of vorticity resulting in an increasing size of the dipole. This can be illustrated with the evolution of the streamline pattern. A plot of $\psi(x, y)$ in Fig. 12(a) shows that two different regions can be recognized: the outside region where fluid moves around the dipole like the flow around a moving cylinder, and the inside region with closed streamlines for each dipole half. The dipole itself is completely surrounded by one closed streamline, called the separatrix. Fluid initially inside this separatrix will remain there and is advected with the dipole, fluid outside the separatrix flows (in a frame comoving with the dipole) around the dipole and towards the right. Due to lateral diffusion the vorticity distribution of the dipole grows in size and so does the region inside the separatrix. The conserva-

tion of mass thus demands that fluid that was initially outside the separatrix is entrained into the dipole. In Fig. 12(b) these processes are schematically illustrated. The dashed regions represent dyed fluid that is initially trapped inside the dipole. At a later stage the separatrix is represented by the dotted circle (2), and undyed fluid originally in the ring (1) is entrained from the rear into the dipole. Figure 12(c) presents a cross section through the dipole with distributions of the vorticity and the (corrected) streamfunction. The position of the separatrix is indicated by the arrows and one can observe that due to viscosity vorticity has leaked through the separatrix, as illustrated in the two circles. This fluid outside the separatrix, in Fig. 12(b) schematically indicated by the region between the circles (2) and (3), will not be transported by the dipole and is, due to the translation of the dipole, eventually left behind and forms a tail-like structure behind the dipole. Finally, it should be emphasized that for a small Reynolds number flow (like the runs with $Re=500$ and 1000) viscosity will smooth out the kink in the vorticity cross section and the (ω, ψ) relationship becomes less nonlinear.

Flór and van Heijst²⁹ have found scatter plots similar to that in Fig. 10 (i.e., linear and nonlinear) for dipoles created by the collapse of a laminar or turbulent jet in a stratification. Turbulent (or pulsed) injections always resulted in dipoles with a nonlinear (ω, ψ) relationship whereas dipoles resulting from laminarly injected fluid possessed a much more linear (ω, ψ) relationship. Note that the Reynolds number for the dipoles from turbulent injections is considerably larger than for the laminar injection case. The authors suggested that the nonlinearity might disappear when the vorticity of the dipole is corrected for the vertical density structure, by using the potential vorticity in the scatter plot. However, more recently 2D numerical simulations have shown that also an initially exact Lamb dipole, i.e., a 2D vorticity structure with a linear

FIG. 12. (a) Streamline pattern of a dipole in a frame comoving with the dipole. (b) Schematic representation of the flow in and around an expanding dipole. See text for more details. (c) Cross section through the dipole with distributions of the vorticity (solid line) and the corrected stream function ψ (dashed line).

FIG. 13. (a) Vorticity contours of the dipole formed by the interaction between two shielded monopoles for $Re=2000$ and $F=1.0$. (b) Cross section of the vorticity (scaled by the maximum value) through the dipole, and (c) (ω, ψ) -scatter plot of the dipole in (a) with $U=0.046$.

(ω, ψ) relationship, eventually results in a dipole with a non-linear (ω, ψ) relationship.^{30,31} This finding supports the idea that the nonlinearity cannot be ascribed to the density structure inside the dipole, but is most likely due to the effects of (lateral) diffusion of the vorticity distribution and consequent entrainment of (nearly) irrotational fluid into the dipole.

