Blind identification and allocation of multivariate disturbances

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Abstract—A second order statistics based blind identification technique is used to recover the physical sources from disturbances acting on a multivariable system. The results are then used to find the physical location of disturbance sources in an active vibration isolation platform. Furthermore, implications for multivariable controller design are discussed.

I. INTRODUCTION

In multivariable systems, the same disturbance, due to a single physical cause (source), can enter in more than one controlled variable. A typical example can be found in motion control, where multiple degrees of freedom of a controlled plant suffer from the same disturbance sources, e.g., pump, floor and machine vibrations. The sources cannot be measured directly, only an unknown mixture of these sources is observed at a certain place in the feedback loop. In order to study the physical nature of the disturbances, one has to recover both the sources and the mixture of these sources.

In this work, we show that this is equivalent to solving a blind identification problem. Blind identification problems appear in information theory, direction of arrival problems and array processing, see [3], [5] for a survey. Blind identification methods often rely on higher order statistics of the observed signals. An example of this is the independent component analysis technique used in [15]. Using higher order statistics implies that Gaussian sources cannot be retrieved. Also, as estimates of higher order statistics have high variance, long data sets are required. The novelty of our contribution is that we show that for time colored sources, only a set of second order statistics are required to solve the blind identification problem. Hence, the method from [1] can be used to solve the blind identification problem within some indeterminacies.

As opposed to other disturbance modeling techniques, e.g. [14], we are able to find a structured disturbance model. Herein, the contribution of each source can be studied individually. Also, the direction of each source is identified which is crucial for multivariable control design, as suggested in [9], [11, p. 85]. Furthermore, we show that with some additional assumptions, the physical location of sources can be recovered. This offers the possibility to trace down the sources and reduce their influence through mechanical redesign. The theory is demonstrated on a non trivial 6 × 6 MIMO active vibration platform.

In the next section, we discuss how multivariate disturbances can be recovered from closed loop measurements. From thereon, we introduce the blind identification method used in this work. This is then applied to the active vibration isolation platform. Herein, disturbances are allocated using the proposed methodology. Finally the implications on multivariable feedback control design are discussed.

II. MULTIVARIATE DISTURBANCES

We consider a plant $G$ with $n$ inputs and $n$ outputs which is controlled by a feedback controller $K$ in the architecture depicted in Fig. 1. The disturbance vector $d$ enters the loop at the input of the plant. In this paper, we assume that the plant is invertible and known within negligible uncertainties. Hence, disturbances at the output of the plant can be considered at the input of the plant (and vice versa).

The error $e$ equals

$$
e = S_d G d$$

where $S_d = (I + G K)^{-1}$ is the output sensitivity function. From an initial experiment, a batch of observations of the error can be obtained, $e(t) \in \mathbb{R}^n$ for $t = 0, ..., T_s N$, where $T_s$ denotes the sample time and $N + 1$ is the number of samples. With $e(t)$ given, one can reconstruct $d(t)$ by using the inverse of the output sensitivity, $d(t) = G^{-1} S_d^{-1}(q) e(t)$, with $q$ the differential operator, $q u(t) = \frac{\partial u(t)}{\partial t}$. The disturbance at each channel $d_i, i = 1, ..., n$ results from a mixture of sources that are to be identified $s_j, j = 1, ..., m$, where $m \leq n$ and sources that are not identified $w_i, i = 1, ..., n$. The challenge is to find 1) the number of physical disturbances ($m$), 2) the physical disturbance sources ($s(t)$), and 3) the matrix that mixes the sources (the relevant part of $G_d$).

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III. BLIND IDENTIFICATION

As we do not know the mixing matrix $G_d$ and have no knowledge about the sources $s(t)$, we face a blind identification problem. Key in this analysis, is the use of the $\tau$-lagged covariance matrix, defined for a multivariate signal $x(t)$,

$$R_x(\tau) = E\{x(t)x(t-\tau)^T\}$$

(2)

where $E\{\cdot\}$ is the statistical expectancy. The components of $x(t)$ are uncorrelated if $R_x(0)$ is diagonal.

