High-power cryocooling

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Chapter 1

Introduction

1.1 Cryogenic cooling

Cryogenics can be described as the branch of physics dealing with the production and effects of very low temperatures [1]. There is no specific definition of 'very low temperatures'. People use the boiling points of nitrogen (77 K), air (79 K), or natural gas (111 K) as the limit. Also the limit of 100 K and lower is often used.

Domestic refrigeration techniques used in household or industrial applications such as condensation-evaporation systems are not considered cryogenic. One could say cryogenic cooling is cooling that requires more advanced techniques to reach and maintain those temperatures. Cryogenic cooling can be realized by using so-called cryocoolers.

Cryocoolers are machines that supply refrigeration with a working gas that goes through a specific thermodynamic cycle. All these cycles use compression and expansion of gasses to transport energy from one state to the other. Compression of a working gas causes it to heat up. This heat is removed. Expansion of the gas causes it to cool down. This reduction of temperature is used for refrigeration.

Compression can be realized by a piston, scroll/screw or even nonmechanical. Expansion can also be done several ways. First of all over a flow restriction, in a so-called Joule-Thomson cooler, where the working fluid is throttled over a flow restriction. This expansion is isenthalpic. In Claude and reverse-Brayton coolers a turbine is used instead of the flow resistance. This way, the efficiency of the cooling cycle is increased because useful work can be extracted from the gas. This class of cryocoolers is called ‘recuperative’, since the downstream fluid is recuperated (or precooled) by the upstream fluid.

The other class of cycles is called ‘regenerative’. In a regenerative cycle, the working fluid is transferred from compression to expansion in a periodical manner. The gas is moving back and forth through the machine. Between compression and expansion, heat is regenerated in a so-called regenerator. The Stirling coolers, pulse-tube refrigerators, and Gifford-McMahon (GM) coolers use a regenerative cycle.

The application of cryogenic cooling is quite wide-spread. One can find applications for basically any sector. The biggest cryogenic installations are used for the separation of air. The main components of air are nitrogen (N\textsubscript{2}) and oxygen (O\textsubscript{2}). This separation of air on such a large scale is done by cryogenic distillation, in which the cold source is a cryocooler. On such large scales Linde or Claude cycles are used. Primary product of these large-scale air separation
plants is oxygen. Oxygen is being used in the (petro-)chemical and steel producing industry, for instance for fueling melting furnaces. The liquid nitrogen that is formed is sold in large quantities and is in most cases a very good alternative for cryocoolers as a source of cold. Apart from the use as coolant, nitrogen is also used for mixing natural gas to its proper caloric value.

Secondly, a very large user group of cryogenics is found in the medical industry. Cryogenics is used directly for storing biological material. Artificial Insemination (AI) centres use liquid nitrogen to store bulls semen. Countries as India have large live-stock breeding programs and hence a large number of AI-centers. Another application is storage of medical gasses. Medical facilities often require large storage capacities of clean air and oxygen. In many situation storing oxygen in liquid form is much easier than storing it in high-pressure cylinders.

Another application of cryogenics in the medical industry is found in Magnetic Resonance Imaging (MRI) equipment. The high magnetic fields required in these machines are generated by superconducting magnets. These magnets operate at liquid-helium temperatures (4K). Often cryocoolers are used to recondense helium, or to provide active shield cooling.

In the military and aerospace sector cryocoolers are used to cool sensors and electronics. Infrared sensors are cooled with cryocoolers to improve the signal-to-noise ratio of the image. These are often miniature coolers with cooling powers of approximately 1 W.

### High-temperature superconductors

A relatively new application for cryocoolers is the cooling of High-Temperature Superconductors (HTS). A superconducting material has zero electric resistance. High-temperature means in the range of 25 K - 138 K. Because of the relatively high transition temperatures, these materials are easy to cool. Liquid nitrogen can be used as coolant, instead of liquid helium.

There are two advantages of superconductors. First of all, because of the zero resistance, there are no losses in transport of electricity. This is beneficial in applications such as cables, and transformers. Secondly, the current density in a superconductor can be much higher than in a normal conductor. As a result, devices with superconductors can be much smaller than their conventional equivalents. Typical applications where this is important are motors, generators, and cables. When space is a factor, superconductors are preferably cooled well below the transition temperature as this increases the maximum current density even more. The same holds for the maximum magnetic field. This can be higher at lower temperatures, which is important for magnets and Superconducting Magnetic Energy Storage (SMES). Typical operating temperatures and cooling-power ranges are indicated in figure 1.1[2].

Superconductors need to be cooled and cooling requires energy. This is sometimes referred to as the cryogenic penalty. The consequence is that superconductivity only appears to be interesting in large applications where the energy required for refrigeration is less than the energy that is saved by using the superconductor.

Cooling systems for superconductors are commercially available. Stirling Cryogenics & Refrigeration BV supplies a range of cooling systems for different applications [2]. The cooling source are currently Stirling cryocoolers. An example of such a system is shown in figure 1.2. This system is used to cool a superconducting cable, using a forced circulation of subcooled liquid nitrogen in a closed cycle.

### 1.2 Stirling and pulse-tube refrigerators

The cryocoolers that are the subject of this thesis are Stirling and pulse-tube refrigerators. Stirling coolers are based in the Stirling cycle, that was developed by the Scottisch reverend Robert Stirling, who patented a hot-air engine in 1816. An important improvement of Stirlings
CHAPTER 1. INTRODUCTION

Figure 1.1: Typical HTS power applications and their respective temperature and cooling power ranges. FCL stands for Fault-Current Limiter, a superconducting safety device.

Figure 1.2: A forced-flow liquid cooling system (LPC-2 FF). In the background two SPC-1 cryocoolers are visible, that provide the cooling power. The blue vessel is a pump cryostat that holds all the process equipment for pumping and pressurizing the liquid nitrogen. The control cabinet is placed on the left. This installation provides a cooling power of 1700 W at 77 K down to 1100 W at 65 K.
engine was what he called the *economizer*, now known as the *regenerator*. A very important aspect of the Stirling engine over for instance internal combustion engines is that the heat source is external. This means that a Stirling engine can operate on any source of heat. This is the reason that Philips started investigating the Stirling engine in the 1940's as a versatile source of electricity, in order to expand its business in electronic devices such as radio's worldwide, including areas where no reliable electricity infrastructure was available [3].

In that research it was found that the Stirling cycle could be reversed. Instead of using a temperature difference to create work (engine), work can be used to create a temperature difference (refrigerator). These machines were able to reach a temperature low enough to produce liquid air and nitrogen. A wide range of machines was designed, from less than 1 W to 25 kW of cooling power. The high-power industrial Stirling cryocoolers are today still produced by Stirling Cryogenics & Refrigeration BV.

Pulse-tube refrigerators (PTR’s) [4] are comparable to Stirling refrigerators, with the main difference the absence of the moving displacer. This results in two big advantages: less vibrations and improved reliability. They were first described by Gifford and Longsworth [5] in the mid 1960's. In their basic *pulse-tube refrigerator* heat transfer between the gas and wall of an empty, closed tube resulted in a refrigeration effect. The most important improvement was done by Mikulin et al. [6]. With these *orifice pulse tubes*, temperatures below 100 K were quickly realized [7]. Further development of new types such as the *multi-stage, double-inlet, and inertance tube* PTR further reduced the minimum temperature and increased the efficiency of PTR’s. Currently, the lowest temperature reached with a PTR is 1.3 K [8].

In the low-temperature range of around 4 K, PTR’s are already an alternative to conventional coolers. In the high temperature range around 77 K, PTR’s are lacking in efficiency, and are therefore usually chosen in applications where reliability and absence of vibrations is more important. This is especially the case in the high-power range of 100-1000 W, where the only commercially available PTR [9], has only half the efficiency of the state of the art Stirling cryocoolers [10].

1.3 Requirements for ’new-generation’ cryocoolers

The cooling of high-temperature superconductors is distinctive for the approach towards cooling in practical situations. The use of cryogenics outside lab-environments yields strict requirements. If cryogenic cooling is required in a certain application, it should be transparent, reliable and efficient. In case of HTS, the industry has indicated its requirements for coolers [11]-[14]. The requirements that are requested for these ’new-generation’ cryocoolers are:

- **High reliability**: the reliability of the cryocooler should not be the limiting factor in the reliability of the application, such as electric power grids. Associated with reliability is *availability*. Availability is more related to an entire system than to a single cooler. A way to increase availability of a cooling system is to use redundancy. If a single cooler has a reliability of 90%, i.e. the chance that it does not fail within its lifetime, a system of two coolers will have a theoretical availability of 99%, under the assumption, however, that the failure modes of the coolers during its *lifetime* are random processes. Development of coolers has mainly focussed on improving reliability by removing moving and rubbing parts. Development of the PTR is an important example, but also the development of compressors using gas [15] or flexure bearings [16] has been a mayor improvement;

- **Low maintenance**: this is related to reliability and availability in a paradoxal way. Maintenance to a cooler improves reliability, as it is less likely that coolers will fail. However,
maintenance also means that a cooler has to be taken out of operation and the application needs to warm up. Current Mean Time Between Maintenance (MTBM) for cryocoolers is of the order of 8000 hours \[10\][17], or approximately 1 year of continuous operation. Maintenance is mainly required for cleaning and replacing worn parts. Developments such as the gas and flexure-bearing compressors significantly reduce the number of wearing parts, and because they are usually oil-free, also reduce the need for cleaning. Requirements for MTBM are approximately 25000 hours, or 3 years of continuous operation;

- **Cooling power**: because of the previously mentioned cryogenic penalty, HTS devices are only interesting if the reduction in electrical losses outweighs the increase in power consumption due to the refrigeration system. Therefore, there will be a lower limit to the size of the HTS applications. Depending on the application, the required cooling power at 77 K is between 100 W and several kilowatts;

- **Efficiency**: the efficiency of a cryocooler, or the Coefficient of Performance (COP), is the ratio of cooling power and input power. The higher the efficiency, the lower the power consumption and consequently the operational cost. Efficiency is often expressed relative to the theoretical maximum efficiency (Carnot). For a cooler operating between 300 K and 77 K, this maximum efficiency is 35 \%. State of the art Stirling cryocoolers have efficiencies of approximately 10 \%, or approximately 30 \% of Carnot. This is also what is required for cryocoolers for HTS applications;

- **Cost**: this is a trivial requirement. Cost should be as low as possible. The cost requirement is related to the other requirements, since availability relates to cost if redundancy is used. Efficiency is related to cost as it determines the cost of operation. In literature, requirements for cost are mentioned of \$25 per watt cooling power at 77 K. This is the capital investment only, and does not seem to take cost of operation and lifetime into account. Even though current available cryocoolers are a few factors more expensive than this requirement, cost optimization has not been taken into account in this thesis.

## 1.4 Scope of this thesis

This thesis describes the investigation of high-power cryocoolers operating in the 77 K temperature range. It consists of seven chapters. In chapter 2 the theory of Stirling and pulse-tube refrigerators is explained. First, the relevant principles of thermodynamics and gas dynamics are given. The equation of state for ideal and nearly-ideal gases is described, together with relevant other gas properties. The laws of thermodynamics are applied to the two types of cryocoolers, and the differences between the two types are explained.

An important component in cryocoolers, the regenerator, is described in more detail, together with a proposed method for optimizing this regenerator in the design phase of a cryocooler. Finally, loss mechanisms in the pulse tube are explained. The heat pumping mechanism is based on the transport of enthalpy through an open tube, the pulse tube. Disturbances in the flow in the pulse tube lead to disturbances in the transport of enthalpy, and hence to a reduction in performance. Several mechanisms responsible for such disturbances are described in the last paragraph of chapter 2.

Chapter 3 describes several models for Stirling and pulse-tube refrigerators. The so-called Stirling model is introduced. The Stirling model is a design tool for Stirling cryocoolers that has been in use at Stirling and Philips for several decades. It is a very powerful tool that enables accurate prediction of the performance of Stirling cryocoolers. It was used as the basis for the so-called Stirling pulse-tube model (SPTM). First a simple one-dimensional first-order harmonic
model for pulse tubes is described, before the detailed description of the SPTM. The SPTM also is a one-dimensional, first-order harmonic model. It uses the accuracy of the Stirling model for the calculation of the regenerator and heat exchangers.

The loss mechanisms in the pulse tube that were introduced in chapter 2 cannot be calculated using the one-dimensional model. These losses are caused by two or three-dimensional flows in the pulse tube. They are simulated with two numerical models. The initial investigation was done with a commercially available code. This code is easy to use, but numerically not very accurate nor efficient. Therefore, a dedicate numerical model was developed. This development was part of a second PhD project, and its basics are described in the final paragraph.

Chapter 4 describes a more practical aspect of pulse tubes. The heat pumping mechanism in a pulse tube is generated by a flow resistance on the warm end of the pulse tube. The dissipation in this flow resistance is essential for the operation of the PTR. In an orifice, the flow and pressure are always in phase. A PTR can operate much more efficiently if this flow and pressure would be out of phase, with the pressure leading the flow. Several methods of realizing this phase difference are described, with special attention on the so-called inertia tube. This passive phase shifter basically is a long, narrow tube. The flow resistance of this tube ensures sufficient dissipation, the inertia of the moving gas ensures the required phase shift.

In chapters 5 and 6 the experimental results of two PTR’s are presented. Both setups use a crankshaft-driven compressor from a Stirling SPC-1 cryocooler. This enables easy integration of the PTR with the compressor. Setup 1, described in chapter 5 is also built with the heat exchangers and regenerator of the SPC-1 Stirling cryocooler. Setup 2, described in chapter 6 is an optimized PTR. With this PTR, significant cooling power was realized. The experimental results from setup 2 are used to validate the Stirling pulse-tube model. This validation is described in the last paragraph of chapter 6.

Finally, in chapter 7, two industrial cryocoolers are proposed, one PTR, based on setup 2, and one Stirling-type cryocooler. Both cryocoolers will be driven by an oil-free, linear-motor driven compressor. Such a compressor is essential to fit the requirements for maintenance and reliability. It was found that, for an industrial high-power cryocooler, the free-displacer Stirling cryocooler is to be preferred over a PTR. The reason is the efficiency of the cooler. The advantage of a PTR, less vibrations, is less important in high-power applications.
Bibliography


Chapter 2

Introduction to Pulse-Tube and Stirling Refrigerators

2.1 Thermodynamics and gas dynamics

2.1.1 First and second law of thermodynamics

Thermodynamics describes macroscopic systems that interact with other systems, and describes the processes that occur in this interaction. Important quantities are temperature and pressure, but also energy and work. Thermodynamics is always applied to systems (so-called thermodynamic systems). These systems are described by the laws of thermodynamics.

The first law of thermodynamics describes conservation of energy. For an open system it can be written as

\[ \dot{U} = \sum_k \dot{Q}_k + \sum_k \dot{H}_k - \sum_k p_k \dot{V}_k + P. \]  

(2.1)

This equation says that the rate of change of internal energy \( \dot{U} \) of a system is equal to the sum of all heat flows \( \dot{Q}_k \) and enthalpy flows \( \dot{H}_k \) crossing the system boundary \( k \), and work done by the changing volume of the system \( p_k \dot{V}_k \) (for instance due to a moving piston, which sees a pressure \( p_k \)), and any other form of external power applied to the system, \( P \), such as electrical power. The kinetic and potential energy are omitted since the expected gas velocities are too low to have significant kinetic energy, and the physical size of a cryocooler is small enough to neglect the influence of gravity. Convention is that a flow into the system is positive. The enthalpy flow is equal to the molar flow multiplied by the molar enthalpy

\[ \dot{H} = n \dot{H}_m, \]  

(2.2)

with the molar enthalpy equal to

\[ H_m = U_m + p V_m, \]  

(2.3)

where \( U_m \) is the molar internal energy and \( V_m \) the molar volume. The notation \( \dot{X} \) describes a flow of a property of state, the notation \( \dot{X} \) describes its time derivative. Even though their dimensions are the same, the physical meaning is different. For other properties which are not properties of state, the notation \( \dot{Y} \) is used for its flow.
The second law of thermodynamics is written as

\[ \dot{S} = \sum_k \frac{\dot{Q}_k}{T_k} + \sum_k \dot{S}_k + \dot{S}_{ir}, \dot{S}_{ir} \geq 0. \]  

(2.4)

This equation says that the rate of change of entropy of a system is equal to the sum of heat flows \( \dot{Q} \) into the system divided by the temperature \( T \) at which this heat enters the system, the sum of entropy flows \( \dot{S} \) into the system, and the rate of entropy production \( \dot{S}_{ir} \) by irreversible processes in the system. The entropy flow is calculated similar to the enthalpy flow

\[ \dot{S} = \dot{n}S_m. \]  

(2.5)

In a cryocooler we are dealing with closed-cycle processes, in the steady state. At the end of a cycle, the system is in the same state as it was in the beginning, hence all state quantities will have the same value. It is therefore more interesting to look at time average values of quantities, instead of the instantaneous values. Both the internal energy and the entropy are state variables, so for the time averages we can say that \( \bar{U} = 0 \) and \( \bar{S} = 0 \). For the time-averaged values of enthalpy flow and entropy flow we can say

\[ \bar{H} = \frac{1}{t_c} \int_0^{t_c} \dot{n}H_m(p,T) \, dt, \]  

(2.6)

\[ \bar{S} = \frac{1}{t_c} \int_0^{t_c} \dot{n}S_m(p,T) \, dt, \]  

(2.7)

with \( t_c \) the cycle time. As \( \dot{V} = 0 \), equations 2.1 and 2.4 then become

\[ 0 = \sum \bar{Q} + \sum \bar{H} + P, \]  

(2.8)

\[ 0 = \sum \frac{\bar{Q}}{T} + \sum \frac{\bar{S}}{T} + \dot{S}_{ir}. \]  

(2.9)

The closed cycle processes in a cryocooler can then be described as shown in figure 2.1. Using external power \( P \) heat is transported from a low temperature \( T_l \) to high temperature \( T_h \). During this process, an amount of entropy \( \dot{S}_i \) is produced.

According to the first law, the amount of work is equal to the difference between the two heat flows

\[ P = \bar{Q}_h - \bar{Q}_l. \]  

(2.10)

The second law yields

\[ \frac{\bar{Q}_h}{T_h} = \frac{\bar{Q}_l}{T_l} + \dot{S}_i \]  

(2.11)

The so-called Coefficient Of Performance (COP) for a cryocooler is the ratio between useful cooling power and input work.

\[ COP = \frac{\bar{Q}_l}{P}. \]  

(2.12)
Combining 2.10 and 2.11 yields

\[ \text{COP} = \frac{\dot{Q}_l}{\frac{\dot{Q}_l}{T_l} \left( \frac{T_h}{T_l} - T_l \right) + \dot{S}_i T_h} \]  

(2.13)

If there are no irreversible processes in the cycle, no entropy is produced. The COP can then be written as

\[ \text{COP}_C = \frac{T_l}{T_h - T_l} \]  

(2.14)

This coefficient of performance is also called the Carnot efficiency of a cryocooler. This is the maximum efficiency attainable with a cryocooler.

2.1.2 Gas properties

The thermodynamic state of a gas is described by its equation of state. For an ideal gas, this equation is

\[ pV_m = RT, \]  

(2.15)

where \( R \) is the molar ideal gas constant. For a real gas, we can include a nonideality parameter \( Y \)

\[ pV_m = \frac{RT}{Y}. \]  

(2.16)

Both equations above use molar properties. If we use mass units, the equation of state for the nonideal gas becomes

Figure 2.1: Heat flows, entropy flows and power in a closed cycle thermodynamic machine. By adding power to the system, heat is transported from a low temperature to a high temperature.
\[ \rho = \frac{pY}{R_M T} \]  

(2.17)

With the gas constant \( R_M \) per unit of mass. Equation of state 2.16 can be formulated in terms of virial coefficients \( B_k \) [1],[2]

\[ \frac{pV_m}{RT} = 1 + \sum_k \frac{B_k(T)}{V_m^k}. \]  

(2.18)

The molar heat capacity at constant pressure is written as

\[ C_p = \left( \frac{\partial H_m}{\partial T} \right)_p, \]  

(2.19)

the molar heat capacity at constant volume as

\[ C_V = \left( \frac{\partial U_m}{\partial T} \right)_V. \]  

(2.20)

Furthermore we define

\[ H_p = \left( \frac{\partial H_m}{\partial p} \right)_T \]  

(2.21)

which describes the dependence of pressure for the molar enthalpy. For an ideal gas, \( H_p \) is zero, hence the enthalpy for an ideal gas only depends on temperature, not on pressure.

If we write equation 2.16 as \( V_m = RT/pY \), and differentiate with respect to \( p \) for constant entropy, we can write

\[ \left( \frac{\partial V_m}{\partial p} \right)_{s_m} = -\frac{RT}{pY} + \frac{R}{pY} \left( \frac{\partial T}{\partial p} \right)_{s_m} - \frac{RT}{pY^2} \left( \frac{\partial Y}{\partial p} \right)_{s_m}, \]  

(2.22)

or

\[ \left( \frac{\partial V_m}{\partial p} \right)_{s_m} = -\frac{V_m}{p} + \frac{V_m}{T} \left( \frac{\partial T}{\partial p} \right)_{s_m} - \frac{V_m}{Y} \left( \frac{\partial Y}{\partial p} \right)_{s_m}, \]  

(2.23)

or

\[ \left( \frac{\partial V_m}{\partial p} \right)_{s_m} = -\frac{V_m}{p} (1 - K + \alpha), \]  

(2.24)

with

\[ K = \frac{p}{T} \left( \frac{\partial T}{\partial p} \right)_{s_m} \]  

(2.25)

and

\[ \alpha = \frac{p}{Y} \left( \frac{\partial Y}{\partial p} \right)_{s_m}. \]  

(2.26)

For nearly ideal gases, we can write

\[ n = \frac{1}{1-K+\alpha}, \]  

(2.27)

such that \( n \) does not depend (strongly) on \( p \). In that case, we can write for an adiabatic process

\[ \left( \frac{\partial p}{\partial V_m} \right)_{s_m} = -n \frac{p}{V_m} \]  

(2.28)
or
\[ pV_m^n = \text{constant}. \] (2.29)

For an ideal gas (\( Y = 1 \)), this leads to the well-known Poisson equation
\[ pV_m^n = \text{constant}, \] (2.30)

with
\[ \gamma = \frac{C_p}{C_V} \] (2.31)

and
\[ \gamma = \frac{1}{1 - K}, \] (2.32)

with \( K = R/C_p \)

### 2.2 Stirling Cryocoolers

#### 2.2.1 General principle

Stirling cryocoolers are based on the Stirling cycle, which consists of two isothermal and two isochoric processes. A Stirling cryocooler consists of the so-called compression space and the expansion space (figure 2.2). Each space has a piston. The spaces are separated by a regenerator. A regenerator is a porous medium that can store and release heat from and to gas flowing through it. In the ideal case the gas flowing through it will always have the same local temperature as the regenerator material (matrix), without changing the temperature of the matrix material itself.

The four processes are shown in figure 2.2. In the initial state, the compression space is at its maximum volume, and the expansion space at minimum volume. All the gas is in the compression space. The compression space is at high (ambient) temperature \( T_h \), the expansion space at a lower temperature \( T_l \). The cycle consists of the following steps:

I-II Isothermal compression. The compression piston moves to the right, while the expansion piston stays at its left-most position. The gas is compressed. Because this compression is isothermal, an amount of heat \( (Q_h) \) has to be removed from the system. The volume of the system is reduced from \( V_1 \) to \( V_2 \).

II-III Isochoric displacement. Both pistons move to the right. As the gas moves through the regenerator, it is cooled down to temperature \( T_l \). Heat \( (Q_2) \) is transferred from the gas to the regenerator.

III-IV Isothermal expansion. The expansion piston moves to the right, while the compression piston stays in its right-most position. The volume is increased from volume \( V_2 \) back to \( V_1 \). The gas is expanded. Because the expansion is isothermal, an amount of heat \( (Q_l) \) is transferred to the gas.

IV-I Isochoric displacement. Both pistons move to the left. Upon passing through the regenerator, the gas is reheated \( (Q_4) \) to temperature \( T_h \). The system is now back in its original state.
Figure 2.2: The four steps in the Stirling Cycle. Step 1 (I-II): All the gas is in the compression space. As it is compressed, heat is removed. Step 2 (II-III): As the gas is displaced through the regenerator, it is cooled down to temperature of the expansion space, $T_i$. Step 3 (III-IV): All the gas is now in the expansion space. When it is expanded, heat has to be added to the gas. During this stage, the cooling power is generated. Step 4 (IV-I): When the gas is displaced back to the compression space, it is reheated to the compression space temperature $T_h$. 
In figure 2.3, these four steps are given in a p-V diagram. The volume $V$ is the volume of the entire system.

The first law for a closed system can be written as, assuming ideal gas

$$dQ = nC_VdT + p\,dV.$$  \hfill (2.33)

For an isothermal process this becomes

$$dQ = pdV,$$  \hfill (2.34)

so for the amount of heat added during isothermal compression or expansion from a volume $V_a$ to $V_b$ at fixed temperature $T$, we can write

$$Q_{iso} = \int_{V_a}^{V_b} pdV = nRT \int_{V_a}^{V_b} \frac{dV}{V} = nRT \ln \frac{V_b}{V_a}. \hfill (2.35)$$

So the amount of heat that has to be removed during compression is

$$Q_h = nRT_h \ln \frac{V_1}{V_2}, \hfill (2.36)$$

and the amount of heat added to the gas during expansion is

$$Q_l = nRT_l \ln \frac{V_1}{V_2}. \hfill (2.37)$$

The amount of heat transferred to the regenerator matrix during processes 2 and 4 are equal but opposite

$$-Q_2 = Q_4 = nC_p(T_h - T_l). \hfill (2.38)$$

The coefficient of performance is then (using equations 2.36 and 2.37)

$$COP = \frac{Q_l}{P} = \frac{Q_l}{Q_h - Q_l} = \frac{T_l}{T_h - T_l}, \hfill (2.39)$$

which is equal to the Carnot efficiency, given by equation 2.14.

This maximum efficiency is possible because there are no irreversible processes in the ideal Stirling cycle. Isothermal heat transfer is a reversible process, as is the heat transfer to and from the ideal regenerator. Furthermore, the work generated by the expansion process is recovered. The corresponding energy and entropy flows in an ideal Stirling cryocooler are shown in figure 2.4.

The p-V diagram from a practical Stirling-cycle cryocooler will look different from the one shown in figure 2.3. First of all, this is due to the movement of the pistons, which is harmonic, instead of step-wise as assumed above. More important, the cycle will not be ideal due to the processes not being ideal. Several sources for losses can be identified:

- The regenerator is not ideal. There will be heat conduction in axial direction. The heat transfer between gas and regenerator matrix is not perfect, so the finite temperature difference between gas and matrix material will lead to entropy production. The regenerator has flow resistance. A pressure drop over the regenerator will result in losses.

- Volumes in the system which are not part of compression and expansion spaces store gas. The gas in these void volumes has to be compressed and expanded, leading to additional mass flows. This will also lead to losses, as these mass flows are associated with pressure drops, for instance in the regenerator.
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Figure 2.3: p-V Diagram of the Stirling cycle. The pressure in the system is plotted as a function of the entire system volume V. The compression takes place at temperature $T_h$, displacement at volume $V_2$, expansion at temperature $T_l$ and displacement at volume $V_1$.

Figure 2.4: Time-average heat flows, entropy flows and powers in an ideal Stirling cryocooler. Entropy flows are shown in the top half of the picture, enthalpy and heat flows in the bottom half. The compression power $P_c$ is removed as heat from the system at temperature $T_h$. The associated entropy is removed from the system. Entropy flows through the regenerator from the cold end to the warm end. The expansion of the gas leads to the transfer of heat to the system at temperature $T_l$, with its associated entropy flow. The expansion power $P_e$ is recovered from the system.
A practical heat exchanger will always have a temperature difference between heat exchanger and medium, and a pressure drop. This leads to the production of entropy.

The losses in the regenerator are the major losses in the system. The regenerator and its associated losses will be discussed further in paragraph 2.4.

### 2.2.2 Piston-displacer cryocoolers

Several configurations are possible for practical Stirling cryocoolers (figure 2.5). The construction as shown in the previous paragraph is the two-piston, in-line construction (figure 2.5a). Driving these pistons would require a complex drive mechanism, especially for the expansion piston which would require a cold piston. Therefore, a Stirling cryocooler is often built as a so-called 'piston-displacer' type (figure 2.5b). There is a single piston that does both compression and expansion. A so-called displacer moves the gas between compression and expansion space. In principle moving the displacer does not require work. The only work is caused by pressure differences between compression and expansion space, for instance due to flow resistance in the regenerator. It can be driven by the same crankshaft as the piston, with a drive rod fed through the compression piston.

A different method of driving the displacer is using the gas forces acting upon it. Such a cooler is called the free-displacer type (figure 2.5c). The displacer is mounted on a spring instead of a drive shaft. The advantage is that the piston and displacer only have to be connected by a gas line. This gas line can have a significant length, so the cold part (cold head) can be mounted separate from the compressor. The gas forces form, together with the force from the spring and damping, a mass spring system. The equation of motion for the displacer is given by

$$M_d \frac{d^2x}{dt^2} = -k_{sp}x - f_{fr} \frac{dx}{dt} + A_d(p_c - p_e)$$  \hspace{3cm} (2.40)

with $M_d$ the displacer mass, $x$ the displacer position, $k_{sp}$ the spring stiffness, $f_{fr}$ a friction coefficient, $A_d$ the displacer surface and $p_c$ and $p_e$ the pressures in the compression space and expansion space respectively. In a design tool for free displacer Stirling cycle cryocoolers, equation 2.40 has to be solved together with the other equations in the model. More on this can be found in chapter 7.

### 2.3 Pulse-Tube Refrigerators

#### 2.3.1 General Principle

The most important aspect of a pulse-tube refrigerator (PTR) is the absence of moving parts in the cold end of the cooler. In figure 2.6 the typical layout of a pulse tube is given. A compressor is used to provide the necessary compression and expansion of the gas. The compression work is removed as heat from the first heat exchanger, the after cooler (AC). Between the compressor and the cold end of the pulse tube a regenerator is placed. This regenerator has the same function as in a Stirling cryocooler. Gas flowing from the compressor towards the pulse tube is precooled by the regenerator. When this gas flows back, it is reheated.

The cold heat exchanger (CHX) is where the cooler is interfaced with the application that is to be cooled. Heat is extracted from this application by the CHX. At the warm end of the pulse tube, the hot heat exchanger (HHX) extracts heat from the system.

The simplest form is the so-called basic pulse tube. In this type of PTR the pulse tube is closed after the HHX [5]. The cooling mechanism is based on the heat exchange between the
Figure 2.5: Different types of Stirling cryocoolers. The in-line type (a) has a compression piston and an expansion piston, separately driven. The displacer-type (b) has a single piston and a displacer that moves the gas between expansion and compression spaces. The displacer shaft feeds through the compression piston, where it is driven by the same crank-shaft as the piston. The free-displacer type (c) also has a piston and displacer, but the displacer is now mounted on a spring instead of a drive rod. This makes it into a mass spring system, driven by gas forces.

Figure 2.6: Layout of a single orifice, Stirling-type pulse-tube refrigerator (PTR). A compressor is used to provide the compression and expansion of the gas. The heat of compression is removed in a heat exchanger called the after cooler (AC). A regenerator is placed between the compressor and the pulse tube. In the cold heat exchanger (CHX) heat is extracted from the application. In the hot heat exchanger (HHX) heat is extracted from the gas. The last components are the orifice and the buffer.
gas and the wall. The performance of such a PTR is poor. An important improvement in the performance of the PTR was found by Mikulin et al. [6]. They placed a flow resistance and buffer at the hot end of the pulse tube, as shown in figure 2.6. Now all the gas in the pulse tube contributes to the heat pumping effect, not only the gas near the wall. In a flow resistance - often called ‘orifice’ - gas flow and pressure are in-phase.

The cooling mechanism of an orifice-type PTR is explained in figure 2.7. Indicated is the first cycle. The mechanism follows the following steps:

I-II The system starts at ambient temperature and low pressure. The compression piston is in the left-most position. The piston moves to the right while the orifice is closed. In the isothermal compression space, heat is extracted from the gas by the AC. After flowing through the regenerator and CHX, the gas enters the pulse tube. In the pulse tube the temperature of the gas increases as the pressure increases. The gas that enters the pulse tube does so at the cold heat exchanger temperature which, initially, is at room temperature.

II-III Now the orifice is opened with the piston position fixed. Because the system is at high pressure, gas now flows through the orifice into the buffer. Heat is removed as hot gas flows through the HHX. Because of the gas leaving the tube, the pressure will decrease again. The temperature of the gas near the CHX and in the rest of the tube will drop below room temperature.

III-IV The piston moves to the left. The orifice is closed. The temperature and pressure decrease. Cold gas leaves the tube through the CHX, where heat is extracted and cooling is generated.

