Generation of fast electrons by breaking of a laser-induced plasma wave

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(Received 4 July 2000; published 23 January 2001)

A one-dimensional model for fast electron generation by an intense, non evolving laser pulse propagating through an underdense plasma has been developed. Plasma wave breaking is considered to be the dominant mechanism behind this process, and wave breaking both in front of and behind the laser pulse is discussed. Fast electrons emerge as a short bunch, and the electrostatic field of this bunch is shown to limit self-consistently the amount of generated fast electrons.

DOI: 10.1103/PhysRevE.63.026406 PACS number(s): 52.38.—t, 41.75.Ht, 52.27.Ny

I. INTRODUCTION

Fast particle generation, the production of multi-MeV electrons during the propagation of an intense (1019 W/cm2) laser pulse through an underdense plasma, is an aspect of laser-plasma interaction that has recently attracted a lot of attention, in both theoretical and experimental circles. The effect has been observed in many experiments [1–6], as well as in numerical simulations [7,8].

At the heart of this phenomenon lies the capture of plasma electrons by the laser-induced wake wave. Once captured, these electrons are accelerated by the wakefield, just as in a laser wakefield accelerator. Breaking of the wake wave has been suggested as the dominant mechanism for electron capture [2], although Raman backscattering might play a role in preaccelerating a small fraction of the background plasma electrons, which are then more easily captured and accelerated by the wake wave [5,6,9]. Closer investigations of electron capture due to wave breaking can also be found in Refs. [10,11]. Electron capture in a plasma wave induced by a relativistic electron bunch has been studied in Ref. [12].

In this paper, we develop an analytical one-dimensional (1D) model for the behavior of plasma waves, driven by the ponderomotive potential of a short [L = O(λp)] intense laser pulse. We assume that the laser pulse envelope does not evolve during its propagation through the plasma, and study only the plasma response to the pulse; as a consequence of this, the laser pulse intensity will not increase due to instabilities. This assumption is motivated by the observation that very short laser pulses show only minor envelope modification during their propagation through a plasma [13,14], whereas longer pulses show significant pulse shape modification due to, among other things, Raman scattering. We explore several scenarios that lead to the breaking of the plasma wave, and derive the necessary conditions for this to happen. We show that the fast electrons emerge as a very short bunch, whose electric field will limit the amount of generated fast electrons in a self-consistent way. From this, we derive the maximal charge density of the bunch as a function of laser pulse intensity.

II. ONE-DIMENSIONAL NONLINEAR PLASMA WAVES

In this paper, we start from an infinitely extended, homogeneous plasma, through which a laser pulse propagates in the z direction. The laser and plasma frequencies are denoted by ω and ωp, respectively, and the plasma is assumed to be underdense: ωp < ω. We denote the group velocity of the laser pulse, which is at the same time the phase velocity of the wake wave, by vϕ, and the corresponding Lorentz factor by γϕ.

The following model assumptions concerning the plasma are made: the plasma is cold and temperature effects are neglected; the plasma electrons are initially at rest; the plasma background density n0 changes on a very large length scale, so n0 will be considered a constant. Concerning the laser pulse, we assume that it is one-dimensional and circularly polarized, and that its envelope does not change as it propagates through the plasma; as a consequence, instabilities in the pulse evolution due to Raman scattering are not investigated here. Furthermore, we assume that γϕ is sufficiently large so all the terms of O(1/γϕ2) or smaller can be neglected.

After separating fast and slow time scales, the slow, longitudinal motion of the electron fluid is governed by the following equations (the Coulomb gauge is used here):

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(nv_z) = 0, \tag{1}
\]

\[
\frac{\partial^2 \Phi_z}{\partial z^2} = -4\pi(n_0 - n)e, \tag{2}
\]

\[
\frac{\partial p_z}{\partial t} = \frac{\partial}{\partial z}(e\Phi_z - \gamma mc^2), \tag{3}
\]

where n denotes the electron density, v_z and p_z the electron speed and momentum, γ = (1 + p_z^2/(mc)^2 + (e|A_\parallel|^2)/mc^2)^1/2 the Lorentz factor, |A_\parallel| the amplitude envelope of the laser pulse, and Φ_z the electrostatic wakefield potential. The other symbols have their usual meanings.