In order to solve the blind identification problem, several assumptions must be made. Here, we make use of the following assumptions,

A 1: The sources are mutually statistically independent
A 2: Each source is a different time colored phenomena
A 3: The mixing process $G_d$ is constant in time
A 4: The sources and noise are uncorrelated

The first assumption A1 states that the sources are statistical independent physical phenomena. Two variables are called independent when knowing the value of one, does not provide any information about the other. In that case, their joint probability density function equals the product of the marginal probability density functions, [5]. Sources that are independent are also uncorrelated, but not vice versa. The second assumption A2 assumes that the sources are time colored. Then, the lagged covariance $R_x(\tau)$, is positive semi-definite for non-zero $\tau$. The sources are different so that the lagged covariance is different for at least one value of $\tau$. Assumption A3 states that $G_d$ is a static transfer function, at least at the frequencies of interest. This is justified when sources have a narrow band spectrum and locations of the sources do not change in time. For ease of notation, we assume the sources are zero mean.

We divide the sources into two classes, namely the sources $s(t)$ which are to be identified, and the noise signals $w(t)$ which are not identified directly. Depending on the nature of $w(t)$, great simplifications in the blind identification problem can be realized. We assume that the noise and the sources are uncorrelated, A4. The disturbance model becomes,

$$d(t) = \begin{bmatrix} G_s & G_w \end{bmatrix} \begin{bmatrix} s(t) \\ w(t) \end{bmatrix}$$

(3)

The blind identification procedure consists of two steps; Step A) principal component analysis and scaling. Step B) independent component analysis.

A. Principal component analysis

The objective of principal component analysis (PCA) is to find the minimal number of uncorrelated components $z(t)$ in the observed disturbances $d(t)$. In addition, the uncorrelated components are scaled to unit covariance. This procedure is also known as whitening, [1]. Hence the objective is to find a possibly non-square matrix $W$ so that,

$$z(t) = Wd(t)$$

(4)

with, $R_z(0) = I$. Using (3), the covariance of the disturbance equals,

$$R_d(0) = G_sR_s(0)G_d^T + G_wR_w(0)G_d^T$$

(5)

The issue is that the structure at the right hand side of this equation is to be determined while only $R_d(0)$ is known. The unknown singular value decompositions of the source and noise parts, is defined as,

$$G_sR_s(0)G_d^T = U_s\Sigma_sU_s^T, \quad G_wR_w(0)G_d^T = U_w\Sigma_wU_w^T$$

(6)

where $R_s(\tau), R_w(\tau)$ are symmetric. Next, the singular value decomposition of the covariance of $d(t)$ is studied, so that,

$$R_d(0) = U_d\Sigma_dU_d^T = U_s\Sigma_sU_s^T + U_w\Sigma_wU_w^T = \begin{bmatrix} U_{ds} & U_{dw} \end{bmatrix} \begin{bmatrix} \Sigma_{ds} & 0 \\ 0 & \Sigma_{dw} \end{bmatrix} \begin{bmatrix} U_{ds}^T \\ U_{dw}^T \end{bmatrix}$$

(7)

The $i^{th}$ singular value of $R_d(0)$, $\sigma_{di}$, namely the $i^{th}$ diagonal element of $\Sigma_d$, equals the square of the variance of the $i^{th}$ principal component in decreasing order. The best rank $m$ approximation of $R_d(0)$ is achieved by considering the $m$ dimensional subspace related to the first, hence largest, $m$ principal components. The costs of this dimension reduction is small when $\sigma_{ds}^2(\Sigma_{dw}^{-1})^{-1/2}$ is large. When the sources dominate the noise signals, one finds the subspace of the sources as, $G_sR_s(0)G_d^T \approx U_{ds}\Sigma_{ds}U_{ds}^T$.

In the case that the noise space outside the source space can be approximated as $\Sigma_{dw} \approx \rho^2 I_{n-m}$. And one can assume, e.g., on physical bases, that $\Sigma_{ds} = \Sigma_s + \rho^2 I_{m}$, one can determine the variance of the spatially white noise space as $\rho^2 \approx \frac{1}{n-m} \text{tr}(\Sigma_{dw})$. Hence, an unbiased estimate of the source variances can be obtained using, $G_sR_s(0)G_d^T \approx U_{ds}(\Sigma_{ds} - \rho^2 I_{m})U_{ds}^T$. This strategy is justified in the special case that $G_w = I_n$ and $R_n(\tau) = \rho^2 I_n\delta_{\tau}$. This special structure is commonly assumed in array processing applications, [12]. Herein, all sensors in the array are assumed to suffer from sensor noise with the same covariance. In control applications, these assumptions may be justified when all channels (e.g. all sensors) have the same noise variance. This is a crude assumption in most applications.