IV-V The orifice is opened again, with fixed piston position. The pressure in the system is now low, so gas flows from the buffer via the HHX into the pulse tube. It does this at the room temperature. Also cold gas flows through the CHX towards the compressor.

As this cycle is repeated, the regenerator and cold heat exchanger will cool down to the desired temperature.

In practice, the PTR will not operate stepwise as described above, but all processes will occur more or less harmonically. The temperature of gas parcels entering and leaving the pulse tube at the cold and hot heat exchanger are shown in figure 2.8.

The PTR described above is a so-called Stirling-type PTR. The compression and expansion of the gas is done directly by the reciprocating piston of a compressor. Typical working frequencies of such a PTR are 10 Hz-60 Hz. It is also possible to compress and expand the gas indirectly, using valves that connect the pulse tube intermittently to a high- and low pressure buffer (figure 2.9). These buffers are kept at these pressures by a compressor that can run on frequencies higher than the frequency of the PTR itself. Typical operating frequency of the PTR in this case is 1 Hz-2 Hz. This is a so-called Gifford-McMahon type PTR (GM-type). Instead of two separate valves, usually a single rotary valve is used [7],[8].

The orifice provides a flow on the hot end of the PTR that is in phase with the pressure. The second orifice as proposed by Zhu et al. [2] enables other phase relations. This second orifice is placed between the hot end of the pulse tube and the compressor. Gas can flow directly between the compressor and the pulse tube, both influencing the phase relation and lowering the mass flow through the regenerator. Any asymmetry in flow resistance in this second orifice can result in a steady mass flow through the system (DC-flow) [10]. This flow can be adjusted by adding so-called minor orifices [11] or anti-parallel placing of identical orifices [12]. The second orifice is the most used phase shifter in low frequency, GM-type pulse tubes (figure 2.10a).
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Figure 2.7: Cooling mechanism of an orifice-type pulse-tube cryocooler. The line inside the pulse tube indicates the temperature profile. In the initial situation (I) the system is at ambient temperature and low pressure. When the piston is moved to the right (II) the orifice stays closed and the gas is compressed and pressure and temperatures are increased. When the orifice is opened with fixed piston position, hot gas moves through the hot heat exchanger towards the buffer. Heat is removed, and pressure decreases again (III). When the piston moves back to the left (IV), while keeping the orifice closed, pressure and temperatures decrease. Cold gas moves through the CHX towards the regenerator. When the orifice is opened with fixed piston position, gas flows from the buffer into the pulse tube and the compression space (V).
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Figure 2.8: Temperature of gas entering and leaving the pulse tube at the cold and hot heat exchanger. A gas particle enters the pulse tube from the cold heat exchanger at temperature $T_l$ and leaves at a temperature below $T_l$. Gas that enters the pulse tube from the hot heat exchanger at temperature $T_h$ returns at a higher temperature.

Figure 2.9: A Gifford-McMahon type PTR. Compression and expansion takes place by intermittently connecting the PTR to a high and low pressure reservoir, using valves. A compressor is used to keep the reservoirs at their respective pressures.
Figure 2.10: Different types of phase shifters for the PTR: second orifice (a), inertance tube (b), four-valve (c), and warm expander (d).

For high-frequency pulse tubes, a so-called inertance tube is often used. Its effect was first observed by Kanao et al. [13]. In a long and narrow tube, the combination of flow resistance, capacitance and inertia of the gas is used for influencing the gas flow (figure 2.10b). These inertance tubes do not suffer from DC flows. Inertance tubes are treated in more detail in chapter 4.

Other phase mechanism that exist are active phase shifters. In case of a GM-type pulse tube, a second set of valves (or a second rotary valve) can be placed between the hot end and the compressor (2.10c). The correct timing of the valve opening and closing applies the correct gas flow [14]. This is called a 'four-valve PTR'. A variant with two buffers is called 'active buffer' [15]. In high frequency PTR’s a so-called ‘warm expander’ can be used [16]. A second piston is used to apply the correct gas flow (figure 2.10d). If this second piston is part of a precisely tuned mass spring system, it can be made to move passively, equivalent to the displacer motion in a free-displacer Stirling cryocooler [17]. This ‘free warm expander’ needs to incorporate a certain amount of dissipation (e.g. due to friction), since this is required for the working mechanism of the PTR (see below).

The location where the phase between the pressure variations and velocity in the system is of most importance is the cold heat exchanger and regenerator. Since the pulse tube itself also has a significant influence on the phase relation, one might say that all these components together form the actual phase shifter network.

The time-averaged energy and entropy flows in the pulse tube can be determined by applying the first and second law of thermodynamics to the components of the pulse tube. This is shown in figure 2.11. The working gas is assumed to be an ideal gas.
The pressure drop due to flow resistance in the phase shifter leads to the production of entropy $\dot{S}_o$. The gas in the buffer is at constant temperature and pressure, so $S_m(p, T) = \text{const.}$ and the entropy flow on the buffer side of the orifice is zero. The gas in the pulse tube compresses and expands adiabatically. This means that there is no net transport of entropy through the pulse tube. Therefore, the entropy produced in the orifice can only flow in the direction of the hot heat exchanger, where it is removed from the system by removing heat. The amount of heat removed from the hot heat exchanger must thus be

$$\dot{Q}_{\text{HHX}} = T_h \dot{S}_o. \quad (2.41)$$

The connection between the orifice and HHX is isothermal, so there is no enthalpy flow. Therefore, the amount of heat removed from the HHX should be equal to the enthalpy flow through the pulse tube, and the heat entering in the cold heat exchanger

$$\dot{Q}_{\text{CHX}} = \dot{H}_t = \dot{Q}_{\text{HHX}}. \quad (2.42)$$

The entropy associated with the heat transferred in the cold heat exchanger is then transported through the regenerator. In the compression space, all processes occur adiabatically and reversible. Therefore no entropy is transported through the compression space, so the heat removed from the after cooler is

$$\dot{Q}_{\text{AC}} = \frac{T_h}{T_1} \dot{Q}_{\text{CHX}}. \quad (2.43)$$

An ideal regenerator is isothermal. This means that no enthalpy is transported through it. The compression work must then be equal to the heat removed from the after cooler.
The coefficient of performance of the system is equal to (equation 2.12)

\[
COP = \frac{Q_{CHX}}{P} = \frac{T_i}{T_h}
\]

The theoretical maximum $COP$ of a PTR is thus lower than the Carnot efficiency. This is due to the losses in the phase shifter network which are essential to the pulse-tube operation. According to equations 2.41 and 2.42 there can only be cooling power if entropy is produced in the phase shifter - if a passive phase shifter is used. This is one of the explanations why Stirling-type cryocoolers achieve higher coefficients of performance than pulse-tube cryocoolers.

For a cryocooler operating at 77 K, $COP_C = 77/(300 - 77) = 0.35$, while $COP = 77/300 = 0.26$. For pulse tubes operating at very low temperatures ($\approx 4$K) the difference is negligible.

### 2.4 Regenerators

#### 2.4.1 Introduction

The regenerator is a component that the PTR and Stirling cryocoolers have in common. It is also the most critical component for the efficiency of the cryocooler. The requirements for an ideal regenerator are:

- infinite heat capacity: the temperature of the material does not change due to the heat that is stored in and released from the matrix material;
- perfect heat transfer: heat is transferred from the gas to the regenerator and vice versa without any temperature difference. Only then the heat transfer is a reversible process;
- no pressure drop: the gas can flow freely through the regenerator. This transport does not require any work;
- no heat conduction in axial direction: heat conduction from the hot to the cold end of the regenerator is an irreversible process that decreases the performance of the cryocooler;
- no void volume: any gas that is stored in the regenerator does not take part of the compression and expansion cycle. Expanding and compressing this gas leads to increased mass flow, which in turn might lead to additional losses;
- the gas in the regenerator is ideal.

A regenerator is built using some kind of porous material. This porous material is usually stacked wire screens or spherical particles of a material with high heat capacity. For temperatures down to 70K this material is for instance stainless steel or phosphor bronze. For lower temperatures down to 10K, lead is used. For temperatures around 4K and lower, magnetic materials are often used.

Optimizing a regenerator means choosing the right combination of materials, pore size/filling factor, wire or sphere diameter and outer dimensions. The goal is to meet the above criteria as closely as possible. This will always lead to a compromise. The first and second requirement result in a certain size of the regenerator. Optimizing for these two requirements results in a large regenerator, with a very fine structure. The size is important for the total heat capacity, the
structure determines the heat transfer area. The larger this area, the smaller the temperature difference between gas and matrix. However, this conflicts with the other three requirements. A large size regenerator will have a large void volume. The fine structure results in more friction between gas and matrix, so there will be a high pressure drop. Size does not directly have an influence on heat conduction but the geometry that is ideal for heat conduction is not ideal for pressure drop. For a given volume of regenerator, a low pressure drop would require a short and wide regenerator. Low heat conduction requires a long and narrow regenerator.

2.4.2 Governing equations

The processes in a non-ideal regenerator can be written in terms of conservation of energy, mass, and momentum.

Heat is transferred from the gas to the matrix material and vice versa. This results in a change of internal energy of both the regenerator material and the gas, resulting in a change of temperature. In a cryocooler, pressure will be oscillating. This in- and decrease of pressure causes the amount of gas that is stored in the void volume of this piece of regenerator to change, leading to a change in mass flow. Any gas that flows through the regenerator will result in a pressure drop.

An analytical description of the regenerator has been given by De Waele et al. [18],[19]. The equations below are part of this description. The molar flux through the regenerator is given by

\[ j = \frac{\bar{n}}{A_r}, \quad (2.45) \]

with \( \bar{n} \) the total molar flow, and \( A_r \) the area of cross section of the regenerator. Mass conservation can be formulated as

\[ \frac{\partial j}{\partial t} = -\frac{\partial N}{\partial t}. \quad (2.46) \]

The rate of change of the amount of moles per unit of volume \( N \) is equal to the gradient of the molar flow density \( j \). The molar density is given by

\[ N = \frac{1 - f}{V_m}, \quad (2.47) \]

with \( f \) the filling factor.

The basic equation for the conservation of energy (per unit of volume) of the total regenerator - gas plus matrix - is

\[ \frac{\partial U}{\partial t} = -\frac{\partial h}{\partial t} - \frac{\partial q}{\partial t}. \quad (2.48) \]

It means that the rate of change of internal energy per unit volume of the regenerator is equal to the difference in the gradients of enthalpy flux and heat flux. The enthalpy flux is equal to

\[ h = jH_m, \quad (2.49) \]

the molar flux times the molar enthalpy. For the heat flux we write

\[ q = q_g + q_m, \quad (2.50) \]

with the heat conduction through the gas and matrix written as
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\[ q_x = -\kappa_x \frac{\partial T_x}{\partial l}, \quad \text{with } x = g, m \]  \hfill (2.51)

\( \kappa \) is the thermal conductivity. The description leads to two differential equations describing the temperature of the gas and matrix, respectively

\[
\frac{(1-f)C_p}{V_m} \frac{\partial T_g}{\partial t} = -jC_p \left( \frac{\partial T_g}{\partial l} - j \left( \frac{\partial V_m}{\partial T_g} \right) \frac{\partial p}{\partial l} \right) + \frac{(1-f)T_g}{V_m} \left( \frac{\partial V_m}{\partial T_g} \right) \frac{\partial p}{\partial l} + \beta(T_r - T_g) + \frac{\partial}{\partial l} \left( \kappa_g \frac{\partial T_g}{\partial l} \right), \]  \hfill (2.52)

and

\[
C_r \frac{\partial T_r}{\partial t} = \beta(T_r - T_g) + \frac{\partial}{\partial l} \left( \kappa_r \frac{\partial T_r}{\partial l} \right). \]  \hfill (2.53)

\( \beta \) is the volumetric heat transfer coefficient and \( T_r \) the matrix temperature. Equation 2.52 describes the rate of change of gas temperature \( T_g \) in the regenerator. The terms on the right-hand side describe respectively the influence of:

- convective heat transfer due to flow of gas in presence of a temperature gradient;
- throttling of gas through the flow resistance of the matrix (Joule-Thomson effect);
- temperature change of the gas due to compression;
- heat exchange between gas and matrix;
- heat conduction through the gas.

Since there are no convective terms in the matrix material, the rate of change of temperature of the matrix \( T_r \) (equation 2.53) is determined by the heat transfer between gas and matrix, and heat conduction in the matrix material only.

Equations 2.52 and 2.53 are coupled through the heat exchange term

\[ \dot{Q}_{ex} = \beta(T_r - T_g). \]  \hfill (2.54)

In this first-order description of heat transfer, the volumetric heat transfer coefficient \( \beta \) is related to the surface heat exchange coefficient \( \alpha \) by

\[ \beta = \alpha_{hx} F_s, \] \hfill (2.55)

with \( F_s \) the wetted surface per unit of volume (specific surface area). The heat transfer coefficient \( \alpha_{hx} \) can be determined using standard empirical relations in terms of Nusselt number or NTU approximations \[20],[21]. In the Stirling model and Stirling pulse-tube model described in the next chapter, these empirical relations have been determined from extensive measurements on actual regenerators \[22],[23],[24]. The specific surface \( F \) is determined by the matrix geometry, and depends on parameters as filling factor and wire/sphere diameter.

The pressure drop can be described in first order using Darcy’s equation

\[ \frac{\partial p}{\partial l} = -\mu z v, \] \hfill (2.56)

with \( v \) the velocity and \( \mu \) the dynamic viscosity. In terms of molar flux and molar volume this equation becomes
\[ \frac{\partial p}{\partial l} = -\mu z r jV_m. \] (2.57)

The specific flow impedance \( z_r \) can be determined using empirical relations found in literature \([4],[20],[21]\). In the Stirling and Stirling pulse-tube model described in the next chapter, these empirical relations have been determined from extensive measurements on actual regenerators.

When this set of coupled differential equations is solved, expressions will be found for molar flow \( j \), pressure \( p \), and gas and matrix temperatures \( T_g \) and \( T_r \) as functions of time and position. These properties together describe the complete gas dynamics inside the regenerator. One possible method of solving this set is the so-called harmonic approximation. This is described in the next chapter.

The loss occurring in the regenerator can be expressed in terms of enthalpy flow and entropy production. The average enthalpy flow for an ideal gas can be calculated with

\[ \bar{H}_{reg}(x) = \frac{A_r}{t_c} \int_0^{t_c} j(x,t)C_p T_g(x,t) dt. \] (2.58)

Entropy production rate per unit of volume of the regenerator \( \sigma_{ir} \) is given by \([18]\)

\[ \sigma_{ir} = \beta \frac{(T_r - T_g)^2}{T_g T_r} + \kappa_{eff,r} \left( \frac{\partial T_r}{\partial l} \right)^2 + \kappa_{eff,g} \left( \frac{\partial T_g}{\partial l} \right)^2 + \mu z r (jV_m)^2. \] (2.59)

Total entropy production rate is then given by

\[ \dot{S}_{ir} = A_r \int_0^{L_r} \sigma_{ir} dl, \] (2.60)

and the average total entropy production rate is

\[ \bar{S}_{ir} = \frac{1}{t_c} \int_0^{t_c} \dot{S}_{ir} dt. \] (2.61)

An ideal regenerator has zero enthalpy and heat flow, and no entropy production. Terms that lead to entropy production are finite heat exchange, heat conduction, and flow resistance. These are the four terms in equation 2.59, with the heat conduction through the gas and matrix written as two separate terms. The thermal conductivity for both matrix (\( \kappa_{eff,r} \)) and gas (\( \kappa_{eff,g} \)) are written as effective values, as the actual value will differ from the bulk value due to matrix geometry and flow effects.

The irreversible entropy production leads to a reduction of efficiency (\( COP \)) of the cooling machine.

In figure 2.12 the energy and entropy flows through a nonideal regenerator are shown. This is applicable to both the pulse-tube cryocooler and the Stirling cryocooler. The energy flow through the regenerator is the sum of the enthalpy flow by the gas and heat conduction through the gas and the regenerator matrix

\[ \bar{E}_{reg} = \bar{H}_{reg} + \bar{Q}_{cg} + \bar{Q}_{cm}. \] (2.62)

This energy flow leads to a reduction in available cooling power.
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Figure 2.12: Time averaged energy flows and entropy flows in a non-ideal regenerator. The energy flow through the regenerator is the sum of enthalpy flow through the gas, and heat conduction through the gas and matrix. Entropy is produced by irreversible processes.

\[ \dot{Q}_h = H_t - E_{\text{reg}}. \]  
\[ \dot{Q}_h = \dot{Q}_l + S_{\text{ir}}. \]  
\[ P = \frac{\dot{Q}_h}{T_h} = \frac{\dot{Q}_l}{T_l} + S_{\text{ir}}. \]  
\[ COP = \frac{\dot{Q}_l}{P} = \frac{\dot{Q}_l}{T_h \dot{Q}_l + T_h S_{\text{ir}} + E_{\text{reg}}}. \]

Regenerator optimization using entropy production rates

To optimize regenerator performance, the optimal compromise between the losses has to be found. The most common way to do this, is to minimize the amount of entropy that is produced in the regenerator, yielding a maximum coefficient of performance of the entire cryocooler. Similar work is presented by Will [25]. From equation 2.59 we can identify the individual contributions to the total entropy production rate per unit of length. These individual contributions are

\[ \frac{\partial S_{\text{cg}}}{\partial l} = A_r \kappa_{g, \text{eff}} \left( \frac{\partial T_g}{\partial l} \right)^2, \]  
\[ \frac{\partial S_{\text{cm}}}{\partial l} = A_r \kappa_{r, \text{eff}} \left( \frac{\partial T_r}{\partial l} \right)^2. \]
\[
\frac{\partial \dot{S}_l}{\partial l} = z_r \frac{\mu \left( \frac{n V_m}{A_r} \right)^2}{A_r T_g},
\]

(2.69)

and

\[
\frac{\partial \dot{S}_h}{\partial l} = \beta A_r \frac{(T_r - T_g)^2}{T_r T_g}.
\]

(2.70)

The influence of the matrix geometry can be described using the hydraulic diameter \( d_h \) and specific surface area \( F_s \) [21], which are related for porous media through

\[
d_h = \frac{4(1 - f)}{F_s}.
\]

(2.71)

For tubes, the hydraulic diameter is equal to the tube diameter, for spheres the hydraulic diameter is equal to \( d_h = 2/3d_s \) and for perfectly stacked screens the hydraulic diameter is equal to

\[
d_h = d_w \left( 1 - \frac{f}{f} \right).
\]

(2.72)

Under certain assumptions, a qualitative impression can be given about the influence of the design parameters of the regenerator. These assumptions are that the working fluid is ideal and the matrix heat capacity is infinite, so the matrix temperature is not a function of time. Furthermore we also neglect the influence of the nonzero void volume (assume \( f = 1 \)). Finally we assume linear temperature profiles in the gas and matrix. Equation 2.52 then simplifies to

\[
T_r - T_g = \frac{C_p^* n}{\beta A_r} \frac{d T_g}{d l},
\]

(2.73)

so equation 2.70 becomes

\[
\frac{\partial \dot{S}_h}{\partial l} = \frac{C_p^* n^2}{T_r T_g \beta A_r} \left( \frac{d T_g}{d l} \right)^2.
\]

(2.74)

For simplicity, we further assume that material properties are constant and since the average temperature profiles in the gas and matrix are equal, we combine the heat conduction contributions in a single term. Finally, we need equations to express the heat transfer coefficient \( \beta \) and the flow resistance \( z_r \) in terms of the hydraulic diameter. In this case we use typical empirical relations for steady flow, as found in literature [4]. The basis of these empirical relations is the Reynolds number

\[
R_e = \frac{\rho v d_h}{\mu}.
\]

(2.75)

Usually, the pressure drop is calculated using the friction factor \( c_w \) in

\[
\Delta p = c_w \frac{l_r}{d_h} \frac{1}{2} \rho u^2.
\]

(2.76)

For laminar flow in straight tubes, the friction factor is given by

\[
c_w = \frac{64}{R_e}.
\]

(2.77)
Combining equations 2.56, 2.76 and 2.77 yields

\[ z_r = \frac{32}{d_h^2} \]  

(2.78)

The empirical relation for straight tubes will result in an underestimation of the flow resistance, since the actual flow path is very irregular. It is used here however for the ease of calculation. The heat transfer coefficient \( \beta \) is calculated using equation 2.55 and the Nusselt number

\[ N_u = \frac{\alpha d_h}{\kappa g} \]  

(2.79)

which can be estimated using the empirical relation [21]

\[ N_u = 0.68 P_r^{0.6} P_t^{0.33}. \]  

(2.80)

The Prandtl \( P_r \) number can be approximated by \( P_r = 0.7 \). Typical parameters for a regenerator for a Stirling-type cryocooler are shown in Table 2.1.

**Entropy calculations** In figure 2.13 the contributions of the different terms in the entropy production per unit of length for the regenerator from Table 2.1 are shown, together with the total entropy production, as functions of temperature. In this case this is equivalent to the position along the regenerator length, since the temperature gradient was assumed constant. The contribution of heat conduction turns out to be very small. The contribution of the heat transfer losses is the largest. The contribution of the pressure drop losses are significant only at the high-temperature side of the regenerator. The total entropy production rate in this regenerator, obtained by integrating the values of figure 2.13, is 3.8 J/sK, of which 80 % is due to heat transfer losses 16 % due to pressure drop, and only 4 % is due to heat conduction.

Optimizing a regenerator means finding the optimum combination between the different contributions, so that the total entropy production is minimal. In the following graphs, the influences of several design parameters are shown. The parameters that are varied are regenerator length, diameter, and gauze geometry.

In figure 2.14, the influence of hydraulic diameter is shown. This can be influenced by both wire diameter and filling factor. Within limits, these two aspects are interchangeable. A lower hydraulic diameter means a finer mesh. For low hydraulic diameter, the fine structure causes
a large pressure drop but high heat transfer. For large hydraulic diameter, the flow is less obstructed so the pressure drop is low. Heat transfer is more difficult, so the entropy production due to heat exchange is high. In figure 2.15, the influence of the aspect ratio is shown. The aspect ratio is the ratio between length and diameter of the regenerator. The volume is kept constant. For low aspect ratio, the regenerator is short and wide. The velocity in the regenerator is low, so the pressure drop is low. As a result of the low velocity, the heat transfer coefficient is also low, resulting in a large entropy production. In a short and wide regenerator, heat conduction is also large.

It can be seen in figure 2.13 that the influence of each loss mechanism differs at different temperatures. At low temperature, the heat exchange contribution is very large, while the contribution of pressure drop in this region is negligible. This means that a homogeneous regenerator will not correspond with the optimum performance. In this example, the regenerator performance could be further improved by using material with lower hydraulic diameter in the cold section of the regenerator. To visualize this, the entropy production rate per unit of length as a function of hydraulic diameter is shown in figure 2.16, for two locations of the regenerator - the warm end at 300 K and the cold end at 100 K. At the cold end of the regenerator, the minimum entropy production rate is achieved at a much lower hydraulic diameter than at the warm end. In this case, a regenerator consisting of different matrix geometry would outperform a homogeneous regenerator. Similar optimizations could be done for other parameters such as aspect ratio or materials properties.

discussion  The optima according to figures 2.14 and 2.15 do not correspond exactly to the actual regenerator as used in the Stirling-type cryocooler. Most of this difference is due to the assumptions made in this estimation. Void volume has a large influence on the local mass flow in the regenerator, influencing the entropy production due to flow resistance. The specific flow
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Figure 2.14: Influence of hydraulic diameter on the total entropy production in the regenerator. At a smaller hydraulic diameter, the matrix structure is finer, improving heat exchange. At higher hydraulic diameter, the matrix is coarser, reducing the pressure drop.

Figure 2.15: Influence of the aspect ratio $l_r/d_r$ on the total entropy production rate in the regenerator. The volume of the regenerator is kept constant. At low aspect ratio, the regenerator is short and wide, improving pressure drop. At high aspect ratio, the regenerator is tall and narrow, improving heat exchange. Heat conduction is only significant at very low aspect ratios.
Figure 2.16: Entropy production rates per unit of length in the regenerator as functions of hydraulic diameter, at the cold end of the regenerator (100 K) and the warm end (300 K). At the cold end, the minimum in entropy production is reached at a much smaller hydraulic diameter than on the warm end.

impedance is underestimated. Temperature dependence of material properties such as viscosity and heat conduction also has an influence.

But there are other aspects in the optimization that are not taken into account in this optimization. Heat conduction appears to be negligible. So far only heat conduction through the regenerator matrix itself has been taken into account. This matrix will in practice be placed in a casing, with a particular wall thickness. This wall conducts heat, and as the wall thickness will increase with the regenerator diameter, so will the total heat conduction. And finally, economical aspects could also be taken into account in the optimization. The improvement in performance when using the very fine wire as suggested in Figure 2.16, might not outweigh the additional cost of such material.

The optimization as performed above is not the only way to optimize a machine. Apart from reaching the maximum efficiency of a cooling machine, usually there also is a requirement for absolute cooling power. The choice of regenerator does influence this cooling power, for instance via the pressure amplitude that will be available in the pulse tube or expansion space of a Stirling-type cryocooler. The consequence of using a more efficient regenerator could be that a compressor with a much higher swept volume is required, increasing the overall complexity and cost of the cooling machine. Another method of optimizing a machine is therefore the optimization of energy flows rather than of entropy production. This is how optimization with the so-called "Stirling Models" is done. This is explained in the next chapter.

2.5 Losses in the pulse tube

2.5.1 Introduction

In the description of the PTR so far, ideal conditions have been assumed for the pulse tube. It was assumed that the processes in the pulse tube are reversible, there is no heat conduction,
and that the flow pattern is one-dimensional. This is also called ‘plug flow’. As a result, the hot and cold ends are perfectly separated by the adiabatic zone in between. This zone is sometimes called the ‘gas piston’.

In a real system, the processes in the pulse tube itself are not ideal, so losses occur. Heat conduction leads to entropy production in the same way as in the regenerator (equation 2.59). Furthermore, heat transfer between the gas and the wall will also lead to entropy production. This surface heat-pumping effect is responsible for basic pulse-tube operating [5]. Finally, the gas piston may be disturbed by the flow. This could be caused by turbulence, or a second-order mass flow, called streaming. Mixing of the gas is an irreversible effect, reducing the efficiency of the cooler. Turbulence, surface heat pumping, and streaming will be discussed in the next paragraphs.

2.5.2 Turbulence

In a steady flow, for instance through a pipe, the onset of turbulence is determined by the Reynolds number (equation 2.75). For steady flow, the limit for laminar flow is $R_e = 2300$.

In oscillatory flow, the prediction of the onset of turbulence is different. Because of the oscillatory motion, the forces vary in time. Acceleration and deceleration allow the flow to show transition to turbulence and relaminarization during each cycle. A lot of experimental information is available from literature, for instance [26]-[30]. Brereton and Mankbadi’s review [30] summarizes available data in 4 regimes. Division of these regimes is determined by the Reynolds number expressed in the viscous boundary layer thickness

$$R_e\delta = \frac{\rho u \delta_{\mu}}{\mu}, \quad (2.81)$$

with

$$\delta_{\mu} = \sqrt{\frac{2\mu}{\rho \omega}}, \quad (2.82)$$

and the ratio between the channel radius $R$ and boundary layer thickness $\delta_{\mu}$. The four flow regimes are shown in figure 2.17. Two of the four regimes show only partial turbulence. In the perturbed laminar regime small perturbations to the main flow occur during flow acceleration. During deceleration flow relaminarizes. In the intermittently turbulent regime more energetic turbulence occurs during decelerations, while the flow returns to laminar during acceleration.

In a typical large-scale pulse tube, as presented further on in this thesis, typical values are $\delta_{\mu} \approx 0.24$ mm-0.73 mm, so $R/\delta_{\mu} \approx 80-30$, and $R_e\delta\approx 80-40$, which is well within the perturbed laminar region. Because of the transition back to laminar flow every half cycle, it is not expected that large disturbances that cause mixing will form. A final answer to that assumption, and the effect on pulse-tube performance, can only be given after detailed (numerical) simulations of pulse-tube flow behavior.

The same is true for effects such as surface heat pumping and streaming. They will have an influence on pulse-tube performance, but how large the influence is depends on the flow patterns and temperature profile in the pulse tube. Those can only be determined with detailed numerical simulations. In the next two paragraphs, surface heat pumping and streaming will be discussed further.

2.5.3 Surface heat pumping

Surface heat pumping is the result of heat transfer between gas and a wall, in an oscillatory flow in presence of a temperature gradient, or in presence of an oscillating pressure. It is also
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Figure 2.17: Laminar and turbulent flow regimes in oscillatory flow (recreated from [30]) Based on Reynolds number and boundary layer thickness, the flow can be completely laminar, fully turbulent, or partially turbulent with periods of relaminarization during acceleration or deceleration of the flow.

sometimes called shuttle heat transfer. The mechanism was described by Gifford and Longsworth [31]. It is the driving mechanism behind the operation of the basic pulse-tube refrigerator [5]. The mechanism is shown in figure 2.18. To the left of this system, a piston is periodically compressing and expanding the gas. This causes an oscillating pressure and movement of the gas parcel. The system is closed on the right side. The gas parcel is located within the thermal boundary layer of the wall. The four steps are:

I Initially, the gas parcel is at low pressure on the left side. As the pressure increases, the gas parcel is adiabatically compressed and moves to the right. Because of the adiabatic compression, the gas parcel heats up.

II The parcel is now stationary on the right. Here it is cooled by the wall.

III As the pressure decreases again, the parcel moves back to the left. Because of the adiabatic expansion, the temperature of the parcel decreases.

IV Heat is transferred from the wall to the gas. The gas parcel is returns to its original state.

As a net result, heat is transported from the left to the right. The surface heat pumping only takes place within the thermal boundary layer. Far away from the boundary layer, the gas behaves adiabatic. On the wall, the gas has the same temperature as the wall, so there is no enthalpy flow there. The amount of heat transfer depends on the pressure amplitude, frequency, gas type, and temperature profile in the wall. In figure 2.18, the temperature of the wall is assumed constant, so that the energy flows from left to right. If there is a temperature gradient in the wall, the temperature difference between the gas parcel and the wall decreases. When the temperature difference in the parcel is equal to the temperature difference in the wall, no heat is transferred. If the gradient is even larger, the direction of energy transport is from right to left. This effect is shown in figure 2.19, where the temperature of the parcel and the temperature
Figure 2.18: The mechanism of surface heat pumping. I: The gas parcel is compressed adiabatically, so it heats up and moves to the right. II: heat is transferred from the gas parcel to the wall. It cools down again. III: The gas parcel is expanded, cools down and moves to the right. IV: The gas parcel is reheated by the wall. The result is a net heat flow from left to right.

The profile in the wall are shown. The gradient at which the direction changes is called the critical temperature gradient. The distance that the particle travels for a given pressure difference can be calculated using

\[ \frac{dl}{dp} = \frac{\gamma l}{p} \]  

(2.83)

the temperature variation for given pressure variation is

\[ \frac{dT}{dp} = (\gamma - 1) \frac{T}{p} \]  

(2.84)

so the critical gradient is

\[ \nabla T_{\text{crit}} = \frac{\gamma - 1}{\gamma} \frac{T}{l} \]  

(2.85)

In orifice pulse tubes, usually the temperature gradient is larger than the critical gradient. In that case, heat is transported from the hot end of the pulse tube to the cold end, so the surface heat pumping effect is a loss. A typical critical gradient is 400 K/m, a typical gradient for an orifice pulse tube at 70 K is approximately 1200 K/m. Analytical solutions for the net energy flows exists only for basic pulse tubes and standing wave thermoacoustic devices. A solution for an orifice pulse tube is very complicated especially since the oscillation amplitudes are very large, so it will have to be found numerically.
2.5.4 Streaming

The term ‘streaming’ describes several phenomena. It usually refers to a steady mass flux or velocity, usually of second order, that is superimposed on the larger first-order oscillating flow. The type of streaming that is important for pulse tubes is a boundary layer driven streaming, often referred to as ‘acoustic streaming’ or ‘Rayleigh streaming’. It is called acoustic streaming because it is often observed in acoustic systems. Even though the pulse tube is not an acoustic system, it is present there as well. A review of different types of streaming can be found in [34].

The streaming is caused by the viscous boundary layer. The effect is illustrated in figure 2.20. When a gas parcel in the viscous boundary layer moves in one direction, it will have a different temperature than when it moves back. As a result, the drag it experiences is different too. As a result there is a net displacement of the gas parcel.

Because the total mass flow over the tube cross section should be zero, a mass flux in the boundary layer must be compensated by a mass flux in the center of the tube. The result is a torus-like vortex (figure 2.21). This vortex has been experimentally visualized by Shiraishi et al. [35]. An example can also be seen in figure 3.12.

The streaming mass flux depends on several parameters. Pressure amplitude, velocity and temperature in the boundary layer play an important role. Because the thickness of the thermal boundary layer and viscous boundary have are about equal, the above mentioned surface heat

---

**Figure 2.19:** Temperature position trajectories for gas particles in contact with the wall. The dotted line is the gas temperature, the straight line the wall. In the top trajectory, the temperature gradient in the wall is larger than the critical gradient, so enthalpy flow is from hot to cold. In the bottom trajectory it is the opposite.