We apply the usual scaling transformations: t' = ωp t, z' = (ωp/λc) z, v' = v/λc, p' = p(λc), Φ'_z = eΦ_z/(mc^2), A'_\parallel = eA_\parallel/(mc), and drop the primes for convenience. In the scaled variables, γ = (1 + p'_z^2 + U, where U = A'_\parallel^2 denotes the laser pulse envelope. We also introduce the new coordinates
\(\zeta = z - v_{ef}t, \; \tau = t\). In line with our model assumptions, the wave is stationary in the frame moving with the pulse, i.e., \(\partial/\partial\tau = 0\).

At this point, it is convenient to introduce the variable \(\pi = \pi_0 p_z\). As follows from the equations of motion, \(\pi = 1 + \Phi_f\). From its definition, it follows that \(\pi\) has a \(U\)-dependent lower bound, \(\pi = \pi_0(U) := (\sqrt{1 + U})/\gamma_{\varphi}\) for all \(p_z\). Furthermore, \(\pi\) reaches its lower bound for a certain value of \(p_z\), \(\pi = \pi_{\varphi}(U)\) for \(p_z = p_{cr} := \gamma_{\varphi} v_\varphi \sqrt{1 + U}\). As a result of this, the mapping \(\pi \rightarrow p_z\) is double valued:

\[
p_{\varphi} = \gamma_{\varphi}^2 \left[ v_\varphi \pi \pm \sqrt{\pi^2 - (1 + U)/\gamma_{\varphi}^2} \right],
\]

where \(p_{\varphi} < p_{cr}\) and \(p_{\varphi} > p_{cr}\). Since we wish to study regular plasma waves first, where the particle velocity is below the wave velocity, we take \(p_{\varphi} = p_{\varphi}^-\) for the moment.

After substituting \(v_\varphi = p_{\varphi}/(\gamma_\varphi)\), the equation for \(\pi\) takes the form

\[
\frac{d^2 \pi}{d\xi^2} = \frac{1}{2} \left( \frac{\partial\pi}{\partial\xi} \right)^2 + V(\pi, U) = \gamma_{\varphi}^2 \left[ \pi - v_\varphi \sqrt{\pi^2 - (1 + U)/\gamma_{\varphi}^2} \right].
\]

This differential equation describes anharmonic oscillations of the variable \(\pi\). (See also Ref. [15] for a similar equation, although with a slightly different \(V(\pi, U)\), or Ref. [16] for a similar equation with ion motion included.) It is analogous to the Newton equation for a virtual particle with “coordinate” \(\pi\), moving in a 1D “potential well” \(V(\pi, U)\), where \(\xi\) is the “time” and \(U(\xi)\) acts as a “time”-dependent parameter. Continuing this analogy, we define the “velocity” of the virtual particle as \(d\pi/d\xi\), and its “kinetic energy” as \(\gamma_{\varphi}^2 (d\pi/d\xi)^2\). Although somewhat artificial, the description of plasma oscillations in terms of oscillations of a virtual particle will provide much insight in this case, especially when studying necessary conditions for the occurrence of wave breaking.

It should be noted that a maximum (minimum) of \(\pi\) corresponds to a minimum (maximum) of the local electron density, with minimal (maximal) electron fluid momentum \(p_z\) and zero electric field. The minimum of \(V(\pi, U)\) as a function of \(\pi\) corresponds to a point with maximal \(\partial\pi/\partial\xi\), i.e., maximal electric field, zero electron fluid momentum \(p_{\varphi}\), and \(n = n_0\).

When \(\pi = \pi_{\varphi}(U)\), we have \(\partial V/\partial\pi \rightarrow -\infty, \; n \rightarrow \infty\), and \(p_{\varphi} = p_{cr}\) for some plasma electrons, so \(\pi = \pi_{\varphi}(U)\) is a bifurcation point. These electrons then overtake the wake wave, which is called wave breaking [19,20]. It can be concluded from the equations of motion that wave breaking is bound to happen if a sufficiently large oscillation is excited in the plasma.

III. EXCITATION OF 1D PLASMA WAVES

Following the approach of the previous section, longitudinal plasma waves are described as oscillations of a virtual particle with “coordinate” \(\pi(\xi)\), in a potential well

![FIG. 1. Excitation of a plasma oscillation by a rectangular pulse](image)

\(V(\pi, U)\). The mechanism behind the excitation of such oscillations by strong laser pulses will be considered below.

