Application of the modal transmission-line method to radiowave propagation through building walls

S.V. Savov and M.H.A.J. Herben

Abstract: The modal transmission-line (MTL) method is used to analyse the transmission coefficient of building walls, which is an important parameter for the accurate planning of microcellular mobile networks. For that, the buildings walls are modelled as multilayer periodic structures. After a description of the extended MTL theory, the newly developed MTL method is applied to reinforced-concrete walls. From a comparison of the MTL results with previously published theoretical results obtained by the finite-element method, it is concluded that the MTL method is very well suited because it is accurate as well as computational-time efficient.

1 Introduction

For accurate planning of cellular networks, new radiowave propagation models for the estimation of losses on the radio path must be used. For conventional macrocells usually the position of the base station is higher than the average height of the surrounding buildings, but for microcells this height is very often below the rooftops of the buildings. In the microcellular concept, the shielding properties of the surrounding buildings are used to confine the radiated field. There are several different mechanisms of propagation, such as line-of-sight propagation, reflection, and diffraction. These three mechanisms are included in most methods based on ray optics [1] or physical optics [2]. However at UHF, transmission of radiowaves through buildings can be the dominant propagation mechanism [3, 4]. Neglecting the contributions of these waves in the planning of a network may result in an unacceptable degree of intercell interference. There are very few publications available on the shielding effectiveness of buildings and they are based on rather simplified models.

In the present paper the transmission through walls is studied in more detail, using one special property typical for many of them: the periodicity. This property allows using a full-wave method, the so-called modal transmission-line (MTL) method, for an accurate and at the same time computationally efficient prediction of the transmitted field [5].

This paper begins with a description of the MTL method following [6, 7]. An extension of the theory is made by using matrix notations as in [8]. Then the theory is applied to the computation of electromagnetic wave transmission through reinforced-concrete walls. To apply the MTL method to such walls, they are modelled as multilayer periodic structures. To check the accuracy of the MTL results, they are compared with previously published finite-element method (FEM) results [9].

2 Extended MTL method with plane wave excitation

The geometry of the two-dimensional scattering problem is shown in Fig. 1a. In this paper only the TE case will be considered, but the method can also be applied to the TM case. A plane electromagnetic wave with unit amplitude is incident on a periodic lossy dielectric structure, with thickness \( h = h_1 + h_2 + \ldots + h_L \) and period \( d = d_1 + d_2 \), at an angle of incidence \( \theta \). A time convention for the harmonic field \( \exp(-j\omega t) \) is used and suppressed. The problem space consists of three regions (a, b and c) and region b is subdivided into \( L \) layers. In each region there are two types of solutions, namely incident and reflected waves with respect to the \( z \)-direction. It is convenient to use a global \( x \) and a local \( z_l \) co-ordinate for every layer \( x_l = x, z_l = z - (h_1 + \ldots + h_{l-1}) (l = 0, 1, 2, \ldots, L + 1) \) (0 \( \leq z_l \leq h_l \)) (here \( h_0 = 0 \) is assumed). The tangential fields \( \{E_{z,f}, H_{z,f} \} \) in the \( \lambda \) region can be written in terms of periodic Floquet space harmonics [8]

\[
E_l(x, z_l) = \sum_{n=\infty}^{\infty} \{ f_{l,n} \exp(ik_{x,l}z_l) + b_{l,n} \exp(k_{x,l}z_l) \} \\
\times \sum_{m=\infty}^{\infty} a_{l,m}\exp(ik_{x,l}x)
\]

\[
-H_l(x, z_l) = \sum_{n=\infty}^{\infty} g_{l,n} \exp(ik_{x,l}z_l) - b_{l,n} \exp(k_{x,l}z_l) \\
\times \sum_{m=\infty}^{\infty} a_{l,m}\exp(ik_{x,l}x)
\]

where \( f_{l,n} \) are the amplitudes of the incident waves and \( b_{l,n} \) are the amplitudes of the reflected waves, \( a_{l,m} \) are the amplitudes to be determined, \( k_{x,l} \) are the modal longitudinal wavenumbers, and \( k_{x,l} \) are the modal transversal wavenumbers. Because there is a single incident plane wave in the region (a), \( f_{0,n} = \delta_{n0} (\delta_{nm} \) the Kronecker delta