**Figure 2.20:** Principle of streaming. A gas parcel experiences different drag during different flow directions, for instance due to the temperature dependence of the viscosity. The result is a net displacement.
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Figure 2.21: Torus-like vortex in the pulse tube as a result of a-symmetrical viscous drag in the boundary layer.

Figure 2.22: Asymmetry in the development of the boundary layer. When gas flows out of the flow straightener towards the right, the velocity profile is uniform. When the gas flows back, the boundary layer has developed. The asymmetry leads to a constant second order mass flow in the pulse tube.

... pumping effect will also influence the streaming. To simulate streaming in arbitrary geometries, numerical simulations are necessary.

An analytical description of the streaming mass flux in an infinitely long geometry is given by Olson and Swift [36]. They give an expression for the streaming mass flux outside the boundary layer. This mass flux is expressed in terms of gas velocity, density, and pressure amplitude. The mass flux is influenced by a cross section change in axial direction, a so-called tapered pulse tube. This was first suggested by Lee et al. [37]. Finally, the mass flux depends on the phase angle between velocity and pressure. This phase angle can be optimized for instance by using an inertance tube as phase shifter. This is what is done by Zia [4].

Another type of streaming was recently proposed by Liang et al. [39]. They showed, using analytical analysis and CFD simulations that the development of the boundary layer itself is asymmetrical, which also leads to streaming. This results in a quadruple torus-like vortex in the pulse tube. The mechanism is shown schematically in figure 2.22. When gas flows through a flow straightener into the pulse tube, the velocity profile is uniform. As the gas flows further into the tube, a boundary layer starts to develop. When the gas flows into the opposite direction, the boundary layer is further developed. As a result, a steady flow pattern develops. From the simulations it was shown that four vortices (figure 2.23) developed, the size of which depend on the trip length of the gas flow back and forth from the flow straighteners.

The magnitude of this streaming depends on factors such as frequency and dimensions of the pulse tube. The influence will be the largest in situations where the gas is displaced over long distances and where boundary layers have sufficient time to develop, such as in low frequency pulse tubes and long and narrow pulse tubes. The difference with Rayleigh streaming is that this effect is not caused by asymmetry in the boundary layer, due to changes of materials properties, but by asymmetry of the entire boundary layer. How the two streaming mechanisms compare quantitatively, is not yet known.

Flow straighteners also play an important role in the suppression of another type of streaming. So-called jet-driven streaming occurs when there is periodic suction and ejection of a viscous fluid through a change in cross section of the flow channel. The mechanism is based on the fact that...
flow patterns are different during suction and ejection. During suction, the flow comes from all directions, while during ejection the flow forms a jet (figure 2.24). In a pulse tube this can occur for instance where the heat exchangers interface with the pulse tube, but also where the orifice or inerter tube interface with the heat exchanger.

Flow straighteners are used to avoid jet flow (figure 2.25). The flow resistance of the straighteners causes a pressure drop. This pressure drop is much larger than the Bernoulli pressure difference

$$\Delta p >> \frac{1}{2} \rho (v_{in}^2 - v_{out}^2),$$

(2.86)

with $v_{in}$ the high velocity in the restriction, and $v_{out}$ the lower velocity in the wider section. In this case, the pressure drop over the straightener is uniform, and the velocity pattern of the gas leaving the straightener is also uniform. Usually porous media such as stacked wire gauzes or sintered metal powder discs are used as flow straighteners.

### 2.5.5 Conclusions

In this paragraph, three important losses in the pulse tube are considered. Turbulence, surface heat pumping, and streaming influence the three-dimensional mass and heat flow in the pulse tube. Because of this three dimensional aspect, and the fact that these effects tend to be very small compared to the dominant processes in the pulse tube, they cannot easily be calculated. Numerical simulations are necessary to visualize their effects and calculate the magnitude. The models that should be capable of doing so are described in the next chapter.
Figure 2.25: Flow straightening transforms the jet-flow into a uniform flow.
Bibliography


Chapter 3
Modeling and Simulation

3.1 Introduction
In this chapter, several tools are described that can be used to design Stirling and pulse-tube cryocoolers. The chapter is divided into two sections. The first section describes a computer program [1],[2] that has been in use for a long time at Stirling Cryogenics & Refrigeration BV and before that at Philips, for the modeling of Stirling machines. Some basic equations, algorithms and features are explained to clarify the design described in chapter 7.

The second section focuses on modeling of pulse-tube refrigerators. Two approaches have been used. First of all, two models are described, based on one-dimensional, first-order harmonic approximations. Because of these approximations, these models are capable of producing results without significant computational costs.

The processes in the pulse tube itself require more extensive modeling than just the one-dimensional, first-order harmonic modeling. The gas dynamics in the pulse tube itself can yield high losses, for instance due to turbulence or other multidimensional effects. For that, two numerical modeling methods are described: one based on commercial Computational Fluid Dynamics (CFD) software and one based on a numerical code developed specifically for PTR’s. The latter provides a significant reduction of computational costs, together with an appropriate numerical description of the relevant processes in the pulse tube [3].

The goal of the development of the simulation models for PTR’s is an integral design tool that uses both the first-order and the numerical model together. The first-order model is used to do the large number of calculations required for the trend calculations and optimization. The numerical model is used for fine-tuning, for instance through the optimization of the geometry of the pulse tube, heat exchangers, etc.

3.2 Stirling Modeling

3.2.1 The Stirling Model
Introduction
The model that is presented in this section was developed by the Philips Research Laboratories as a part of their extensive research program into Stirling engines and coolers. Developing it was not part of this thesis, but the model is described here because it was used in designing
CHAPTER 3. MODELING AND SIMULATION

the Stirling-cycle cryocooler discussed in chapter 7, and was the basis for the Stirling pulse-tube model from the next section.

Several versions of this model were built over time. In this section we treat the basic version. In chapter 7, where the design calculations for the Stirling-cycle cryocooler are mentioned, the so-called branches model will be used.

Central feature in the development of the model was minimizing the required computation time. The development started in the days that computer capacity was limited, a first version of the model was even meant to be calculated by hand (this would take one person three months [4]). Therefore, some important assumptions and simplifications were made. First of all, all properties were considered to be first-order harmonically varying quantities. This avoids having to calculate with time-varying quantities. Furthermore the model assumes all components as one-dimensional.

A second important approximation is the decoupling of mass and energy calculation from the momentum calculation. This implies that the temperature profile and thermal losses are calculated independently of the pressure drop.

The most important feature of the program is the way the actual geometries of different components were taken into account. The model is a complete design tool. To ensure high accuracy despite the necessary approximations, the model was calibrated with extensive measurements. For processes like heat transfer and pressure drop, simple one-dimensional analysis is used, that can for instance be found in [5]. These analyses use empirical relations for heat transfer coefficients, pressure drop, etc. in simple one-dimensional equations. These empirical relations were determined with dedicated experimental setups under operating conditions valid for Stirling-cycle machines [6].

The result is a model that can best be described as the ultimate design tool: a fast calculating model, incorporating all relevant components and physical processes, capable of accurately predicting and optimizing the performance of a Stirling-cycle machine.

Thermodynamic considerations

The greatest value of the Stirling program is the combination of accuracy and calculation speed. There are, however, a few issues that need clarification. First of all, the energy equation is somewhat ad-hoc. Energy flows from different components and sources are combined, to obtain the final, 'net' results. This is the result of the somewhat awkward integration of the temperature and pressure profiles.

The model decouples the equations for energy flows and pressure drop. First, the temperature profile and energy flows are calculated under the assumption that there is no pressure drop. This leads to the so-called undisturbed situation. The influence of pressure drop is then calculated as a correction. The corrections to the different energy flows are presented as losses, in terms of energy flows. These energy flows cannot be described in terms of the first and second laws of thermodynamics as described in the previous chapter. An example is the influence of pressure drop in the regenerator on the cooling power. As a result of pressure drop, the pressure amplitude in the expansion space is smaller than in the undisturbed case. As a result, the work done on the expansion piston will be smaller. The difference is considered as the 'flow loss' on the expansion piston.

If we would use the same method in a pulse-tube model, a pressure drop over the regenerator would lead to a change in pressure amplitude in the pulse tube, which would then lead to a change in the amount of heat that has to be removed from the hot heat exchanger. If one is not aware of the ambiguous use of the term 'loss' it could lead to the wrong conclusions.
CHAPTER 3. MODELING AND SIMULATION

Figure 3.1: Schematic representation of the control volumes used by the Stirling Program. The regenerator has 16 control volumes, each heat exchanger 2. Around the heat exchangers control volumes are present to include void volumes. Together with the compression and expansion space, the total number of control volumes is 28.

The machine is divided into a number of control volumes. This is shown schematically in figure 3.1. The control volume distribution is determined by the geometry. As seen in figure 3.1, there are in total 28 control volumes. The compression space and expansion space are each represented by a single adiabatic volume. Around the heat exchanger a few control volumes are present to account for spaces that are close enough to the heat exchanger to be treated isothermally. Finally, the regenerator is divided into 16 control volumes. These are also treated isothermally.

The geometry is treated one-dimensional, and all time-dependent quantities are approximated to the first order. Within one control volume, all properties are considered homogenous.

In figure 3.2 the flow chart of the Stirling model is shown. The entire model structure comprises of several nested loops. The innermost loop is the iteration loop for undisturbed pressure, mass flow, and temperature. It is called undisturbed because the system is treated as an ideal Stirling machine, i.e. without any losses.

Next the temperature profile and thermal losses (heat exchangers, regenerator) are calculated. This is done using conservation of energy. The calculation starts again with this new temperature profile. This is repeated until the changes in temperature are below a specified tolerance value.

With the known temperature profile and mass flow, the pressure drop over each component is calculated. This pressure drop is treated as a correction to the undisturbed pressure.

With this corrected pressure distribution, mass flows and temperature profiles are iterated one more time. The total hydro- and thermodynamic state of the machine is determined. A next iteration loop can be run to do a series of calculations for optimization purposes.

Figure 3.2: Flow chart of the Stirling Model.

Flow chart and model structure

The undisturbed pressure in the system is described by the following function \[1\]
\[ p(t) = p_0(1 - 2e \cos(\gamma(t))), \]  

(3.1)

with

\[ \gamma(t) = \omega t - \theta_d. \]  

(3.2)

The factor \( e \) is the dimensionless, half pressure amplitude. The phase angle \( \theta_d \) represents the phase difference between undisturbed pressure and displacer position. The mass flow through control volume \( j \) is written as

\[ \dot{m}_j(t) = m_{a,j} \sin(\gamma(t) - \Gamma_j). \]  

(3.3)

The phase angle \( \Gamma_j \) describes the local phase difference between mass flow and undisturbed pressure. Because the pressure is uniform, the undisturbed pressure and mass flows can be calculated using algebraic equations, for a given temperature profile [1].

### Losses

Thermal losses occur in different locations. The imperfectness of the heat exchangers cause thermal losses, but the most significant losses occur in the regenerator.

The heat exchanger losses result in a temperature difference between the gas and the heat exchanger body itself. This means that the cooling power is delivered at a higher temperature than it is generated in the Stirling cycle. This is not directly associated with an enthalpy flow. The relevant quantity is the average gas temperature. At a lower temperature, regenerator losses will be higher and thus cooling power and efficiency will be lower.

The regenerator losses do result in an energy flow. This energy flow consists of an enthalpy flow due to heat transfer phenomena, and a heat flow due to conduction. The total average regenerator loss is assumed to consist of 5 contributions [1],

\[ E_{\text{reg}} = I_\Phi + I_c + I_\Lambda + I_p + I_\kappa. \]  

(3.4)

The five loss mechanisms are given and explained below. The equations are given as is, in order to explain the phenomena they describe. They are calculated for each control volume separately. The materials properties are temperature dependent, and are calculated using the average temperature in the respective control volume. The five effects are:

**\( \Phi \)-effect.** These are the losses due to the finite heat capacity of the regenerator matrix. The contribution of this effect is proportional to the ratio between the heat capacity of the gas and the total heat capacity of the regenerator, per unit of volume,

\[ I_\Phi = (1 - \rho c_T) \cdot \Phi_e \dot{Q}_t, \]  

(3.5)

\[ \Phi_e = (1 - f)\rho_e \frac{c_p}{c_T}, \]  

(3.6)

\[ c_T = f \rho_m c_m + (1 - f)\rho c_p. \]  

(3.7)

With \( \dot{Q}_t \) the cooling power, \( \rho_e \) the gas density in the expansion space, \( c_T \) the total regenerator specific heat capacity (gas+matrix), \( \rho_m \) the density of the matrix material, and \( c_m \) the specific heat capacity of the matrix material. The pressure dependence of specific enthalpy \( c_T \) is the same as equation 2.21, but then for the specific enthalpy rather than the molar enthalpy.
Pressure-dependent enthalpy flow: This is the enthalpy flow due to the pressure oscillations in the regenerator and the nonideality of the gas. This effect does not exist when an ideal gas is used \((c_T = 0)\). It is defined as

\[
I_{c_T} = c_T \rho_e \dot{Q}. \tag{3.8}
\]

This term is proportional to the mass flow and pressure on the entrance of the regenerator. It can be either positive or negative, depending on the sign of \(c_T\). It is also often called the Joule-Thomson effect.

Heat-transfer losses: Losses due to the finite heat transfer coefficient and the temperature gradient in the matrix

\[
I_{\Lambda} = \frac{c_p m \Delta T}{\Lambda + 2 - \Omega}. \tag{3.9}
\]

This term is proportional to the temperature difference \(\Delta T\) over the boundaries of the control volume, the mass flow through the volume, and inversely proportional to the heat transfer coefficient \(\Lambda\). The heat transfer coefficient is equal to the Number of Transfer Units (NTU), and defined as

\[
\Lambda = \frac{\alpha A_{\text{wet}}}{\dot{m} c_p}, \tag{3.10}
\]

where \(A_{\text{wet}}\) is the total wetted surface in the respective control volume. The factor \(\Omega\) is a dimensionless empirical correction parameter.

Pressure effect: Losses due to the finite heat transfer coefficient and heat capacity in combination with pressure oscillations. The contribution to the total enthalpy flow is

\[
I_p = -r_\pi \zeta I_\Phi. \tag{3.11}
\]

The factor \(r_\pi\) is a constant that is inversely proportional to the heat transfer coefficient, the factor \(\zeta\) is proportional to the pressure amplitude.

Heat conduction through the regenerator matrix: Losses due to heat conduction. These are proportional to the temperature gradient over the control volume

\[
I_\kappa = -\kappa_{\text{eff},\pi} A_{\lambda} \frac{\Delta T}{\Delta l}. \tag{3.12}
\]

The heat transfer effect and the heat conduction both depend on the temperature gradient, the other three effects don’t. Thus we can write for the temperature difference over control volume \(j\)

\[
\Delta T_j = \left( \bar{E}_{\text{reg}} - A_j \right) B_j, \tag{3.13}
\]

with

\[
A_j = I_{\Phi,j} + I_{c_T,j} + I_{p,j}, \tag{3.14}
\]

and

\[
B_j = \frac{\Delta T_j}{I_{\Lambda,j} + I_{\kappa,j}}. \tag{3.15}
\]

Equation 3.13 is similar to what De Waele found in the harmonic approximation for the regenerator [16]. This equation is also the basis for the iteration of the temperature profile in the regenerator.

Pressure drop is calculated after the calculation of undisturbed pressure, mass flows and thermal losses. The pressure drop over control volume \(j\) is calculated with
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\[ \Delta p_j = -c_{w,j} l \frac{1}{2} \rho |v_j| v_j. \] (3.16)

The resistance factor \( c_{w,j} \) is calculated for each position separately and depends on the geometry and on the velocity. The empirical relations that are used for this are determined experimentally under operating conditions.

The pressure drop over one particular control volume is considered to have an effect on the pressure on both sides of this volume. In the compression space, pressure is assumed to increase, in the expansion space pressure is assumed to decrease. The influence of each individual control volume on all the other is integrated to obtain the so-called flow losses. The fact that mass flow and pressure are not in phase, is taken into account.

3.3 Pulse-tube modeling

3.3.1 Introduction

Two types of models have been used to predict the performance of a PTR. The first type approximates and linearizes the processes in the cooler. These models are capable of doing a large number of calculations in a short time, so parametric studies can be done. These models use the following approximations

1. All processes are approached by a first-order harmonic approximation;
2. The machine is regarded as one-dimensional.

A first-order harmonic signal can be written in complex notation

\[ X(x,t) = X_0(x) + \frac{1}{2} \left( \tilde{X}(x)e^{i\omega t} + cc \right). \] (3.17)

The property \( X(x,t) \) is described with an average value \( X_0 \) and a complex amplitude \( \tilde{X} \). The real and imaginary part of \( \tilde{X} \) determine the amplitude. The factor \( cc \) is the complex conjugate.

We then assume that the amplitude is small compared to the average value, with the exception of velocity and molar flow whose average values are usually zero. This way all higher-order terms are much smaller than the first-order terms. In any derivation we end up only with average terms and the first-order amplitudes. Any time-dependent quantity can thus be described by four constant values: an average value, an amplitude, a phase, and the frequency. As a result, calculations are fast, so they can be used for optimizations. Also they provide insight in physical processes, because the models are basically simple. Two first-order harmonic models are described. First a basic harmonic model is described that only describes the pulse tube and the regenerator. Second, a more complete model based on the Stirling model is described, the so-called Stirling pulse-tube model. This model includes all the components of the PTR.

There are several drawbacks to this approach. First of all, these models are one-dimensional. This is already a very large simplification. It may be valid for the regenerator and heat exchangers, but for the pulse tube itself it is not valid. Effects such as mass streaming and surface heat pumping are two-dimensional, and the effect of the orientation of the pulse tube as a result of gravity is even three dimensional.

Some work has been done to construct two-dimensional models which describe, for instance, acoustic streaming and surface heat pumping [7]-[12]. It is very difficult, however, to determine
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what the effect of these phenomena is on the system performance. They provide an explanation on how and why they occur, but do not give an exact solution for the resulting thermodynamic losses. It becomes even more difficult to see the influence of these effects on each other since both the mass streaming and the surface heat pumping effect are determined by and have an influence on the thermal and viscous boundary layers.

For these effects a second type of model is used. The processes in the pulse tube are described in terms of the basic conservation laws, without too many simplifications. As a result, these simulations rely heavily on numerical solvers, since the governing equations can no longer be solved analytically. Two approaches for the numerical solution of the problem are followed. First of all, we describe some numerical experiments that were done with commercially available software. Commercial software has excellent pre- and postprocessing capabilities, so a simulation run is very quickly set up. Drawback is that, as a user, one has limited influence on the algorithms and equations to be solved.

A second approach is a dedicated model. This model was developed in a separate PhD project [3] at the Mathematics department of Eindhoven University of Technology as a part of the pulse tube development program that this thesis is also part of. Some approximations are done to use the optimum numerical schemes for the description of the processes in the pulse tube. At present this model is less user friendly, but it has a higher numerical efficiency, accuracy, and stability then the commercial code.

The multidimensional effects have the most effect on the gas in the pulse tube itself. Here is where the influence of turbulence, gas mixing, surface heat pumping, etc. are most important. The processes in the regenerator, heat exchangers, and compressor can best be calculated with a linear model. So we could make a symbiosis of two modeling methods. Use one-dimensional models for regenerator and heat exchangers, a multidimensional numerical model for the processes in the pulse tube, and use both models to generate each others boundary conditions.

3.3.2 Harmonic model

The harmonic model described here is completely based on the theory of De Waele et al. [13]-[16], summarized into a set of equations capable of calculating the processes in the pulse tube and regenerator. It is assumed that the working fluid is an ideal gas.

Tube dynamics

We describe the pressure in the tube as homogenous, and given by

\[ p(t) = p_0 + \frac{1}{2} (\tilde{p}_t e^{i\omega t} + cc) . \] (3.18)

The absolute phase is assigned to this pressure. The complex amplitude \( \tilde{p} \) has only a real part, (\( \tilde{p}_t = p_{t_a} \)). So the pressure would be described by a \( \cos \) – function. The volume flow through the orifice is then described by

\[ V_h(t) = C_1 (p_t - p_b) = \frac{1}{2} C_1 p_{t_a} \cos(\omega t), \] (3.19)

assuming that the buffer pressure \( p_b \) is equal to the average pressure \( p_0 \); \( C_1 \) is the orifice conduction coefficient. The average enthalpy flow through the pulse tube can be written as:

\[ \overline{H_t} = \frac{1}{2} C_1 p_{t_a}^2. \] (3.20)
As we assume that there are no losses in the pulse tube, we can say that the amount of heat removed from the system at the hot heat exchanger and the cooling power at the cold heat exchanger are the same.

The volume flow at the cold end is described as

\[ \dot{V}_l(t) = \dot{V}_h(t) + \frac{1}{\gamma} \frac{V_t p_t}{p_0} \frac{dp_t}{dt}, \]  
(3.21)

with \( V_t \) the volume of the pulse tube. Since all processes in the pulse tube are assumed to be adiabatic, this relation does not depend on the temperature profile. The only important temperatures are the boundary temperatures \( T_l \) and \( T_h \). Equation 3.21 can be written as

\[ \dot{V}_l(t) = \frac{1}{2} \left( C_1 p_{\text{ta}} + \frac{V_t p_{\text{ta}}}{\gamma p_0} \right) e^{i\omega t} + cc, \]  
(3.22)

or, taking only the real part, as

\[ \dot{V}_l(t) = C_1 p_{\text{ta}}(\cos(\omega t) - \alpha_t \sin(\omega t)), \]  
(3.23)

with

\[ \alpha_t = \frac{\omega V_t}{\gamma C_1 p_0}, \]  
(3.24)

and

\[ \phi_t = \arctan(\alpha_t). \]  
(3.25)

Phase angle \( \phi_t \) is the phase shift between hot end and cold end flow. In [17], it is shown that the pulse tube has optimum performance is reached for \( \alpha \approx 1 \), under the assumption that the void volume in the regenerator is negligible.

**Regenerator dynamics**

In the regenerator we have imperfect heat exchange between the gas and the matrix, as well as pressure drop, both with their associated entropy production. In [15] the equations for energy and mass balance are written in the harmonic approximation.

For mass conservation we write

\[ \frac{\partial N}{\partial t} = \frac{\partial j}{\partial t}, \]  
(3.26)

With \( N \) the amount of moles of gas per unit volume and \( j \) the molar flux. After substituting \( N = \frac{(1-f)}{V_m} \) and harmonic variables in equation 3.26, we get [17]

\[ \frac{1}{(V_{m0} + \frac{1}{2}(\tilde{V}_m e^{i\omega t} + cc))} \frac{1}{2i\omega} \left( \tilde{V}_m e^{i\omega t} + cc \right) = \frac{dj_0}{dt} + \frac{1}{2} \left( \frac{dj}{dt} e^{i\omega t} + cc \right). \]  
(3.27)

The zeroth-order terms give an expression for \( j_0 \)

\[ \frac{dj_0}{dt} = 0. \]  
(3.28)

This means that the average molar flux is uniform in position. In steady state we assume that there is no net mass flow through the system (DC-flow). So we can write
\[ j_0 = 0. \] (3.29)

The first-order terms of equation 3.27 lead to
\[ \frac{d\tilde{j}}{dl} = i\omega(1 - f)\frac{\tilde{V}_m}{V_{m0}}, \] (3.30)
or after substituting \( V_m = RT/p \)
\[ \frac{d\tilde{j}}{dl} = i\omega \frac{(1 - f)p_0}{RT_0} \left( \frac{\tilde{T}_g}{T_0} - \frac{\tilde{p}}{p_0} \right). \] (3.31)

For the pressure drop we use
\[ \frac{dp}{dl} = -\mu z r V_m j. \] (3.32)

Because there is no zeroth-order molar flux, there is no zeroth-order pressure drop
\[ \frac{dp_0}{dl} = 0. \] (3.33)
This means that the average pressure in the pulse tube is uniform. The first-order term is
\[ \frac{d\tilde{p}}{dl} = -\mu_0 z r V_{m0} \tilde{j}. \] (3.34)

Conservation of energy yields two separate equations, one for the gas and one for the matrix. For the latter we have (equation 2.53)
\[ C_r \frac{dT_r}{dt} = \beta(T_g - T_r) + \frac{\partial}{\partial l}(\kappa_r \frac{dT_r}{dl}). \] (3.35)
Collecting zeroth-order terms gives
\[ T_{r0} = T_{g0} + \frac{1}{\beta} \left( \frac{\partial \kappa_r}{\partial l} \frac{dT_{r0}}{dl} + \kappa_r \frac{d^2T_{r0}}{dl^2} \right). \] (3.36)
And similarly for the first-order terms
\[ \tilde{T}_g = \tilde{T}_r \left( 1 + \frac{C_{r0}}{\beta} \right) - \frac{1}{\beta} \left( \frac{\partial \kappa_r}{\partial l} \frac{dT_r}{dl} + \kappa_r \frac{d^2\tilde{T}_r}{dl^2} \right). \] (3.37)

Only the zeroth-order term \( C_{r0} \) of the matrix heat capacity is used. The right-hand side of equation 3.36 can be simplified. The heat transfer coefficient \( \beta \) is rather large, and the position dependency in the part between brackets is small, so the second part of the right-hand side can be neglected. This means that the average matrix temperature is equal to the average gas temperature
\[ T_{r0} = T_{g0} = T_0. \] (3.38)
For equation 3.37 a similar approximation is valid, so the second part on the right-hand side can be neglected here as well. Equation 3.37 then reduces to
\[ \tilde{T}_g = \tilde{T}_r \left( 1 + \frac{C_{r0}}{\beta} \right). \] (3.39)
In order to obtain an equation for the gas temperature, conservation of energy for the matrix and gas together is considered

$$\frac{\partial U}{\partial t} = -\frac{\partial h}{\partial l} - \frac{\partial q}{\partial l},$$  \hspace{1cm} (3.40)

with $U$ the internal energy per unit of volume, $h$ the the enthalpy flux, and $q$ the heat flux. Collecting zeroth-order terms gives

$$\frac{dh_0}{dl} + \frac{dq_0}{dl} = 0.$$  \hspace{1cm} (3.41)

So the total energy flux through the regenerator is constant.

In [15] it is shown that the first-order terms of conduction can be neglected. It is also shown that first-order terms of equation 3.40 lead to

$$N_0 C_p \tilde{T}_g + C_0 \tilde{T}_r = i \frac{C_p j_0}{\omega} \frac{dT_0}{dl} + (1 - f) \tilde{p}.$$  \hspace{1cm} (3.42)

This could also be derived from equation 2.52, after eliminating the $\beta(T_g - T_r)$ term and neglecting the heat conduction terms.

We have now in total 4 complex variables: molar flux $j$, pressure $p$, matrix temperature $T_r$, and gas temperature $T_g$. The zeroth order terms are known, or can easily be calculated. The constant average $p_0$ (equation 3.33) pressure is an input parameter, there is no zeroth order flow, so $j_0 = 0$ (equation 3.28) and the average temperature $T_0$ should be chosen such that equation 3.41 is fulfilled. So we have four equations for these four unknowns, equations 3.31, 3.34, 3.37 and 3.42.

This system of equations can be simplified. Substitution of equation 3.39 in 3.42 gives

$$\tilde{T}_g = \frac{-i \frac{C_p j_0}{\kappa_{eff}} g_0 + (1 - f) \tilde{p}}{N_0 C_p + \frac{\pi C_0}{\beta + \omega C_0}}.$$  \hspace{1cm} (3.43)

This expresses the gas temperature as a function of pressure, flow and average temperature (through $g_0 = -\kappa_{eff} dT_0/dl$). The equation for pressure and flow (3.31 and 3.34) are two coupled differential equations. To decouple them from the temperature equation, we assume that the temperature dependence in 3.31 is much smaller than the pressure variation, so it can be neglected. This can be done because in the regenerator the gas temperature will always be close to the matrix temperature. Two differential equations remain:

$$\frac{dj}{dl} = -i \omega \frac{1 - f}{R T_0} \tilde{p},$$  \hspace{1cm} (3.44)

$$\frac{dp}{dl} = -\eta_0 \tilde{V}_w \tilde{j},$$  \hspace{1cm} (3.45)

which can for instance be solved numerically.

Calculation principles and model limitations

A model of the complete pulse tube would be based on the tube and regenerator dynamics described in the previous section. The input parameters are $\omega$, $p_0$, $p_{ta}$, and $C_1$. The boundary conditions are the heat exchanger temperatures $T_i$ and $T_j$. The flows on the warm and cold end of the pulse tube are calculated with equations 3.19 and 3.21. The cold-end velocity then acts
as a boundary condition for the calculation of the regenerator (equations 3.43, 3.44, and 3.45). The quantities we are interested in are the powers removed from the hot heat exchanger and after cooler, cooling power at the cold heat exchanger, and input power of the compressor. In this section we are considering only the time-averaged values of these quantities. Furthermore we assume that the heat exchangers are ideal.

The heat exchangers and compressor power can be calculated using the first and second law of thermodynamics, as described in chapter 2. The relevant energy and entropy flows are indicated in figure 3.3. The powers can be calculated with

\[ \dot{Q}_{HHX} = \ddot{H}_t - \dot{Q}_{ct} - \dot{Q}_{cw}, \]

\[ \dot{Q}_{CHX} = \ddot{H}_t - \ddot{H}_{reg} - \dot{Q}_{ct} - \dot{Q}_{cw} - \dot{Q}_{reg} - \dot{Q}_{cr}, \]

\[ \dot{Q}_{AC} = T_h \left( \frac{\dot{Q}_{CHX}}{T_l} + \dot{S}_{ir} \right), \]

\[ P = \dot{Q}_{AC} + \ddot{H}_{reg} + \dot{Q}_{c} + \dot{Q}_{cr}. \]

The conduction losses through the gas in the pulse tube and the pulse-tube wall \( \dot{Q}_{ct} \) and \( \dot{Q}_{cw} \) can be calculated from the heat exchanger temperatures and an appropriate length scale (tube length modified with the gas penetration length near the boundaries). The gas and matrix heat conduction \( \dot{Q}_{cg} \) and \( \dot{Q}_{cr} \) through the regenerator are calculated with the average temperature profile in the regenerator.

The only two quantities that are not yet known are regenerator enthalpy flow \( \dot{H}_{reg} \), and regenerator entropy production. For the time average enthalpy flux we use

\[ \ddot{h}_t(l) = \frac{C_p}{T_e} \int_0^l j(l, t) T_g(l, t) dt, \]

and for the entropy production rate per unit of volume \( \sigma_{ir} \) (is equation 2.59).
σ_{ir} = \beta \frac{(T_r - T_g)^2}{T_r T_g} + \frac{\kappa_{g,\text{eff}}}{T_g^2} \left( \frac{\partial T_g}{\partial l} \right)^2 + \frac{\kappa_{r,\text{eff}}}{T_r^2} \left( \frac{\partial T_r}{\partial l} \right)^2 + \mu z_r \frac{(jV_m)^2}{T_g},

(3.51)

with \kappa_{g,\text{eff}} and \kappa_{r,\text{eff}} the effective thermal conductivity of the gas and matrix respectively, \mu the viscosity, and \mu z_r the specific flow impedance. Equations 3.50 and 3.51 can be rewritten in the harmonic approximation as well. In equation 3.51 we see four sources of entropy. The first term is entropy production due to imperfect heat exchange between gas and matrix, the second and third represent heat conduction through gas and matrix respectively, and the last term is entropy production due to flow resistance.

The only remaining unknown is the average temperature in the regenerator. This is found iteratively until equation 3.41 is fulfilled. Another option is the derivation of an algebraic equation as is done in [16].

There are a few limitations to this approach. Apart from the obvious limitations due the one-dimensional approach, a few assumptions were done that limit the validity of this model. The most important one is the ideal gas assumption. Helium is usually treated as an ideal gas, but especially at lower temperatures this is no longer the case. Including real-gas properties would include additional terms in the governing differential equations that give additional sources of entropy production. One of these sources is the so-called Joule-Thomson effect, which is the result of the specific enthalpy of the gas being dependent on pressure. Finally, the model described depends on accurate correlations for heat transfer and pressure drop over regenerator and heat exchangers. Empirical relations from literature [5][18], could be used, but it is also possible to use the empirical relations used in the Stirling model described above. This is done in the so-called Stirling pulse-tube model, described in the next section.

3.3.3 The Stirling pulse-tube model

Introduction

The basis of the Stirling Pulse-Tube Model (SPTM) is the same as the Stirling Model described in section 3.2.1. Since most of the components from a Stirling cryocooler are also found in a pulse-tube refrigerator, it is relatively easy to adapt the Stirling model to calculate pulse tubes. There are still a piston, a regenerator, and heat exchangers. Together with the accurate description of the performance of components such as the regenerator, an accurate design tool is the result.