For simplicity, we assume the laser intensity envelope \(U(\xi)\) is rectangular:

\[
U(\xi) = \begin{cases} U_0, & -L < \xi < 0 \\ 0, & \text{otherwise.} \end{cases}
\]

This allows us to write down the energy equation for the virtual particle:

\[
\frac{\partial}{\partial \xi} \left[ \frac{1}{2} \left( \frac{\partial\pi}{\partial\xi} \right)^2 + V(\pi, U) \right] = 0,
\]

where one can see \(V(\pi, U)\) as a function of \(\pi\) in two parametric regimes, \(U = 0\) and \(U = U_0\). Oscillations are excited by moving the virtual particle away from the bottom of the “potential well,” located at \(\pi = \pi_{\varphi}(U) = \sqrt{1 + U}\). This will be done as follows (roman numbers correspond to those in Fig. 1).

(I) In front of the pulse, the plasma is at rest, so we start at \(\pi = 1 = \pi_{\varphi}\) for \(U = 0\).

(II) At \(\xi = 0\), \(U\) increases to \(U_0\), while \(\pi\) remains constant. Since the minimum of \(V(\pi, U_0)\) is at \(\pi = \sqrt{1 + U_0}/2 > 1\), the virtual particle will start to oscillate between \(\pi = 1\) and \(\pi = \pi_{\varphi}(U) > 1\), where \(\pi_{\varphi}(U)\) denotes the other solution to \(V(\pi, U) = V(1, U)\), and is given by

\[
\pi_{\varphi}(U) = \gamma_{\varphi}^2 \left( \frac{1}{\sqrt{\gamma_{\varphi}^2} - 2 v_\varphi \sqrt{1 - (1 + U)/\gamma_{\varphi}^2}} \right).
\]

(III) When the laser intensity drops back to zero at \(\xi = -L\), the potential energy of the virtual particle drops from \(V(\pi(L), U_0)\) to \(V(\pi(L), 0)\).

(IV, V) Since \(\partial^2\pi/\partial\xi^2 > 0\) and \(\pi > 1\), an oscillation in the potential well for \(U = 0\) has been excited.

The total “energy” of the oscillation is given by \(V(1, U_0) - V(1, 0) - V(\pi(L), U_0) - V(\pi(L), 0)\), and since \(V(\pi, U_0) - V(U, 0)\) decreases for increasing \(\pi\), we find that the total oscillatory “energy” at a given pulse intensity \(U_0\) is maximal for \(\pi(L) = \pi_{\varphi}\). For \(\pi(L) < \pi_{\varphi}\), the total “energy” differs from the maximal “energy” as \(\Delta E \sim O((\pi_{\varphi} - \pi(L))^2)\).

One must note that, whereas the front edge of the laser pulse increases the potential energy of the virtual particle, the back edge decreases it; in the special case that \(\pi(L) = 1\), the final energy of the virtual particle will be zero, and
there will be no wake behind the pulse. For this reason, we
define the pulse length L to be optimal if the excited plasma
oscillation has the maximal possible energy for the pulse
intensity \( U_0 \). For the remainder of this paper, L is always
assumed to be optimal.

If the total “energy” level of the oscillation after passage
of the pulse exceeds \( V(\pi_0(0),0) = \gamma_e \), then we will have
\( \pi(\xi) = \pi_0(0) \) for some \( \xi \), while \( |\partial \pi/\partial \xi| > 0 \). [This means
that, in Fig. 1, the end point \( \pi_0(0) \) will be reached.] As
mentioned above, the wave breaks at this point. For the
maximal electrostatic field associated with a wave on the
verge of breaking, we find, using \( \partial \pi/\partial \xi = (\omega / \omega_p)E_z \) and
\[ E_{\text{max}} = (\omega_p / \omega) \sqrt{2(\gamma_e - 1)}, \]
which coincides with the wave breaking limit obtained by
Akhiezer and Polovin [18].

IV. FAST PARTICLE GENERATION
AS WAVE BREAKING

Thus, we know that the wake wave driven by the laser
pulse breaks if \( \pi(\xi) = \pi_0(0) \) at some point, causing the
production of electrons with speed \( v_\perp > v_f \). These electrons
may be captured and accelerated by the wave wake itself,
attaining a final energy up to \( 2\gamma_e^2 - 1 \). This mechanism
is believed to be responsible for fast particle generation by
intense laser pulses [2]. We are going to investigate two possi-
ble scenarios for reaching the wave breaking point: (i)
wave breaking at the front edge of the laser pulse, and (ii)
wave breaking during the first plasma oscillation behind the
pulse.