The influence of the Froude number on the dipole formation has been investigated by comparing the previous results for $Re=500$ and $Re=2000$ with similar simulations where $F=1.0$ (in all four cases: $\Lambda=0.3$ and $d/L=2.5$). For $Re=500$ no striking differences between the dipole formation processes with the two Froude numbers ($F=0.3$ and $F=1.0$) could yet be observed, but for $Re=2000$ the differences become more profound. As was discussed in Beckers *et al.*,⁸ the effect of stretching of a vortex during its viscous decay becomes stronger for higher Froude number flows, and the dipole formation process appears to be affected by stretching of the vortex cores. In Fig. 13 a vorticity contour plot of the dipole (obtained for $Re=2000$ and $F=1.0$), with the associated vorticity cross section and (ω, ψ) -scatter plot, are shown. Especially the cross section of the vorticity in Fig. 13(b), which clearly shows a kink around the center of the dipole, indicates that the dipole is not a very compact structure, which results in a nonlinear (ω, ψ) relationship, see Fig. 13(c). It was found that the difference between the $F=0.3$ and $F=1.0$ simulation is a result of differences in the deshielding process. Due to stretching of the vortex cores during their decay, which is strongest in the center of the vortex core and decreases rapidly in strength with increasing distance from the vortex core,⁸ the vorticity shields become relatively weak. Therefore, the shields are less effective in bringing the vortex cores closer together and a less compact dipole is formed, with a nonlinear (ω, ψ) relationship (see also Ref. 10).

Summarizing we can conclude that two mechanisms exist that are responsible for the formation of a noncompact dipole (with a kink in the vorticity profile). For increasing Reynolds number, while keeping the Froude number constant, entrainment of nearly irrotational fluid into the dipole becomes more effective in forming less compact dipoles, because diffusion is less effective in smoothing the vorticity profile. For increasing Froude number, and now keeping the Reynolds number constant, kink formation in the vorticity profile is (indirectly) enhanced by vortex stretching. In both cases lateral diffusion of vorticity will oppose kink formation in the vorticity profile. The trend to form noncompact dipoles

for increasing Re and F is also clearly visible in Table II (see Sec. VI E).

C. The steepness parameter α

Thusfar we only considered interactions between monopoles with a steepness parameter $\alpha=2$, because this value appeared to be most relevant for the laboratory experiments. It is, however, an interesting question how the value of α of the interacting shielded monopoles might influence the dipole formation process. A relatively weaker ring of opposite vorticity, compared to the core vorticity, leads to a less compact dipole (see, e.g., the role of F on the compactness of dipoles as discussed in the preceding section). Therefore it seems obvious that monopoles with relatively strong vorticity shieldings (i.e., with higher values of α) will form more compact dipoles. By comparing the interactions between two monopoles with either $\alpha=2$ or $\alpha=4$, for various initial separation distances, it was found that the larger value of α indeed enhances the formation of a compact dipole, but in return the increasing instability of the ring of opposite vorticity can also hamper the dipole formation, depending on the initial value of d/L . This will be illustrated by two examples.

For a relatively small initial distance d/L the rings of opposite vorticity are easily shed off and a dipole is formed. In fact, by comparing the cases with $d/L=3.0$ for $\alpha=2$ and $\alpha=4$ it was found that the interaction between the monopoles with the higher steepness parameter resulted in a much more compact dipole; it even had a linear (ω, ψ) relationship and the velocity of the dipole was much higher than for the one formed by the $\alpha=2$ monopoles. The reason for this difference is easily understood: the rings of opposite vorticity have higher (absolute) vorticity values and therefore they have a much larger pushing effect on the two vortex cores. Consequently, the cores are pushed closer to each other and a more compact (thus faster) dipole is formed. A larger steepness parameter, however, does not always result in the formation of a more compact dipole. When the initial mutual distance between the monopoles becomes larger, the inherent instability of the monopoles with $\alpha>2$ can drastically affect the interaction process. This process can be illustrated for the case of two monopoles with $\alpha=4$ and $d/L=3.6$. Although no artificial perturbation was added to the vorticity distributions of the monopoles, their mutual interaction apparently triggers the instability of both monopoles. The vorticity

FIG. 14. The dipole formation process (same simulation as shown in Fig. 8, i.e., for $d/L=2.5$, $\Lambda=0.3$, $Re=2000$, $F=0.3$, and $Sc=10$) illustrated by isosurfaces of $|\omega_z|=0.04$. The reader views the structure slightly from above the vortex symmetry plane ($z>0$), looking in the negative x and in the positive y direction.

shields split up into two satellite vortices and the cores become elliptical. Then the satellites are seriously deformed as they pass between the two vortex cores. This behavior prevents the cores from forming a dipole.