This adaptation was made and is described in this chapter. Large parts of the model are indeed kept as they were in the Stirling model. Only for the calculation of pressure and mass flow, some changes were made. In the Stirling Model, undisturbed pressure and mass flow were calculated, and flow losses were calculated as a correction to these values. One assumption was made that the difference between undisturbed and disturbed mass flow had little to no influence on the powers, since mass flow is dictated by both pistons motion. In a pulse tube, the second ‘piston’ is replaced by the gas flow through the orifice, which depends on the actual pressure, not the undisturbed. This required a revision of the pressure and mass flow calculation. This is described in more detail in the following sections.

In the SPTM, it is also assumed that the geometry is one-dimensional. Also, for all time dependent quantities, a first-order harmonic approximation is used.

Flow chart and model structure

In figure 3.4 the flow chart of the Stirling pulse-tube model is given. Compared to figure 3.2 three things are changed. First a section ‘Temperature profile tube’ is added. In this block the
average temperature profiles for the wall of the pulse tube and gas are calculated. This is done in a similar way as the temperature profile in the regenerator.

Another thing that is added is an iteration loop. The pressure in the system is no longer calculated analytically, but is determined in an iteration loop. The method used in the Stirling program where undisturbed pressure and flow losses are calculated separately is not valid, since there is a coupling between undisturbed pressure and losses due to the flow through the orifice as mentioned above. Because this additional iteration loop is needed, the entire 'undisturbed' method is left out, and pressure drop is integrated in the mass flow calculations. Therefore the block 'flow losses' is removed from the flow chart. How this new pressure-calculation algorithm works is described in the next section. The relevant powers from the heat exchangers are calculated in the block 'energy balance'. Similarly to the harmonic model, the hot and cold heat exchanger power are calculated from equations 3.46 and 3.47. The enthalpy flow through the tube is calculated by integration of mass flow, heat capacity, and temperature, similar to equation 3.50. The compression power is calculated by integrating pressure and piston velocity, which is then used to calculate the after-cooler power with equation 3.48.

The system is divided into a number of control volumes. Some volumes are considered adiabatic, some isothermal. There is also a distinction between isothermal volumes without a temperature gradient (e.g. heat exchangers), and isothermal spaces with a gradient (regenerator). The size of the control volumes is fixed, and determined by the geometry. In total 48 control volumes are used. The regenerator and pulse tube are each divided into 16 control volumes. Two control volumes are used per heat exchanger. The rest is used for the compression space and void volumes. These void volumes are spaces that are not part of a specific component of the PTR, but they still contain gas that is part of the cycle. An example of such a volume is the connection between the orifice and the hot heat exchanger.

The number of control volumes appears rather small, given the computing power that is available nowadays. This is partly due to the fact that we use the Stirling model as a basis. But more important it is not necessary to use more volumes. Within one control volume, all properties are considered uniform. The control volume are not used to accurately resolve gas behavior such as is done in numerical modeling techniques. They are used as a computational basis for a more or less analytical approach, comparable to what is described in the previous section; in the harmonic model we describe the pulse tube by a single volume. If, in the future, the 16 volumes appear to be insufficient, for instance if we would like to include streaming effects, the number can be increased easily.

Since it is based on the Stirling Model, the pulse-tube model uses the same method and nomenclature in describing parameters, processes and quantities.

**Mass and pressure**

The pressure in control volume $j$ is described by the following first order harmonic equation
The phase angle $\gamma_j(t)$ and pressure amplitude $p_{a,j}$ are different for each control volume $j$. The average pressure is the same for all control volumes. The phase angle is referenced to the position of the compression piston

$$V_c = \frac{V_{ca}}{2}(1 + \cos(\omega t)),$$

and

$$\gamma_j = \omega t - \theta_{p,j}.$$  

Mass flow through the left boundary of control volume $j$ is described as

$$m_j = m_{a,j} \cos(\gamma_j(t) - \Gamma_j).$$

Next, we will derive equations for mass flow through, and pressure drop over a control volume. We treat them separately, first we calculate mass flow while neglecting pressure drop. Secondly, pressure drop is calculated using this mass flow. We consider control volumes of constant size. This is valid for all control volumes, except the compression space. The mass flow through control volume $j$ and the phase difference with respect to pressure can be calculated as follows.

Equation of state 2.17

$$\rho = \frac{pY}{R_M T}$$

is used, which can be rewritten as a so-called process, using equation 2.29, as

$$\frac{p}{\rho^n} = C,$$

where $C$ is a (local) constant, and the power $n$ is determined by the type of process. For isothermal processes, we use $n = 1$. For adiabatic processes, we use $n = 1/(1 - K + \alpha)$ (equation 2.27). Together with this equation, we use the continuity equation (mass conservation)

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0.$$  

The differential operator $\frac{D}{Dt}$ is the total time-derivative, including a convection term: $\frac{D}{Dt} + \mathbf{v} \cdot \nabla$. In one dimension, this becomes

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial l} (\rho \mathbf{v}) = 0.$$  

Substituting the equation of state gives

$$\frac{\partial}{\partial t} \left( \frac{pY}{R_M T} \right) + \frac{\partial}{\partial l} \left( \frac{pY \mathbf{v}}{R_M T} \right) = 0.$$  

This equation can be simplified. Because we are only interested in the first order terms, we only have to take the zeroth order terms of part between brackets into account. Furthermore we assume that we can neglect the time-dependence of the non-ideality factor $Y$. This is done because this factor is already close to 1, and will not vary much within one cycle. For isothermal control volumes, the time-dependence of temperature also disappears. We can thus write
This can be written as
\[ \frac{\partial}{\partial l} \left( \frac{Y_0 v}{T_0} \right) = -Y_0 \frac{\dot{p}}{T_0 p_0}, \] (3.61)
which can be integrated to
\[ v_{j+1}(t) = T_{0,j+1} Y_{0,j+1} v_j(t) - \frac{\dot{p}_j}{p_0} T_{0,j+1} T_{0,j} \int_{x_j}^{x_{j+1}} Y_0(l) \, dl, \] (3.62)
where \( x_j \) and \( x_{j+1} \) denote the boundaries of control volume \( j \). The length of the control volume is equal to \( l_j = x_{j+1} - x_j \). If we assume that \( Y \) does not change within one control volume, and that the temperature profile over the control volume is linear, the integral can be solved
\[ v_{j+1}(t) = T_{0,j+1} v_j(t) - \frac{\dot{p}_j}{p_0} T_{0,j+1} T_{0,j} \ln \left( \frac{T_{0,j+1}}{T_{0,j}} \right). \] (3.63)
With this equation, we can calculate the mass flow entering control volume \( j + 1 \). If we multiply equation 3.63 with the surface area \( A \) and the density in volume \( j + 1 \),
\[ \rho_{0,j+1} = \frac{p_0 Y_{0,j+1}}{R M T_{0,j+1}} \] we get
\[ \dot{m}_{j+1}^* = \dot{m}_j - \frac{\dot{p}_j}{p_0} A \frac{Y_{0,j+1}}{T_{0,j+1}} T_{0,j} \ln \left( \frac{T_{0,j+1}}{T_{0,j}} \right). \] (3.64)
When there is no temperature gradient over the control volume, this limits to
\[ \dot{m}_{j+1}^* = \dot{m}_j - \frac{\dot{p}_j}{p_0} A \frac{Y_{0,j+1}}{T_{0,j+1}} \] (3.65)
To solve equation 3.57 for an adiabatic volume, we use equation 3.56. The solution is
\[ v_{j+1}(t) = v_j(t) - \frac{1}{n} \frac{\dot{p}_j}{p_0} l_j. \] (3.66)
Multiplying with density to obtain mass flow gives
\[ \dot{m}_{j+1}^* = \frac{T_{0,j+1} Y_{0,j+1} \dot{m}_j}{T_{0,j+1}^* Y_{0,j}} \] (3.67)
The factor \( n \) is defined by equations 2.25-2.27.

The last control volume in the system is the compression space. The volume of the compression space varies in time according to equation 3.53. The moving piston closes one end of the compression space. The other end is the stationary interface with the rest of the pulse tube. The compression space has index \( j = 48 \). The compression space is adiabatic. Equation 3.66 can be applied, if we consider the velocity of the piston as one of the boundary velocities, and the volume to be half the swept volume of the piston. The control volume is then described with
The iteration process starts at element 1, the orifice. In the first iteration step, the pressure amplitude in the first cell \( p_{n,1} \) and pressure phase \( \theta_{p,1} \) are assumed. The mass flow is then calculated using

\[
m_{n,1} = X_1 (p_{n,1} - p_b),
\]

\( \text{cf. equation 3.19.} \) The conductance \( X_1 \) depends on the type of phase shifter. For an orifice, \( X_1 \) is a real number, so \( \Gamma_1 = 0 \). For an inertance tube or another type of phase shifter with an inductive or capacitive behavior, the values for \( X_1 \) and \( \Gamma_1 \) have to be determined using the appropriate models for those phase shifters (see chapter 4).

Using equations 3.64, 3.65, and 3.67 for the respective control volumes, we end up with the mass flow into the compression space. This is compared with the mass flow according to equation 3.68, and in an iteration process the initial guesses for pressure amplitude \( p_{n,1} \) and phase \( \theta_1 \) are changed until this mass flow and phase in the compression space are correct.

### Pressure drops

Pressure drop is calculated in the same iteration loop as mass flow. For the pressure drop between two control volumes we can use

\[
p_{j+1} - p_j = -c_{w,j} \frac{l_j}{2} \frac{1}{\rho_j |v_j|} v_j.
\]

The resistance factor \( c_{w,j} \) is determined from the local geometrical and flow characteristics, similar to the way pressure drop was calculated in the Stirling model.

As mentioned before, mass flow through a control volume and pressure drop over a control volume are calculated separately. This can be done because control volumes with high flow resistance have a small volume (regenerator and heat exchangers), so the mass flow entering the control volume will be about equal to the average mass flow. Large control volumes have only low pressure drop (pulse tube), so the error is small here as well.

### Losses in the pulse tube

In the Stirling pulse-tube model, the same algorithms are used for calculating losses as in the Stirling model. The losses determine the temperature profile over the tube, in both gas and wall. In the tube part, there is interaction between gas and wall, both viscous and thermal. So one could describe the pulse tube as a very bad regenerator. Loss mechanisms due to this interaction are for instance surface heat pumping and mass streaming.

In the current version of the computer program only heat conduction through the wall and the gas are incorporated. The temperature profile that is used is schematically represented in figure 3.5. In the entry zones near the heat exchangers the temperature is assumed constant.

For mass streaming and surface heat pumping losses, models and algebraic equations can be found in [19],[20]. Before including such terms in the model, further numerical experimentation needs to be done.
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Figure 3.5: Schematic representation of the control volumes in the pulse tube with the two volumes near the heat exchangers with zero temperature gradient.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>125 rad/s (20 Hz)</td>
</tr>
<tr>
<td>$V_t$</td>
<td>400 cm$^3$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>30 bar</td>
</tr>
<tr>
<td>$p_{h,1}$</td>
<td>5 bar</td>
</tr>
<tr>
<td>$T_h$</td>
<td>300 K</td>
</tr>
<tr>
<td>$T_l$</td>
<td>80 K</td>
</tr>
</tbody>
</table>

Table 3.1: Basic parameters for the pulse tube used for the comparison between the harmonic and Stirling pulse tube model.

Model comparison

Because the harmonic pulse-tube model (HPTM) and the SPTM operate under the same principles and assumptions, they can be compared. With the outcome of the HPTM described in section 3.3.2, the SPTM can be validated. To do this validation, we assume an arbitrary pulse tube. Its properties are given in table 3.1.

The orifice flow conductance is a variable. The swept volume of the compressor is adapted to obtain the pressure amplitude $p_{h,1}$ as given in table 3.1. In figure 3.6 the enthalpy flows through the tube for both the HPTM and the SPTM are given as functions of the orifice flow conductance $C_1$.

Figure 3.6: Enthalpy flow through the tube as a function of orifice flow conductance for the HPTM and the SPTM, for the pulse tube of table 3.1.
The agreement between the two models is very good. The small difference comes from the influence of the flow resistance of the hot heat exchanger, and some difference in materials properties of the gas. A similar comparison can be made for the phase shift between the mass flow at hot end and cold end. This phase shift can be expressed in the parameter $\alpha$, cf equation 3.24 and equation 3.25. This is given in figure 3.7.

The correspondence for the phase shift is good too. With the comparison made above, one does not have to compare the mass flows at hot and cold end, since they are present in the enthalpy flow and phase shift as well.

A further aspect that we consider is conservation of energy. If we consider the entire cryocooler as one thermodynamic system the work done by the piston should be equal to the total amount of heat removed from the three heat exchangers. In figure 3.8, these are shown as functions of the orifice flow conductance. An arbitrary regenerator geometry is used. One can clearly see that energy is conserved.

Perhaps more interesting is the comparison of the regenerator losses between the two models. This is a difficult comparison however, since in the Stirling pulse-tube model much more components are taken into account. An example calculation for the above mentioned sample pulse tube is given in table 3.2. Here we see the cooling power, shaft power, regenerator loss, and pressure in the compression space, for $T_1 = 80$ K, $T_h = 300$ K, and $C_1 = 10$ mm$^3$/Pas. The regenerator has a diameter of 8 cm, and a length of 8 cm. Other parameters are mentioned in table 3.1.

There appears to be a fairly large difference between the two predictions. The main reason

---

**Figure 3.7:** Phase shift factors $\alpha$ as a functions of orifice flow conductance for the HPTM and the SPTM, for the pulse tube of table 3.1.

**Table 3.2:** Results for the same pulse tube refrigerator, calculated with the harmonic model and the Stirling pulse tube model

<table>
<thead>
<tr>
<th></th>
<th>$\dot{Q}_1$ [W]</th>
<th>$P$ [kW]</th>
<th>$\dot{E}_{\text{reg}}$ [W]</th>
<th>$p_{ca}$ [bar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPTM</td>
<td>860</td>
<td>10.4</td>
<td>380</td>
<td>7.3</td>
</tr>
<tr>
<td>SPTM</td>
<td>660</td>
<td>13.7</td>
<td>730</td>
<td>8.0</td>
</tr>
</tbody>
</table>
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![Figure 3.8: Validation of conservation of energy for the SPTM. Compressor work and the total amount of heat removed from the system through the three heat exchangers as a function of orifice flow conductance.](image)

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Stirling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{in}$ [K]</td>
<td>80</td>
</tr>
<tr>
<td>$T_{out}$ [K]</td>
<td>300</td>
</tr>
<tr>
<td>$E_{reg}$ [W]</td>
<td>380</td>
</tr>
<tr>
<td>$I_\Phi$</td>
<td>-</td>
</tr>
<tr>
<td>$I_c$</td>
<td>-</td>
</tr>
<tr>
<td>$I_p$</td>
<td>-</td>
</tr>
<tr>
<td>$I_A + I_\kappa$</td>
<td>380</td>
</tr>
</tbody>
</table>

Table 3.3: Calculation results for the regenerator. Gas temperatures on the hot and cold end of the regenerator, and individual regenerator loss terms. In the harmonic model, only enthalpy flow due to heat exchange and heat conduction are present.

is that the Stirling model takes the heat exchangers into account. Pressure drop over these heat exchangers accounts for part of the difference in compressor pressure amplitude. More important, they influence the gas temperatures on the both ends of the regenerator. Furthermore, the Stirling model follows a different approach towards the regenerator losses (equations 3.5 - 3.12). The gas temperatures at the inlet and outlet of the regenerator also have an influence on pressure drop. In table 3.3 these effects are summarized.

Taking the results shown in figures 3.6-3.8 into account, the correspondence of the two models is good. This means that the basic principles are implemented consistently. From tables 3.2 and 3.3 it can be seen that the SPTM takes more components and effects into account than the harmonic model, and is therefore a more complete simulation model. Final validation of the Stirling pulse-tube model will be done experimentally. This is described in chapter 6.

3.3.4 Commercial CFD Code

We were among the first to apply Computational Fluid Dynamics (CFD) to pulse-tube refrigerators [21]. Currently, a fair number of CFD codes are commercially available. They are mostly based on the finite volume method, which basically aims at preserving conservation of fluxes of mass, momentum, and energy in small control volumes. So the Navier-Stokes (N-S) equations are solved in the integral formulation. Inside one of these cells, the equations are linearized. By iteration, the full set of coupled N-S equations are solved for the entire collection of control vol-
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<table>
<thead>
<tr>
<th>$L_t$</th>
<th>20 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_t$</td>
<td>5 cm</td>
</tr>
<tr>
<td>$T_1$</td>
<td>70 K</td>
</tr>
<tr>
<td>$T_3$</td>
<td>300 K</td>
</tr>
<tr>
<td>$\omega$</td>
<td>125.6 rad/s (20 Hz)</td>
</tr>
<tr>
<td>$p_0$</td>
<td>30 bar</td>
</tr>
<tr>
<td>$p_{ha}$</td>
<td>5 bar</td>
</tr>
<tr>
<td>$C_1$</td>
<td>10 mm$^3$/Pas</td>
</tr>
</tbody>
</table>

Table 3.4: basic pulse tube parameters used in the numerical simulations

umes. In addition to the basic N-S equations, additional equations are modelled, e.g. describing turbulence and fluid behavior near walls. For a detailed description of the finite volume method, we refer to e.g. [22], and the user manual of the CFD code that was used [23].

**Basic model setup, geometry and meshing**

We have set up a simulation of the time-dependent flow inside the pulse tube. We assumed an ideal regenerator and ideal heat exchangers, which determine the boundary conditions. The geometry is then a simple cylinder with open ends to allow gas to flow into and out of the tube.

The simulations are considered quasi-steady state. This means that system transients such as cool down, are not taken into account, so the temperatures at the boundaries are constant in time. On a shorter time scale, i.e. within one cycle, the simulation is of course treated time-dependent. The equations are solved for fully-compressible flow, so no simplifications are made with that regard. The advection scheme is high-resolution second order, to ensure an accurate description of the high gradients.

Because of the cylindrical geometry, a structured mesh of hexagonal volumes is used. Care is taken that the mesh spacing is smaller in special regions, such as near the boundaries and near the walls. In the center of the tube grid can be coarser.

The basic parameters of the pulse tube under consideration are given in table 3.4. The initial conditions are estimated using the HPTM. The simulation is run for several cycles until a steady situation is reached, so that the average solution does not change within a small tolerance band. The real time that is thus calculated is typically 0.5 seconds. This is much smaller than the typical time scales for stabilization, as observed from experiments (typically a few minutes). It is therefore not very likely that the steady state is the final steady state that one would observe in practice. The situation that is reached after a number of cycles is the situation where the influence of the initial conditions is stabilized, so the processes that are observed are based on actual physical processes, rather than numerical artifacts.

**Boundary conditions**

In any numerical simulation boundary conditions need to be looked at carefully. In our system there are three boundaries: at $z = 0$ (where the CHX would be), at $z = L_t$ (where the HHX would be), and the wall. Since we assumed that the regenerator and heat exchangers are ideal, they can be described by a fixed-temperature boundary condition. So when gas enters the tube through either end, the temperature of the gas is always equal to the heat-exchanger temperature.

At the cold end a sinusoidally varying pressure is used as the boundary condition. The average pressure is taken 30 bar with an amplitude of 5 bar. At the hot end the velocity is described
with the following equation:

\[ v_h = \frac{C_1}{A_t} (p_h(t) - p_b). \tag{3.71} \]

We only considered the orifice-type pulse tube, so the flow is always in phase with the pressure. The average buffer pressure \( p_b \) is not the same as the average pulse-tube pressure, due to the fact that the density is not constant. The more accurate solution [17] for the buffer pressure is not accurate enough, so it is determined by requiring that the average mass flow over one cycle is zero.

The velocity profiles at the boundaries are taken uniform, but a specific profile could be used to simulate imperfect flow straightening from the heat exchangers. In most cases the default wall-boundary condition was an adiabatic, no-slip wall. This means that the gas velocity is zero on the surface of the wall, and there is no heat transfer between the gas and the wall. In addition to that, we also considered a case with heat transfer.

**Validation with 'ideal' assumptions**

To validate the results from the simulations, we start with a simple setup. The wall is considered adiabatic, the working fluid is considered an ideal gas and materials properties are taken constant. Heat conduction is neglected. The results from this calculation are compared with the results from the harmonic model, as described in section 3.3.2, equations 3.20 and 3.21.

The system is run for approximately 10 cycles to obtain a steady situation. Quantitative data is extracted at both the hot- and cold-end boundaries. The time-averaged values for mass flow and enthalpy flow are determined by extracting these quantities at 40 time steps in the final cycle. In figure 3.9 the average temperatures near the hot- and cold-end boundaries are given, together with the analytical results. In figure 3.10 the same for the total mass flows. In table 3.5 the time-averaged values for enthalpy flow are given. The analytical results are calculated using equation 3.20.

As seen from figures 3.9 and 3.10, the two models show comparable results. In figure 3.10 a slight difference in mass flow can be seen. The difference is caused by the first order approximation of the analytical model versus the higher order CFD model. Since density is not approximated to the first order in the CFD calculations, the mass flow amplitude is slightly higher.

The overshoot of the temperature at the cold end boundary (figure 3.9) has several causes. As mentioned, the average buffer pressure \( p_b \) has to be carefully chosen to make the net mass flow zero. Nevertheless a small DC mass flow of the order \( 10^{-2} \text{ g/s} \) in the negative direction remains, causing the temperature distribution to slowly shift toward the cold end. Another cause is the establishing of a stable temperature profile from the initial temperature distribution. Because of the absence of heat conduction this leads to steep temperature gradients in the region between the ‘gas piston’ and the boundary region. This, together with a third effect called numerical heat conduction or numerical diffusion [23], leads to the smoothing of the steep gradients in the aforementioned region. However, this effect is expected to be small in the case of a structured mesh and a second order advection scheme. Therefore, it is expected that the overshoot would disappear after a larger number of cycles. The influence of the initial condition is also the cause

<table>
<thead>
<tr>
<th></th>
<th>CFD</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm end</td>
<td>1034</td>
<td>1125</td>
</tr>
<tr>
<td>Cold end</td>
<td>1033</td>
<td>1125</td>
</tr>
</tbody>
</table>

*Table 3.5: Time-averaged values for enthalpy flow [W] in axial direction, ideal case.*
Figure 3.9: Temperature profiles as a function of time at the hot and cold end boundaries. Results from CFD simulations and analytical model.

Figure 3.10: Mass flow at the cold and warm end boundaries. Results from the CFD simulations and the analytical model.
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<table>
<thead>
<tr>
<th></th>
<th>CFD</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm end</td>
<td>965</td>
<td>1100</td>
</tr>
<tr>
<td>Cold end</td>
<td>675</td>
<td>1100</td>
</tr>
<tr>
<td>Wall</td>
<td>315</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.6: Time-averaged results for energy flow [W] in the nonideal case. The difference between the hot end and cold end energy flow is due to heat transfer through the wall to the gas.

for the observed differences between CFD and analytical, for the temperature on the warm end, and for the average enthalpy flow (Table 3.5). It appeared that an increase in mesh density and an increase of number of time steps per cycle and iterations per time step had no influence on the effect, so the numerical setup is not the cause.

Nonideal case

The ideal situation, discussed in the previous section, is now abandoned. Equations for the real material properties are inserted in the program, and the gas is treated as a real fluid instead of an ideal gas. The tube wall is no longer treated as an adiabatic surface, but is included in the calculation. An additional energy equation for the wall is solved, including thermal conductivity within the material, and heat exchange with the gas. The boundary conditions for the wall are the heat exchanger temperatures. The initial condition for the wall temperature is a fixed linear temperature profile between the heat exchanger temperatures. The average buffer pressure is found to be slightly different from the ideal case (0.2 kPa less on a 3 MPa level). The same temperature overshoot as in figure 3.9, is found.

In table 3.6 the results for time-averaged energy flow are compared to the analytical results. The results are different from the ideal case, especially at the cold end. This is partly due to the material properties and a different effective volume flow through the orifice. The biggest cause, however, is the heat exchange between the gas and the tube wall. In these simulations the initial condition for the wall temperature is linear. In reality, heat conduction, surface heat pumping and the finite heat capacity of the wall material will lead to a steady distribution which is probably some average between a linear profile and the average gas temperature profile. However, the physical time scales required for reaching a steady state temperature profile are much larger than can practically be calculated using this model, even when a more realistic initial temperature profile would be used.

The solver calculates how much heat is transferred from the gas to the wall. If this wall heat flux is averaged over time, an estimation of the losses can be made. In figure 3.11, the time-averaged wall heat flux is shown on the z-axis. The total heat flux over the tube wall is 9.9 kW/m². There is therefore a net heat flux into the gas of 315 W. The difference between the energy flows into and from the hot and cold end boundary (table 3.6) is caused by this wall heat flux. The remaining difference (≈2%) is probably due to the numerical accuracy.

Mass streaming

Mass streaming is a second-order effect that leads to a steady mass flow in the pulse tube (section 2.54). It is caused by friction between the gas and the tube wall. The amount of friction depends on the viscosity of the gas. The viscosity depends on temperature. As a result of temperature oscillations in the pulse tube, the amount of friction also varies with time. If the amount of friction in one flow direction is different from the opposite flow direction, a steady mass flow in
one direction results. This steady mass flux in the viscous boundary layer is compensated by an opposite mass flux in the centre of the pulse tube. As a result, a vortex develops in the pulse tube. This type of mass streaming is often referred to as acoustic streaming.

The result of mass streaming is one or more torus-like vortices. Since this is a second-order effect, manifested on a much longer time scale than exists within one cycle, it is not visible in momentary velocity profiles. The vortex can be visualized by taking the time-average of the local mass flux. In figure 3.12, a vector plot is given of these averages. The vortex is clearly visible. It is also visible in figure 3.12 that the net mass flux over the cold end boundary (\(z = 0\)) is not completely zero: a small mass flows remains. This means that the system is not yet fully in the steady state. However, this mass flow is so small that it does not lead to large errors in the total enthalpy flow.

Streaming can also be seen from the temperature distribution in the tube. In the absence of streaming, the temperature profiles are flat. Streaming leads to a deformation of these profiles. In figure 3.13 lines of constant temperature in the tube are given. The distribution is not uniform, but distorted as the result streaming.

From the work of Olson and Swift [19] an estimate can be made of the magnitude of the streaming mass flux. The net mass flux in the centre of the tube resulting from our simulations is approximately 0.7 kg/sm². From [19] a flux of approximately 1.8 kg/sm² is expected. The model of Olson and Swift assumes that the tube length is much larger than its radius. This is not valid in the geometry used for our CFD simulations, hence the difference in mass flux. The direction of streaming is similar to what is observed by Shirashi et al. [28].

**Influence of gravity**

The effects mentioned in the previous sections are basically two-dimensional. Within the given geometry, and the CFD software, it is possible to calculate full three-dimensional flow dynamics. One such effect is buoyancy in tilted pulse tubes. It has been reported that, in some cases, tilting a pulse tube leads to degradation of the performance of the system [28]. It appears that the effect

---

*Figure 3.11: Net wall heat flux. Positive means that heat flows from the wall to the gas. The cold end is at \(z = 0\), the hot end at \(z = L_t\).*
Figure 3.12: Vector plot showing net streaming mass flux, in a plane through the center of the tube.

Figure 3.13: Distorted temperature contour plots after 10 cycles, at the beginning of the cycle.
is stronger in low-frequency pulse tubes.

Taking the effects of buoyancy into consideration in the calculations is a matter of specifying
the magnitude and direction of gravity. In this case the tube was considered to be horizontal.
The results from the run mentioned in the previous sections are taken as the initial conditions
for this run. The results are taken from the tenth cycle. In figure 3.14, the time-averaged mass
flux in a vertical plane are shown. In figure 3.15, time-average mass flux in a horizontal plane
are shown. The pattern of the time-averaged vortex is clearly three-dimensional. The largest
influence is visible in the vertical plane (figure 3.14). The influence of the gravity is visible in two
regions. Near the cold heat exchanger a recirculation zone is visible, because the gas leaves the
tube at low temperature (high density), and enters the tube at higher temperature (low density).
This recirculation is not visible on the warm end, because the velocity profile is imposed by the
boundary condition. In the centre of the tube, another vortex is visible, clearly different from
the streaming vortex in the horizontal plane. Similar effects have been visualized experimentally
as well [28].

The influence on the enthalpy flows is small. A reduction of cooling power of approximately 20
W is observed. Further simulations have to be carried out to establish the influence of geometry
and frequency on streaming.

Conclusions

The simulations with the commercial CFD code show that it is possible to simulate the flow in a
pulse tube relatively easily. Streaming and gravitational effects can be shown in an elegant way.
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The model is limited by the boundary conditions. The influence of the heat exchangers and flow straighteners is not included. Furthermore, the assumption for the sinusoidal pressure profile is a first-order approximation. The boundary condition would be more precise if a regenerator and piston motion were included. The incorporation of a buffer would mean that the mass flow over the hot end boundary would be balanced by definition.

The difference between hot end and cold end boundary enthalpy flux pose a bigger problem. To get the proper conservation of energy, the temperature profile in the wall should converge to a steady value at which there is no net heat transfer to the wall. But that process would require a large number of cycles to be calculated, since there is no way of predicting beforehand what this profile is going to be. The current 10 cycle simulation runs already took approximately one week of computing time on a standard 1 GHz PC. It is therefore not useful to use this code in the design process. Furthermore, accuracy of some effects such as gas-wall interaction is limited because of the numerical methods that are used. Therefore, numerical simulations are continued with a new model, which does not have these limitations. This model is described in the next section.

3.3.5 Dedicated numerical model

Introduction

The work with the commercial codes showed that it is possible to simulate processes in the pulse tube in three dimensions. However, this commercial code has also serious limitations. First of all, the user has no influence on the numerical scheme and solution methods. The general purpose solver turned out to be not very efficient in terms of calculation time. This makes that approach not suitable for cryocooler design. Furthermore, the numerical scheme might not be accurate enough. According to Lyulina [3], standard numerical solvers are not very accurate for compressible flows like in the pulse tube. Finally, the code used above is limited in the description of the interaction of the gas with the wall. In the first computational cell near the wall, the solution is found using empirical data [23], which is based on stationary incompressible flow. Full numerical simulation of wall interaction is necessary, but would have made the simulations described above even more time-consuming.

Therefore it was decided that a numerical code was to be developed to accurately model the three-dimensional flow in the pulse tube. This development was done by the Scientific Computing Group of the Mathematics Department of Eindhoven University of Technology, and was carried out as a separate PhD project. However, this was part of a joint project so it will be discussed here very briefly. The results are described in detail in Irina Lyulina’s thesis [3]. In this section, only a summary is given of the model, and its results.

The objectives of the project were

• development of a mathematical model for simulating the heat transport in compressible oscillating gas flow in the tube section of a pulse-tube refrigerator;

• development of suitable numerical methods;

• implementation of the developed model in a simulation tool for calculating the dynamic characteristics of the cooling system.

The purpose is to predict energy flow and losses in the pulse tube. Losses in the regenerator have already been studied [15],[16],[24]-[27], but losses in the pulse tube are not easily quantified. The proposed numerical model is a general model, not limited to the prediction of certain isolated effects. It can estimate the relevant parameters of a cooler, such as temperature, velocity, mass flow, energy flow, heat transfer, and fluid-wall interaction in oscillating flow.
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Figure 3.16: Pulse tube geometry used in the 2-D simulations. Only the pulse tube is considered. The HHX is placed at $z = 0$, CHX at $z = L$.

Numerical principles

The code is based on the solution of the Navier-Stokes equations in one or two dimensions. The details of the numerical model will not be given here. They can be found in [3]. An important aspect of the code is the so-called low Mach-number approximation.

The principle of the low Mach-number approximation is explained by looking at one of the Navier-Stokes equations, the momentum equation. In one dimension, this equation reads in dimensionless formulation [3]

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = -\frac{1}{M_a^2} \frac{\partial p}{\partial x} + \frac{4}{3R_e} \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right).$$

Lyulina introduced the so-called modified Mach number

$$M_a = \frac{\bar{\nu}}{\sqrt{\rho_0/\bar{\rho}}},$$

where $\bar{\nu}$ and $\bar{\rho}$ denote typical values for velocity and density. In a pulse tube, a typical value for $M_a = 10^{-3}$. This leads to the usual assumption to neglect the pressure gradient. However, effects such as streaming are caused by the very small pressure gradients and would thus be neglected as well. Lyulina therefore used the low Mach-number approximation to include these gradients without having to use very high numerical accuracy. The low Mach-number approximation is based on a series expansion of the variables in terms of powers of $M_a^2$.

Another important aspect of the numerical model are the so-called flux-limiters. They preserve the steep temperature gradients in the pulse tube, in combination with local grid refinement techniques.

Geometry, computational domain, and meshing

A one-dimensional and a two-dimensional geometry are used. In one dimension, the wall is not taken into account. In figure 3.16, the computational domain for the two-dimensional geometry is shown. It has axial symmetry. Only the pulse tube itself is considered. The cold heat exchanger is at $z = L$, the hot heat exchanger at $z = 0$. This is different from the rest of this thesis, but it follows the notation of Lyulina [3]. The boundary between gas and wall is at $r = R_0$, the outer wall radius at $r = R_1$. The pulse tube under consideration is the same as with the commercial code. Its characteristics are summarized in table 3.4. At $t = 0$ the pressure in the pulse tube is average and starts to rise.


