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Citation for published version (APA):

Krauskopf, B., Wieczorek, S., & Lenstra, D. (2000). Different types of chaos in an optically injected semiconductor laser. *Applied Physics Letters*, 77(11), 1611-1612. <https://doi.org/10.1063/1.1310213>

DOI:

[10.1063/1.1310213](https://doi.org/10.1063/1.1310213)

Document status and date:

Published: 01/01/2000

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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- The final published version features the final layout of the paper including the volume, issue and page numbers.

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Different types of chaos in an optically injected semiconductor laser

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(Received 14 April 2000; accepted for publication 18 July 2000)

With advanced tools from bifurcation theory routes to chaos via period doubling and the break-up of tori are identified. This allows us to distinguish between different types of chaos in terms of the output characteristics of the laser. We also find locking to a periodic solution inside a region of chaos. This information is important for applications requiring chaotic signals, such as encryption schemes. © 2000 American Institute of Physics. [S0003-6951(00)05737-5]

As complex nonlinear dynamics are of increasing interest for applications, such as cryptography^{1,2} and computing,³ the question arises to locate and study sources of chaotic output. Laser systems are natural candidates because they are indeed known to produce chaotic output and have the additional advantage of operating on very short time scales. Arguably the most accessible such laser system is a semiconductor laser subject to external optical injection. It is known that optical injection produces an enormous variety of phenomena, including chaotic output,⁴⁻⁸ and the injection laser was recently considered for chaotic synchronization.^{9,10}

An optically injected semiconductor laser is described well by the three-dimensional single-mode rate equation model⁸

$$\begin{aligned}\dot{E} &= K + \left(\frac{1}{2}(1 + i\alpha)n - i\omega\right)E, \\ \dot{n} &= -2\Gamma n - (1 + 2Bn)(|E|^2 - 1).\end{aligned}\quad (1)$$

Here E is the complex electric field, n is the population inversion, K is the injected field strength, and ω the detuning of the injected field from the solitary laser frequency. Furthermore, α is the linewidth enhancement factor, B is the cavity lifetime, and Γ is the damping rate. To keep this exposition simple we restrict to the case $\alpha=2$, but similar routes to

FIG. 1. The bifurcation diagram in the (K, ω) plane. The stable locking region is shaded.

FIG. 2. Period-doubling route to chaos; $K=0.62$ and from (a) to (f) $\omega = 0.3, 0.5, 0.7, 0.71, 0.78$, and 1.1 .

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orbit appears (b), which then bifurcates to an attracting torus (c) when the curve T^2 in Fig. 1 is crossed. The power is modulated, but the frequencies of the period doubling are still prominent in the spectrum. The dynamics on the torus then locks to a period-six orbit (d). After the locking the torus is already breaking up (e). The ensuing chaotic attractor then grows, the power series becomes more chaotic and the spectrum even broader (f). Notice that there is still clear evidence of the frequencies that appeared in the period doubling.

In summary we identified and mapped out in detail different routes to chaos, exemplified by a sequence of period doublings, and the break-up of two different tori. The importance of this work lies in providing clear information of where certain types of chaotic dynamics can be found in a laser system that is experimentally quite accessible. In particular, we pointed out different output characteristics of the lasers, corresponding to chaotic attractors born in different ways. The chaotic attractor after period-doublings does not have a very broad spectrum and the frequency peaks typical for period doubling are still very prominent. These peaks disappear and the spectrum broadens only after further bifurcations leading to the growth of the attractor. On the other hand, the spectrum after the break-up of a torus appears to be quite broad. Here, a possible pitfall for applications is the occurrence of locking on the broken-up torus. We found a loss of chaos over a certain parameter range due to locking onto a period-three orbit. We expect this to be observable in experiments.

The variability of chaotic attractors presented here is important for applications requiring a chaotic signal with particular properties. For example, for chaotic encryption schemes where the parameter values provide the encryption key¹⁰ one needs to be confident to switch between suitable types of chaos. A loss of chaos due to locking must be avoided to ensure privacy. Investigating this further remains an interesting challenge for future research.

The research of S.M.W. was supported by the Foundation for Fundamental Research on Matter (FOM), which is financially supported by the Netherlands Organization for Scientific Research (NWO). B.K. thanks the British Council and NWO for support under the UK-Dutch Joint Scientific Research Programme.

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