In the first scenario, the envelope \( U(\xi) \) increases from 0
to its maximum value \( U_0 \), while at the same time, \( \pi \) stays at
its original value of 1. (See steps I and II in Fig. 1.) The
Corresponding value of \( \pi_0(U) \) increases from \( 1/\gamma_\perp \)
to \( (1+U_0)/\gamma_e \). If \( \pi_0(U) \) reaches the initial value of \( \pi = 1 \),
the singularity at \( \pi = \pi_0(U) \) will be encountered right at the
front edge of the pulse. The threshold for this mechanism of
wave breaking is

\[ U_0 = \gamma_e^2 - 1. \]  

This scenario could dominate fast electron generation at high
laser intensities, but it does not explain the intensity thresh-
olds found in experiments and simulations, which are com-
monly one order of magnitude lower than predicted.

In the second scenario, \( U_0 \) is assumed to be below \( \gamma_e^2 - 1 \), so there will be no wave breaking at the front edge
of the pulse. Instead, a plasma wave will be excited that is
strong enough to break behind the pulse. As shown in the
previous section, this will happen as soon as the total energy
of the oscillation exceeds \( \gamma_e \). This situation can be reached
as follows. Define \( \pi_1 \) as the other solution for \( \pi \) of
\[ V(\pi,0) = V(\pi_1,0) \):

\[ \pi_1 = \gamma_e^2 (2 - 1/\gamma_e) \pi_0(0) = \gamma_e^2 (2 - 1/\gamma_e^2). \]

Clearly, the energy of the system will reach the wave break-
ing threshold if at point III in Fig. 1 \( \pi(L) \geq \pi_1 \).

If \( L \) is optimal, i.e., \( \pi(L) = \pi_0(U_0) \), wave breaking will occur if \( \pi_0(U_0) \geq \pi_1 \), or

\[ U_0 \geq 2(\gamma_e - 1) \frac{\gamma_e - 1}{4 \gamma_e^2 (\gamma_e + 1)}, \]

which, since \( \gamma_e \) is assumed to be large, will be approximated
by \( U_0 \geq 2(\gamma_e - 1) \).

For any \( \gamma_e > 1 \), we find that the wave breaking limit for
\( U_0 \) in the second scenario is smaller than in the first scenario,
so this second scenario is at least partly responsible for fast
electron generation at intensities below \( \gamma_e^2 - 1 \), and it might
define the intensity threshold for the existence of this phe-
nomenon.

As an example, we see that for \( n_0/n_{cr} = 0.02 \) and laser
wavelength \( \lambda_0 = 800 \) nm, the first scenario predicts wave
breaking for \( U_0 \approx 49 \), corresponding to a power threshold of
\( 1.1 \times 10^{20} \) W/cm\(^2\). The second scenario predicts a power
threshold of \( 2.6 \times 10^{19} \) W/cm\(^2\). Simulations by Nagashima
et al. [17] predict a threshold of approximately \( 10^{19} \) W/cm\(^2\)
for this case. Malka et al. [4] conducted simulations with
\( n_0/n_{cr} = 0.05 \) and \( \lambda_0 = 1 \) m \( \mu \)m, and found an intensity
threshold of \( 10^{18} - 10^{19} \) W/cm\(^2\), where our first scenario
predicts \( P = 2.7 \times 10^{19} \) W/cm\(^2\) and the second predicts \( P = 1.0 \times 10^{19} \) W/cm\(^2\).

One should also note that several other mechanisms exist
that may cause fast electron generation, such as Raman back-
scattering (see Ref. [9]) and thermal effects (see Ref. [20]).
The associated intensity thresholds are slightly lower than
those derived above.

V. BEHAVIOR OF THE BROKEN WAVE

The simple one-fluid model discussed here is suitable for
the description of the plasma wave only until it breaks. For
the description of the plasma wave after breaking, the model
needs to be extended as follows. The plasma electrons
captured by the wake wave can ultimately reach an energy of
\( \gamma_f = 2\gamma_e^2 - 1 \), and a corresponding velocity \( v_f \) with
\( v_\perp < v_f < 1 \). However, both \( v_\perp \) and \( v_f \) are close to 1; for example, if
\( \gamma_e = 7.1 \), we have \( 1 - v_\perp = 0.01 \), and \( 1 - v_f = 5 \times 10^{-5} \). This
means that captured plasma electrons will remain virtually
immobile with respect to the wakefield, and bunch up just in
front of the wake breaking point. This process cannot be
described by our simple one-fluid model, so we need to ex-
tend the model to include the effect of a very short bunch of
captured plasma electrons, moving with the wakefield. In the
extended model, the relative density perturbation caused by
fast electrons is approximated by a “sheet” of charge, lo-
ated at \( \zeta = \xi_n \), close to the value of \( \zeta \) at which the wave
broke (about half a plasma wavelength behind the back edge
of the pulse). As we shall see, the charge density in the sheet
will increase until a certain limit has been reached. Then a
new stationary state will be reached, consisting of a laser
pulse, a plasma wave in the wake of the sheet, a charge sheet
of a certain density located in the wake wave, and a (regular)
plasma wave in the wake of the sheet. This line of reasoning is supported by the simulation results of Nagashima et al. [17], which show a sharp spike in the electron density leaving the plasma and staying closely behind the laser pulse.