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<table>
<thead>
<tr>
<th>Number of points in z direction</th>
<th>101</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of points in r direction</td>
<td>45</td>
</tr>
<tr>
<td>Number of points in the wall</td>
<td>5</td>
</tr>
<tr>
<td>Number of time steps per cycle</td>
<td>200</td>
</tr>
<tr>
<td>Number of cycles</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3.7: Numerical parameters for the 2-D simulations. The grid spacing is reduced near the wall.

<table>
<thead>
<tr>
<th>Position</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot end</td>
<td>1267.7 W</td>
</tr>
<tr>
<td>Middle</td>
<td>1267.6 W</td>
</tr>
<tr>
<td>Cold end</td>
<td>1267.3 W</td>
</tr>
<tr>
<td>Analytical</td>
<td>1250 W</td>
</tr>
</tbody>
</table>

Table 3.8: Time averaged enthalpy flow in three positions of the pulse tube, compared with the analytical result.

In table 3.7, the grid specification for the 2-D simulation are shown. The total number of cells is approximately 5000. One cycle with 200 time steps takes approximately 10 minutes computation on a standard 2.66 GHz PC. This is more efficient than the commercial code calculations, where one cycle with 200 time steps and 30000 cells took approximately 24 hours.

Simulation results, 1-D case

The first validation of the code was done on a one-dimensional geometry. In this geometry, there is no influence of the wall. Only boundary conditions at \( z = 0 \) and \( z = L \) have to be used. The velocity on the hot end is given by equation 3.71. On the cold end, a pressure boundary condition is used. The temperature boundary conditions on the hot and cold ends depend on the direction of flow. A boundary condition only has to be applied when gas flows into the tube. In this case the gas temperature equals the heat exchanger temperature.

In figure 3.17 the temperature profiles at the beginning of the cycle, after 100, 500, and 1000 cycles are shown. On the warm end the temperature profile due to the flow in and out of the heat exchanger is visible. The final profile develops by the combination of enthalpy flow and heat conduction. When the profile does not change much anymore, a steady state is reached. In this case, this takes approximately 1000 cycles.

In figure 3.18 the temperatures near the hot and cold-end boundaries are given as functions of time. Also plotted in this figure are the result of the harmonic model and the three-dimensional commercial code. The 1D results show the same overshoot as the 3D results, but slightly smaller. This could be the result of the flux limiters. The temperature profile after a larger number of cycles is not given, so whether this overshoot is caused by real heat conduction, numerical heat conduction or the initial conditions is still not known. Other simulations [29] show a similar overshoot.

In table 3.8, time averaged enthalpy flow is given for three different positions in the pulse tube, compared to the expected analytical values. The agreement here is very good.

Simulation results, 2-D case

For two-dimensional flow, additional boundary conditions are required. Radial velocity \( v \) is zero on all the boundaries and the symmetry line. The axial velocity on the hot end is given by equation 3.71. On the cold end, a pressure boundary condition is used. The temperature
CHAPTER 3. MODELING AND SIMULATION

Figure 3.17: Temperature profile in the pulse tube at the beginning of the cycle, after 100, 500 (thin lines) and 1000 cycles (bold line). The dotted line is the initial condition.

Figure 3.18: Temperature as a function of time on the cold and hot end of the pulse tube, calculated with the harmonic model, the 3D commercial numerical model and the 1D numerical model.
boundary conditions on the hot and cold ends depend on the direction of flow. A boundary condition only has to be applied when gas flows into the tube. In this case the gas temperature equals the heat exchanger temperature.

In figure 3.19 temperatures and velocity fields at four different times in the third cycle are shown. The thermal boundary layer near the wall is clearly visible. The radial velocity component is very small and not visible in the vector plots. In figure 3.20 the temperature profile, averaged over the tube cross section, is shown. The overshoot observed in the one-dimensional calculations is visible only in the first few cycles.

An important aspect of the two-dimensional model is the possibility to study interaction between the gas and the wall. The driving mechanism of streaming, temperature dependent viscosity, has not yet been implemented in the code, nor has streaming been investigated in the numerical experiments. Heat transfer with the wall, responsible for shuttle heat transfer, has been investigated. In figure 3.21, the different heat flows are shown. In figure 3.22 the interface heat flow, gas axial heat flow, wall axial heat flow, and enthalpy flow as a function of axial position are given. Figure 3.22a can be compared to figure 3.11. Direction and shape of the heat transfer are comparable. The elapsed time in figure 3.22a is much larger. This explains the difference in magnitude. It should be noted that the positive direction of heat flux is defined in a different way in the two figures. In figure 3.11 positive means heat flows from the wall to the gas, in figure 3.22 it is the opposite direction.

The enthalpy flow through the gas is dominant and equal to approximately 1280 W. Total heat conduction through the wall, and interface flow is of the order of 2 W, and conduction through the gas is even smaller, approximately 0.5 W.

**Discussion**

The dedicated numerical model is a general model capable of simulating low Mach-number, oscillating flows and heat transfer. It is applied to the pulse tube. In two dimensions, it can predict velocity, temperature, mass flow and energy flow, including radial thermal and viscous effects. It is numerically not expensive. Compared to the commercial code, it can calculate one cycle in minutes, where the commercial code required hours, for comparable numbers of computational cells. The model is not yet complete, however. It is limited to ideal gas with laminar flow. It currently uses a uniform grid with a limited number of points in the boundary layer, approximately 8 as visible in figure 3.19. More important it does not yet include temperature dependent material properties, such as thermal conductivity, heat capacity, and viscosity. The latter is important for the prediction of mass streaming. This has not been investigated with the simulations done so far. Recent improvements of the model [30] have included temperature-dependent viscosity, but the streaming that was found was very small. This proves that it is possible to simulate streaming, but further investigations will have to be done regarding the validity of these predictions.

An expansion to three dimensions could also be done. It is, however, to be determined if the model will remain numerically as efficient as it is right now. Other improvements to the model include the possibility of simulating other geometries, such as tapered pulse tubes [19]. Also turbulence modeling could be included. Finally, the model should either include the other components of the PTR as well, or be included itself in the Stirling pulse-tube model, to include the entire cooler in the simulations. The proposed integral design tool has thus not yet been realized.
Figure 3.19: Two-dimensional temperature and velocity fields for four moments in the third cycle.
CHAPTER 3. MODELING AND SIMULATION

Figure 3.20: Surface-average temperatures at the cold and warm end of the pulse tube, for the first 50 cycles.

Figure 3.21: Time-averaged energy flows in the pulse tube as calculated in the two-dimensional numerical model.
Figure 3.22: Time-average interface heat flux, gas axial heat flow, wall axial heat flow and axial enthalpy and energy flows as functions of position, after 2000 cycles.
3.3.6 Conclusions

Four different pulse-tube models were discussed. The first two are based on one-dimensional, first-order harmonic approximations. These are fast models, capable of doing a large number of calculations in a short time. This enables parametric studies and optimizations of geometries.

There are effects that are not calculated with the first order models. These are higher-order effects, that are often two or three dimensional. To simulate these, numerical modeling was used. Initial simulations were done with commercially available CFD code. The results showed good agreement with the first-order models, and also visualized three-dimensional effects such as streaming. The accuracy of this simulation is limited by the numerical methods used in the code. Furthermore, simulations with this code are computationally very expensive and therefore not very useful in the design process.

To overcome these limitations, a new numerical model was designed in a separate PhD project. This resulted in a numerical model capable of simulating low Mach-number oscillating flows, in two dimensions. This model was applied to the pulse tube. The results are promising. However, at the moment the code is not complete enough to be able to predict all the phenomena in the pulse tube. An important example is the use of time and temperature dependent viscosity, which is one of the driving forces in streaming.

Further work has to be done to complete this numerical model. Even in two dimensions, the results are promising, and do not show the limitations from the commercial code. After finalizing the code, it should be integrated in the SPTM to build a complete, integrated design tool. The SPTM could then be used to do the optimization and parametric study, the optimum result could then be used to generate the boundary conditions for the numerical model. This could finalize the design by optimizing the geometry of the pulse tube and possible other components to minimize the losses not predicted by the first-order model.

In order to design a pulse-tube cryocooler, the following procedure is thus proposed. After the specifications have been determined, the SPTM will be used to find a global optimum for the geometry of the PTR. Important aspects for this calculations are length and diameter of the regenerator and pulse tube, dimensions of the heat exchanger, fill pressure, frequency, and compressor dimensions. The results of this optimization can then be used to generate the boundary conditions for the dedicated numerical model, in particular the pressure amplitude and flow on the warm end of the pulse tube. The results of the numerical model can then be used to establish whether the geometry has to be fine-tuned, for instance by changing the cross section of the pulse tube, the dimensions of the heat exchangers, or tapering of the pulse tube in order to reduce streaming.
Bibliography


[23] CFX-5.4 solver manual, CFX international, AEA Technology plc., pp. 102-104.


Chapter 4

Inertance tubes and other phase shifters

4.1 Introduction

The PTR’s in the form as discussed in this thesis, operate because of an enthalpy flow in the pulse tube, induced by a dissipating element at the hot end of the pulse tube. In the most basic form, this is an orifice. This orifice is a resistive flow controlling device. The flow resistance leads to entropy production, entropy production leads to the desired enthalpy flow (see chapter 2).

Pulse-tube refrigerators tend to have a lower efficiency than Stirling cryocoolers. Part of that is fundamental, because pulse tubes need dissipation at the hot end for their cooling mechanism. This results in a theoretically lower maximum efficiency than the Carnot efficiency, cf. equation 2.44.

Another cause for a lower efficiency is the flow in the regenerator. For a Stirling cryocooler flow and pressure inside the regenerator will be approximately in phase. For a pulse tube this is not the case.

If we consider an orifice pulse tube, with fixed pressure amplitude $p_{ta}$, and average pressure $p_0$, the hot end flow is described by

$$\dot{V}_h = C_1 p_{ta} e^{i\omega t}. \quad (4.1)$$

The complex conjugate that is used to calculate the real part of $\dot{V}_h$ (cf. equation 3.17) is omitted for sake of readability. So the hot end flow is in phase with the pressure. For the cold end, the flow will be, according to equation 3.21,

$$\dot{V}_1 = C_1 p_{ta} \left( 1 + \frac{iV_1 \omega}{\gamma C_1 p_0} \right) e^{i\omega t}. \quad (4.2)$$

The average enthalpy flow is only proportional to the component of the flow that is in phase with the pressure. For the orifice pulse tube this enthalpy flow is given by

$$\overline{H} = \frac{1}{2} C_1 p_{ta}^2.$$

So because of the phase shift over the pulse tube, the cold-end velocity, and thus the velocity inside the regenerator is higher than required for the enthalpy flow.
The phase shift and resulting increased mass flow are caused by the ‘mass storing’ effect of the pulse tube. The flow at the cold end will always be leading the flow at the hot end. This effect can be compared to the effect of a capacity in electrical circuits. The increased mass flow leads to losses, since the regenerator losses are proportional to the square of the mass flow.

The losses can be reduced by changing the phase at the hot end. If we would replace the orifice flow conductance $C_1$ with a complex conductance $X_1$, consisting of an in-phase part $C_1$ and an out-of-phase part $S_1$

$$X_1 = C_1 - iS_1,$$

the cold-end volume flow will become

$$\dot{V}_1 = p_{th} \left( C_1 - iS_1 + \frac{i\omega V_t}{\gamma p_0} \right) e^{i\omega t}. \quad (4.4)$$

If we then choose $S_1$ as

$$S_1 = \frac{\omega V_t}{\gamma p_0}, \quad (4.5)$$

the cold-end flow will be in phase with the pressure. Since the phase shift over the pulse tube is usually around $45^\circ$, phase shift between pressure and flow at the inlet of the phase shifter should be approximately $45^\circ$ in the opposite direction. This means that the flow should lag the pressure. If the phase shift over the regenerator is also taken into account, it turns out that this phase shift should actually be closer to $60^\circ$. In that case, the flow and pressure are in phase in the middle of the regenerator, instead of on the side of the cold heat exchanger.

To get this out-of-phase flow, special flow controlling devices can be used. The first one that was reported is the second inlet [2]. This is a second orifice, connected between the hot end of the pulse tube and the compressor. The principle of this second orifice is that part of the mass flow, that is required for the pressure variations in the tube, is flowing around the regenerator, thus reducing the regenerator losses.

Disadvantage of this second orifice is the possibility of a DC flow through this orifice[3]. A small amount of DC flow can be helpful, but usually it has to be avoided. Double-inlets are mostly used in low-frequency (GM-type) PTR’s.

Another possibility is to determine this phase shift actively. In low-frequency pulse tubes this can be done with a second rotary valve (four-valve method [7] or active buffer [8]). In a high-frequency pulse tube the so-called warm expander can be used. This is a piston that is placed at the hot end of the pulse tube. Its principle is given in figure 4.1. Instead of the orifice and a buffer, a heat exchanger and piston are added to the cooler. In chapter 2, it was shown that an entropy flow towards the hot heat exchanger was necessary to generate the enthalpy flow through the pulse tube. In figure 4.1, the enthalpy flows, entropy flows and power are shown.

The entropy flow $S_\alpha$, necessary for the enthalpy flow through the pulse tube, is now generated in an additional heat exchanger. Heat $\dot{Q}_{\text{exp}}$ has to be added to this heat exchanger. Because the connection between the two heat exchangers is isothermal, this heat is removed from the system as power $P_{\text{exp}}$ on the expansion piston. The amount of heat removed from the hot heat exchanger $\dot{Q}_{hhx}$ is equal to the amount of heat added to the expander heat exchanger. So there is no net heat transfer from these two heat exchangers to the surroundings. Together, they act like a regenerator.

The motion of the warm expander can be controlled to provide optimum performance of the cryocooler. In addition to that the power can be used, for instance by connecting the expander
piston to the same crankshaft as the compressor. The additional moving element eliminates most of the advantages that a pulse tube has over a Stirling cryocooler. Thermodynamically, the warm expander pulse tube is equivalent to a Stirling cooler. The big difference is that the expansion piston is now placed at room temperature. The expander can be actively driven [9]. It can also be a free piston [10]. In that case, a dissipative element has to be added to the tuned mass spring system.

The type of phase shifter that is mostly used in high-frequency, Stirling-type pulse tubes is the so-called inertance tube. This passive phase shifter uses the combination of inertia and resistance to achieve the desired enthalpy flow and phase shift. This phase shifter will be further investigated in the next paragraph.

### 4.2 Inertance tubes

#### 4.2.1 General principles

The inertance tube is a long, narrow tube. Three phenomena are responsible for its desired flow behavior. First of all, there is flow resistance due to the friction of the gas with the wall of the tube. Second, because of the mass of gas in the tube, there is inertia. This inertia results in a phase shift between flow and pressure. A third effect is due to the gas-storing capacity of the tube. This capacitive effect is negative. It counteracts the desired phase shift.

The beneficial effects of inertance tubes were observed by several people [4],[5],[6]. Only recently, effort is put into the modeling of inertance tubes. Two of these models will be described in the next paragraph, and reviewed for best use in the Stirling pulse-tube model.

To design an optimal inertance tube, one has to find the combination of length and diameter of the tube that gives the minimal mass flow in the regenerator for the desired cooling power.

#### 4.2.2 Inertance-tube models

There are two classes of inertance-tube models. The most common are the models that describe the inertance tube in terms of an impedance, in analogy with an electrical impedance [12]. Other models are more extensive and solve the system of Navier-Stokes equations, usually by using numerical tools. Examples are given by [13] and [14].
CHAPTER 4. INERTANCE TUBES AND OTHER PHASE SHIFTERS

Figure 4.2: Electrical analogy of the lumped-parameter model. The inertance tube has a resistive, an inductive, and a capacitive contribution. The capacitive contribution is divided in two parts, one near the pulse tube and one near the buffer.

The impedance models are the most suitable for use with the Stirling pulse-tube model. The inertance tube is considered as a network of the individual contributions (resistance, inertia, capacitance). The resulting impedance $Z_i$ is determined analogous to an electrical network. The resulting impedance describes the relation between pressure $p_i$ on the inlet of the inertance tube and volumetric flow $\dot{V}$

$$p_i(t) = Z_i \dot{V}(t). \quad (4.6)$$

For harmonic models, this is equivalent to a description in terms of mass flow, since density is considered only a function of average temperature and pressure. All the models described here assume ideal gas. This is correct because the gas in the inertance tube is room temperature helium, which closely approximates an ideal gas. In the harmonic approximation, the average pressure in the pulse tube and buffer are equal. Therefore, only the time-dependent part of the pulse-tube pressure is substituted in equation 4.6.

**Lumped-parameter model**

The simplest of the impedance models is the so-called lumped-parameter model. This model [15],[3] separates the three terms that influence the behavior into separate contributions. In figure 4.2 the equivalent electrical network is shown. The total capacitance of the inertance tube is divided in two. One part near the pulse tube and one part near the buffer.

The capacity of the inertance tube is separated into a part that is considered part of the pulse tube, and a part that is part of the buffer. For the resistive term we have

$$R_i = \frac{64f_r \rho_0 L_{it}}{\pi^3 D_{it}^5}. \quad (4.7)$$

The friction factor $f_r$ is the Fanning friction factor. In [3], the following relation for this friction factor is used

$$f_r = 0.046Re^{-0.2} \quad (4.8)$$

but other correlations can be used as well, depending on wall roughness and flow regime.

For the capacitive term

$$C_i = \frac{A_{it} L_{it}}{\gamma \rho_0}. \quad (4.9)$$
Figure 4.3: Electrical analogy of the transmission-line model for the inertance tube. The difference with the lumped-parameter model is that the tube is divided in a large number of small segments and that the individual components are calculated per unit of length.

For the inductive term

\[ L_i = \frac{\rho_0 L_{it}}{A_{it}}. \]  

(4.10)

The resulting impedance of this model is then:

\[ \frac{1}{Z_{it}} = \frac{1}{i\omega C_i/2} + \frac{1}{R_i + i\omega L_i + \frac{1}{i\omega C_{buf} + C_i/2}}. \]  

(4.11)

Usually the inertance tube is small compared to the buffer volume and the pulse-tube volume. In this case the capacitive terms of the inertance tube can be neglected. Equation 4.11 then reduces to

\[ Z_{it} = R_i + i\omega L_i + \frac{1}{i\omega C_{buf}}. \]  

(4.12)

Transmission-line model

In [3] the behavior of an inertance tube is considered equivalent to an electrical transmission line. Instead of one combination of the three impedance contributions, the inertance tube is considered as a string of infinitely small rcl segments, as shown in figure 4.3. This way, any error introduced by placing a contribution in the ‘wrong’ place is integrated out.

The ‘type’ of cell is somewhat changed. The capacitance is placed between the resistive and inductive contributions. The impedance of each contribution is now described as an impedance per unit of length. For resistance, capacity, and inductance we have then respectively

\[ r_i = \frac{64 f_i V \rho_0}{\pi^3 D_{it}^6}, \]  

(4.13)

\[ c_i = \frac{A_{it}}{\gamma \rho_0}, \]  

(4.14)

\[ l_i = \frac{\rho_0}{A_{it}}. \]  

(4.15)

Similar to equations 4.7, 4.9, and 4.11. The equivalent impedance can be found by integrating over all the impedance cells. The result is [3]

\[ Z_{it} = \frac{Z_{buf} + Z_0 \tanh(k_i L_{it})}{Z_0 + Z_{buf} \tanh(k_i L_{it})}. \]  

(4.16)
The characteristic impedance $Z_0$ of the line is given by

$$Z_0 = \sqrt{\frac{r + i \omega l}{i \omega c}}. \quad (4.17)$$

The buffer impedance is given by

$$Z_{buf} = \frac{1}{i \omega C_{buf}}, \quad (4.18)$$

with $C_{buf}$ calculated according to equation 4.9. The wave vector $k_i$ is given by

$$k_i = \sqrt{(r + i \omega L) \omega c_i}. \quad (4.19)$$

Usually the difference between the lumped-parameter model and the transmission-line model will be small. Only in situations where the capacitive contribution to the impedance is large compared to the resistive and inductive contributions, a difference can be expected.

### 4.2.3 Model results

The lumped-parameter model and the transmission-line model are based on the same mechanisms, so they are comparable. Especially if the capacitance of the inertance tube is very small, the results are the same. The accuracy of the models is influenced by several factors. First of all, the accuracy of the friction factor is very important. Different friction factors exist for different flow regimes. Furthermore, the geometry will have a large influence on the accuracy of the predictions, for instance due to its effect on the onset of turbulence. Finally, there will be thermal effects in the inertance tube. Depending on the ratio between the thermal penetration depth and the inertance tube diameter, the gas will behave either isothermal or adiabatic. Conclusions on validity can therefore only be drawn after comparison with measurements.

Comparison between models and experiments found in literature show good agreement only for relatively small inertance tubes (inner diameter 1-1.5mm) and low enthalpy flows ($\approx 30$ W) [17], [18]. For larger systems correspondence is not good enough for accurate predictions [18].

In the figures below, some calculations are shown. Unless noted otherwise, the average pressure is 30 bar, pressure amplitude is 3 bar, and the frequency is 25 Hz. First the phase between flow and pressure, and the enthalpy flow as functions of inertance tube lengths are given, for an inertance tube of 4 mm (figure 4.4) and 6 mm (figure 4.5) inner diameter. The average pressure is 30 bar, the pressure amplitude 3 bar, and the frequency 25 Hz.

The differences between the lumped model and transmission-line model are negligible. The differences occur only at longer lengths. The influence of frequency is shown in figure 4.6. For an inertance tube of 6 mm inner diameter, and a length of 3.5 m, the phases and enthalpy flows are given for different frequencies. Since the results for the lumped model and transmission line model are so small, only the lumped model results are given.

Because the $\omega L$ term increases with frequency, the phase shift increases. As a result of the increased absolute flow, the friction factor also increases. As a result, the in-phase part of the flow decreases and the enthalpy flow decreases.

In figure 4.7 the effect of the pressure amplitude in the pulse tube is shown. At increasing pressure amplitude, the flow into the inertance tube increases. The resistive term depends on the pressure amplitude through the influence of the friction factor. Because the resistive component of the impedance increases with increasing flow, its contribution becomes more important. As a result, the phase difference decreases. The enthalpy flow increases with pressure amplitude. This increase is slightly less than quadratic.
Figure 4.4: Phase shifts between flow and pressure and enthalpy flows through the pulse tube as functions of $L_R$, for an interance tube of 4 mm inner diameter. The solid lines represent the enthalpy flow, the dotted lines the phase. The round markers are the results for the lumped model, the square markers the results of the transmission line model.

Figure 4.5: Phase shifts between flow and pressure and enthalpy flows through the pulse tube as functions of $L_R$, for an interance tube of 6 mm inner diameter. The solid lines represent the enthalpy flow, the dotted lines the phase. The round markers are the results for the lumped model, the square markers the results of the transmission line model.
Figure 4.6: Phase shift between flow and pressure and enthalpy flow at the inlet of the inertance tube as functions of frequency, for $D_{it} = 6$ mm and $L_{it}=3.5$ m. The solid line represents the enthalpy flow, the dotted line the phase. Calculated with the lumped parameter model.

Figure 4.7: Phase shift between flow and pressure and enthalpy flow at the inlet of the inertance tube as functions of pressure amplitude, for $D_{it} = 6$ mm and $L_{it}=3.5$ m. The solid line represents the enthalpy flow, the dotted line the phase. Calculated with the lumped parameter model.
4.2.4 Discussion and conclusions

The lumped-parameter model and the transmission-line model give almost the same results. The small difference between the two models does not justify the additional complexity of the transmission-line model. Therefore, the simplest model, the lumped-parameter model, is best suited for use in the Stirling pulse-tube model. The accuracy of the model will have to be determined by experiment.

As mentioned, the lumped-parameter model is a rather simple representation of the inertance tube. More elaborate models exist as well. The thermoacoustic transmission-line model [17] uses thermoacoustic theory to explain the behavior of the inertance tube. Especially the thermal effects are taken into account in more detail. This model however depends even more on empirical data than the above mentioned models, for instance for its description of the influence of turbulence. Another method of modeling has been proposed by Schunk et al. [18]. They propose a so-called distributed-component model. Such a model is comparable to a transmission-line model, but instead of integrating the impedance beforehand, the inertance tube is divided in several discrete sections that each have their own impedance and flow. As a result, the influence of the distribution of the individual components is taken into account, but more importantly, also the influence of local flow is considered. This might yield more accurate results for the resistive component, in case of a large capacitance.

All these models have in common that they rely on empirical data to calculate flow resistance and heat transfer. Accurate empirical data for the inertance tube is not available. Therefore, these more elaborate models are not per definition more accurate.

From figures 4.6 and 4.7 it can be seen that the phase shift and enthalpy flow depend on frequency and pressure amplitude. An optimized PTR requires an optimized phase shift. It can therefore be difficult to operate a pulse tube outside of its optimum point. That might be of interest when a PTR is used in an application where the required cooling power is variable. In such a case, the cooling power of a cooler is usually controlled by changing frequency or compressor amplitude. The resulting change in phase shift is likely to lead to a small reduction of efficiency.
Bibliography


Chapter 5
Pulse-tube experiments - Setup 1

5.1 Introduction
In Stirling-type PTR’s, two types of compressors can be used, linear-motor and crank-shaft driven. A linear-motor driven compressor needs to be matched to the PTR to reach optimum performance. It is important to operate such a compressor at or very near to its natural resonance frequency. It is therefore inconvenient to use such a compressor in a system where one wants to vary e.g. frequency and pressure.  

We chose to use a crankshaft driven compressor, because it is very easy to combine with a PTR. Especially because we want to operate the system at different frequencies, pressures, and temperatures. The result is a very flexible system with which it is easy to gain experience in operating the pulse tube itself, and gathering data to validate the Stirling pulse-tube model.  

The first test setup described in this chapter and the second setup, described in the next chapter, both use the compressor of the Stirling SPCG1 cryocooler. In the SPCG1, the crankshaft drives both the compression piston and the displacer piston. In our test setups, the displacer is removed and the hole in the piston, through which the displacer drive rod usually feeds, is closed.  

The results of this first setup are mainly qualitative. Initial experiments mainly focussed on the general operating principles. The setup was designed with a preliminary version of the simulation tool, and the predictions were not accurate at the time.  

5.2 Experimental setup
The cross section of this setup is shown in figures 5.1 and 5.2. Apart from the compressor, we also used the heat exchangers and regenerator of the Stirling SPC-1 cryocooler. Because the after cooler and regenerator are annular, the pulse tube itself is also annular. As a result the system was coaxial with a vacuum space in the middle and on the outside. Above the hot heat exchanger, the annular flow channel then flows to a central connection for the phase shifter. In table 5.1 the data are given for the geometry of the different components. In figure 5.3 the outer part and inner part of the pulse tube are shown. In figure 5.4 an overview of the test setup is shown. The pulse tube is mounted on top of the compressor. The hot heat exchanger, orifice, buffer space, and vacuum space are not mounted yet.  

The volume of the pulse tube can be reduced by placing two bushes in the flow channel. That way, the volume can be reduced from 520 cm³ to 295 cm³.
Figure 5.1: Complete cross section of test setup 1. It can be divided in two parts. The bottom part is the compressor, the top part is the pulse-tube cooler itself.
Figure 5.2: Details of the pulse-tube cryocooler (setup 1). The components are annular, resulting in a coaxial system with a vacuum space in the centre. From top to bottom: position of a pressure sensor, connection of the phase shifter, connection to the inner vacuum, cooling water connection, hot heat exchanger, flow straightener, the pulse tube itself, bushes to reduce the pulse tube volume, positions of thermometers, flow straightener, cold heat exchanger, electrical heater, position of the temperature sensor, regenerator, inner vacuum space, after cooler, position of the pressure sensor, compression space, and piston. The phase shifter and the outer vacuum space are not shown.
### Table 5.1: Dimensions of setup 1.

<table>
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<tr>
<th>Component</th>
<th>Amplitude</th>
<th>Diameter</th>
<th>Frequency</th>
<th>Slit width</th>
<th>Slit depth</th>
<th>Slit number</th>
<th>Height</th>
<th>Cross section</th>
<th>Wire diameter</th>
<th>Filling factor</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston</td>
<td>26 mm</td>
<td>80 mm</td>
<td>15-30 Hz</td>
<td>260 cm³</td>
<td></td>
<td>120</td>
<td>58 mm</td>
<td></td>
<td>0.4 mm</td>
<td>5 mm</td>
<td>304L</td>
</tr>
<tr>
<td>After cooler and Hot heat exchanger</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Cold heat exchanger</td>
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<td>45 mm</td>
<td></td>
<td></td>
<td>0.4 mm</td>
<td>120</td>
<td>58 mm</td>
<td></td>
<td>0.4 mm</td>
<td>5 mm</td>
<td>304L</td>
</tr>
<tr>
<td>Regenerator</td>
<td>47 mm</td>
<td>50 cm²</td>
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<td>0.055 mm</td>
<td>35 %</td>
<td>119</td>
<td>45 mm</td>
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<td>0.4 mm</td>
<td>7 mm</td>
<td>304L</td>
</tr>
<tr>
<td>Pulse tube</td>
<td>70 mm</td>
<td>106 mm</td>
<td></td>
<td>520 cm³</td>
<td></td>
<td>120</td>
<td>104.5 mm</td>
<td></td>
<td>0.4 mm</td>
<td>5 mm</td>
<td>304L</td>
</tr>
<tr>
<td>Pulse tube with bushes</td>
<td>80 mm</td>
<td>100 mm</td>
<td></td>
<td>295 cm³</td>
<td></td>
<td>120</td>
<td></td>
<td></td>
<td>0.4 mm</td>
<td>5 mm</td>
<td>304L</td>
</tr>
</tbody>
</table>
Table 5.2: Estimated performance of the heat exchangers in the first test setup. The estimated temperature difference and pressure drop over the heat exchangers is based on expected velocities in the slits, geometry, and standard empirical relations.

<table>
<thead>
<tr>
<th></th>
<th>$v$ [m/s]</th>
<th>$R_e$</th>
<th>$N_u$</th>
<th>$\alpha_{hx}$ [W/m²K]</th>
<th>$f_c$</th>
<th>$Q_{typ}$ [W]</th>
<th>$\Delta T$ [K]</th>
<th>$\Delta p$ [mbar]</th>
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</thead>
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<tr>
<td>HHX</td>
<td>13</td>
<td>$2.2 \cdot 10^4$</td>
<td>60</td>
<td>$1.3 \cdot 10^4$</td>
<td>$9.9 \cdot 10^{-3}$</td>
<td>1000</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>CHX</td>
<td>18</td>
<td>$2.5 \cdot 10^4$</td>
<td>270</td>
<td>$2.4 \cdot 10^4$</td>
<td>$6.1 \cdot 10^{-3}$</td>
<td>1000</td>
<td>0.5</td>
<td>52</td>
</tr>
<tr>
<td>AC</td>
<td>85</td>
<td>$1.5 \cdot 10^4$</td>
<td>160</td>
<td>$4.2 \cdot 10^4$</td>
<td>$6.7 \cdot 10^{-3}$</td>
<td>10000</td>
<td>3</td>
<td>360</td>
</tr>
</tbody>
</table>

Figure 5.3: Pictures of setup 1. Left the outer part, showing the location of the regenerator, cold heat exchanger and pulse tube. Right the centre bush, with the SRBF bush and two flow straighteners mounted.

All heat exchangers are made of high-purity copper for its high thermal conductivity. All other parts were made of stainless steel, except the filling bushes. They were first made of cotton laminated phenol-formaldehyde resin (SRBF or PF), and later replaced by stainless steel ones.

A needle valve was used as orifice. The flow straighteners were made of sintered slices of stainless steel gauzes, similar to the material used in the regenerator.

An estimation for heat exchanger performance is given in table 5.2. Using empirical relations, for instance as used in regenerator calculations or inertance tube calculations, we can estimate the performance of the heat exchangers. A typical velocity in the pulse tube can be calculated from the expected orifice flow conductance and pressure amplitude. A typical velocity in the after cooler is estimated from the swept volume of the compressor. The pressure drop and temperature difference over the heat exchangers are small, so these heat exchangers are expected to perform sufficiently.
5.2.1 Data acquisition and instrumentation

The most important sensors are pressure and temperature sensors. The temperature sensors used are platinum resistance sensors (Pt-100) for the cold heat exchanger temperature and the cooling water temperatures. The thermometer for the cold heat exchanger is calibrated by the supplier to an accuracy of 0.05 K. The sensors for the cooling water are matched to less than 0.05 K with respect to each other, to accurately measure the temperature difference between inlet and outlet water flow. All resistive thermometers are measured using the four-wire method. Thermocouples (type-K) are used to measure temperature profiles over the tube wall and other places where less accuracy is needed (≈ 1 K).