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symbols) and \( b_{0,n} = r_n \) (modal reflection coefficients), while in the region (c) there are no reflected waves which means \( b_{L+1,n} = 0 \) and \( f_{L+1,n} = t_n \) (modal transmission coefficients). The longitudinal wavenumbers have the following discrete spectrum:

\[
k_m = k_0 \sin \theta + m \frac{2\pi}{d}, \quad (m = 0, \pm 1, \pm 2, \ldots) \tag{2}
\]

The corresponding transverse wavenumbers (propagation constants) in the case of a homogeneous layer with a dielectric constant \( \varepsilon_0 \) are \( k_{m,n} = \sqrt{\epsilon_0 k_0^2 - k_{m,n}^2} \) for propagating modes and \( k_{m,n} = i \sqrt{\epsilon_0 k_0^2 - k_{m,n}^2} \) for evanescent modes. The modal admittances are

\[
y_{m,n} = k_{m,n}/k_0 \eta_0 \tag{3}
\]

where \( k_0 \) is the free-space wavenumber and \( \eta_0 \) is the free-space impedance. In every layer the complex electric constant \( \epsilon_0(x) \) is a periodic function with the same period \( d \), which means that it can be expressed in terms of Fourier series:

\[
e_\ell(x) = \sum_{m=-\infty}^{\infty} g_{\ell,m} e^{2\pi imx/d}, \quad g_{\ell,m} = \frac{1}{d} \int_0^d e_\ell(x) e^{-2\pi imx/d} dx \tag{4}
\]

In case of dichroic dielectric layers \( e_\ell(x) = \{ \epsilon_{\ell,1}, 0 < x < d/2; \epsilon_{\ell,2}, d/2 < x < d \} \) the following coefficients of the Fourier series are obtained:

\[
g_{\ell,0} \equiv \eta_n = \frac{d_1 \epsilon_{\ell,1} + d_2 \epsilon_{\ell,2}}{d},
\]

\[
g_{\ell,n} = \int_{2\pi}^{2\pi} \left[ 1 - e^{-\epsilon_{\ell,n}^2 \eta_0} \right] (n \neq 0) \tag{5}
\]

Next, for every layer a square matrix \( [P] \) is defined with elements \( P_{m,n} = k_{0,m,n}^2 - k_{m,n}^2 \). Then starting from Helmholtz time-harmonic wave equation for the tangential electric field \( E_\ell(x, z) \) in the \( n \)th layer, represented by (1), the eigenvalue equation for the modal propagation constants can be derived as

\[
det([P] - k_{m,n}^2[I]) = 0 \tag{6}
\]

where \([I]\) is an identity matrix. Generally, the eigenvalues are complex numbers with positive real and imaginary parts. The associated eigenvectors form a matrix \([A]\) with elements \( a_{0,m}\). To solve (6) numerically, the infinite matrices should be truncated so that \( -N_1 \leq n \leq N_1 \), which means only a finite number of \( N \) modes are used in (1), where \( N = N_1 + N_2 + 1 \). An estimate of the number of the modes \( N \) is given in the following Section. Note that if the layer is homogeneous all the coefficients in the Fourier series are zero except \( g_{\ell,0} = \varepsilon_0 \), which simply reduces the \( [P] \) matrix to the value \( k_{m,n}^2 \) already given, and the matrix \([A]\) to \([I]\).

The diagonal matrices \( [k]\) (with elements \( k_{m,n} \)) and \( [Y, d] \) (with elements \( y_{m,n} \)) correspond to the equivalent transmission-line representation of the problem, shown in Fig. 1b. The main difference with the classical transmission-line (TL) theory for multiple homogeneous layers is that here appropriate matrix characteristics \( \{ [k], [Y, d] \} \) are used.