When none of the above arguments hold, and no clear decay of the singular values of $R_d(0)$ is visible, it is questionable if reduction of the dimension of the disturbance signal space is justified. If the dimension is reduced, performance of the blind identification procedure may decrease significantly, as shown in [8]. Blind identification procedures that make no use of a post-processing step as principal component analysis, such as [7], can then be considered.
In the following, we assume that we can use \( G_s R_s(0) G_s^T \approx U_{ds} \Sigma_{ds} U_{ds}^T \), so that the signal dimension can be reduced to \( m \). Next, the issue is to find the whitening matrix \( W \in \mathbb{R}^{m \times n} \) so that,

\[
\begin{align*}
R_z(0) &= WW_d(0)W^T \quad (8) \\
&= W U_{ds} \Sigma_{ds} U_{ds}^T W^T = I
\end{align*}
\]

when we assume, without loss of generality, that \( R_s(0) = I \), we find that,

\[
W = \Sigma_{ds}^{-\frac{1}{2}} U_{ds}^T \quad (9)
\]

Now, the directions of the dominant disturbances are contained in \( U_{ds} \). The most dominant disturbance lies in the direction of the first column of \( U_{ds} \). Note that when \( m < n \), all signals in the subspace orthogonal to \( W \), i.e., signals in the image of \( U_{ds} \), are not considered in future steps of the blind identification procedure.

The covariance matrix \( R_z(0) \) does not change when \( z(t) \) is transformed with any unitary matrix \( U \). Due to this freedom, the uncorrelated components \( z(t) \) can still result from a mixture of the sources with any unknown unitary matrix \( U \),

\[
z(t) = U s(t). \quad (10)
\]

Even though the components are uncorrelated, their behavior can be much different from the behavior of the physical sources \( s(t) \). The next step in the blind identification procedure is to reduce this freedom by using a stronger statistical condition, namely statistical independence.

B. Independent component analysis

In this step, the uncorrelated components \( z(t) \) are transformed to components that are mutually statistical independent. As independence is a stronger statistical condition, the residual freedom in the blind identification problem reduces. As the sources are time colored signals, their \( \tau \) lagged covariance, \( R_s(\tau) \) is non-zero. The case is studied where the noise signals are small and fast compared to the source signals. Hence \( R_s(\tau) \gg R_n(\tau), \tau > 0 \), so that,

\[
R_d(\tau) = G_s R_s(\tau) G_s^T, \quad \tau > 0. \quad (11)
\]

In [12] it is argued, that when the lagged covariance matrix is diagonal for multiple lags, the components are independent. Starting from the whitened components \( z(t) \in \mathbb{R}^m, R_z(0) = 0, R_z(\tau) \geq 0, \tau > 0 \), the objective is to find a unitary matrix \( U \) so that,

\[
R_s(\tau) = U^T R_z(\tau) U \quad (12)
\]

is diagonal for a set of \( \tau_k > 0, \tau_k = \{\tau_1, ..., \tau_{N_k}\} \).

This is a unitary simultaneous diagonalization problem that can be solved within two indeterminacies; namely sign and permutation of the columns of \( U \). We express these indeterminacies with the matrix \( P \) which is the product of a permutation matrix and a phase matrix. The solution \( VP = U \), with unknown \( P \), is the approximated eigenstructure of \( R_z(\tau_k) \) for \( \tau_k = \tau_1, ..., \tau_{N_k} \). A solution exists if at least one covariance matrix \( R_s(\tau_k) \) has distinct diagonal values. As long as this covariance matrix is in the set for \( \tau_k = \tau_1, ..., \tau_{N_k} \), the sources can be separated. Hence, increasing \( N_k \), will improve signal separation, especially for more broadband disturbances. The unitary simultaneous diagonalization can be formulated as an optimization problem. Here, we use the joint approximate diagonalization of eigenmatrices (JADE) solver from [4] to find \( V \).

Knowing both \( W \) and \( V \), the physical sources can be recovered up to the indeterminacies \( P \) as,

\[
\hat{s}(t) = Ps(t) = V^T Wd(t). \quad (13)
\]

We define \( \hat{G}_s = W^T V \). The indeterminacies \( P \) imply arbitrary ordering and arbitrary sign of the recovered sources.