Pressure sensors are mounted in the compression space, pulse tube, and buffer. A fourth sensor is placed on the crank case of the compressor to monitor gas filling of the system. Further sensors are a mass flow meter for the cooling water. A magnetic pickup on the drive shaft is used for frequency measurements and triggering of pressure measurements. Electrical current and voltage are measured to determine the electrically applied power. The power supply unit (PSU) for this heater is controlled by an analog signal from the PCI measurement card. A frequency inverter is used to control the frequency of the motor.

The read-out of the sensors is done by two systems. An accurate data-acquisition/multiplexer unit is used to measure all slowly varying properties with a maximum sample frequency of approximately 0.2 Hz, and a second high-speed PCI-based measurement system is used for real-time dynamic pressure measurements, with a sample speed of 200 kHz. Both systems are connected to a computer running LabView software for local collection and visualization and remote archiving of the measurement data. A schematic representation of the data-acquisition system is given in figure 5.5.
Figure 5.5: Data-acquisition system. A fast, PCI-based measurement card is used to measure dynamic pressure and to capture the piston position. A slower, but more accurate data acquisition system is used to measure all the other quantities. An analog output channel of the PCI-card is used to control the power supply used for the heater.
5.3 Experimental results

5.3.1 Introduction

A cryocooler can be characterized in several ways. Quantities that are of interest are temperature, cooling power and input power. The ratio of cooling power and input power gives the efficiency or $COP$. Another parameter of interest is temperature. Minimum or no-load temperature is one characteristic, but the temperature where a certain amount of cooling power is produced is also of interest. For cryocoolers in the liquid-nitrogen temperature range, the cooling power is usually given at a temperature of $T_l = 77 \text{ K}$. Since the relation between cooling power and temperature (the so-called cooling curve) is often nearly linear, we can also express the performance in terms of its minimum temperature and the so-called 'Angle Of Inclination' ($AOI$), the slope of the cooling curve.

Most results in this chapter are presented relative to so-called 'standard conditions'. These are somewhat arbitrarily chosen conditions, based on typical operating conditions of a Stirling-cycle cryocooler with the same compressor. Standard conditions are an operating frequency of 25 Hz (1500 RPM) and an average pressure of 28 bar. This is by no means an optimized operating point, but just mentioned for reference purposes.

The usual experimental procedure starts with pumping down the vacuum space to a pressure below $10^{-4}$ mbar, and purging the pulse tube. This is done by flushing with clean helium gas. The pulse tube is filled up to 10 bar, after which the gas is released again. This is repeated 6 times. Resulting helium purity should then be of the order of 1 ppm.

The orifice is then set at a typical value and the machine is started. A cool down typically took 1-2 hours to reach a stable minimum temperature. After that, the setting of the orifice is changed to obtain the setting where the temperature is minimal. After that a cooling curve is measured. A stable cool down is shown in figure 5.6.

Cooling curves are measured for different pressures and frequency. The measurement of each curve starts with optimizing the orifice setting for minimum temperature. During the measurement of the cooling curve, the orifice setting is not changed anymore.

5.3.2 Instabilities

The initial measurements were done with an orifice. For the first measurements the cool down did not reach a stable low temperature as shown in figure 5.6. They all showed large instabilities with both oscillatory as well as chaotic behavior. The initial no-load temperature was around 200 K. In figure 5.7, an example of the oscillatory instabilities is shown. In this figure, the temperature at several axial positions on the pulse tube are plotted versus time. The instabilities were visible in all temperatures along the pulse tube wall, showing oscillatory variations, but also variations in their mutual temperature difference. The characteristic time period of the oscillation is about 3 minutes. An example of the chaotic instability is shown in figure 5.8.

The appearance of the instabilities was quite puzzeling. A probable cause for the oscillations is feedback from the orifice. The valve that is used as orifice is made of a large brass block. It turned out that the valve became rather hot during operation of the PTR. This heat comes from the hot heat exchanger and heat pumping in the connecting channel. The heating causes the flow resistance of the valve to change. To reduce these effects the valve was replaced by a capillary tube. We expected that the influence of temperature on flow resistance should be lower, due to the reduced influence of thermal expansion of the valve components. Even though the oscillatory behavior was reduced, the chaotic behavior remained.

Next the pulse tube volume was reduced by placing two bushes in the pulse tube. This enabled the placement of flow straighteners and optimized the flow transition between heat exchangers.
Figure 5.6: A typical cool down of setup 1.

Figure 5.7: Oscillatory instabilities of the temperatures on the pulse-tube wall. Each line represents a temperature at different axial position along the pulse-tube wall. The line showing the lowest temperature is placed nearest to the cold heat exchanger. Time $t=0$ is chosen at an arbitrary moment during the measurement.
and pulse tube. Reduction of volume should increase the pressure amplitude in the pulse tube, thus reduce the minimum temperature. The volume was reduced from 520 cm$^3$ to 295 cm$^3$. The first cool down with the reduced volume showed a minimum temperature of 98 K. This was not reproduced, however, and all the consecutive measurements continued to show instabilities.

A large number of experiments were done to investigate whether operating conditions like pressure and frequency had any influence on the instabilities. This was not the case. However, placing thin-walled stainless steel tubes as axial flow straightening in the channel (figure 5.9), and thus reducing the hydraulic diameter of the flow channel, made the instabilities disappear. A disadvantage was the additional shuttle and conduction losses which lead to a reduced performance.

A final improvement was found in the material used for the bushes. Originally solid SRBF was used. This material is made of cotton fibres in an phenolic resin. The cotton fibres protruded from the surface making it very rough. When these SRBF bushes were replaced with stainless steel ones, the instabilities disappeared and a minimum temperature of approximately 120 K was reached, without the need for axial flow straightening as shown in figure 5.9. The mechanism causing the instabilities is still not known, but we assume that the surface roughness due to the fibres has been the cause of the instabilities. The surface roughness itself could be the source of turbulence, but is also possible that nonhomogeneous distribution of the fibres caused a streaming-like effect. This nonhomogeneous distribution can be seen in figure 5.3b.

The minimum temperature was further reduced by polishing the surface of the stainless steel bushes. The minimum temperature of 117 K reached with these stainless steel bushes was higher than the single stable measurement of 98 K from the SRBF ones. This is probably caused by the void volume behind them, leading to lower pressure amplitude and some additional pumping losses. The position of this additional void volume is indicated in figure 5.10.
The cool down curve for this stable measurement is shown in figure 5.6, the corresponding wall temperatures can be seen in figure 5.11.

5.3.3 Measurements at different pressures and frequencies

In figure 5.12 the cooling curve for standard conditions is given. The minimum temperature is 117 K, and the \( AOI \) is 9.8 W/K.

In figure 5.13 \( \dot{Q}_1 \) versus \( T_1 \) is given for three average pressures. The orifice setting is optimized for minimum temperature for each pressure separately. For lower pressures, the minimum temperature is higher and the \( AOI \) is lower. This is caused by the lower pressure amplitude since a lower average pressure yields a lower pressure amplitude (equation 3.20). The dependence is not quadratic, however, because the orifice flow conductance is higher at lower average pressures, according to equation 3.24. The pressure amplitude in the pulse tube for these three average pressures is shown in figure 5.14.

At lower average pressure, the density of the gas is lower. This means that less heat has to be transferred per unit of time. This results in lower regenerator losses, because the amount of heat that has to be transferred per unit of time is also lower. This explains why there is little variation in minimum temperature.

In figure 5.15 \( \dot{Q}_1 \) versus \( T_1 \) is given for different frequencies. Again, at each separate frequency the orifice setting is optimized for minimum temperature. The \( AOI \) is higher for higher frequencies. The \( AOI \) for 16 Hz measurement seems higher than expected, based on the other curves. This is probably due to a measurement error, possibly because of incorrect orifice setting.

A higher frequency gives a higher pressure amplitude, and hence a higher cooling power. The pressure amplitude is given in figure 5.16. Furthermore, a higher frequency requires a higher orifice flow conductance, also leading to a higher cooling power.

5.3.4 Inertance tube

The setup was also fitted with two inertance tubes. The first tube had an inner diameter of 8 mm and a length of 2 m (tube 1), the other had an inner diameter of 6 mm and a length of 1 m (tube 2). These dimensions were calculated with the lumped parameter model for the inertance tube, see chapter 4. The two tubes should have approximately the same enthalpy flow, but different
CHAPTER 5. PULSE-TUBE EXPERIMENTS - SETUP 1

Figure 5.10: Void volumes behind the stainless steel bushes that are used to reduce the volume of the pulse tube.

Figure 5.11: Temperatures on different axial positions of the pulse-tube wall, for a stable measurement, a few hours after cool down. The minimum temperature for this measurement was 117 K.
Figure 5.12: $\dot{Q}_i$ versus $T_i$ for standard operating conditions. The line is a linear fit through the measurement points.

Figure 5.13: $\dot{Q}_i$ versus $T_i$ for three different average pressures, operating frequency is 25 Hz.
Figure 5.14: $p_a$ versus $T_l$ for three different average pressures.

Figure 5.15: $\dot{Q}_1$ versus $T_l$ for four frequencies; average pressure is 28 bar.
phase shifts. The calculated results for these two tubes are given in table 5.3. These values are calculated for a pressure amplitude of 3.5 bar, based on the measurements with the orifice pulse tube.

The cooling curve at standard operating conditions is given in figure 5.17. The minimum temperature does not change dramatically, but the AOI does. The AOI for inertance tube 1 is approximately 15 W/K, a 50 % increase over the orifice type pulse tube. As predicted, the inertance tube with the phase shift closer to 45° has a better performance.

The influence of inertance tubes is further investigated on the second setup, described in the next chapter.

5.4 Discussion and conclusions

At first, this setup showed large instabilities. Much effort was put into investigating and eliminating this chaotic and unstable behavior. Initial measurements were done with the large pulse tube volume of 520 cm³. The no-load temperature was 200 K and most measurements showed large instabilities. Therefore the pulse tube volume was reduced by placing two filling bushes, reducing the volume to 295 cm³. These bushes also improved the flow transition between the pulse tube and the hot and cold end heat exchanger. The very first measurement with these
bushes showed a no-load temperature of 98 K. Unfortunately this result could not be reproduced. Axial flow straightening reduced the instabilities, but they were only completely removed when the SRBF bushes were replaced by smooth-walled, stainless steel bushes. The best performance was a reproducible minimum temperature of 117 K, with an $AOI$ of 9.8 W/K.

The mechanism, responsible for the instabilities, has not been clearly identified, but it is assumed that the wall roughness due to protruding fibres from the SRBF was a source of some sort of turbulence.

With the stable machine cooling curves were measured with different pressures and frequencies. These measurements showed the trends that could be expected. Even though the minimum temperature is not yet what is to be expected of a cryocooler, we were capable of producing a few hundred watt of cooling power.

Some initial experiments with inertance tubes were also done. As expected, using an inertance tube instead of an orifice increased the performance of the cooler. The no-load temperature did not change much, but the $AOI$ increased from 9.8 W/K to 15 W/K.

Because of all the compromises that were made, and the apparent reduction in performance because of them, we did not use these measurements to validate the Stirling pulse-tube model. A new test setup was constructed, which should be able to reach lower temperatures. This setup was also constructed with better flow straightening and less chance of turbulence. The experimental results are described in the next chapter.
Bibliography

Chapter 6

Pulse-tube experiments - Setup 2

6.1 Introduction

The design goal of the second setup was a no-load temperature below 77 K, and and some finite cooling power at 77 K. A preliminary version of the Stirling pulse-tube model was used to do some basic calculations. These showed that the pulse-tube volume could be reduced from 295 cm$^3$ to 200 cm$^3$, thus increasing the pressure amplitude in the system. Furthermore, the model was used to estimate the performance of the heat exchangers and regenerator. The details of the preliminary calculations will not be given but the results of the final version will be discussed later on in this chapter (section 6.4).

6.2 Experimental setup

Based on the experience from setup 1, new heat exchangers, regenerator and pulse tube were designed. The components were no longer annular, but cylindrical. Regenerator, heat exchangers and pulse tube were optimized for minimum temperature, using a preliminary version of the Stirling pulse-tube model.

In figure 6.1 the cross section of this setup is shown. The cylindrical flow channel is an improvement over the first setup. The flow channel is now in-line without transitions in flow channel geometry between components. The inner surface of the pulse tube was polished. The dimensions are given in table 6.1.

The heat exchangers are made of cylindrical solid copper parts with holes drilled through them. Around these copper parts a water jacket removes the heat (after cooler and hot heat exchanger), or a resistive heater coil supplies heat (cold heat exchanger). This heat exchanger construction was chosen because of the simple fabrication method. The number and diameter of holes were chosen with heat transfer, pressure drop and flow distribution in mind. Calculations on the performance of the heat exchangers in the entire cooler were done using the Stirling pulse-tube model. An estimate of their performance in terms of heat exchange can be made using the following equation for heat exchange

$$\dot{Q} = \alpha_{hx} A_{wet} \Delta T.$$  (6.1)

The heat transfer coefficient $\alpha$ can be estimated using an empirical relation for the Nusselt number, for instance [1]
Figure 6.1: Cross section of the second setup. From top to bottom: connection to the phase shifter, flow straightening, hot heat exchanger, pulse tube, cold heat exchanger, regenerator, and after cooler. The pulse tube is mounted on the same compressor as used in the first test setup. A vacuum space is used to insulate the cryocooler from ambient heat leak (not shown).
CHAPTER 6. PULSE-TUBE EXPERIMENTS - SETUP 2

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aftercooler</td>
<td>Number of holes: 271</td>
</tr>
<tr>
<td></td>
<td>Height: 50 mm</td>
</tr>
<tr>
<td></td>
<td>Hole diameter: 1.5 mm</td>
</tr>
<tr>
<td></td>
<td>Heat exchanger diameter: 80 mm</td>
</tr>
<tr>
<td>Cold heat exchanger</td>
<td>Number of holes: 127</td>
</tr>
<tr>
<td></td>
<td>Height: 45 mm</td>
</tr>
<tr>
<td></td>
<td>Hole diameter: 1.0 mm</td>
</tr>
<tr>
<td></td>
<td>Heat exchanger diameter: 40 mm</td>
</tr>
<tr>
<td>Hot heat exchanger</td>
<td>Number of holes: 61</td>
</tr>
<tr>
<td></td>
<td>Height: 40 mm</td>
</tr>
<tr>
<td></td>
<td>Hole diameter: 2.0 mm</td>
</tr>
<tr>
<td></td>
<td>Heat exchanger diameter: 40 mm</td>
</tr>
<tr>
<td>Regenerator</td>
<td>Cross section: 50 cm²(bottom)-12.5 cm²(top)</td>
</tr>
<tr>
<td></td>
<td>Height: 80 mm</td>
</tr>
<tr>
<td></td>
<td>Height of tapered part: 20 mm</td>
</tr>
<tr>
<td></td>
<td>Wire diameter: 0.040 mm</td>
</tr>
<tr>
<td></td>
<td>Filling factor: 31 %</td>
</tr>
<tr>
<td>Pulse tube</td>
<td>Inner diameter: 40 mm</td>
</tr>
<tr>
<td></td>
<td>Height: 160 mm</td>
</tr>
<tr>
<td></td>
<td>Volume: 201 cm³</td>
</tr>
</tbody>
</table>

Table 6.1: Dimensions of setup 2.

\[ N_u = \frac{\xi(R_e - 1000)P_r}{1 + 12.7\sqrt{\xi(P_r^{2/3} - 1)}} \left(1 + \left(\frac{d_h}{l}\right)^{2/3}\right), \]  

(6.2)

with

\[ \xi = \frac{8}{(1.82 \log R_e - 1.64)^2}. \]

with \(d_h\) the hydraulic diameter of the flow channel, \(l\) the length, and the Prandtl number \(P_r\) approximated \(P_r = 0.7\). The Reynolds number is defined by equation 2.75. The heat transfer coefficient can be calculated from the Nusselt number by

\[ N_u = \alpha h d_h. \]  

(6.3)

The pressure drop can be calculated with equation

\[ \Delta P = c_w \frac{l}{d_h} \frac{1}{2\rho v^2}. \]  

(6.4)

a suitable empirical relation for the friction factor, for instance the fanning friction factor used in chapter 4

\[ c_w = 4 f_r = 4 \cdot 0.046 R_e^{-0.2}. \]  

(6.5)

In table 6.2, the results of the estimation are given.

Not only heat transfer from the gas to the copper was taken into account, but also the heat transfer through the copper. This can be calculated using the diffusion equation for constant thermal conductivity

115
Table 6.2: Estimated performance of the heat exchangers in setup 2. The estimated temperature difference and pressure drop over the heat exchangers is based on expected velocities in the slits, geometry, and standard empirical relations.

<table>
<thead>
<tr>
<th></th>
<th>$v$ [m/s]</th>
<th>$R_e$</th>
<th>$N_u$</th>
<th>$\alpha_{hx}$ [W/m²K]</th>
<th>$c_w$</th>
<th>$Q_{typ}$ [W]</th>
<th>$\Delta T$ [K]</th>
<th>$\Delta p$ [mbar]</th>
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<tbody>
<tr>
<td>HHX</td>
<td>13</td>
<td>6.2·10³</td>
<td>23</td>
<td>1.8·10³</td>
<td>3.2·10⁻²</td>
<td>300</td>
<td>2.3</td>
<td>4</td>
</tr>
<tr>
<td>CHX</td>
<td>18</td>
<td>67·10³</td>
<td>140</td>
<td>9.5·10³</td>
<td>2.0·10⁻²</td>
<td>100</td>
<td>0.1</td>
<td>120</td>
</tr>
<tr>
<td>AC</td>
<td>85</td>
<td>15·10³</td>
<td>56</td>
<td>4.9·10³</td>
<td>2.7·10⁻²</td>
<td>2000</td>
<td>5.5</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 6.3: Estimated temperature difference in radial direction for the three heat exchangers.

<table>
<thead>
<tr>
<th></th>
<th>$Q_{typ}$ [W]</th>
<th>$\phi$ [W/cm²]</th>
<th>$f$</th>
<th>$\Delta T$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHX</td>
<td>300</td>
<td>6</td>
<td>0.85</td>
<td>1.8</td>
</tr>
<tr>
<td>CHX</td>
<td>100</td>
<td>1.8</td>
<td>0.83</td>
<td>0.6</td>
</tr>
<tr>
<td>AC</td>
<td>2000</td>
<td>8</td>
<td>0.90</td>
<td>9</td>
</tr>
</tbody>
</table>

\[ \kappa \nabla^2 T = \phi, \]  
(6.6)

with $\phi$ a heat source term. In this case, it is determined by the amount of heat that is transferred to the heat exchanger, per unit of volume of the heat exchanger. If we assume that the temperature will only depend on the radius, equation 6.6 written in cylindrical coordinates becomes

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \frac{\phi}{\kappa}. \]  
(6.7)

The solution is

\[ T(r) = T_0 + \frac{\phi}{4\kappa} r^2. \]  
(6.8)

This makes the temperature difference between center and edge of a heat exchanger equal to

\[ \Delta T = \frac{\phi R_{hx}^2}{4\kappa}. \]  
(6.9)

With $R_{hx}$ the radius of the heat exchanger. To compensate for the influence of the holes on the heat flow resistance of the heat exchanger body, we can use a modified thermal conductivity $\kappa' = f\kappa$, with $f$ the filling factor of the heat exchanger. The results of this calculation for the three heat exchangers is given in table 6.3, using the other specifications for the geometry in table 6.1. The temperature difference in radial direction over the after cooler is rather large, since the heat has to be transported over a relatively large length.

The pulse tube has a diameter of 40 mm, while the diameter of the piston is 80 mm. The transition from the large to the small diameter takes place in the regenerator. This way we minimize the risk of turbulence due to the transition. Furthermore, the smaller cross section on the cold end of the regenerator counteracts the effects of the lower velocity (due to lower density) on the heat transfer between the gas and the regenerator matrix. Because of the low velocity this does not lead to a significant pressure drop.

The setup is modular in a way that individual components can be changed. Therefore, seals are placed between each component. At room temperature, standard rubber O-ring seals are used.
(e.g. between pulse tube and hot heat exchanger, and between the after cooler and regenerator). There are no rubber O-ring seals between helium and vacuum. In cold places, indium seals are used. A photograph of setup 2 is shown in figure 6.2.

### 6.3 Experimental results

#### 6.3.1 Orifice Pulse tube

Due to the smaller total gas volume of the machine, and the resulting pressure amplitude, the average pressure for the ‘standard’ conditions was lowered from 28 bar for setup 1 to 26 bar for setup 2. Standard frequency remained 25 Hz. The orifice setting was optimized for minimum temperature. A typical curve for minimum temperature versus orifice setting is given in figure 6.3.

The orifice used in this test setup is a needle valve. The value of $C_1$ is calculated from the measured pressures in the pulse tube and buffer. Combining equations 2.30 and 3.20 results in
CHAPTER 6. PULSE-TUBE EXPERIMENTS - SETUP 2

Figure 6.3: Temperature versus orifice flow conduction, setup 2.

\[ C_1 = \frac{\omega V_b p_{ba}}{\gamma p_{ta} p_0} \]  

(6.10)

So the orifice flow conductance can be calculated from the frequency, buffer volume \( V_b \), buffer pressure amplitude \( p_{ba} \), pulse-tube pressure amplitude \( p_{ta} \), and average pressure \( p_0 \). All quantities that can be measured. Once the optimum setting has been determined, the valve is fixed. At this setting, a cooling curve is measured. For standard conditions, the cool down is shown in figure 6.4 and the cooling curve is given in figure 6.5.

The influence of the orifice flow conductance \( C_1 \) can be expressed in terms of the phase shift factor \( \alpha_t \) (equation 3.24). The minimum temperature is found for \( C_1 = 10 \text{ mm}^3/\text{Pas} \). For that value, the phase shift factor is \( \alpha_t = 0.72 \), which is close to the expected \( \alpha_t \approx 1 \) (paragraph 3.3.2).

The minimum temperature is 63 K, cooling power at 77 K is approximately 50 W. The electrical input power at is 3.2 kW, so the \( \text{COP} \) at 77 K is 1.6 %. This is 4.5 % of Carnot. Since the theoretical ideal \( \text{COP} \) of a pulse tube is less than the Carnot efficiency (equation 2.44), it is better to express the \( \text{COP} \) relative to this ideal \( \text{COP} \). In this case \( \text{COP}/\text{COP}_C \) equals 6 %.

Measurements at different pressures and frequencies

Several measurements were done at different pressures and frequencies, similar to the measurements of the first setup. For each set of conditions, the orifice setting is optimized for minimum temperature, before measuring the cooling curve. In figure 6.6 \( \dot{Q}_1 \) is given versus \( T_l \), for different average pressures. The \( A_OI \) is proportional to the average pressure (figure 6.7), as in setup 1. This is caused by the pressure amplitude. At a higher average pressure, the pressure amplitude is higher as well. This is indicated in figure 6.8, which gives the pressure amplitude at the minimum temperature of the respective curve. The pressure amplitude varies linearly with the
**Figure 6.4:** Cool-down curve and first few orifice optimization steps, for setup 2.

**Figure 6.5:** $\dot{Q}_l$ versus $T_i$ for standard conditions, for setup 2.
average pressure. According to equation 3.20

$$\bar{H}_t = \frac{1}{2} C_1 \rho_0^2,$$

this means that the enthalpy flow through the tube is also higher. One would expect that the enthalpy flow would vary quadratic. However, each curve has a different orifice setting. The orifice setting determines the phase shift over the pulse tube, though the phase shift parameter $\alpha$ (equation 3.24)

$$\alpha_t = \frac{\omega V_t}{\gamma C_1 \rho_0}.$$

The optimum performance is expected for $\alpha_t \approx 1$. This means that the optimum flow conductance is proportional to the reciprocal of the average pressure. This results in the enthalpy flow being linear with average pressure again, as shown in figure 6.9.

The cooling curves for different frequencies are given in figure 6.10. We see the same trends as in setup 1. There is a slight variation in no-load temperature, and the $AOI$ again is proportional to the frequency.

The $AOI$ versus frequency is given in figure 6.11. The $AOI$ is proportional to the frequency. Again, pressure amplitude and orifice setting are determining this trend. In figure 6.12 the pressure amplitude is plotted against frequency. The higher pressure amplitude is caused by the higher flow rate, which is proportional to frequency.

From equation 3.24 it follows that at a higher frequency, the orifice flow conductance should also be higher. This is shown in figure 6.13. The resulting enthalpy flow through the pulse tube is also shown in figure 6.13. These results are at the minimum temperature, so this enthalpy flow is equal to the losses in the PTR.
CHAPTER 6. PULSE-TUBE EXPERIMENTS - SETUP 2

Figure 6.7: Angle of inclination versus average pressure, cylindrical test setup.

Figure 6.8: Pressure amplitude in the pulse tube versus average pressure at no-load temperature.
Figure 6.9: Optimum orifice flow conduction and resulting enthalpy flow through the pulse tube versus average pressure, setup 2.

Figure 6.10: $\dot{Q}_1$ versus $T_1$ for different operating frequencies, average pressure is 26 bar, setup 2.
CHAPTER 6. PULSE-TUBE EXPERIMENTS - SETUP 2

Figure 6.11: The AOI versus operating frequency for setup 2.

Figure 6.12: Pressure amplitude in the pulse tube versus the operating frequency, setup 2.
6.3.2 Inertance-tube experiments

For the first inertance tube experiments, the orifice was directly replaced by an inertance tube. The first inertance tube was of inner diameter 4 mm, and a length of 700 mm. The result was a very unstable measurement, with a no-load temperature of approximately 160 K.

The cause of this instability was the interfacing between hot heat exchanger and inertance tube. The inertance tube ended directly above the hot heat exchanger. Flow straightening was placed between the inertance tube and the hot heat exchanger (figure 6.14). This however turned out not to be enough for stable measurement.

The gas velocities are quite high. According to the lumped parameter model, an inertance tube of \( D_{it} = 4 \text{ mm} \) and \( l_{it} = 700 \text{ mm} \) and \( p_{ta} = 3.5 \text{ bar} \) gives a velocity of approximately 280 m/s. The momentum of such a stream is high, leading to jet-driven streaming. The dynamic pressure \( p_d = 0.5 \rho v^2 \) is 1.5 bar. A way to straighten that is to use a resistive element with at least the same amount of static pressure drop. However, that will be a significant contribution to the total flow impedance on the hot end. As a result, it will be very difficult to design and use an optimized inertance tube in the current setup. It would be better to reduce the velocity before the straightener by means of a diffuser. A diffuser reduces the velocity through a channel of gradually increasing cross section. It is important that the angle of the diffuser is not too large. If a diffuser changes cross section too rapidly, such as in this setup (figure 6.14), the flow separates and the diffuser does not work properly.

Instead of adding more flow straightening, we choose to re-place the orifice between the inertance tube and the hot heat exchanger (figure 6.15). The orifice is expected to act as a flow diffuser for the jet from the inertance tube, because the flow channel in the valve body is so complex that a jet cannot be formed. Furthermore, it enabled adjustment to find the optimum performance for each inertance tube. With this solution, stable measurements were performed and no-load temperatures of approximately 50 K were reached.
A complication in these measurements is that the exact influence of the orifice is not known. The orifice adds a resistive component to the entire flow impedance of the orifice-inertance tube combination. The orifice itself acts nonlinear in such a way that it turned out to be impossible to predict its resistance, for instance as a function of the number of turns that the valve is opened. Therefore, the results above can not be analyzed completely quantitatively. A qualitative analysis can be done. The lumped-parameter inertance tube model is used for this.

Several inertance tubes were tried. In table 6.4 their dimensions are given, together with the expected phase shift and enthalpy, based on a pressure amplitude of 3.5 bar, and calculated with the lumped parameter model. The inertance tubes with a high enthalpy flow have relatively low contribution of the resistive part. When such a tube is used, it might be the case that the flow is so high, that the assumed pressure amplitude of 3.5 bar is not reached.

All these inertance tubes were used together with an orifice. The orifice setting was optimized for minimum temperature. The 8 mm tube did not show any difference with the orifice alone. The cooling curves for the 4 mm and 6 mm inertance tubes are given in figure 6.16. The best results were obtained with the 3.5 m long tube. The cooling power at 77 K is approximately 160 W. The measured electrical input power is 3.6 kW, and $COP = 4.5\%$.

The best-operating inertance tube has a length of 3500 mm. From table 6.4 it can be seen that the expected phase shift of this tube is $48^\circ$, and the enthalpy flow is 384 W. Because of the additional resistive component, the actual phase shift will be less.

For this inertance tube, measurements have been done with an additional pressure transducer

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Figure 6.14: Detail of the flow straightening between the hot heat exchanger and the connection for the inertance tube.

Figure 6.15: Placement of pressure sensors around the orifice, inertance tube and buffer.
Table 6.4: Calculated phase shift and enthalpy flows for different inextance tubes, for standard operating conditions and a pressure amplitude of 3.5 bar.

<table>
<thead>
<tr>
<th>Diameter and Length [mm]</th>
<th>Phase Shift</th>
<th>Enthalpy Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x700</td>
<td>19</td>
<td>631</td>
</tr>
<tr>
<td>6x1000</td>
<td>26</td>
<td>1405</td>
</tr>
<tr>
<td>6x1350</td>
<td>31</td>
<td>1086</td>
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<tr>
<td>6x2000</td>
<td>38</td>
<td>744</td>
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<td>48</td>
<td>384</td>
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<tr>
<td>6x5000</td>
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<td>219</td>
</tr>
<tr>
<td>8x2000</td>
<td>42</td>
<td>1486</td>
</tr>
</tbody>
</table>

Figure 6.16: $\dot{Q}_1$ versus $T_1$ for different inextance tubes, for standard conditions.
Figure 6.17: Measured pressures in the compression space, pulse tube, inlet of the inertance tube and buffer as functions of time. The inertance-tube dimensions are $D_{it}=6$ mm and $l_{it}=3.5$ m. The cold-end temperature is 49 K.

The measured pressures in the compression space, pulse tube, inlet of the inertance tube and buffer are shown in figure 6.17. This is the measurement with $T_l = 48$ K. In figure 6.18, the pressure amplitude in the buffer ($p_{ba}$) and at the inlet of the inertance tube ($p_{ia}$) are given as functions of $T_l$. The measured $p_{ha}$ is used to calculate the impedance of the inertance tube, using the lumped-parameter model. The pressure variation in the buffer can be calculated, using the electrical analogy of figure 4.2

$$p_b = p_i \frac{1/j\omega (C_{buf} + C_i/2)}{R_i + j\omega L_i + 1/j\omega (C_{buf} + C_i/2)}.$$  \hspace{1cm} (6.11)

Equations 4.7, 4.9, and 4.10 are used for $R$, $C$, and $L$ respectively. The volume flow into the buffer is equal to

$$\dot{V}_b = i\omega C_{buf} p_b.$$  \hspace{1cm} (6.12)

The flow into the buffer is thus 90° out of phase with the pressure.

In figure 6.19, the measured and calculated buffer pressure amplitudes, and phase differences between volume flow into the buffer and inertance tube inlet pressure are given. The difference between measured and calculated pressure amplitude is less than 10%. The difference in phase is approximately 15°, or approximately 30%. This is less accurate than the other predictions and needs further investigation.

The pressure difference over the orifice is in phase with the flow through it. With this pressure difference we can determine the phase difference between flow through the inertance tube and
Figure 6.18: Measured pressure amplitudes at the inlet of the inertance tube and in the buffer as functions of $T_1$.

Figure 6.19: Measured and calculated pressure amplitudes and phases in the buffer, as functions of the pressure amplitude at the inlet of the inertance tube. The inertance tube has $D_i=6$ mm and $l_i=3.5$ m. Calculations are based on the lumped parameter model. The lines represent the calculated values, the markers the measured values.
pressure drop over the inertance tube. The results are given in figure 6.20. The difference between measurements and predictions is approximately 30° or 50%. The difference in phase between figures 6.19 and 6.20 is caused by the capacitance of the inertance tube.

The results show that the lumped parameter model gives results for buffer pressure amplitude. The predictions for phase are however not accurate. In [2] it is mentioned that measuring differential pressure over a resistive element is not accurate. Inertia of the gas in a resistive element can lead to errors. This could be a reason for the large difference between measured and calculated phase differences. This can be avoided by using flow meters that measure the flow directly, for instance with a hot-wire anemometer. Therefore, the results mentioned above should be considered with caution. More accurate experiments should be done to provide quantitative data for the inertance tube model validation.

6.3.3 Conclusions and discussion

With the second pulse-tube setup, it was possible to perform stable and reproducible measurements. With the pulse tube operated with an orifice, measurements were done at different temperatures, pressures and frequencies.

Inertance-tube measurements were also done. Stable measurements were possible when the inertance tube is combined with an orifice, thus avoiding jet-driven streaming in the pulse tube. With an inertance tube of 6 mm inner diameter and 3.5 m length, a cooling power of 160 W at 77 K was reached, with a COP of 4.5%. Only one other PTR with such performance is known to exist [4].

Because of the presence of the orifice in the inertance tube measurements, it is difficult to determine which inertance tube model provides the most accurate results. For such a validation,
further experiments are necessary.