D. The 3D structure of the dipole

The 3D numerical simulations of the interactions between monopoles, and the subsequent formation of dipoles, offer the possibility to investigate the 3D structure of a dipole in a stratified fluid in more detail. For one specific case (a dipole formed from the interaction between two monopoles with $Re=2000$, $F=0.3$, $\Lambda=0.3$, and $d/L=2.5$) the 3D structure of the flow field is illustrated in Fig. 14. The particular shape of the dipole during its evolution is visualized by drawing isosurfaces of the vertical vorticity. The fluid

TABLE I. Overview of the resolution $N_x \times N_y \times N_z$ ($L_x \times L_y \times L_z = 20 \times 10 \times 4$) of the numerical experiments with oppositely signed vortices, and an assessment of the well-resolvedness of the computed flow. For equally signed vortices the resolution and well-resolvedness of the flow is similar as shown below (e.g., $N_x \times N_y \times N_z = 129 \times 129 \times 65$ with $L_x \times L_y \times L_z = 10 \times 10 \times 4$).

		$N_x \times N_y \times N_z$	Well-resolvedness
Re=500	$F \leq 0.65$,	$65 \times 33 \times 33$	good
	$F = 1.0$,	$129 \times 65 \times 65$	good
Re=1000	$F \leq 1.0$,	$129 \times 65 \times 65$	good
	$F \leq 0.65$,	$129 \times 65 \times 65$	good
Re=2000	$F = 1.0$,	$257 \times 129 \times 65$	good
	$F \leq 0.65$,	$257 \times 129 \times 65$	sufficient
Re=5000	$F = 1.0$,	$257 \times 129 \times 65$	fair
	$F = 0.30$,	$257 \times 129 \times 65$	sufficient
Re=10 000	$F \geq 0.65$,	$257 \times 129 \times 65$	fair

motion in a stratified fluid is predominantly 2D and the distribution of ω_z therefore gives a good representation of 'the shape' of the dipole (or more precisely, the 3D distribution of the Q2D flow field). In Fig. 14 the isosurfaces of $|\omega_z|=0.04$ are plotted for $t=0, 20, 40$ and 60 . This particular isosurface corresponds with the outermost vorticity contour in Fig. 8, so that the reader can easily compare these two figures. One can observe how the vorticity rings are shed off and start to move to the right [Figs. 14(c) and 14(d)], whereas the cores obtain a characteristic boomerang-like shape as the compact dipole is formed and starts to move to the left. For more details of the 3D structure of the dipole we refer to Ref. 27.

E. A regime study of the shielded monopole interaction

Two large sets of numerical simulations have been performed, one concerning the interaction between oppositely signed monopoles and the other between equally signed ones. For both sets we varied the Reynolds number, the Froude number and the initial separation distance: $Re \in \{500, 1000, 2000, 5000, 10\,000\}$, $F \in \{0.30, 0.65, 1.0\}$, and $d/L \in \{2.0, 2.5, 3.0, 3.5, 4.0\}$.

The computations of the interaction of two oppositely signed monopoles were carried out in a box with dimensionless size $20 \times 10 \times 4$ ($L_x \times L_y \times L_z$) with stress-free boundary conditions in the x and y direction and periodic boundary conditions in the z direction. The x coordinate of the initial position of the vortices is $x=5$, and the resulting dipole moves in positive x direction. The influence of the stress-free boundaries is negligible as long as the dipole does not collide with the boundary at $x=20$. The monopoles are sufficiently thin ($\Lambda=0.3$) and located at $z=2$, the bottom and top boundary are thus sufficiently far away. The spatial resolution $N_x \times N_y \times N_z$ for the different runs are summarized in Table I. The time steps used in the simulations are $\Delta t=0.1$ for $Re \leq 2000$ and $\Delta t=0.05$ for $Re \geq 5000$. From Table I it can be concluded that the data obtained from the runs with $Re=5000$, $F=1.0$ and those with $Re=10000$ and $F \geq 0.65$ should be interpreted with care and are therefore indicative (larger Froude numbers also require an increased resolution