The arbitrary permutation, implies that the ordering of the recovered sources is not fixed. In the principal component analysis, it was assumed that \( R_s(0) = I \), which implies that all scalings of the sources are contained in \( G_s \). Alternatively, one may choose to scale the columns of \( G_s \) to unity, as we show in Section IV. This is just a matter of convention and does not play any role in further use of this disturbance model.

IV. EXPERIMENTAL SETUP

An industrial active vibration isolation platform is studied, see Fig. 2. The platform consist of an active mounted table

![Shakers](shakers.png)

Fig. 2. Active vibration isolation platform. Shakers mounted at the table surface generate disturbances.

where actuators apply forces and moments at the center of gravity (COG). Geophones (sensors) measure the velocity of the COG. As the table behaves rigid in the domain of interest, the transfer function matrix of the plant is diagonal. Hence, all six cartesian degrees of freedom can be controlled independently.

Two disturbances are added synthetically to the system, by means of two shakers placed at the surface of the table. Both the location and the time behavior of the disturbances are considered to be unknown. For validation purposes, the acceleration of the shakers is measured. The errors are
measured, so that using (1), the disturbances $d(t)$ can be recovered, see Fig. 3. It is visible that due to the location of the shakers, the shakers excite all controlled axes. In the principal component analysis, we find the singular values of the covariance matrix $R_d(0)$, Fig. 4. We take in account three principal components ($m = 3$). There is no clear distinction between the third and fourth singular value of $R_d(0)$. The influence of other sources in the $m$ dimensional signal subspace of $R_d(0)$ can therefore be significant. We choose to do this, to illustrate the power of the blind identification procedure. In Fig. 5 the three components are shown, as $z(t) = Wd(t), W \in \mathbb{R}^{3 \times 6}$. Next, using independent component analysis the uncorrelated components $z(t)$ are transformed to independent components, Fig. 6. Herein, we used $N_k = 50$ lagged covariance matrices in the simultaneous diagonalization problem, (12). We decompose the recovered matrix $G_s$ in the directions of, $G_s$, and the input gains $\Gamma$, so that $G_s = G_s \Gamma$

$$G_s = \begin{bmatrix} -0.013 & 0.003 & 0.209 \\ -0.010 & -0.007 & 0.582 \\ -0.997 & 0.970 & 0.781 \\ -0.008 & -0.093 & -0.078 \\ -0.081 & -0.224 & 0.026 \\ -0.001 & 0.002 & 0.020 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0.020 & 0 & 0 \\ 0 & 0.013 & 0 \\ 0 & 0 & 0.002 \end{bmatrix}$$

Fig. 3. Reconstructed input disturbance of the active vibration isolation platform.

Fig. 4. Singular values of $R_d(0)$, equals the squared variance per principal component.

Fig. 5. Three principal components $z$ recovered from the observed disturbances.

The $j^{th}$ column of $G_s$ is the direction of the $j^{th}$ independent component. It is visible that the first and second independent component act mostly in $z$ direction. Also, both two sources cause a rotation about the $y$-axis. The third source is a disturbance in both $z$ and $y$ direction, this is due to the motions of the floor in our laboratory for which the table is poorly isolated. In $\Gamma$, it is visible that the first and second source are much larger than the third source.

For validation, we compare the first two independent components with the measurement of the acceleration sensors of the shakers, Fig. 7. We see that, within a scale, sign and permutation indeterminacy, the wave forms match closely, so it is justified to conclude that the independent components describe the behavior of the sources. It is clear that the principal components, Fig. 5, do not posses this property. Hence, for this application, the practical significance of requiring statistical independence is demonstrated.
to finding the vector from the center of gravity to the point where the disturbance source acts, see Fig. 8. As we have

\[ \overrightarrow{M_{COG}} = \overrightarrow{r} \times \overrightarrow{F_{COG}}, \text{ see } [10], \text{ it follows that} \]

\[
\begin{bmatrix}
  d_{R_x}^j \\
  d_{R_y}^j \\
  d_{R_z}^j
\end{bmatrix}
= \begin{bmatrix}
  0 & d_{y}^j & -d_{x}^j \\
  -d_{y}^j & 0 & d_{x}^j \\
  d_{x}^j & -d_{y}^j & 0
\end{bmatrix}
\begin{bmatrix}
  r_{x}^j \\
  r_{y}^j \\
  r_{z}^j
\end{bmatrix}
\] \hspace{1cm} (16)