After truncating the infinite series (1) the following finite-size local matrices in the \( n \)th layer are introduced: square matrices of the transmission \( [T] \) through the \( n \)th layer and the reflection \( [R(\xi_0)] \) from the plane \( z \). The unknown coefficients \( \{ f_{n,0}; b_{n,0} \} \) are determined after imposing the continuity of the tangential components of the fields on the plane interfaces between the regions

\[
E_{l+1}(x, z) = E_l(x, h_1); \quad H_{l+1}(x, 0) = H_l(x, h_1) \tag{7}
\]

For software implementation, a backward recurrence is derived and applied, starting from the semi-infinite layer \((n = L + 1)\) and ending to the semi-infinite layer \((n = 0)\). For the later one the transmission matrix is given by \( [T_0] = [T] + [R] \).

Next, the total reflection \( [R] \) and transmission matrix \( [T] \) are introduced as

\[
[R] = [R_0(0)]
\]

\[
[T] = [T_L][T_{L-1}][\ldots][T_1][T_0] \tag{8}
\]

The column matrices of the field transmission and reflection coefficients are then obtained by \( [t] = [T][u], [r] = [R][u] \), where \( [u] \) is a column matrix of the single-mode excitation \( u_n = b_{n,0} \). Note that the algorithm gives also an opportunity to find the field in every region by (1).

The total power transmission and power reflection coefficients are by definition

\[
P_t = \sum_n |\alpha_n|^2 \frac{y_{n+1,0}}{y_{n,0}} \tag{9}
\]

\[
P_r = \sum_n |\beta_n|^2 \frac{y_{n,0}}{y_{n,0}}
\]

### 3 Application of MTL method to building walls

The investigation continues with the transmission coefficients of periodic walls. First, a single periodic layer wall is considered (Fig. 1a, with \( L = 1 \)). The principal difference between the TL model and MTL model is that in case of a homogenous layer, according to the TL model, the transmitted wave propagates in the same direction as the incident wave, while in case of a periodic layer the MTL model delivers a number of waves, all with their own direction of propagation \( \phi_m \), which can be determined from (2) to be \( \phi_m = -\tan^{-1}(\sin \theta + m\pi/d) \), \( m = 0, \pm 1, \pm 2, \ldots \). It is obvious that the propagation angle of the fundamental mode \((m = 0)\) equals the angle of incidence. The width of the modal scattering pattern is a frequency dependent parameter. In the limiting case \( d/\lambda \rightarrow \infty \) (ray optics) this spectrum becomes very dense, but the main contribution has the fundamental mode. However, in the case of small values of \( d/\lambda \), the spectrum becomes sparse, but more modes contribute to the transmitted power. The condition for the propagating modes is simply \( \left| \sin \theta + m\pi/d \right| \leq 1 \), which is satisfied by \( N_1 \) "negative" modes and \( N_2 \) "positive" modes, so the total number of propagating modes is
\[ N = N_1 + N_2 + 1. \] A simple estimation of \( N \) in the case of dielectric periodic layers can be obtained from

\[ N \approx \lceil 2d/\lambda \rceil - 1 \quad (10) \]

where the symbol \( \lceil x \rceil \) means the largest integer, less than \( x \). For accurate calculations some evanescent modes have also to be included in \( N \). This explains why the total number \( N \) is nearly linear proportional to the normalised period.

To demonstrate the validity of (10), Fig. 2 presents the scattering pattern of a single periodic dielectric slab with parameters: \( d_{11} = d_{21}, e_{11} = 5, e_{21} = 3, f = 1.9 \text{GHz} \), obtained by the MTL model. The results shown in Fig. 2 hold for the case of \( d = b = 2 \), for an oblique incidence with \( \theta = 45^\circ \). The number of propagating modes \( N = 4 \) is indeed in good agreement with (10) and the maximum power here is transmitted into the direction of the fundamental mode (which is not always true). Fig. 3 shows the corresponding field distribution behind the wall, which clearly demonstrates the periodic interference pattern of the transmitted plane waves.