TABLE II. Overview of the numerical experiments with two oppositely signed, shielded vortices. CD denotes the formation of a compact dipole and NCD the formation of a noncompact dipole. No dipole is formed for $Re=500$ and $d/L=4.0$. Additionally, the separation distance of the two vortex cores of the dipole, in terms of the initial vortex radius, has been tabulated (the values with a $\dagger$ represent upper limits because the minimum vortex separation is most likely found for $t > t_{\text{end}}$). The estimated error of the separation distance is approximately 0.1 for $Re \leq 2000$ and 0.2 for $Re \geq 5000$.

		$\frac{d}{L}=2.0$	$\frac{d}{L}=2.5$	$\frac{d}{L}=3.0$	$\frac{d}{L}=3.5$	$\frac{d}{L}=4.0$
Re=500	$F=0.30$	CD, 1.3	CD, 1.7	CD, 2.1	CD, 2.5	-, -
	$F=0.65$	CD, 1.5	CD, 1.7	CD, 2.0	CD, 2.6	-, -
	$F=1.0$	CD, 1.5	CD, 1.6	CD, 2.0	CD, 2.5	-, -
Re=1000	$F=0.30$	CD, 1.4	CD, 1.5	CD, 1.7	CD, 1.9	CD, 2.8 †
	$F=0.65$	CD, 1.4	CD, 1.6	CD, 1.7	CD, 1.9	CD, 2.8 †
	$F=1.0$	CD, 1.6	CD, 1.6	CD, 1.8	CD, 2.0	CD, 2.8 †
Re=2000	$F=0.30$	CD, 1.4	CD, 1.5	CD, 1.7	CD, 1.7	CD, 2.2
	$F=0.65$	CD, 1.4	NCD, 1.6	NCD, 1.8	CD, 1.8	CD, 2.2 †
	$F=1.0$	NCD, 1.5	NCD, 1.6	NCD, 1.7	NCD, 1.9	CD, 2.4 †
Re=5000	$F=0.30$	CD, 1.4	NCD, 1.6	NCD, 1.7	NCD, 1.8	CD, 1.8
	$F=0.65$	NCD, 1.6	NCD, 1.6	NCD, 1.6	NCD, 1.9	NCD, 2.1
	$F=1.0$	NCD, 1.6	NCD, 1.7	NCD, 1.9	NCD, 2.0	NCD, 2.3
Re=10 000	$F=0.30$	NCD, 1.5	NCD, 1.7	NCD, 1.7	NCD, 1.8	NCD, 1.9
	$F=0.65$	NCD, 1.6	NCD, 1.8	NCD, 1.8	NCD, 2.0	NCD, 2.4
	$F=1.0$	NCD, 1.8	NCD, 1.8	NCD, 1.8	NCD, 2.1	NCD, 2.6

which was impossible with the available computer resources). The length of the time integration depends on the initial vortex separation $t_{\text{end}}=200$ for $d/L=2.0$ and 2.5 , $t_{\text{end}}=300$ for $d/L=3.0$ and 3.5 , and $t_{\text{end}}=400$ for $d/L=4.0$.

The computations of the interaction of two equally signed monopoles were carried out in a box with size $10 \times 10 \times 4$ with a similar resolution as indicated in Table I. The length of the time integration was for all runs $t_{\text{end}}=100$, and the interaction with the mirror images in neighboring boxes (note that stress-free boundaries are used) remained small. For both sets of simulations the steepness parameter was kept constant at the value $\alpha=2$. This value is rather close to the experimentally found value. Higher values of α could be of interest due to enhanced instability of the ring of opposite vorticity (see Sec. VIC) but are not considered in these two sets of simulations.