With the additional contribution of the charge sheet, the differential equation for $\pi$ becomes [compare to Eq. (5)]:

$$\frac{\partial^2 \pi}{\partial \xi^2} = -\frac{\partial}{\partial \pi} V(\pi, U) + Q \delta(\xi - \xi_0),$$

(13)

where $Q$ represents the saturated surface electron density of the sheet. We find that for both $\xi > \xi_0$ and $\xi < \xi_0$ the energy equation (7) still holds, while the presence of the sheet causes a jump in $\partial \pi / \partial \xi$ at $\xi = \xi_0$, i.e., a jump in the electrostatic field. One should keep in mind that the extraction of fast electrons from the plasma leaves a positive charge cancel in front of the pulse, so the presence of the charge sheet will not be felt for $\xi > \xi_0$.

The scenario of plasma wave excitation is depicted in Fig. 2.

(I–V) A plasma oscillation is excited similarly to the scenario without the charge sheet. Its energy level is higher than the energy $V(1/\gamma, 0) = \gamma$ of the end point of the potential well.

(VI) After passage of the charge sheet at $\xi = \xi_0$, $\partial \pi / \partial \xi$ has decreased by $Q$, and since $\partial \pi / \partial \xi > 0$ for $\xi > \xi_0$ we find that the ‘kinetic energy’ of the virtual particle has also decreased. Since $\pi = \pi(0)$ at $\xi = \xi_0$, the ‘kinetic energy’ just before passage of the charge sheet is given by $\frac{1}{2}(\partial \pi / \partial \xi)^2 = V(\pi(0), 0) - V(\pi(0), 0)$. As long as $Q$ is small, one still has $\partial \pi / \partial \xi > 0$ after passage of the charge sheet, and the growth of $Q$ will continue; however, as soon as $Q^2 = 2[V(\pi(U), 0) - V(\pi(0), 0)]$, fast electron generation, i.e., increase of $Q$, will stop.

(VI, VII) In the stable situation where the charge sheet prevents further wave breaking, there is a regular plasma oscillation behind the sheet.

From

$$V(\pi(0), 0) = \gamma,$$

(14)

$$V(\pi(U), 0) = \gamma^2 [X - \varphi \sqrt{X^2 - 1/\gamma^2}];$$

(15)

where

$$X = \frac{\pi(0)}{\gamma} = 2 - \frac{1}{\gamma} - 2 \varphi \sqrt{1 - \frac{1 + U}{\gamma^2}}.$$  

(16)

we find that the fast electron generation will stop as soon as

$$Q \approx \sqrt{(U - U_{cr}) + \frac{2(U - U_{cr})}{2 + U_{cr}} + \frac{(U - U_{cr})^2}{2 + U_{cr}}^2}.$$  

(17)

Quite obviously, $U = U_{cr} = 2(\gamma - 1)$ is the necessary condition for the formation of a charge sheet to happen. Since there will be no further charge buildup past this point, the above expression provides an upper limit for $Q$. The total surface charge density of the generated fast electrons will be $\left(n_0 e / \varphi c \omega_p\right) Q$. (See Fig. 3.)

The behavior of $Q$ vs $U_0$ in Fig. 3 is in good qualitative agreement with simulation results by Nagashima et al. [17].

Note that $U \gamma^2$ should remain sufficiently small, otherwise wave breaking at the front of the pulse occurs, and the approximations made in the above equations are no longer valid.

VI. CONCLUSIONS

In summary, we developed a one-dimensional model for fast electron generation by a strong, nonevolving laser pulse propagating through an underdense plasma. Fast particles emerge as a result of wave breaking caused by captured plasma electrons to bunch up just in front of the wave breaking point. The model describes the excitation of plasma oscillations by the ponderomotive force of the laser pulse, and provides intensity thresholds for the occurrence of wave breaking. It also includes, in a self-consistent way, the capture and acceleration of plasma electrons during wave breaking. The predicted intensity thresholds agree with well-known analytical results, and the amount of fast electrons as a function of laser intensity is in qualitative agreement with the results of recent particle-in-cell simulations [17].