The observed cooling powers are among the highest reported with PTR’s so far. These measurements are valuable in themselves. They will also be used to validate the Stirling pulse-tube model, described in the next paragraph.

6.4 Stirling Pulse-Tube Model Validation

In the validation of a model, its predictions are compared with experiments. This is done with the measurements on the orifice pulse tube refrigerator presented in the previous paragraph. In this section only the measurement at standard conditions are mentioned. Measurements at different pressures and frequencies lead to similar conclusions.

In figure 6.21 the measured minimum temperature is compared to the predicted minimum temperature. For low values of \( C_1 \) the correspondence between measurements and predictions is good.

The predicted optimum is at \( C_1 = 16 \text{ mm}^3/\text{Pas} \), while the measured optimum is at \( 11 \text{ mm}^3/\text{Pas} \). The fact that the predicted temperature is lower than the measured temperature, means that cooling power is lost. Based on the measured \( \text{AOI} \), the loss is of the order of 60 W in the predicted optimum. There must therefore be losses that are not yet incorporated in the model.

In this point, the predicted enthalpy flow through the pulse tube (equation 3.20) is \( H_t = 200 \text{ W} \).

In figure 6.22 the measured and predicted pressures in pulse tube and compressor are given. The correspondence is very good. This means that the pressure drop calculation (equation 3.70) and the empirical data for \( c_w \) are correct. The difference in pressure does not explain the difference between predicted and measured minimum temperature. Most likely the culprits are second order effects such as mass streaming or surface heat pumping. These effects will be
stronger at higher mass flows in the system, and mass flows are higher at larger orifice openings. They can however not yet be adequately predicted by the simulation tools described in chapter 3.

Further comparison between measurements and predictions is done at the optimum setting of the orifice as determined by measurement ($C_1 = 11 \text{ mm}^3/\text{Pas}$). In figure 6.23 the measured cooling power is given as function of $T_l$. The line shows the predicted results, the markers show the measured results. In the calculations the measured average pressure and orifice flow conductance were used as input parameters. This explains why the calculated line is not completely straight. The difference between measurements and predictions is about $10 - 20 \text{ W}$, or $5 - 10 \%$, over the entire temperature range. The most important terms determining the cold heat exchanger power are the enthalpy flow through the pulse tube and the energy flow through the regenerator (equation 3.47). Both of these are thus considered to be accurately calculated.

In figure 6.24 the heat removed from the hot heat exchanger is given, as determined by the temperature rise of the cooling water. It is mainly determined by the enthalpy flow through the pulse tube (equation 3.46), which has to be removed from the system at the hot heat exchanger. Again the markers are the measured results, the line the predicted results. The difference here is typically $50 \text{ W}$. The difference can however be easily explained. The hot heat exchanger is not perfect. From the measurements and from the simulations one can see that the gas leaves the heat exchanger at a temperature higher than the heat exchanger. In this case, it means that the temperature of the buffer rises above the ambient temperature. Because of this high temperature, heat is transferred from the buffer to ambient air by radiation and convection. An estimation based on empirical relations [1] for free convection shows that $50 \text{ W}$ can easily be removed. A measurement was done with the buffer wrapped in thermal insulation (Thermaflex). This measurement showed a higher heat load on the hot heat exchanger and a higher gas temperature in the buffer.
Figure 6.23: Measured and predicted cooling power as a function of cold end temperature, standard conditions and $C_1 = 11 \text{mm}^3/\text{Pas}$. The markers represent the measured values, the line the calculated values.

At higher $T_l$, the total heat load on the warm end of the system increases. Therefore the temperature of the gas leaving the hot heat exchanger and the temperature of the buffer increase as well. This explains the increase in difference at higher temperatures in figure 6.24.

In figure 6.25 the amount of heat removed from the after cooler is given. The measured values are larger than predicted, by about 200 W or 10%. This is consistent with the observed accuracy on the other measured quantities.

To check this consistency, the heat removed from the after cooler can be compared to the mechanical input power. This is not measured directly, but can be checked by comparing the pressure and phase shift in this pressure. The amount of heat removed from the after cooler is almost equal to the power input to the compression piston. From figure 3.3 we can see that their relation is

$$P = Q_{AC} + H_{reg} + Q_{cond}. \quad (6.13)$$

The conduction losses and regenerator loss enthalpy flow are in total of the order of 100 W. The time average shaft power can be calculated with

$$P = \frac{1}{t_c} \int_0^{t_c} p_c A_p v d t, \quad (6.14)$$

with $t_c$ the cycle time, $p_c$ the pressure above the piston, $A_p$ the piston surface, and $v$ the velocity of the piston. Since all properties are harmonic quantities, this equation can be written as:

$$P = \frac{1}{2} V_c \omega p_{ca} \sin \theta_p. \quad (6.15)$$
Figure 6.24: Measured and predicted amount of heat removed from the hot heat exchanger as a function of cold end temperature, for standard conditions. The markers represent the measured values, the line the calculated values.

Figure 6.25: Measured and predicted amount of heat removed from the after cooler as a function of cold end temperature, for standard conditions. The markers represent the measured values, the line the calculated values.
In this equation, $\theta_p$ is the phase difference between pressure and piston position. The pressure amplitude $p_{ca}$ is the pressure amplitude in the compression space. All quantities in equation 6.15 can be measured. In figure 6.26 and 6.27 the pressure amplitudes in the compression space and pulse tube are given. The measured pressure amplitude agree very well with the predicted amplitude. The increase in pressure amplitude with increasing pressure is due to the increased gas density. This influence can for instance be seen in equations 3.65 and 3.67. The second part on the right-hand side of both equations indicates that for a given pressure amplitude, the amount of mass that is stored during the cycle is inversely proportional to the temperature. So for a given mass flow (due to the fixed compressor swept volume), the pressure amplitude will be higher for a higher temperature.

In figure 6.28 the phase difference between pressure and piston position is given. This phase shift is the phase shift by which the pressure leads the piston position. The method of measurement of the phase angle gives an error of approximately $5^\circ$.

The correspondence between measured and predicted pressure and phase in the compression space is very good. Taking the measurement accuracy into account, it is consistent with the observed difference in heat removed from the after cooler of 10%. Other causes of the observed difference could be heat sources not included in the model, such as friction between the piston and cylinder.

The electrical input power into the motor is larger than the heat removed from the after cooler. The measured electrical input power was approximately 3.2 kW for the entire cooling curve. The difference of 1 kW with the amount of heat removed from the after cooler is due to heat dissipated in the electrical motor and the frequency inverter.
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**Figure 6.27:** Measured and predicted pressure amplitude in the pulse tube as a function of the cold end temperature, for standard conditions. The markers represent the measured values, the line the calculated values.

**Figure 6.28:** Measured and predicted phase shift between pressure and piston position as a function of cold end temperature for the compression space and the pulse tube, for standard conditions. The markers represent the measured values, the line the calculated value.
6.4.1 Discussion and conclusions

The observed differences between measured and calculated performance is of the order of 10% or better. This accuracy enables accurate prediction of PTR performance, and validates the Stirling pulse-tube model as a design tool. The only large differences occur in the prediction of the optimum orifice setting. This requires further investigation.

Small differences remain, which are caused by minor differences in experimental circumstances and ideal modeling circumstances. The following aspects can play a role:

- The heat conduction through the regenerator wall will be higher than predicted due to the presence of flanges. The difference will be 5 - 10 W.

- The temperature of the gas after the hot heat exchanger is higher than the heat exchanger body. This will have a small influence on its flow conductivity. Because the measurement of this temperature is not very accurate - it is done with a thermocouple - an error of a few K can be expected here, leading again to a possible error of 5 - 10 W.

- A third possible source for differences are temperature gradients in the heat exchangers in the radial direction. Because heat is removed from or added to the heat exchanger from the outside, there is a temperature gradient in radial direction. For the cold and hot heat exchanger, the expected difference between center and edge are between 0.5 K and 1 K. For the after cooler the difference is 10 K (see table 6.3).

- The exact temperature of the heat exchangers is not known beforehand. In particular the after cooler and hot heat exchanger will be cooled by water. The temperature of the heat exchanger itself will be determined by the cooling water temperature and the effectivity of the heat transfer between cooling water and heat exchanger. This heat transfer is not part of the model.

These influences are so small, and so specific for a certain test setup that their influence was not included in the model as presented here. However, adding correction coefficients to the performance of hardware components is an option.

6.5 Discussion and conclusions

With the pulse-tube setup 2 as described in this chapter, stable and reproducible measurements were done. The pulse tube was operated with an orifice and an inertance tube. With an inertance tube a minimum temperature of 49 K was reached. A cooling power of 160 W at 77 K was realized. This was for the first time with this type of machine, and among the highest cooling powers with PTR’s in this temperature range.

The cooling power and efficiency that were obtained with the inertance tube are high. Such performance is to be expected from a commercial cryocooler. The electrical input power for the above mentioned cooling power was 3.6 kW. As such, the \( \text{COP} \) is 4.5%, which is 13% of the Carnot efficiency. The losses in the electrical motor and frequency inverter are approximately 1 kW, so the net mechanical input power is approximately 2.6 kW. If this pulse tube were to be driven by an electrical motor with (nearly) 100% efficiency, the \( \text{COP} \) would be 6%, or 17% of the Carnot efficiency. Only one other pulse tube is known [4] that has comparable performance.

In figure 6.29 the performance is compared to the data from the review of Ter Brake et al. [5]. In figure 6.29 the specific power is plotted as a function of cooling power. Specific power is the input power needed per watt cooling power. This is the reciprocal of the \( \text{COP} \). Setup 2 is added in the encircled area. It can be seen that the PTR is among the best performing
cryocoolers currently available. Other high-capacity PTR’s there were recently published that show similar cooling powers have lower efficiencies or have deliver this cooling power at higher temperature. GM-type PTR’s deliver respectively 120 W at 74 K, with 7 kW electrical input [6], 100 W at 80 K with 6 kW of input [7] and 120 W @ 165 K with 4.8 kW [8]. Stirling-type PTR’s that were published recently show similar cooling powers at higher COP’s than the GM-types [9],[10]. They have reported performances of respectively 120 W at 80 K with 4.4 kW p-V power and 120 W at 80 K with 2.4 kW p-V power. The p-V power is equal to the net mechanical input power. Compared to our performance of 160 W at 80 K with 2.6 kW input power, our cooler provides a higher cooling power with a better COP, respectively 2.7 % and 5 % versus 6 %.

The measurements with the orifice were used to validate the Stirling pulse-tube model. The comparison between the orifice pulse-tube measurements and the Stirling pulse-tube model showed good agreement. Cooling power and heat removed from the hot heat exchanger and after cooler show agreement between measurements and predicted within 10 %. The predicted pres-
sure amplitudes in the pulse tube and the compression space show agreement within 5%. This accuracy enables accurate prediction of PTR performance, and validates the Stirling pulse-tube model as a design tool.

A relatively large difference was found between the measured and calculated optimum orifice setting. The model predicted a larger orifice setting than measured. Furthermore, the difference between no-load temperature as a function of orifice flow conductance increased with increasing orifice opening. This suggests that loss mechanisms not yet described by the model degrade the pulse tube performance at larger flows in the pulse tube. These losses increase with increasing mass flow. Further experimentation and (numerical) modeling should provide more insight in these effects.

Inertance-tube measurements were done to validate the different inertance-tube models as described in chapter 4. The measurement procedure made accurate validation impossible. Because an orifice is placed between the inertance tube and pulse tube, no accurate data for the pulse-tube side of the inertance tube could be measured. The measured buffer pressure shows good correspondence with both the lumped-parameter and the transmission-line model, within approximately 10%. Experiments where the flow on the inlet of the inertance tube is directly measured, for instance using hot-wire anemometry, have to be done.
Bibliography


Chapter 7

Design of industrial cryocoolers

7.1 Introduction

In this chapter, two cryocoolers are proposed: One Stirling and one pulse-tube cryocooler. Both will use oil-free compressors and have the same cooling power range. The proposed pulse-tube refrigerator is based on setup 2, described in chapter 6. The principles of both coolers are shown schematically in figure 7.1. From experiments, it was concluded that it is possible to build a large-scale pulse-tube refrigerator, with a cooling power in the order of 200 W at 77 K. Our experiments showed a $COP$ of approximately 5 %, similar to comparable machines found in literature [1].

To make full benefit of a pulse-tube refrigerator, it should be driven by a compressor that is also low vibration and low maintenance. In practice, that means a dual opposed piston, oil-free compressor, with linear-motor driven pistons. The design of such a compressor requires development of bearing techniques that eliminate oil lubrication, such as flexure or gas bearings.

In a Stirling cooler similar types of bearings could also be applied to the displacer. Typical power levels, masses and amplitudes are lower in a displacer than in a compressor piston. Such a Stirling cryocooler would have many of the advantages of a PTR. It has high reliability and low maintenance. Cooling power ranges are comparable for both types of cryocoolers. The main advantage of a PTR would then be the absence of a moving displacer. The main advantage for a Stirling cooler the high efficiency. In many industrial applications, the latter would be the most important.

7.2 General requirements

Typical requirements for cryocoolers for industrial applications have been given in detail in chapter 1. These requirements and their consequences are repeated below:

1. high reliability;
2. low maintenance;
3. medium to large scale cooling power;
4. high efficiency;
5. low cost.
CHAPTER 7. DESIGN OF INDUSTRIAL CRYOCOOLERS

(a) Pulse-tube refrigerator.  
(b) Free-displacer Stirling cryocooler.

*Figure 7.1*: Schematic representations of the pulse-tube refrigerator (a) and the free-displacer Stirling cryocooler with linear dual-opposed piston compressor.

**Ad1 and 2**: These are the most apparent requirements for a new-generation cryocooler for industrial applications. A cryocooler should operate for an extended period, without any interruption for maintenance. Typical Mean Time Between Maintenance (MTBM) is 6000 to 8000 hours, or approximately 1 year of continuous operation [2],[3]. After this period, a cryocooler will have to be taken out of operation, warmed up, and serviced. This usually means that the system, that such a cooler is part of, also needs to be taken out of operation. For some applications, this is unacceptable. Numbers for mean time to failure (MTTF) are difficult to give, as failure mechanisms differ between types of cryocoolers and their associated failure modes, and are usually determined by random processes [4]. The MTBM for high-power industrial applications should be at least 25000 hours, or approximately 3 years of continuous operation.

There are two main reasons for maintenance. One is cleaning of the system, the other replacement of worn-out parts. Both reasons should be addressed. Cryocoolers are closed-cycle machines. Any pollution comes from within the system itself. This can be either from the oil that is used for lubrication or materials that are formed due to wear. Eliminating pollution due to oil can only be realized if the oil is completely removed from the machine. Elimination of wear can be achieved if any contact between moving parts is avoided.

These requirements are necessary for both the compressor and the cold head. The reliability of a moving displacer should be at least equal to the reliability of a pulse-tube cold head. Therefore, the cold head should be of the so-called free-displacer type. A free displacer is driven pneumatically. It is basically a tuned mass-spring systems, that achieves the correct amplitude and phase by carefully chosen mass and spring stiffness [6].

**Ad 3**: A typical cooling power is in the order of 100–1000 W. This allows either a significant liquefaction rate, or the cooling of HTS devices for large power applications. In the first prototype, the design goal is 200 W at 77 K.

**Ad 4**: Stirling Cryocoolers are currently state of the art with respect to efficiency, in the medium to large scale cooling power ranges. Typical COP are 10 %, or approximately 30 % of
CHAPTER 7. DESIGN OF INDUSTRIAL CRYOCOOLERS

Figure 7.2: Principle of a moving-magnet linear motor. The outer part is stationary, the inner part is moving. Permanent magnets create a permanent magnetic field. They are magnetized in opposite directions. The stationary coils are positioned in this magnetic field. A current through the coils produces a Lorentz force, moving the motor. The coils are wound in opposite directions. A magnetic yoke on the inside and outside closes the magnetic circuit.

the COPs according to Carnot. The total cryocooler efficiency also includes the efficiency of the electric motors. Conventional rotary motors have efficiencies higher than 95 %, linear motors have efficiencies of approximately 75 - 80 %. The system design should take this additional electrical loss into account.

Ad 5: A cooler should be cost effective. Cost requirements are present in several stages of cryocooler design and production. They have, however, not been taken into account in this chapter.

Additional requirements are related to the application of the cryocooler in a larger system. The machine should be easily integrated in the system, hence the total size should be kept as small as possible. It is also favorable if the cooler can be operated in any orientation. Furthermore, the cooler should not introduce too much mechanical vibration in the system. While some application might be more susceptible to vibrations than others, vibrations cause additional mechanical loads that could influence the total lifetime of the system. The common way to minimize vibrations is to build the compressor with two opposed pistons. The pistons are the heaviest moving components in the Stirling cryocooler. When they move in opposite directions, ideally no vibrations remain.

7.3 Compressor integration

If a linear motor driven compressor is used, integration of the compressor with the rest of the cooler requires special attention. Linear motors are usually based on permanent magnets and electromagnets. The Lorentz force, which drives the motor, is proportional to the current through the coils. The wire in the coils has resistive losses. If the resistive losses are minimized, the efficiency of the motor is maximized. In a so-called moving-magnet linear motor (figure 7.2) the magnets are placed on the moving part and the coils on the stationary part. In a moving-coil motor the coils would be placed on the moving part, which requires moving electrical leads.

In the section below we are considering the adaptation of a cryocooler to a linear compressor. The characteristics of the cooler are known. They can for instance be found experimentally, or by simulation, so we know (among others) the swept volume of the compressor, the pressure in the compression space, and their phase difference.

The piston is schematically shown in figure 7.3. The piston mass is $M_p$; the motor force is $F_m$; the position of the piston is $x_p$. A spring force $F_{sp}$ exists from a mechanical spring. This spring force is proportional to the piston position $x_p$ and a spring constant $k_s$. The pressure
Figure 7.3: Schematic representation of the forces on a linear motor driven compressor (free piston compressor). The piston (mass $M_p$) position is indicated by $x_p$. The piston is moved by a linear motor, causing a force $F_m$ on the piston. In the compression space, the pressure $p_c$ exerts a force on the piston surface. In the buffer space the pressure is $p_b$. A spring force $F_{sp}$ exists on the piston.

The difference over the piston ($p_c - p_b$) leads to a force on the piston. Friction is neglected. The point of most efficient operation of a linear compressor can be determined using the equation of motion of the piston, which is

$$-k_s x_p + F_m - (p_c - p_b) A_p = M_p \ddot{x}_p. \quad (7.1)$$

The pressure, movement, and forces are described with first order harmonic functions

$$x_p(t) = x_{pa} \cos(\omega t), \quad (7.2)$$

$$F_m(t) = F_{ma} \cos(\omega t - \phi_m), \quad (7.3)$$

$$p_c(t) = p_0 + p_{ca} \cos(\omega t - \theta_p), \quad (7.4)$$

and

$$p_b(t) = p_0 - p_{ba} \cos(\omega t). \quad (7.5)$$

The volume displacement of the compressor is equal to

$$V_p(t) = V_{pa} \cos(\omega t), \quad (7.6)$$

with

$$V_{pa} = x_{pa} A_p. \quad (7.7)$$

Equation 7.1 gives

$$(-k_s x_{pa} - A_p p_{ba}) \cos \omega t + F_{ma} \cos(\omega t - \phi_m) - p_{ca} A_p \cos(\omega t - \theta_p) = -M_p x_{pa} \omega^2 \cos \omega t. \quad (7.8)$$

This equation can be split in a part that is in phase with the piston position, and a part that is out of phase. The in-phase component gives
\[-k_s x_{pa} - A_p \rho_{ba} + F_{ma} \cos \phi_m - p_{ca} A_p \cos \theta_p = -M_p x_{pa} \omega^2, \]  
(7.9)

the out-of-phase component gives

\[F_{ma} \sin \phi_m - p_{ca} A_p \sin \theta_p = 0. \]  
(7.10)

The values for \(V_{pa}, p_{ca}, \theta_p, \) and \(\omega\) are fixed input parameters. \(A_p\) and \(x_{pa}\) are related through equation 7.7, so only one of them can be freely chosen. Because the buffer volume behind the piston acts as a gas spring, the \(p_{ba}\) is proportional to \(x_{pa}\). The buffer-spring stiffness is

\[k_b = \frac{p_{ba} A_p}{x_{pa}} = \frac{\gamma p_0 A_p^2}{V_{b0}}, \]  
(7.11)

with \(V_{b0}\) the average buffer volume. The buffer volume is assumed adiabatic, so the correlation between \(p_{ba}\) and \(x_{pa}\) can be calculated using Poisson’s equation (2.30). The quantities that thus can be varied are \(F_{ma}, \phi_m, A_p, k_s, M_p,\) and \(V_{b0}\).

From equation 7.10, the required force from the linear motor is

\[F_{ma} = \frac{p_{ca} A_p \sin \theta_p}{\sin \phi_m}. \]  
(7.12)

For a given \(A_p, F_{ma}\) is minimal for \(\phi_m = \pi/2\). For this value of \(\phi_m\) the motor force disappears from equation 7.9, which reduces to

\[-k_s x_{pa} - k_b x_{pa} - p_{ca} A_p \cos \theta_p = -M_p x_{pa} \omega^2. \]  
(7.13)

This is the equation of motion for an undamped mass spring system consisting of three springs, the mechanical with stiffness \(k_s\), the buffer gas spring with stiffness \(k_b\), and a so-called cycle-gas spring with stiffness \(k_c\), equal to

\[k_c = \frac{p_{ca} A_p \cos \theta_p}{x_{pa}}. \]  
(7.14)

The spring stiffnesses \(k_c\) and \(k_b\) can be varied by varying \(A_p\) and \(V_{b0}\). For \(\phi_m = \pi/2\) we must choose them such that

\[\omega^2 M_p = k_s + k_b + k_c. \]  
(7.15)

This is equal to the natural resonance condition of an undamped mass spring system. We thus need to design the compressor such that its natural resonance frequency equals the operating frequency. The piston diameter is usually one of the first parameters that needs to be frozen in the design phase. This means that \(k_c\) is determined. If we then vary the piston mass and the spring stiffnesses \(k_b\) and \(k_s\) to reach resonance such that equation 7.15 is fulfilled, the required force from the linear motors is minimal. The power delivered to the piston is then equal to

\[P = \frac{1}{t_c} \int_0^{t_c} F_m x_p \, dt, \]  
(7.16)

or

\[P = -\frac{1}{2} \omega p_{ca} A_p x_{pa} \sin \theta_p. \]  
(7.17)

For \(\phi_m = \pi/2\), the motor force is only used to perform work. This minimizes the required force from the motors, thus minimizing the required current through the coils, thus minimizing the resistive losses in the coils. This maximizes the linear motor efficiency.
In some cases, there is no mechanical spring. In that case, the optimum buffer volume is

\[ V_b = \frac{\gamma p_0 A_p^2}{\omega^2 M_p - k_c} \]  

(7.18)

A physical solution only exists for

\[ M_p > M_{p,\text{min}}, \]

with

\[ M_{p,\text{min}} = \frac{k_c}{\omega^2} \]  

(7.19)

If \( M_p = M_{p,\text{min}} \), the buffer volume needs to be infinite. The system is then resonant on the cycle spring stiffness alone. If the piston is heavier, its mass can be compensated by the additional buffer gas spring. If the piston is lighter, resonance at the required frequency is not possible.

The resonance conditions show the point where the minimum amount of force is required to move the pistons. It is assumed that the electrical current and thus the electrical dissipation in the linear motor coils is proportional to this force, so that the motors are running at maximum efficiency in this setting. However, for this to be true, it is required that motor force and electrical current are in phase. If this is not the case, an additional equation describing the electrical impedance of the motor coils has to be taken into account to find the most efficient operating point of the linear motors.

### 7.4 Pulse-tube cryocoolers with linear drive

The experiments with setup 2 showed state of the art performance. Therefore, its design will be used as the basis. First, the integration of the pulse tube from setup 2 with a linear compressor will be discussed. Second, a proposal will be given for a version with a cooling power of 250 W, as given by the general requirements. Finally, recommendations will be given for further optimization.

#### 7.4.1 Compressor integration

In figure 7.4, the pressure in the compression space and the position of the piston are shown for the measurement with the inerter tube of \( D_{it} = 6 \) mm and \( l_{it} = 3.5 \) m. Substituting this experimental data in equation 7.4, gives \( p_{ca} = 3.98 \) bar and \( \theta_p = -38.9^\circ \). The swept volume of the SPC-1 compressor is 260 cm\(^3\). In a linear-drive compressor this swept volume can be generated with pistons of different diameters and amplitudes. In table 7.1, the piston amplitude, cycle spring stiffness (equation 7.14), and minimum piston mass (equation 7.19) are given for a compressor with piston diameters of 60 mm, 70 mm, and 80 mm respectively. The total swept volume is made by two pistons. The amplitude is that of one of the two pistons. If a compressor with 70 mm pistons is used, each weighing 6 kg, the buffer volume behind the pistons, as derived by equation 7.18, should be approximately 0.8 liter. The power per piston is then 1.3 kW. This is consistent with the measured after-cooler power of 2.6 kW (chapter 6).

#### 7.4.2 Scaling and further optimization

To scale up the pulse tube cryocooler from 160 W to 250 W at 77 K, one should increase the areas of the cross sections by the same ratio. The regenerator diameter increases from 80 mm to 100 mm, and the pulse-tube diameter from 40 mm to 50 mm. The inerter tube should also
be scaled in such a way that for given pressure amplitude, the flow increases without changing the phase angle. A suggestion is given in table 7.2.

The power per piston will increase from 1.3 kW to 2 kW. The required buffer volume for piston resonance can only be determined when the piston geometry \((M_p, A_p, x_{pa})\) is known.

There are several aspects on the improvement of the performance of the pulse tube. The geometry of the pulse tube could be optimized to reduce second-order effects such as mass streaming. Further development in the numerical models is required for this. In literature, several solutions are given. Olson and Swift [5] have derived that streaming is zero if

\[
\frac{1}{A} \frac{dA}{dx} \approx \frac{\omega_{pa}}{p_0 v_t} (0.75 \cos \theta_t + 0.64 \sin \theta_t) - 0.0058 \frac{1}{T_0} \frac{dT_0}{dx}.
\]  

(7.20)

In this equation, \(v_t\) is the amplitude of the gas velocity in the pulse tube and \(\theta_t\) is the phase angle between pressure and velocity. S.I. units have to be used. Two methods for reducing streaming can be seen from equation 7.20. The pulse tube can have a slight diameter change along its axis \((dA/dx \neq 0)\). This so-called tapering counter-acts the driving mechanism behind

<table>
<thead>
<tr>
<th>(D_p) [mm]</th>
<th>(x_{pa}) [mm]</th>
<th>(k_c) [N/mm]</th>
<th>(M_{p, \text{min}}) [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>23</td>
<td>38.1</td>
<td>1.54</td>
</tr>
<tr>
<td>70</td>
<td>17</td>
<td>70.6</td>
<td>2.86</td>
</tr>
<tr>
<td>80</td>
<td>13</td>
<td>120.5</td>
<td>4.88</td>
</tr>
</tbody>
</table>

Table 7.1: Piston amplitude, cycle spring stiffness, and minimum piston mass, for the pulse tube test setup 2, for different piston diameters.
the streaming. Another possibility is to influence $\theta_t$. The optimum angle is approximately 45°, thus comparable to the angle for optimum regenerator performance. This effect should also be further investigated using the numerical models.

Further optimization of regenerator geometry could also be possible. The total regenerator volume could be increased. However, as suggested in chapter 2, the wire diameter and filling factor could be different for different positions in the regenerator. Since the exact phase and impedance of the inertance tubes is not yet known, it is not possible to give a prediction for the proposed pulse tube. However, if we use the orifice pulse tube as an example, we can use the Stirling pulse–tube model for a prediction. The setting under consideration is $T_l = 80$ K and standard conditions (figure 6.23). The first 2 cm of regenerator near the cold heat exchanger are replaced with gauzes of 25 $\mu$m. In table 7.3 the calculated total energy flow through the regenerator, the enthalpy flow due to heat exchange and the difference in pressure amplitude on both sides of the regenerator are shown, together with the cooling power, input power and $COP$.

Because of the smaller wire diameter, the regenerator losses $E_{\text{reg}}$ decrease. This is mostly due to the reduction of the losses due to finite heat exchange. As a result of the decreased energy flow, the cooling power increases. On the other hand, the pressure drop over the regenerator increases and the input power increases. The net result is that the $COP$ is increased.

### 7.5 Stirling Cryocoolers

#### 7.5.1 Design calculations

In the following paragraphs, the design of the ‘Cryosphere’, the free-displacer Stirling cryocooler of Stirling Cryogenics & Refrigeration BV is described. In the first paragraph, the adaptations to the Stirling Model are described. In the second paragraph, the starting point and optimization parameters are determined, and in the last paragraph the actual optimization is described.

An important step in the optimization of a system is determining what parameters need to be optimized. In this case, we focus on performance of the cryocooler. The cooling power is given by the basic requirements. The $COP$ needs to be maximized. The goal for the cooling power is 200 W. For safety margin, the optimizations were done with 250 W (a margin of 25%). Efficiency needs to be maximized, with an expectation of 10%. If a typical electrical motor efficiency is assumed to be 80%, the $COP$ of the Stirling cycle should be at least 12.5%.

---

Table 7.2: Proposed inertance-tube dimensions for a 250 W pulse tube.

| $l_t$ [m] | $D_t$ [mm] | $|Z|$ [Pas/mm$^2$] | $\arg(Z)$ |
|----------|------------|------------------|----------|
| 160 W pulse tube | 3.5 | 6 | 0.105 | 49.8° |
| 250 W pulse tube | 3.0 | 6.7 | 0.068 | 49.7° |

Table 7.3: Expected increase in performance of the orifice pulse-tube refrigerator when the regenerator is optimized by replacing a part near the cold end with finer gauzes.

<table>
<thead>
<tr>
<th></th>
<th>$\bar{E}_{\text{reg}}$ [W]</th>
<th>$\bar{H}_{\text{reg,hx}}$ [W]</th>
<th>$\Delta p_a$ [mbar]</th>
<th>$\bar{Q}_l$ [W]</th>
<th>$P$ [W]</th>
<th>$COP$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>existing reg.</td>
<td>142</td>
<td>96</td>
<td>530</td>
<td>72</td>
<td>2020</td>
<td>3.6</td>
</tr>
<tr>
<td>optimized reg.</td>
<td>118</td>
<td>74</td>
<td>740</td>
<td>114</td>
<td>2200</td>
<td>5.0</td>
</tr>
</tbody>
</table>
CHAPTER 7. DESIGN OF INDUSTRIAL CRYOCOOLERS

![Flowchart of the free displacer Stirling model.](image)

**Figure 7.5:** Flow chart of the free displacer Stirling model. Compared to figure 3.2 a third iteration loop is added to solve the equation of motion for the displacer.

![Schematic representation of the forces on the free displacer.](image)

**Figure 7.6:** Schematic representation of the forces on the free displacer. The compressor is not shown. The compressor provides a pressure $p_c$ in the compression space. The displacer is mounted on a displacer shaft. In a buffer space to the left of the compression space, this displacer shaft is mounted on a spring with a certain stiffness (providing force $F_{sd}$) and an optional motor providing additional force $F_{ext}$. The displacement of the displacer is $x_d$.

**Adaptations to the Stirling Model**

In its standard form, the Stirling Model as described in chapter 3 calculates a Stirling cycle based on a given compressor and displacer movement. In a free-displacer Stirling cooler, the displacer motion is not fixed a priori. Therefore, an additional equation has to be solved in the model, the equation of motion for the displacer. In our case this was solved by adding an additional iteration loop to the model. In figure 7.5 the flow chart of the Stirling model for a free displacer is shown. A third iteration loop (as compared to figure 3.2) is added to solve the equation of motion for the displacer.

The forces acting on the displacer are schematically shown in figure 7.6. The compressor provides a pressure $p_c$ in the compression space. Due to flow resistance in the regenerator and heat exchangers the pressure $p_e$ in the expansion space differs from the pressure in the compression space. The pressure drop over the channel connecting the cold head with the compressor is assumed negligible. The displacer is mounted on a displacer shaft. This displacer shaft is mounted on a spring, providing a force $F_{sd}$ proportional to the displacer position $x_d$. This spring is placed in a buffer space at average pressure $p_{db}$. The displacer has a mass $M_d$. Since one of the requirements of the cooler was wear-free operation, friction losses can be neglected.
For the system described in figure 7.6 the equation of motion then becomes

$$-k_s x_d + F_{ext} - (p_c - p_{	ext{bd}})A_{ds} + p_c A_d - p_e A_d = M_d \ddot{x}_d,$$

(7.21)

with $A_d$ the surface of the displacer, $A_{ds}$ the surface of the displacer shaft, and $F_{ext}$ an external force. This term can be used if we would like to actively drive the displacer. If we assume that pressure and motion can be described by first-order harmonic functions, we can separate this equation in sin and cos-terms. These two equations can be solved, with only the displacer amplitude and phase as unknowns. In the Stirling model this is done iteratively.