We first consider the interaction of two shielded monopoles with oppositely signed core vorticity. For the characterization of the resulting dipole (if any) we make a distinction between the appearance of a compact and a noncompact dipole. The decision to call the resulting dipole either compact or noncompact is always somewhat arbitrary. In our comparison it is based on the vorticity cross section through the centers of the dipole, like those shown in Figs. 11(c) and 13(b), at the moment of minimum separation of the vortex cores. The results of the simulations are summarized in Table II, where we have also indicated the minimum core separation of the resulting dipoles. The trend for increasing Reynolds and Froude numbers is clear: all runs with $Re=500$ and 1000 show the formation of compact dipoles except for $Re=500$ and $d/L=4.0$ (the vortex cores are almost completely disappeared by diffusion of vorticity before they come sufficiently close together), and the simulations with $Re=5000$ and $10\,000$ show that nearly all resulting dipoles

are noncompact. The data for $Re=2000$ show the importance of the Froude number on the formation of compact dipoles: low Froude numbers yield compact dipoles and for $F \approx 1$ mainly noncompact dipoles are formed.

Finally, the fair resolution of the runs with $Re=5000$ and $F=1.0$ and those with $Re=10\,000$ and $F \geq 0.65$, resulting in small perturbations of the ideal vorticity and density fields, is most likely responsible for the formation of asymmetric dipoles, similar as found in the experiments (see Figs. 3 and 7). Also an increased dissipation of kinetic energy of the coherent structures is observed which might be associated with the presence of internal waves (especially for $F=1.0$) and the transition to turbulence. The data from these runs should, however, be considered with some care. Although we believe that well-resolved simulations will also show the formation of noncompact dipoles, the measured minimum core separation could change and the loss of kinetic energy could be less. Additionally, it is expected that the asymmetry of the resulting dipoles will be reduced.

The second set of simulations concerns the interaction of two shielded monopoles with equally signed core vorticity. A characterization of the resulting vortices is provided in Table III where we make a distinction between the formation of strong asymmetric dipoles (AD), weak asymmetric dipoles (WD) and no formation of dipoles but only the gradual destruction of the ring of opposite vorticity (DR). An illustration of the three different cases is shown in Fig. 15 for runs with $Re=1000$ and $F=0.65$. The distinction between the three types of interaction products is somewhat arbitrary, but it is found suitable to show the trends in the interaction scenario.

No merging was observed for the range of Reynolds and Froude numbers used in the numerical experiments, even not for the experiments with an initial separation of the vortex

TABLE III. Overview of the numerical experiments with two equally signed, shielded vortices. AD denotes the formation of a couple of clear asymmetric dipoles which travel away from each other, WD denotes the formation of a couple of weak, nontravelling dipolar structures, and DR denotes ring destruction (see Fig. 15). In the latter case no dipoles are formed. Finally, DT denotes the formation of a weak deformed tripolar structure. Additionally, the change of the separation distance of the two vortex cores, in terms of the initial vortex radius, measured for $t = 100$ has been tabulated (the error is approximately 0.3).