Note that the matrix at the right hand side of this equation has rank 2. Hence, it is only possible to find the shortest distance \( r_j \) to a line on which the \( j \)th source is located. When we assume that the source is located at the surface of the table, the vector \( \overrightarrow{r}_{x,y} \) from the center of gravity to the source location can be uniquely determined, Fig. 8. In (16), the ratio between the elements in \( d_{s,j} \) (15) is important, it suffices to take in account the direction of \( d_{s,j} \), which equals the \( j \)th column of \( \overrightarrow{G}_s \) (14). Hence, we recovered the vector \( \overrightarrow{r}_{x,y} \) from the center of gravity to each source, and the direction of the source (equals direction of \([d_{x}^j, d_{y}^j, d_{z}^j]^T\) at COG).

The first two sources, recovered in Section IV, are used to demonstrate this allocation procedure. For ten different measurements, the estimated locations are depicted in Fig. 9. The actual location of the shakers is marked with the diamonds. The estimation accuracy improves when the number of lagged covariance matrices (\( N_k \)) is increased.

VI. IMPLICATIONS FOR CONTROL DESIGN

The insights after blind identification offer new opportunities for multivariable control design. Here we discuss possibilities for using the knowledge from blind identification for physical interpretation and redesign of a multivariable feedback controller.

A. Changes in the disturbances

After blind identification, one can study the contribution of each source in the disturbance \( d(t) \). Hence, the benefit of eliminating a particular source in order to improve machine performance can be studied. Here, we measure the performance with a norm on \( e(t) \). The disturbances can be factored as contributions from the independent components \( d_s(t) \) and contributions from the noise space \( d_w(t) \),

\[ d(t) = d_s(t) + d_w(t), \] \hspace{1cm} (17)
with \( d_j(t) = G_s \hat{s}(t) \). The contribution of each \( j \)-th independent component \( \hat{s}_j(t) \) to the disturbance equals,

\[
d_j(t) = g_{dj} \hat{s}_j(t).
\]

(18)

Herein \( g_{dj} \) equals the \( j \)-th column of \( G_s \), so that \( d_j(t) = \sum_{j=1}^{n} d_{sj}(t) \). Using (1), the error as a result of each component \( \hat{s}_j(t) \) is then,

\[
e_{sj}(t) = S_o(p)G(p)d_{sj}(t)
\]

(19)

The benefit of eliminating the \( j \)-th source can be obtained from calculating \( \|e(t) - e_{sj}(t)\|_p \), for any \( p \)-norm.

B. Redesign of feedback controller

Given the structured disturbance model, one has the ability to redesign the feedback controller to reject the disturbances related to the individual sources. Defining \( v_j(t) = G(p) d_{sj}(t) \), and using (19), we have that

\[
e_{sj}(t) = S_o(p)v_j(t).
\]

(20)

As \( S_o(p) \) is a transfer function matrix, the size of \( e_{sj}(t) \) depends on the gain at the input direction of \( S_o(p) \) corresponding to the direction of \( v_j(t) \). When the direction of \( v_j(t) \) is fixed, one may consider shaping the sensitivity function so that attenuation is high in that direction while attenuation in orthogonal directions is decreased. Hence, design freedom is exploit that has no scalar analogue, [9]. An \( H_\infty \) controller design that demonstrates this is discussed in [2].

VII. CONCLUSIONS

It is demonstrated that the spatial diversity and time evolution of disturbances can be used to identify both the disturbance sources and the way they are mixed in a multivariable controlled system. Component wise analysis offers great insight in the physical nature of disturbances and facilitates allocation of sources and improved multivariable controller design choices. The benefit of using directional information of disturbances for improved weighting filter selection in \( H_\infty \) control design, is discussed in our recent paper, [2].

More advanced blind identification techniques, that have less stringent assumptions on the disturbance model are currently under investigation. In [13] more general blind identification techniques are discussed. Also, the case of underdetermined mixtures (\( m > n \)), in [6], is subject to future research.

REFERENCES