### Stirling calculations and optimization

Designing a cryocooler from scratch leaves a large number of degrees of freedom. Parameters such as compressor and displacer strokes and diameters, frequency, operating pressure, void volumes, and heat exchanger properties have a big influence. Finally, the regenerator affects the overall system efficiency. As a starting point, typical quantities were extracted from existing (prototype) cryocoolers [7], [8], [9], [10]. The results are shown in table 7.4.

These parameters can be divided in two groups; cycle parameters and geometrical parameters. Cycle parameters are compressor and displacer swept volumes, average pressure, and operating frequency. Geometrical parameters are regenerator, heat exchanger dimensions, and void volume. Void volumes are e.g. the connection space between the compression space and the after cooler. These volumes should be kept as small as possible.

The importance of choosing the correct optimization parameters is indicated below. We assume that we can freely choose the displacer amplitude and the phase shift. For the machine mentioned in table 7.4 the cooling power and COP are calculated as functions of the phase shift $\phi_d$ between compressor and displacer movement (figure 7.7), and as functions of the displacer swept volume, for two different compressor swept volumes (175 cm$^3$ and 200 cm$^3$). In figure

<table>
<thead>
<tr>
<th>Cycle parameters</th>
<th>Compressor swept volume</th>
<th>175 cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacer swept volume</td>
<td>45 cm$^3$</td>
<td></td>
</tr>
<tr>
<td>Average pressure</td>
<td>25 bar</td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>30 Hz</td>
<td></td>
</tr>
<tr>
<td>Phase shift</td>
<td>45 $^\circ$</td>
<td></td>
</tr>
<tr>
<td>Geometry parameters</td>
<td>Regenerator length</td>
<td>60 mm</td>
</tr>
<tr>
<td></td>
<td>Regenerator cross section</td>
<td>30 cm$^2$</td>
</tr>
<tr>
<td></td>
<td>Regenerator filling factor</td>
<td>35 %</td>
</tr>
<tr>
<td></td>
<td>Regenerator wire diameter</td>
<td>35 $\mu$m</td>
</tr>
<tr>
<td></td>
<td>After cooler - number of slits</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>After cooler - slit depth</td>
<td>10 mm</td>
</tr>
<tr>
<td></td>
<td>After cooler - slit width</td>
<td>0.4 mm</td>
</tr>
<tr>
<td></td>
<td>After cooler height</td>
<td>50 mm</td>
</tr>
<tr>
<td></td>
<td>Cold heat ex. - number of slits</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Cold heat ex. - slit depth</td>
<td>5 mm</td>
</tr>
<tr>
<td></td>
<td>Cold heat ex. - slit width</td>
<td>0.4 mm</td>
</tr>
<tr>
<td></td>
<td>Cold heat ex. height</td>
<td>50 mm</td>
</tr>
<tr>
<td></td>
<td>Void volume</td>
<td>200 cm$^3$</td>
</tr>
</tbody>
</table>

Table 7.4: Starting point for the design calculations for the free-displacer Stirling cryocooler.
Figure 7.7: $\dot{Q}_l$ and COP versus $\phi_d$. Two sets of calculations are shown, for compressor swept volumes of 175 cm$^3$ and 200 cm$^3$ respectively. The displacer swept volume is 45 cm$^3$. The optimum values for efficiency and cooling power are reached at different phase-shift angles.

7.7 the displacer swept volume is kept constant at 45 cm$^3$, in figure 7.8 the phase shift is kept constant at 45$^\circ$. Both cooling power and efficiency show optimum values. They do not coincide. A machine that has to be optimized for maximum cooling power within a certain size would have different cycle parameters as a machine optimized for maximum efficiency.

**Limiting the number of parameters to optimize** If the entire set of parameters from table 7.4 were used to optimize the machine, we could try to find an ultimately optimized design. This would however lead to practical problems. Certain parameters can not be chosen completely independent of other parameters. The average pressure is limited by materials strength, the heat exchanger design also depends on the geometry of the heat exchanger on the other side of it - e.g. the cooling water side. The frequency is limited by capabilities of drive electronics, but also influences noise and vibrations in the system. Therefore, the following choices were made.

- Average pressure: a practical value for average pressure is 30 bar. This is a typical value for cryocoolers of this size. At this pressure level, it will be relatively easy to construct for pressure strength.

- Frequency: Running the machine at 50 Hz would be easy for the drive electronics. No frequency conversion from the 50 Hz main frequency would be necessary. Optimization indicates a lower frequency. The lower the frequency, the lower the flow losses. A lower frequency also means that compressor and displacer swept volumes would have to be larger, increasing the overall size of the system. Therefore a frequency of approximately 25 Hz is chosen. Slight variations of a few Hz are possible for the tuning of the compressor and the displacer.

- Geometrical parameters: A similar philosophy as used in the pulse tube design from chapter 5 can be followed here. The starting point values from table 7.4 are not that different from the heat exchangers that are currently used in the SPC-1 cryocooler. Furthermore, heat exchanger
geometry does not have a large influence on the performance (within limits). Therefore, it was decided that the cold heat exchanger and after cooler of the SPC-1 will be used for the new cryocooler. Main advantage is the economy of scale in manufacturing of the cryocooler. The regenerator that is currently used in the SPC-1 cryocooler will also be used. It will be oversized for the cooling power of the new machine, but for the moment the economy of scale outweighs the possible reduction of materials cost. Despite the small aspect ratio $l/d$, heat conduction losses are small compared to other losses.

Finally, void volume is mentioned as a geometrical parameter. Optimization would lead to the trivial solution with zero void volume. In practice, the total void volume is the result of the entire machine construction, not a requirement. Therefore, the void volume will also be kept out of the optimizations process, and assumed constant at 200 cm$^3$.

The parameters that can freely be chosen are given in table 7.5, together with their optimum values.

**Free-displacer calculations**

The optimum setting from table 7.5 has the displacer swept volume and phase as two separate, independent parameters. The calculations are basically done as if the displacer would still be driven by a crankshaft. In a free-displacer cryocooler, they cannot be chosen freely, but are determined by the cycle itself as given by equation 7.21. The parameters that can be varied to influence these forces are displacer diameter, displacer shaft diameter, displacer mass, and spring stiffness. The latter can also be expressed as a resonance frequency

$$f_d = \frac{1}{2\pi} \sqrt{\frac{k_s}{M_d}}. \quad (7.22)$$
CHAPTER 7. DESIGN OF INDUSTRIAL CRYOCOOLERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor swept volume</td>
<td>196 cm³</td>
</tr>
<tr>
<td>Displacer swept volume</td>
<td>36 cm³</td>
</tr>
<tr>
<td>Phase shift</td>
<td>54 °</td>
</tr>
<tr>
<td>Frequency</td>
<td>25 Hz</td>
</tr>
<tr>
<td>Average pressure</td>
<td>30 bar</td>
</tr>
<tr>
<td>Cooling power</td>
<td>250 W</td>
</tr>
<tr>
<td>Shaft power</td>
<td>1896 W</td>
</tr>
<tr>
<td>COP</td>
<td>13.2 %</td>
</tr>
</tbody>
</table>

Table 7.5: Optimum cycle parameters for a Stirling-type cryocooler. The heat exchangers and regenerator from an SPC-1 cryocooler are used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor swept volume</td>
<td>194 cm³</td>
</tr>
<tr>
<td>Displacer swept volume</td>
<td>36 cm³</td>
</tr>
<tr>
<td>Displacer resonance frequency</td>
<td>25.2 Hz</td>
</tr>
<tr>
<td>Displacer rod diameter</td>
<td>20 mm</td>
</tr>
<tr>
<td>Phase shift</td>
<td>54.6 °</td>
</tr>
<tr>
<td>Cooling power</td>
<td>250 W</td>
</tr>
<tr>
<td>Shaft power</td>
<td>1898 W</td>
</tr>
<tr>
<td>COP</td>
<td>13.2 %</td>
</tr>
</tbody>
</table>

Table 7.6: Optimum operating conditions for the free-displacer Stirling cryocooler.

The diameter and mass are limited by choices already made - using the SPC-1 cold head. Displacer mass can be chosen, but is limited by its construction. Construction gives a minimum mass, from that the displacer can only be made heavier by adding mass. Spring stiffness and shaft diameter can be chosen freely, the latter can of course not be larger than the displacer itself.

The moving mass of the displacer assembly is 1.5 kg. If we perform the optimization procedure with the requirement of a free displacer, we get different results than when calculating with a driven displacer (table 7.5). The results for the free displacer are given in table 7.6. With the proper tuning of the displacer mass spring stiffness, the free displacer cryocooler operates as good as the driven displacer Stirling cryocooler.

7.5.2 The Cryosphere, a free-displacer Stirling cryocooler

The optimizations described above were the basis for the construction of a machine. In this section we describe the development of the actual prototype. A few things have to be taken into account. First of all, the translation of an ideal cooler design into an actual machine has consequences for the design. Certain geometrical parameters will not be possible to vary in reality. Examples are the void volumes in the system and the mass of the displacer. Furthermore, actual properties of components might be different from the design criteria. An example is the resonance frequency of the displacer. It is possible to get very close to the design criteria, but production tolerances will always lead to small differences. The design of the cooler should not be sensitive to such tolerances.

Other aspects of the machine will also require iteration between the original design and construction. For instance the calculations predict the required swept volume of the compressor. A particular swept volume can be obtained by choosing the amplitude and diameter of the
pistons. But other aspects such as the linear-motor design also have requirements with respect to piston amplitude and diameter.

First the final design will be compared to the actual geometry. After that, a few aspects of the mechanical design will be discussed. Even though the mechanical design is not part of this thesis, some aspects are important for the general requirements for such a cryocooler. These will be described as a logical follow-up.

Final design calculations

The design as mentioned in table 7.6 has been integrated as good as possible into the design of the compressor. The practical implementation of the cryocooler has changed some design quantities. The consequences for the geometry of the system are given in table 7.7. The increase in void volume and slightly different resonance frequency has led to a small decrease in efficiency. The calculated optimum still complies to the original design goals. With the calculation results, the equation of motion 7.1 for the compressor pistons can be solved. From the calculation, we find that $p_c \cos \theta = -6.06 \text{ bar}$ and $x_p = 16.1 \text{ mm}$, so $V_b = 1.34 \text{ liter}$. The power per piston is 970 W.

The results of the calculations with these quantities are given in table 7.8.

Realization

In figure 7.9 a cross section is given of the Cryosphere. It shows the dual compressor, with the pistons and their linear motors. In figure 7.10, a close-up of one of the compressor halves is shown. The linear motors consist of a moving magnet part - the middle part of the pistons - and a stationary coils part. The coils are not shown in detail, but are placed around the cylinders. The extremities of the pistons are the bearings. These are self-acting hydrodynamic bearings that use the helium gas itself as a lubricant. The pistons rotate. A special surface pattern on the bearings pumps helium gas into the gap between the piston and cylinder wall. Here the pressure is increased, providing a centering force. This lifting force keeps the rotating piston from touching the cylinder wall, thus eliminating friction and wear [11]. The choice for gas bearings over flexure bearings was made because of the smaller diameter and longer maximum piston amplitude.

<table>
<thead>
<tr>
<th>Void volume</th>
<th>330 cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacer resonance frequency</td>
<td>24.6 Hz</td>
</tr>
<tr>
<td>Displacer moving mass</td>
<td>0.920 kg</td>
</tr>
<tr>
<td>Piston moving mass (per piston)</td>
<td>7.2 kg</td>
</tr>
<tr>
<td>Piston diameter</td>
<td>70 mm</td>
</tr>
</tbody>
</table>

Table 7.7: Details of the geometry of the Cryosphere

| Cooling power | 250 W |
| Shaft power | 1937 W |
| COP | 12.9 % |
| Compressor swept volume | 249 cm³ |
| Displacer swept volume | 38 cm³ |
| Phase shift | 47.3 ° |

Table 7.8: Calculation results for the Cryosphere
Figure 7.9: Cross section of the Cryosphere with its main components. A close up of one of the compressor halves is given in figure 7.10, and of the cold head in figure 7.11.
Figure 7.10: Details of one compressor half. The piston consists of two gas bearings, with the permanent magnets of the linear motor in between. The coils of the linear motors are placed around the moving magnet part, outside the helium space. The rotation motor is placed in the back of the piston. Behind the piston is the buffer space.
The clearance of the piston in the cylinder is optimized to get sufficient sealing and stable bearings. Since these bearings are nontouching, there is no wear and no significant friction. Clearance between the pistons and their cylinders is of the order of 10 $\mu$m. This gap is small enough to provide sufficient sealing. The amount of clearance also determines the required alignment of the individual compressor parts. Ceramic materials are used for the bearings and cylinder walls in order to minimize the friction during the starting and stopping of the piston rotation.

Electrical rotation motors are placed in the end of the pistons, again with permanent magnets placed in the piston and the coils stationary. Placing coils in the stationary part has two advantages. First of all, there is no electrical connection between stationary parts and moving parts, and the coils can easily be placed outside the pressurized-helium space. This eliminates the risk of outgassing - polluting the working gas - and eliminates the need for electrical feed throughs to the pressurized-helium space.

The center part of the cryocooler houses the displacer assembly. The displacer and the expansion space are shown in the bottom of figure 7.9. A detailed view of the displacer assembly is shown in figure 7.11. The displacer is placed on a long displacer shaft. This displacer shaft goes into a buffer space, using a clearance seal. This clearance is maintained by two sets of leaf springs, so-called flexure spring, that provide both the necessary axial stiffness for the displacer movement, and a high radial stiffness to maintain all clearance seals that are of the order of 10 $\mu$m.

**Experimental setup**

The goal of the experimental program that is foreseen for the Cryosphere is to determine the correct functionality of all the individual components of the cooler and the thermal performance...
of the entire cooler. Within this thesis, only a few initial measurements have been performed. Only the performance of the individual components has been verified. The flexure bearings on the displacer and the gas bearings on the pistons both worked. However, due to difficulties with the alignment of the pistons and their cylinders no measurements on the thermal performance of the entire machine were done. The experimental setup and the measurements that are foreseen for these future experiments are described below.

The cross section of the experimental setup is shown in figure 7.12. The measured quantities and the positions at which they are measured are shown. To measure cooling power, an electrical heater is attached to the outside of the cold head. The electrical power used by this heater simulates an applied cooling load. The temperature of the cold head is measured. To determine the COP of the system, the electrical input power to the linear motors is measured. Cooling water flow and temperature difference between water inlet and outlet are used to determine the amount of heat removed in the after cooler. These three powers together determine the overall energy balance of the cooler.

The dynamic pressure in the compression space and buffer spaces, together with the positions of the displacer and compressor pistons are measured e.g. to determine the resonance conditions of both the displacer and compressor pistons. The pressure in the expansion space is not measured since the added void volume by connecting a pressure transducer is expected to seriously influence the system dynamics. The cold head is placed in a vacuum space to isolate the cold head from parasitic heat loads (not shown in figure 7.12).

The dynamics of the entire cooler is complex due to the interaction between the systems around both the displacer and the compressor pistons. Both systems need to run on their design points for the cooler to run at its optimum conditions. Determining these conditions is one of the goals of the experiments. The following experiments are foreseen:
• Determine dynamic properties of the individual components; in particular of the displacer moving mass, displacer resonance frequency, and piston mass. This can be done early in the manufacturing process. An example is the resonance frequency of the displacer assembly of 24.6 Hz, mentioned in the previous paragraph. This is also the phase where the individual components such as the gas bearings and the flexure bearing springs are tested.

• The next step is to determine the performance of the cold head with the associated displacer dynamics at different charge pressures and frequencies. The results from these measurements are also used to validate and fine-tune the simulations used to design the cooler.

• The optimum setting found in the previous step is taken as a basis for optimizing compressor dynamics. The buffer volume can be changed by adding or removing material from the buffer space plugs. Further optimizing will be done by slightly changing average pressure and operating frequency. These two steps will then have to be repeated iteratively to determine the optimum point.

The last two steps can be repeated for different operating temperatures of the cooler. At first, the cooler will be operated at the liquefaction temperature of nitrogen, 77 K. Since the cooler will have to operate at different temperatures, future experiments will also have to determine the optimum performance of the cooler at those other temperatures.

Scaling the free-displacer Stirling cryocooler

In the future, free displacer Stirling cryocoolers are foreseen with cooling powers in the range of 500 W - 1000 W. The way to increase the cooling power is to increase the swept volume of compressor and displacer, and the size of the regenerator. Because the description of the cooler is basically one-dimensional, doubling the cooling power will be done by doubling the cross section area of the cooler. This is equivalent to placing two coolers in parallel.

However, because the heat exchangers and regenerator of a 1 kW cooler have been used, the performance of the current Cryosphere design can be increased. This can be done by only increasing the mechanical input power, by means of increasing the compressor swept volume. The effect of the increase of the compressor amplitude has been calculated and is shown in figures 7.13 and 7.14. For each compressor amplitude the displacer resonance frequency is optimized for maximum $COP$. All other geometry is kept constant. The resulting cooling power at 77 K and displacer amplitude are calculated. The displacer amplitude increases almost linearly with the compressor amplitude. Since the pressure amplitude also increases with increasing compressor amplitude, the cooling power increases.

It can be concluded that this cooler is capable of delivering more cooling power at a higher efficiency than its present target. This is the result of the choices that were made in the early design stage, in particular the choice to use the existing SPC-1 regenerator and heat exchangers.

7.6 Discussion and conclusions

In this chapter, two industrial cryocoolers were proposed. One is a pulse-tube cryocooler, the other a Stirling-type cryocooler. The pulse tube was based on the experimental setup described in chapter 6 (setup 2). The Stirling-type is newly designed. There are several requirements for an industrial cryocooler. Some of these requirements are determined by the intended application. Many of these applications, for instance the cooling of superconductors, require high reliability
Figure 7.13: Calculated cooling power at 77 K and efficiency as functions of compressor amplitude.

Figure 7.14: Calculated displacer amplitude and optimum resonance frequency as functions of compressor amplitude.
and low maintenance. A cooler in an industrial application cannot easily be taken out of operation for maintenance, because this would mean that the entire application would have to be shut down.

High reliability means that there should be as little failure modes as possible. This means that pulse-tube refrigerators would be preferred. The absence of moving parts at low temperatures means that less can break.

However, low maintenance is also important. The maintenance required on cryocoolers is cleaning of dust and particles due to wear, but, more important, of oil used for lubrication. To get low MTBM, the cleaning should be avoided altogether. This means that oil-free compressors have to be used. This requires special bearing techniques such as flexure bearings or gas bearings, both combined with non-touching clearance seals.

For both cryocoolers, we chose a cooling power of 250 W, at a \( \text{COP} \) that is approximately 10\%. If we take the experimental results of chapter 6 into consideration, the proposed PTR should be able to reach this cooling power, but not with a \( \text{COP} \) of 10\%. Some improvement over the reported \( \text{COP} \) of 4.5\% is possible if more efficient electric motors are used and if individual components of the pulse tube are further optimized. This is consistent with the \( \text{COP} \) of other cryocoolers found in literature [1].

The Stirling-type cryocooler is expected to reach a \( \text{COP} \) of 10\%, based on the predictions and on the performance of the existing Stirling cryocoolers [3]. Furthermore, by using flexure springs on the displacer, there is no wear in the cold end of the cooler. Unfortunately, the performance of the cooler was not tested experimentally as part of this thesis.

With a suitable compressor, both types of cryocoolers will have low maintenance and high reliability. The main advantage of the pulse tube is the reduction of vibrations. The main advantage of Stirling cryocoolers is that the \( \text{COP} \) is expected to be up to twice as high as that of a PTR. Similar trends can be seen in [4]. For high-power industrial applications, this makes a Stirling-type cryocooler such as the Cryosphere the first choice. That is also why it was proposed to first build a Stirling cooler. The pulse tube that was proposed here could then easily be combined with the newly designed compressor.

With the compressor of the Cryosphere, it should be possible to operate the 160 W pulse tube of chapter 6 (setup 2) as well. Scaling the pulse tube to 250 W requires a compressor with a larger swept volume.
Bibliography


[10] SPC-1 design specifications, Internal documentation of Stirling Cryogenics & Refrigeration BV.

Chapter 8

Conclusions and future work

8.1 Conclusions

For stand-alone applications industrial cooling is usually provided by so-called cryocoolers. These machines cool the application directly, in a closed system, so without the need for a supply of cryogenic liquids such as nitrogen or helium. In the liquid-nitrogen temperature range ($\approx 77$ K), and the cooling power range of 100 - 1000 watt, cryocoolers based on the Stirling cycle are used traditionally.

Stirling-cycle cryocoolers have two moving elements: a piston and a displacer. The piston is oil lubricated. As a result, these machines require regular maintenance to remove oil contamination and replace worn parts. The mean time between maintenance (MTBM) typically is 6000 hours, or 250 days. New applications, for instance the cooling of superconducting power systems, require a significantly longer MTBM.

In the last two decades, so-called pulse-tube refrigerators (PTR) have been investigated intensely. They operate according to a cycle, comparable to the Stirling cycle, but without a moving part at low temperature. This eliminates a number of factors that determine the MTBM.

In this thesis cryocoolers with a long MTBM for 77 K and 100-1000 W of cooling power are investigated. In order to design a pulse-tube cryocooler, computer programs were written. These programs were based on two types of models. A one-dimensional, first-order harmonic model was used to create a program that enabled fast calculation. Because of the short calculation times of such models, large numbers of variants can be simulated. This enables optimization and trend calculation. The structure of an existing Stirling Model was used to implement such a one-dimensional, first-order harmonic model in a design tool, capable of simulating pulse tube performance.

To calculate the performance of a pulse tube in more detail, numerical models were used. It was shown that three-dimensional numerical simulations can be used to visualize two and three-dimensional flow in the pulse tube. These effects cannot be simulated with the first-order model, but can have a large influence on the performance of the pulse tube. At first two- and three-dimensional simulations were done with commercially available software. However, they needed very long calculation times. The need to reduce calculation times lead to the development of a dedicated numerical model in a separate PhD project.

Two prototype PTR’s were built. They both used a crank-shaft driven compressor, capable of operating the PTR over a wide range of operating conditions. As a final result cooling powers and efficiencies were realized that are required for a cryocooler for industrial applications. The
observed cooling power and efficiency were among the largest for this type of cooler. However, losses were observed that are related to multidimensional flow, such as turbulence and thermal and viscous boundary-layer effects. Furthermore, the efficiencies of our PTR’s were found to be below the state of the art in existing Stirling cryocoolers.

The measurements on the PTR’s were also used to validate the Stirling pulse-tube model. The conclusion of this validation is that the Stirling pulse-tube model predicts the performance of the PTR within 10%. Only the prediction of the optimum orifice setting did not agree very well. The difference between measured and predicted no-load temperature increased with increasing orifice flow conductance. This is consistent with the above-mentioned multidimensional losses in the pulse tube.

An important component of the PTR is the phase shifter. It was found that it is possible to reach optimum performance by placing a so-called inertance tube on the hot end of the pulse tube. This inertance tube combines the effect of flow resistance and inertia to obtain the proper phase shift between pressure and flow. The prediction of the performance of this inertance tube is important. Several models available from literature can be used within the structure of the Stirling pulse-tube model. Because of limited experimental data, the best inertance-tube model was not yet established. However, from the available experimental data, it appears that the simplest model, the so-called lumped-parameter model is already sufficiently accurate.

A compressor with gas bearings and flexure springs was designed and studied. With such a compressor, both pulse-tube and Stirling-type refrigerators can be built with increased MTBM. The main advantage of the pulse tube is the absence of vibrations; the main advantage of the Stirling principle is the intrinsic high efficiency. Based on these findings, a pulse-tube and a Stirling refrigerator were proposed for future investigation.

8.2 Future work

The results presented in this thesis show possibilities for further work. The two proposed cryocoolers from chapter 7 must be constructed and tested. First, the newly designed compressor together with the free-displacer Stirling cryocooler (the Cryosphere) should be tested. The pulse tube from chapter 6, setup 2, could then be integrated with the compressor to create the proposed PTR and further developed into commercial machines.

The modelling of PTR’s also requires further attention. The dedicated numerical model shows promising results as-is. However, it should be expanded so it can accurately predict the multidimensional flow in the pulse tube such as streaming and turbulence. Aspects that are important for this prediction are nonideal gas, temperature-dependent gas properties, and a more detailed geometry including the interface effects of heat exchangers and flow straighteners.

More experimental work needs to be done to establish a correct inertance-tube model. The models that were considered so far all use empirical data for the flow resistance. Further experiments can provide empirical data to accurately predict the impedance of the inertance tube. This inertance tube model is used as a component in the Stirling pulse-tube model.

When both the Stirling pulse-tube model and the numerical model are complete, the integrated design tool can be built. This integrated tool combines the first-order harmonic model for fast optimization with the numerical model for fine-tuning. Both models can generate each others boundary conditions. The result is a design tool that can be used to predict and optimize PTR performance fast and accurately.
# Nomenclature

**Roman symbols**

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**Greek symbols**

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<tr>
<td>h</td>
<td>hydraulic or high-temperature</td>
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<td>hx</td>
<td>heat exchange</td>
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<td>i</td>
<td>inertance</td>
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<td>int</td>
<td>interface</td>
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### Superscripts

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<td>Abbreviations</td>
<td>Description</td>
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<td>---------------</td>
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<tr>
<td>AC</td>
<td>after cooler</td>
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<td>AI</td>
<td>artificial insemination</td>
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<td>AOI</td>
<td>angle of inclination</td>
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<td>cc</td>
<td>complex conjugate</td>
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<td>CFD</td>
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<td>COP</td>
<td>coefficient of performance</td>
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<td>FCL</td>
<td>fault-current limiter</td>
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<td>GM</td>
<td>Gifford-McMahon</td>
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<td>HHX</td>
<td>hot heat exchanger</td>
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<td>HPTM</td>
<td>harmonic pulse-tube model</td>
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<td>HTS</td>
<td>high-temperature supercondutivity</td>
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<td>MRI</td>
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<td>MTBM</td>
<td>mean time between maintenance</td>
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<tr>
<td>MTTF</td>
<td>mean time between failure</td>
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<tr>
<td>NTU</td>
<td>number of transfer units</td>
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<td>PTR</td>
<td>pulse-tube refrigerator</td>
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<td>REG</td>
<td>regenerator</td>
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<td>SMES</td>
<td>superconducting magnetic energy storage</td>
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<tr>
<td>SPC</td>
<td>Stirling process cooler</td>
</tr>
<tr>
<td>SPTM</td>
<td>Stirling pulse-tube model</td>
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Summary

For stand-alone applications industrial cooling is usually provided by so-called cryocoolers. These machines cool the application directly, in a closed system, so without the need for a supply of cryogenic liquids such as nitrogen or helium. In the liquid-nitrogen temperature range (≈77 K), and the cooling power range of 100 – 1000 watt, cryocoolers based on the Stirling cycle are used traditionally.

Stirling-cycle cryocoolers have two moving elements: a piston and a displacer. The piston is oil lubricated. As a result, these machines require regular maintenance to remove oil contamination and replace worn parts. The mean time between maintenance (MTBM) typically is 6000 hours. New applications, for instance the cooling of superconducting power systems, require a significantly longer MTBM.

In the last two decades, so-called pulse-tube refrigerators (PTR) have been investigated intensely. They operate according to a cycle comparable to the Stirling cycle, but without a moving part at low temperature. This eliminates a number of factors that determine the MTBM.

In this thesis cryocoolers with a long MTBM are investigated. In order to design a pulse-tube cryocooler, computer programs were written. Two prototype crankshaft-driven PTR’s were built where high-power cooling could be investigated over a wide range of operating conditions. As a final result cooling powers and efficiencies were realized that are required for a cryocooler for industrial applications. Losses were found that are related to multidimensional flow, such as turbulence and thermal and viscous boundary-layer effects. Furthermore, the efficiencies of our PTR’s were found to be below the state of the art in existing Stirling cryocoolers.

In this work also a compressor with gas bearings and flexure springs was designed and studied. With such a compressor, both Stirling-type and pulse-tube refrigerators can be built with increased MTBM. The main advantage of the Stirling principle is the intrinsic high efficiency; the main advantage of the pulse tube is the absence of vibrations. Based on these findings, one Stirling and one pulse-tube cryocooler are proposed for future investigation.
Samenvatting

In op zichzelf staande, industriële applicaties wordt koeling meestal verzorgd door zogenaamde cryokoelers. Deze machines koelen een applicatie direct, in een gesloten systeem, zonder dat daarvoor een externe aanvoer van cryogene vloeistoffen, zoals stikstof of helium, nodig is. In het temperatuurgebied van vloeibare stikstof, (∼77K), en de vermogensrange van 100 tot 1000 watt, worden traditioneel cryokoelers gebaseerd op de Stirlingcyclus gebruikt.

Een Stirlingkoeler bevat twee bewegende onderdelen: een zuiger en een verdringer. De zuiger wordt gesmeerd met olie. Daarom is regelmatig onderhoud nodig om verontreinigingen als gevolg van deze olie te verwijderen en om versleten onderdelen te vervangen. Het onderhoudsinterval is typisch 6000 uur. Nieuwere toepassingen, zoals het koelen van supergeleidende systemen in de energievoorziening, vereisen een significant langer onderhoudsinterval.

In de afgelopen twintig jaar is er uitgebreid onderzoek gedaan aan zogenaamde pulsbuiskoelers. Deze werken volgens een thermodynamische cyclus die vergelijkbaar is met de Stirlingcyclus, maar dan zonder bewegende delen in het koude gedeelte van de koeler. Dit vermijdt een aantal factoren die het onderhoudsinterval beperken.

In dit proefschrift worden cryokoelers met een lang onderhoudsinterval onderzocht. Om een pulsbuis-koeler te kunnen ontwerpen zijn computermodellen geschreven. Twee prototypen van pulsbuiskoelers zijn gebouwd met een krukas-aangedreven compressor. Hiermee kan hoog- vermogen koeling onderzocht worden over een grote variëteit van werkscondities. Als resultaat zijn er koelvermogens en rendementen behaald die nodig zijn voor industriële, hoog-vermogen cryokoelers. Er zijn echter ook verliezen gevonden die veroorzaakt worden door complexe meerdimensionale stromingen, zoals turbulentie en thermische- en viskeuze-grenslaagverschijnselen. Bovendien zijn de rendementen van pulsbuiskoelers nog steeds lager dan die van gangbare Stirlingkoelers.

Verder is er in dit proefschrift een compressor beschreven die werkt met gas- en bladveerlagers. Met zo’n compressor kunnen zowel pulsbuis- als Stirlingkoelers gebouwd worden met een lang onderhoudsinterval. Het grote voordeel van Stirlingkoelers is het intrinsiek hoge rendement. Het voordeel van een pulsbuis is de afwezigheid van trillingen. Op basis van deze bevindingen worden tot slot twee koelers gepresenteerd voor verder onderzoek.
Nawoord


Ik wil mijn oud-collega’s bij Stirling bedanken voor de prettige samenwerking. In het bijzonder wil ik Tijn, Rik, Rob, Maarten, Jac, Armand en Francesco bedanken voor alle gezellige gelegenheden samen, waaronder de lunchwandelingen met ijsjes van de IKEA. Waar de zwarte knop voor dient blijft tot op heden een mysterie. Erik bedankt voor je hulp met het proefschrift, prettige samenwerking, maar vooral ook voor de motivering het nu eindelijk eens af te maken. Martien bedankt voor het uitwerken van beide prototypes en al deze honger en dorstopwekkende verhalen. Maar uiteraard ook de collega’s die ik niet bij naam genoemd heb, ik bedanken voor de samenwerking, zowel voor de realisatie van de resultaten in dit proefschrift, maar ook voor alle andere leuke dingen. En ‘mijn’ studenten Tijn, Paul, Paul, Mark, Pier, Cor en Katrijn, voor hun nuttige bijdragen aan dit werk.


Alle leden van de groep Lage Temperaturen van de faculteit Technische Natuurkunde wil ik bedanken voor hun hulp, gastvrijheid, de goede koffie en de gezellige groepsuitjes waar ik steeds toch weer op mee mocht. In het bijzonder wil ik Leo van Hout bedanken voor zijn hulp met het oplossen van het vacuümlek in de eerste testopstelling.

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Curriculum Vitae

Daniel Willems was born on May 14, 1975 in Roermond, the Netherlands. He finished his high school in 1993, and in the same year started studying Applied Physics at Eindhoven University of Technology. The masters-thesis work was done at the European Organization for Nuclear Research (CERN), in Geneva, Switzerland, with the subject "Calorimetric measurements and analysis on a full scale model of the LHC arc dipole cryostat". He obtained his Master of Science degree in August 1999.

He started his PhD work in January 2000 on High-Power Cryocooling. The work was done at Stirling Cryogenics & Refrigeration BV in Son, the Netherlands. He started working for Stirling as a Research and Development Engineer in January 2004. In February 2007 he continued his career in cryogenics at Thales Cryogenics BV in Eindhoven, the Netherlands, as Research Engineer.