		$\frac{d}{L} = 2.0$	$\frac{d}{L} = 2.5$	$\frac{d}{L} = 3.0$	$\frac{d}{L} = 3.5$	$\frac{d}{L} = 4.0$
Re=500	$F = 0.30$	DT, 1.2	AD, 2.0	WD, 1.5	DR, 0.9	DR, 0.4
	$F = 0.65$	DT, 1.9	AD, 2.0	WD, 1.5	DR, 0.9	DR, 0.4
	$F = 1.0$	AD, 2.3	AD, 2.0	WD, 0.9	DR, 0.6	DR, 0.4
Re=1000	$F = 0.30$	AD, 3.2	AD, 2.8	AD, 1.7	DR, 0.6	DR, 0.4
	$F = 0.65$	AD, 3.3	AD, 2.4	WD, 1.4	DR, 0.6	DR, 0.2
	$F = 1.0$	AD, 2.8	AD, 1.9	WD, 1.2	DR, 0.6	DR, 0.0
Re=2000	$F = 0.30$	AD, 4.1	AD, 2.8	AD, 1.6	DR, 0.9	DR, 0.4
	$F = 0.65$	AD, 3.9	AD, 2.3	AD, 1.4	DR, 0.6	DR, 0.2
	$F = 1.0$	AD, 2.6	AD, 1.6	WD, 1.1	DR, 0.5	DR, 0.1
Re=5000	$F = 0.30$	AD, 4.5	AD, 3.0	AD, 1.8	DR, 0.8	DR, 0.2
	$F = 0.65$	AD, 3.7	AD, 2.1	AD, 1.2	DR, 0.6	DR, 0.2
	$F = 1.0$	AD, 2.4	AD, 1.6	WD, 1.3	DR, 0.6	DR, 0.1
Re=10 000	$F = 0.30$	AD, 4.7	AD, 3.0	AD, 1.6	DR, 0.8	DR, 0.2
	$F = 0.65$	AD, 3.6	AD, 2.1	AD, 1.4	DR, 0.7	DR, 0.2
	$F = 1.0$	AD, 2.6	WD, 1.2	WD, 1.4	DR, 0.7	DR, 0.1

cores as small as $d/L = 2.0$ (although for all Reynolds and Froude numbers considered here, a tendency to merging can be observed during the first stage of the interaction process). All simulations with $d/L \leq 3.0$ showed the formation of two (weak) asymmetric dipoles, which move in opposite direction away from each other. An exception should be made for two runs with $\text{Re} = 500$, $F = 0.30$ and 0.65 , and both with initial separation $d/L = 2.0$, where the resulting dipoles are so weak that they do not move away in opposite directions. A strongly deformed, weak tripolar structure appears as a result of smoothing of the vorticity gradients between the two vortex cores (obviously, this should not be considered as merging). In the case $d/L \geq 3.5$ the ring of opposite vorticity is simply removed by diffusion (the dominant process for $\text{Re} \leq 2000$), or by a combined effect of advection and diffusion for the runs with higher Reynolds numbers. The end result consists of two equally signed vortices with a slightly larger separation distance compared to their initial separation distance. Additionally, the line connecting the vortex core centers is slightly rotated with respect to the y axis ($\approx 10^\circ$ for $d/L = 3.5$ and $\approx 3^\circ$ for $d/L = 4.0$).

From Table III we can conclude that increasing the Reynolds number up to approximately 5000 promotes the formation of relatively strong asymmetric dipoles: the dipole halves are stronger and the part originating from the ring of opposite vorticity survives longer due to decreasing influence of viscosity. As a result the separation distance becomes larger for increasing Reynolds number (especially for $F = 0.30$), and for all Froude numbers it is observed that the rotation of the line connecting the vortex cores with respect to the one based on the initial position of the vortex cores increases. The decreasing importance of viscosity when Re increases also explains the tendency to form rather strong asymmetric dipoles instead of weak dipoles for $d/L = 3.0$. Increasing the Froude number, on the other hand, diminishes

the role of the ring of oppositely signed vorticity due to enhanced stretching of the vortex core. On average weaker dipoles are formed resulting in a smaller separation distance of the vortex cores (see Table III). This effect is accompanied by a larger rotation of the line connecting the vortex core centers, a process intimately linked with the smaller separation distance. For $\text{Re} > 5000$ a tendency to form weaker dipoles is observed again, especially for $F \geq 0.65$. It seems that inherently three-dimensional effects (internal waves, transition to turbulence) enhance dissipation of kinetic energy of the flow resulting in weaker dipoles and weaker core vortices. As a result the separation distance of the vortices decreases for increasing Re and F . A more extensive numerical study is necessary to analyze this part of the parameter range with well-resolved numerical simulations, which is unfortunately not possible yet.