Next, the case of a reinforced-concrete wall is considered. It is particularly important to compare the performance of a reinforced-concrete wall with an homogeneous concrete wall. In [9] the finite-element method (FEM) is used for analysing such a structure, which is also a full-wave method, but computationally time consuming. To reduce the number of the layers in the MTL model it is necessary to replace the circular wire-grid with a square one with a size \( d_{12} \) equal to the diameter of the wires. Then the reinforced-concrete wall can be modelled as a simple three-layer structure \( (L = 3 \) in Fig. 1a). Computer simulations showed that this simplification is valid. The parameters of the model are extracted from [9]: \( d = 5 \text{cm}, h_2 = d_{12} = 3 \text{mm}, \) the dielectric constant of the concrete \( e_r = 7 + i0.3, \) the conductivity of the steel \( \sigma = 1.11 \times 10^7 \text{S/m}, \) the frequency \( f = 1.8 \text{GHz}, \) and normal incidence is considered.

Fig. 4a shows the transmission coefficient (dB) of both a reinforced-concrete wall (solid line) and a concrete wall without wires (dashed line) against the normalised thickness \( h/\lambda \). Here, \( N = 51 \) modes are needed, which is much more than the estimation \( N = 2 \) found by (10), because of the presence of a conductor, leading to an extremely large difference between \( e_{11} \) and \( e_{22} \). Further increasing the value of \( N \) does not significantly affect the results. An interesting observation from Fig. 4a is that for some wall thickness (like \( h = 0.5 \lambda \)) the transmission coefficient through the concrete wall is largest in the presence of a metal grid. Further, it appears that due to the resonant behaviour of

\[ h \quad | \quad \text{Transmission coefficient in dB} \]

\[ (h/\lambda) \]
the structure it may occur that the transmitted power increases with increasing wall thickness. Fig. 4a also clearly demonstrates that the MTL result for the reinforced-concrete wall is very close to the FEM result [9] (circles). This good correspondence of the results also justifies the replacement of the circular wire grid with a square one.

Next, a similar wall but now with a larger period $d=15\text{ cm}$, is considered. Fig. 4b shows that now the transmission coefficient of the reinforced-concrete wall is always smaller than for the homogeneous wall. Also, this example clearly demonstrates the accuracy of the MTL method. Finally, Fig. 5 shows the angular response of a reinforced-concrete wall with thickness $h=4\text{ cm}$, period $d=20\text{ cm}$, wire diameter $d_{w}=8\text{ mm}$, and dielectric constant $\varepsilon_{r}=7+j0.2$, at a frequency of $f=0.9\text{ GHz}$. Once again, the MTL results (solid line) are very close to the FEM results (circles) as extracted from [9].

![Graph]

**Fig. 5** Transmission coefficient of reinforced-concrete wall against angle of incidence for $f=0.9\text{ GHz}$:

- MTL
- FEM [9]

### 4 Conclusions

The modal transmission-line method has been successfully applied to the analysis of propagation through building walls, which is important for the accurate planning of microcellular mobile networks. For that a special property typical for many walls is exploited namely the periodicity. In addition to the transmission losses, the MTL method also yields the spectral distribution of the transmitted radio-waves, from which the complete field distribution behind the wall can be obtained. It is clear from the theoretical description of the MTL method that this method can also be used to determine the reflection properties of walls. The accuracy of the MTL method was examined here by a comparison with the FEM method for reinforced-concrete wall. This study justifies the conclusion that for the modelling of propagation of plane radio-waves through multilayer periodic building walls, the MTL method is very well suited because it is accurate as well as computationally by time efficient.

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### 6 References