VII. SUMMARY

In this paper the interaction between two shielded monopoles in a stratified fluid has been investigated, both by numerical simulations and by laboratory experiments. Two cases are considered, the interaction between oppositely signed vortices and between equally signed vortices.

In general, interactions between two oppositely signed shielded monopoles result in the formation of two dipoles by the simultaneous pairing between the two core vortices and the two shields. The cores form a (non)compact dipole moving in one direction, whereas the shields form a much weaker dipole moving in the opposite direction. The compact dipole results from the fact that the shields are shed off, while pushing the cores closer together. The characteristics of the compact dipoles are investigated in the laboratory experiments. A close comparison between the results of a laboratory experiment and a 3D numerical simulation is made. The initial

FIG. 15. Illustration of the different interaction scenarios ($Re=1000$ and $F=0.65$): (a), (b) the formation of asymmetric dipoles (AD), (c), (d) the formation of weak dipoles (WD), (e), (f) destruction of the ring of opposite vorticity (DR).

condition for this computation is based on an experimentally obtained flow field at the symmetry plane of the vortex ($z=0$) only. The full initial 3D vorticity field and the density perturbation is based on the diffusion model as proposed by Beekers *et al.*⁸ A very close agreement is found which support our conjecture that fully 3D simulations can be initialized with relatively simple isolated monopolar vortices as described by the diffusion model.

Interactions between equally signed shielded monopoles in a stratified fluid will not result in the merger between the two monopoles, because the strong vorticity shieldings pre-

vent this from happening. The vorticity shields roll up and form two satellite vortices. Each of the satellites then pairs with one of the cores and forms a dipole, and the two dipoles move in opposite directions away from each other. The present experiments confirm previous numerical and theoretical results.

The influence of several parameters on the dipole formation process of two interacting oppositely signed shielded monopoles has been investigated. In general it is found that the stronger the rings of opposite vorticity are and the smaller the initial distance between the monopoles is, the

more compact is the dipole that is formed. The compactness of the dipole is measured by the shape of the (ω, ψ) relationship and the vorticity cross-section through the centers of the dipole. Noncompact dipoles have nonlinear (ω, ψ) relationships and show a kink in the vorticity cross section. It is found that, besides the initial vortex separation and the strength of the rings, the compactness of the dipoles is also influenced by the (initial) Reynolds number and Froude number of the flow. Stretching of the vortex cores during the decay of the vortices is enhanced for increasing Froude number flows, and this process makes the rings of opposite vorticity relatively weaker. Consequently, the rings are less effective in bringing the cores closer together and the resulting dipole is less compact for higher Froude number flows. Furthermore, due to lateral diffusion of momentum, the separatrix of the dipole increases and (almost) irrotational fluid becomes entrained in the dipole along its symmetry axis. This causes a kink in the vorticity cross section and a nonlinearity of the (ω, ψ) relationship. For large Reynolds number flows the kink in the vorticity distribution is rather persistent, and consequently the nonlinearity of the (ω, ψ) relationship remains, in contrast to the low Reynolds number case.

With two sets of numerical simulations, one with opposite and another with equally signed shielded monopoles with varying initial vortex separation ($d/L \in \{2.0, 2.5, 3.0, 3.5, 4.0\}$), a wider range of Reynolds and Froude numbers has been investigated. The simulations generally support our analysis of the role of Re and F on the interaction process as shortly summarized above.

The 3D structure of a vortex dipole in a stratified fluid has been visualized by using the results of a numerical simulation of the interaction between two shielded monopoles. It appeared that the dipole consists of two halves that each have a boomerang-shaped distribution of the vertical vorticity due to the strong difference in translation velocity of the dipole at various levels in the fluid. At the symmetry plane the dipole moves fastest and at levels above or below this plane the structure lags behind.

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